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AN INVESTIGATION INTO THE ACCURACY OF WIND FORECASTS PREPARED AT SHANNON AIRPORT FOR THE GREAT-CIRCLE ROUTE SHANNON – GANDER

BY

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An investigation into the Accuracy of Wind Forecasts prepared at Shannon Airport for the Great-Circle route Shannon-Gander.

Summary

The differences between forecast 700 mb. and 500 mb. wind components on the Great-Circle route from Shannon to Gander and the subsequently estimated actual values of these components have been recorded over a two-year period, and the distribution of these differences is tabulated. It is found that approximately 92% of all such forecasts were within 12 knots of the corresponding actuals and about 73% were within 7 knots. The differences in general are found to have an approximately symmetrical distribution about a mean value close to zero, but the distribution is shown to be different from a simple normal distribution. A compound normal distribution is fitted to the observed values and by this means diagrams are prepared showing the expected percentage of forecasts having a given degree of accuracy. The effect on the accuracy of the forecasts of a lack of recent actual upper-air charts is also investigated. It is found that during the Winter half of the year, forecasts, prepared immediately after the analysis of new upper-air charts are appreciably more accurate than those prepared at other times of the day.

1. Introduction

Forecast contour charts of the 700 and 500 mb. levels covering the North Atlantic are made on a routine basis five times daily by the Meteorological Service at Shannon Airport for use in connection with Westbound flights from Shannon. From each of these charts a wind component is measured and recorded for the Great-Circle track from Shannon to Gander in addition to any other tracks which may seem reasonable for Westbound operations, and the value of this component is compared later with the corresponding value of the actual component, estimated by reference to the appropriate actual charts. This paper consists of an analysis of the frequency-distribution of the differences between the forecast and actual Great-Circle components during the period April 1952 to March 1954, and an examination of certain general problems to which this investigation gives rise.

In accordance with the practice generally adopted in studies in forecast verification, the difference between a forecast component and the estimated value of the corresponding actual component will be denoted by the word "error", although this difference could not properly be spoken of as an error in the ordinary sense of the word. In particular, because of the inevitable uncertainty of the value taken for the actual component arising from the sparsity of upper-air observations in the region concerned, the fact that an "error" different from zero is ascribed to any given forecast does not necessarily imply that the forecast in question was in any way unsatisfactory.

2. Nature of the Basic Material

The "wind components" used here refer to the mean geostrophic wind at fixed time, computed from readings of the heights of pressure-surfaces at pairs of fixed points 150 nautical miles on either side of the track. This value may differ from the "component" reported by an aircraft, that is to say, the difference between average groundspeed and airspeed, for a number of reasons, which include small-scale variations in gradients, ageostrophic terms in the actual wind, and the fact that the aircraft value is a time-average and not a space-average. Experience has shown, however, that in most cases the difference between expected and actual flight-time agrees reasonably well with that estimated from the appropriate forecast and actual components.

For the sake of uniformity attention has been confined to Great Circle components, and as a result of this, some large "errors" have been recorded, which would not, in fact, have been experienced by any aircraft. For instance, some large differences between actual and forecast components are associated with a comparatively small error in the forecast position of an intense depression close to the Great-Circle track. In these cases, considerations of both economy and safety will lead to an actual flight track being chosen which runs well north of the low centre, and on this track the difference will normally be much smaller.

Forecast charts for final planning (which alone are considered in this paper) are normally issued about three hours before the beginning of the period of departures for which they are intended to be used. The daily schedule is given in Table 1.

In practice, a continuous watch is kept on all incoming meteorological data including the actual winds reported hourly by aircraft in flight, as described by Rohan (1952) and when necessary amended charts can be issued or amendments made available in the form of new forecast winds for parts of certain tracks. In this paper, only forecasts issued at the scheduled times have been considered. Later amendments have not been taken into account.

The actual components available for verifying these forecasts were those measured daily from the 0300 and 1500 charts. For the 0800, 1300, 1800 and 2200 forecasts, the actual components were estimated by linear interpolation between the adjacent values.

Tables 2 to 5 show the overall distribution of errors for the different months. Errors are grouped in intervals of five knots, specified by the mid-value of the intervals, so that, for instance, the column headed +5 shows the number of errors with values from +3 to +7 inclusive. The positive sign is used to indicate cases where the forecast westerly or headwind component was greater than the actual value, that is, where the forecast was pessimistic for Westbound operations.

The last row in each table shows the expected frequencies calculated from the appropriate normal distribution. It will be seen that the fit is reasonably close in some respects, but not entirely satisfactory. This point is discussed further in Section 4.

The combined totals of the columns headed +5, 0 and -5 in Tables 2 to 5 give the number of forecast components which were within seven knots of the corresponding actual values, and if the columns headed +10 and -10 are added, the number of forecasts within 12 kts. of the actual is obtained. These data are given in the form of percentages of the total number of forecasts in Table 6. From this table it may be seen that on the whole about 92% of all forecasts were within 12 knots of the corresponding actual values, and about 73% were within 7 knots, but that the corresponding percentages are noticeably higher at 700 mb. in the Summer half of the year and lower at 500 mb. in Winter.

The actual distribution of errors is summarized in figure 1, in which the total frequencies from Tables 2 to 5 are expressed as percentages of the total number of forecasts.

3. Persistence, and its effect on the reliability of the statistics

An examination of the distributions shown in Tables 2 to 5 suggests that there is little to be gained by subdividing the data beyond a division into Summer and Winter for each level. We have, therefore, four more or less uniform samples of a little over 1800 values each. These samples, however, will necessarily be much less effective for estimating parameters of the distributions than would samples of 1800 independent values. Successive values of the forecast component are, in effect, a succession of instantaneous values of a more or less continuous variable. The continuity arises from the almost continuous inflow of relevant information, and is maintained, even when there is a change of forecaster, by the practice of internal briefing at the change-over. This results in a noticeable positive correlation between successive values, both of the forecast component and of the error; and in consequence the sampling variance of any statistic calculated from samples of successive values of the error is liable to be greater than would the corresponding sampling variance for a sample of the same size selected at random.

In the case of the mean, the manner in which this relationship between successive errors increases the sampling variance is quite simple. In a sample of n consecutive errors denoted by:-

$$x_1 \; x_2 \,\ldots\ldots\ldots\; x_i \,\ldots\ldots\ldots\; x_n$$

in which the correlation coefficient between x_i and x_{i-j} is r_j, and the standard deviation is σ, the sampling variance of the sum will be

$$\sigma^2\left[n + 2(n-1)r_1 + 2(n-2)r_2 + \ldots\ldots + 2\,r_{n-1}\right]$$

$$= n\sigma^2\left[1 + 2\left(1-\tfrac{1}{n}\right)r_1 + 2\left(1-\tfrac{2}{n}\right)r_2 + \ldots\ldots + \left(\tfrac{2}{n}\right)r_{n-1}\right]$$

If the series of correlation coefficients is convergent, then the series within the square brackets will tend to some finite limit, say, $1/k$ for large n and so, for a large sample the sampling variance of the mean will be asymptotic to σ^2/kn .

In this respect, therefore, a large sample of n successive values is equivalent to a sample of kn values selected at random, where k is a proper fraction which may be regarded as expressing the efficiency of the sample. For the two-term autoregressive series considered by Yule (1927) k will have the value $(1-b)(1-a-b)/(1+b)(1+a-b)$ where a and b are the first and second regression coefficients.

In the present case, it is possible to gain some information concerning the sample efficiency k by examining the variation of the monthly mean value of the error. Each monthly mean is based on about 150 successive values, and is, therefore, an approximately normal variate with mean m and variance $\sigma^2/150k$ where σ^2 and m are the expected variance and mean of individual values for that particular season and level. For each season and level there are 12 monthly means m_i so that the sum:

$$\sum_i\left[150k(m_i - m)^2/\sigma^2\right]$$

where m and σ^2 are estimates based on the entire sample is approximately a chi-square variate with 11 degrees of freedom. Further, if we assume that k is the same for Summer and Winter, we may combine the values of this sum for a single level to yield a chi-square variate with 22 degrees of freedom. In this way, using the chi-square table, we may find confidence limits for a value of k which will satisfy the above assumptions.

The 95% limits of k found in this way were:

700 mb. - .20 to .56

500 mb. - .17 to .48

Strictly speaking, these considerations apply only to the mean of a sample. It is possible that the efficiency with respect to other statistics may be different, and even in the case of the mean, it is possible that the chosen period of one month is not long enough to make valid the assumptions which have been made. It has been intended merely to form some idea of the order of magnitude of the sampling errors involved, and, in this connection, it appears that our four samples of 1800 values may be regarded as roughly equivalent to random samples of about 500 values each. This fact must be borne in mind in connection with any attempt to apply significance tests to statistics calculated from the samples.

4. The General Distribution

A number of statistics calculated for the entire distribution for each season and level are given in Table 7. The quantities in the first four columns are:-

m the algebraic mean error.

σ' the root-mean square error. The same quantity for forecasts prepared at London is referred to as the "Standard error" by Harley (1954)

a the ratio of the mean deviation to the standard deviation (both calculated from the mean)

$\sqrt{\beta_1}$ the skewness, m_3/σ^3 where m_3 is the third moment about the mean, and σ the standard deviation.

The mean, mean deviation and standard deviation have been calculated from the ungrouped frequencies and the third moments have been calculated from the five-knot group frequencies given in Tables 2 - 5. In the following discussion, the possible effect of using linear interpolation for the intermediate hour actuals has not been considered. The data presented in Fig.(4) suggest that this effect will be small.

Regarding the choice of an index of the general level of accuracy, it becomes apparent in Section 5 that the root-mean square error has the advantage of being amenable to algebraic manipulation. It is believed that the root-mean square error is also a reasonable index from the point of view of the operational usefulness of the forecasts. Although this question is one of great complexity, it will be apparent from the nature of trans-Atlantic operations that the effect of an error of, say, 15 knots would be much more than three times as great as that of a five-knot error, and that the figure of nine times as great is more in accordance with reality. For reasons such as this, a set of forecasts which minimises the mean squared error may be regarded as better, from an operational viewpoint, than one which minimises a linear function of the errors.

The quantity σ' given in column 2 of Table 7 may, therefore, be regarded as showing the variation in the general level of accuracy in the forecast component. Variation with season and level are roughly similar to the variation of the standard deviation of the actual component. Using the data of Gillman and Rohan (1954) it is found that in each case σ' is a little less than half the overall standard deviation of actual components for the appropriate season and level. The ratio varies from 0.45 at 500 mb. in Winter to 0.48 at 700 mb. in Summer.

The values of the algebraic mean error given in column 1 of Table 7 show that, on the whole, the forecast components tend to lie on the "pessimistic" side of the corresponding actual, the effect being greater in Winter than in Summer, especially at 500 mb. In this connection it may also be noticed that the skewness of the distribution, given in column 4, is negative in three of the four groups, so that in these three cases, the positive value of m receives a larger contribution from the smaller than from the larger errors.

The quantities in columns 4 and 5 of Table 7 indicated the extent to which the distributions of errors deviate from normality. The sampling distributions of these two statistics for random samples from a normal distribution are tabulated by Geary and Pearson (1938). From these Tables it can be seen that the negative assymmetry at 700 mb. is not significant at any level in Winter, and in Summer the calculated value of $\sqrt{\beta_1}$ would only reach the 5% level of significance for a random sample of 1000 members. At 500 mb. equal and opposite values of $\sqrt{\beta_1}$ are obtained which are only large enough to reach the 1% level of significance for a random sample of 800 members, or the 5% level for a sample of 400. The evidence for lack of symmetry in the distribution of errors provided by values of $\sqrt{\beta_1}$ is, therefore, by no means conclusive. Apart from the effect of persistence, the symmetrical deviations from normality discussed below, might also be expected to increase the sampling variance of $\sqrt{\beta_1}$.

The expected value of a for large samples from a normal distribution is $\sqrt{2/\pi}$ = 0.798 approx. Each of the sample values of a is less than this, three of them substantially so. Referring again to Geary and Pearson's Tables, it is found that the values for a of .770, .760 and .772 are significant at the 1% level for random samples of 310, 170 and 400 members, respectively. There can be little doubt, therefore, that there exists a tendency for the distribution of errors to deviate from normality in the manner usually associated with leptokurtosis, that is to say, a tendency for the frequencies of large and small values to exceed their Gaussian expectations at the expense of the frequencies of moderate values. The operation of this tendency can be seen by comparing the last two rows of Tables 2 to 5, where the expected frequencies in a normal distribution calculated from the sample mean and variance are entered underneath the corresponding actual frequencies.

Concerning the manner in which this deviation from normality is caused, one explanation immediately suggests itself to anyone who has had experience in upper-air forecasting on the North Atlantic. It is well-known that there is a marked contrast in difficulty between the forecasting problems presented by different types of initial situation. In some situations, the forecaster will be able to say from experience that the probability of a serious error in the forecast Shannon-Gander component is very small indeed; in others his confidence in the accuracy of the forecast is much lower. In addition, the opinion is held by some that the presence or absence of difficulty makes a fairly clear-cut distinction between two kinds of situation; in other words, that almost all situations are either very difficult or very easy, and that situations of intermediate difficulty are comparatively rare.

These considerations lead one to expect that it may be possible to explain the observed deviation from normality by supposing that the actual distribution of errors is of a compound nature, and that it is made up of two component distributions having the same mean but different standard deviations. In order to specify such a compound distribution, we shall require three parameters in addition to the mean, namely, the two component standard deviations σ_1 and σ_2 , and the probability p that any given error will belong to the distribution having the larger standard deviation σ_1 . The theoretical distribution is still symmetrical about the common mean, so that in order to estimate these three parameters, we shall require, in addition to the sample standard deviation, two further symmetrical statistics.

The two additional statistics which it was decided to use were the mean deviation and the fourth moment about the mean. The computation would have been a little simpler if the mean cubed deviation had been used instead of the fourth moment, but in view of the difficulty of estimating the grouping bias of the mean cubed deviation (analogous to the Sheppard adjustment for an even moment) it was decided to avoid the use of this quantity. Another possible method would have been to use the second, fourth and sixth moments, but the use of a sixth moment would have given very great weight to the tail frequencies.

The method of estimating σ_1 , σ_2 and p from these three statistics is given in Appendix I, and the estimates obtained are tabulated in the last three columns of Table 7. The resulting compound distributions are shown in the form of cumulative frequency-curves in figures 2 and 3, together with the actual frequencies, calculated from the seasonal totals in Tables 2 to 5. The normal distributions calculated from the sample standard deviations have also been shown for comparison. In these two diagrams, the frequencies have been indicated on a scale of square-roots in order to show the smaller frequencies in greater detail.

These curves express the frequencies in terms of deviations from zero, that is to say, the frequencies of errors of given absolute magnitudes. The frequencies of corresponding deviations from the mean differ only very slightly from these, but of course the difference between the expected frequencies of positive and negative errors with the same limits of magnitude will be quite noticeable.

In view of the uncertainty of the sampling distribution involved, it cannot be claimed that the values obtained for p , σ_1 and σ_2 will necessarily have any physical significance. The procedure may be regarded merely as a method of graduating the observed frequencies for the purpose of interpolation and extrapolation, which is consistent with the physical nature of the material. From this stand-point the procedure may be said to have been successful to the extent that the discrepancies between observed and expected frequencies, when the latter are taken from the compound normal distribution, are no longer obviously systematic.

Finally, it may be mentioned that the level of accuracy indicated by these Tables and diagrams does not necessarily represent the level of accuracy which forecasting at Shannon may be expected to show over a different period. Even since the beginning of the two-year period on which the present investigation has been based, noticeable improvements have taken place in the facilities available to upper-air forecasters working on the North Atlantic. Observations from aircraft have both increased in number and become available from a wider area, and the number of occasions when the forecaster is seriously hampered by lack of routine observations because of communications failures appears to have diminished. In addition, the availability of an increasing amount of upper-air data and continued improvement in forecasting techniques should contribute to greater accuracy in these forecasts in the future.

5. The Effect of Range on Accuracy

It is generally true that, other things being equal, the expected accuracy of a forecast is greater when the range is smaller, that is to say, when the time-interval between the latest actual information used in making the forecast, and the time to which the forecast refers, is shorter. It is a question of some interest to determine to what extent this effect causes a difference in accuracy between the forecasts for the different times mentioned in Table 1. In order to throw some light on this question, a comparison has been made between the accuracy of two sets of forecasts, one of which was prepared immediately after the analysis of new upper-air charts, and the other some hours earlier.

The two sets of forecasts used were those for 0800 and 0300. It is usually possible to complete the analysis of the 1500 actual charts before the 0800 forecast charts are prepared, so that the range of these forecasts is normally 17 hours. The range of the 0300 forecasts, counted from the time of the latest actual charts available is 24 hours.

It must be borne in mind, however, that the contrast in range is not really as great as these figures alone would suggest. Information received up to about 45 minutes before the scheduled time of issue of a forecast can be taken into account, at least to some extent, in its preparation. When the forecasts for 0300 next day are being prepared, analysed surface charts up to 0900 are available, and the 1200 surface chart will have been partially completed. Upper wind observations for a large number of stations at 0900 will also be at hand. In addition, a substantial amount of information is usually available from aircraft observations made during the late morning, amplified in some cases by personal de-briefing of Eastbound navigators. On the other hand, when the 0800 forecasts are being prepared, the latest surface observations are those for 1800; no upper winds later than 1500 are available, and the number of aircraft in the air over the North Atlantic during the early evening is quite small.

For the purpose of this comparison, root-mean square errors of the 0300 and 0800 forecast components at 700 mb. were computed for each of the 24 months of the period. Before the comparison is made, it is necessary to investigate the possibility that the use of linear interpolation for estimating the 0800 actual components may have caused the root-mean square errors of the 0800 forecasts to be underestimated, as this would lead to a spurious increase in the apparent contrast in accuracy between the two sets of forecasts. We may recognise two effects which might give rise to a bias of this kind. Firstly, the interpolated values will, on the average, lie closer to the mean than will the corresponding true actual values, because of the smoothing out of extremes, and this will tend to bring them closer to the forecast values which may also be expected to deviate towards the mean from the actuals. Secondly, the 1500 actual component, which is usually known when the 0800 forecast is being prepared, and with which, therefore, the forecast value will have some positive correlation, may in some circumstances be more closely correlated with the interpolated component for 0800 (which is partly a linear function of the 0300 actual) than with the true 0800 component. On the other hand, random differences between the interpolated and true 0800 component may be expected to increase the apparent error of the 0800 forecast, especially if the true error is small, and this will tend to counteract the other two effects. The operation of this last effect may sometimes be directly observed, when interpolation indicates a large error, but intermediate-hour wind observations and reports from aircraft show that the forecast was substantially correct.

In order to correct the observed values of the root-mean square error of the 0800 forecasts for these effects, it is necessary to make some assumption about the relationship between successive values of the actual component separated by a short interval. The correlation coefficient r_t between values of the component separated by a time-interval t is given by:-

$$r_t = 1 - \frac{\sigma_t^2}{2\sigma^2}$$

where σ^2 is the variance of the general distribution of components and σ_t^2 the mean square change of component for time t . Monthly values of σ_t for time-intervals of 12 and 24 hours over a period of about seven years are given by Gillman and Rohan (1954) and from these values it can be seen that the ratio of σ_{24}^2 to σ_{12}^2 does not deviate greatly from 2. This suggests that, for small time intervals r_t is approximately a linear function of time, and that intermediate values of r_t may be estimated by linear interpolation between $r_0 = 1$ and the value of r_{12} for the month in question, or between r_{12} and r_{24} . It may also be noted that, for an autoregressive series of the type investigated by Yule (1927) the variation of expected r_t with time will be reasonably smooth as long as the time-interval concerned is small compared with any natural periods of oscillation which may be present. Another relevant point is that the use of linear interpolation to estimate an intermediate value of the actual component has been generally accepted as reasonable on the basis of experience, and, assuming that the diurnal variation of the variance of the actual component is negligible (which is consistent with the information available), a linear relationship between r_t and time is the necessary and sufficient condition that this procedure should be valid from a "least-squares" point of view. A proof of this is given in Appendix 2.

Using this method of estimating intermediate values of r_t it is possible to adjust the observed monthly values of the mean square error of the 0800 forecasts for the sources of bias mentioned earlier. The method of calculating the adjusted values is given in Appendix 2. In most cases, the adjusted value was found to be slightly higher than the original, suggesting that the use of an interpolated actual causes a slight tendency to underestimate the magnitude of the errors.

The monthly root-mean square errors of the 700 mb. 0300 and 0800 forecast components are shown in Figure 4. The effect of adjusting the 0800 values for bias caused by linear interpolation is indicated by the short vertical arrows, the original, unadjusted root-mean square error being indicated by the position of the base of the arrow.

The ratio of the 0300 mean square error to that for 0800 varies from 0.51 to 2.08, with a median value of 1.15. It becomes apparent, however, on examining the individual values that the superiority of the 0800 forecasts is confined mainly to the Winter half of the year. For the twelve months of April to September of both years, the median ratio is only 0.95 while for the total of twelve winter months in the two years it is 1.39. It will be seen from table 1 that the ratio between the ranges of the two sets of forecasts is 1.41.

It appears, therefore, that the material examined does not suggest any relationship between range and accuracy under Summer conditions.

In Winter, the situation is different. In the case of all but two of the twelve Winter months - the exceptions being November and December, 1953 - the 0300 forecasts have the greater root-mean square error, and in only one of these two exceptions is the difference appreciable. In the absence of any reliable knowledge of the distribution associated with these data, it would be difficult to assess the reliability of any statistic which we might use to indicate the magnitude of this contrast of accuracy. It is possible, however, to show that the contrast is very probably a real one. To do this, we may consider the distribution of the common logarithms of the ratios of the monthly mean square errors. On the null hypothesis, namely, that there is no tendency for either forecast to be more accurate, this distribution would be symmetrical about zero. In the actual distribution, 11 values are numerically greater than .05; one of these is negative and ten are positive. The probability of so great a deviation from symmetry (in either direction) in a random sample from a symmetrical population would be

$$2 \times (1+11) \times (\tfrac{1}{2})^{11} = .0117$$

so that in this respect the sample falls only slightly short of significance at the 1% level of probability. Using Table VIII(1) of Fisher and Yates (1943), it can be seen that the upper 95% limit of the expected number of negative values numerically greater than .05 as deduced from this sample is 4.6.

In only one of the twelve Winter months examined were the 0300 forecasts substantially more accurate than those for 0800. This was in November, 1953, when the two values of the root-mean square error were 5.3 and 7.5 knots for the 0300 and 0800 forecasts, respectively. On examining the daily sequence of forecast and actual components for November, 1953, it was found that a considerable part of this difference arose from three cases namely the 0800 forecasts on the 10th, 15th and 28th of the month. In each of these cases, it was found that communications difficulties had occurred on the regular channel for transmission of meteorological information from North America to Europe during the previous evening. The effect of this was that much of the upper-air information from the American stations and certain North Atlantic Weather Ships for the 1500 G.M.T. actual charts, on which the 0800 forecasts in question should have been based, were delayed, and the forecast components concerned were measured from forecast charts which were to some extent 32-hour forecasts. This is a rather unusual occurrence. Difficulties in trans-Atlantic communications occur most frequently during the early morning, and the forecasts most usually affected are those for 1800 and 2200. The occurrence of communications difficulties which affect the accuracy of the 0800 forecasts three times in a single month is highly exceptional, and the inferiority of the 0800 forecasts during November, 1953, may be due to this occurrence.

In conclusion, the available evidence suggests that in Winter, a forecast component based on a recent actual upper-air chart tends to be more accurate than one based on a chart which is seven hours older with respect to the time of validity of the forecasts. The ratio between the mean square errors for the two sets of forecasts appears to be of the same order of magnitude as the ratio between the ranges.

Acknowledgments:

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Table 1. Schedule of forecast contour charts of the North Atlantic region for final planning issued daily at Shannon Airport (all times are G.M.T.)

Forecast chart for:	0300 (next day)	0800 (next day)	1300	1800	2200
Issued at:	1630	2200	0300	0800	1300
For departures from:	2000	0100	0600	1100	1600
to :	0100	0600	1100	1600	2000
Latest completed actual upper-air charts used:	0300	1500	1500	1500	0300

Tables 2 and 3. Distribution of errors in Forecast Components on the Shannon-Gander Great Circle route at 700 mb. for the period April, 1952, to March, 1954.

TABLE 2 - Distribution of Errors at 700 mb., Summer

Month	Error (knots)													
	+30	+25	+20	+15	+10	+5	0	-5	-10	-15	-20	-25	-30	Total
April	0	0	1	3	32	67	107	59	26	4	1	0	0	300
May	0	0	1	6	33	67	94	69	32	7	1	0	0	310
June	0	0	0	2	14	61	137	68	17	1	0	0	0	300
July	0	0	1	3	15	77	129	74	7	3	1	0	0	310
August	0	0	0	2	23	81	111	65	22	4	2	0	0	310
Sept.	0	0	0	3	23	89	107	60	17	1	0	0	0	300
Total:	0	0	3	19	140	442	685	395	121	20	5	0	0	1,830
Normal:	0	0	2	23	150	453	638	416	129	18	1	0	0	

TABLE 3 - Distribution of Errors at 700 mb., Winter

Month	Error (knots)													
	+30	+25	+20	+15	+10	+5	0	-5	-10	-15	-20	-25	-30	Total
Oct.	0	4	1	6	32	85	84	53	32	9	3	1	0	310
Nov.	0	0	1	12	32	66	102	63	14	7	1	2	0	300
Dec.	0	3	4	17	39	79	85	51	19	9	3	0	1	310
Jan.	0	0	2	7	28	62	100	72	23	11	4	1	0	310
Feb.	0	0	0	11	28	52	66	82	29	9	1	2	0	280
March	0	0	1	8	32	65	107	64	24	8	1	0	0	310
Total:	0	7	9	61	191	409	544	385	141	53	13	6	1	1,820
Normal:	0	2	13	64	206	407	500	382	181	53	10	1	0	

Tables 4 and 5: Distribution of errors in Forecast Components on the Shannon-Gander Great Circle route at 500 mb. for the period April, 1952, to March, 1954.

TABLE 4 - Distribution of Errors at 500 mb., Summer

Month	Error (knots)													
	+30	+25	+20	+15	+10	+5	0	-5	-10	-15	-20	-25	-30	Total
April	0	0	2	20	46	65	63	46	22	9	3	1	1	278*
May	0	0	0	14	40	52	88	53	35	12	9	1	0	304*
June	0	1	3	5	19	69	104	69	28	2	0	0	0	300
July	0	0	0	8	30	73	100	78	17	1	0	2	1	310
August	0	0	0	1	20	71	100	64	37	12	3	0	0	308*
Sept.	0	2	1	10	27	75	109	52	20	4	0	0	0	300
Total:	0	3	6	58	182	405	564	362	159	40	15	4	2	1,800
Normal:	0	1	11	59	198	395	505	384	187	50	9	1	0	

* Values missing in these months because of accidental loss of records.

TABLE 5 - Distribution of Errors at 500 mb., Winter

Month	Error (knots)															
	+35	+30	+25	+20	+15	+10	+5	0	-5	-10	-15	-20	-25	-30	-35	Total
Oct.	0	3	2	5	16	35	59	87	62	31	8	2	0	0	0	310
Nov.	0	0	1	11	15	35	59	70	67	26	11	4	1	0	0	300
Dec.	0	1	2	7	27	42	58	68	56	29	15	3	1	1	0	310
Jan.	1	1	7	8	13	30	55	79	62	28	20	4	1	1	0	310
Feb.	1	0	4	5	25	42	48	51	58	26	13	3	3	1	0	280
March	0	0	1	6	23	31	51	92	60	34	8	4	0	0	0	310
Total:	2	5	17	42	119	215	330	447	365	174	75	20	6	3	0	1,820
Normal:	0	2	11	42	118	241	362	401	326	195	86	28	7	1	0	

TABLE 6 - Percentages of forecast components within 7 knots and within 12 knots of corresponding actual values

	April to September (both years)			September to April (both years)		
	700 mbs.	500 mbs.	Both levels	700 mbs.	500 mbs.	Both levels
Within 7 kts.	83.2	73.9	78.6	73.5	62.8	68.9
Within 12 kts.	97.4	92.9	95.2	91.8	84.1	87.9

Entire Period

	700 mbs.	500 mbs.	Both levels
Within 7 kts.	78.4	68.3	73.4
Within 12 kts.	94.6	88.5	91.7

TABLE 7 - Distribution Statistics for the data of Tables 2 to 5

		m	σ'	a	$\sqrt{b}$	p	σ_1	σ_2
700 mb.	Summer:	+.256	5.54	.786	-.122	.269	6.95	4.89
	Winter:	+.368	7.12	.770	-.027	.413	9.09	5.26
500 mb.	Summer:	+.275	6.95	.760	-.206	.701	7.99	3.39
	Winter:	+.860	8.94	.772	+.205	.832	9.62	3.63

Appendix 1. Estimation of the parameters of the compound normal distributions.

The sample of N errors is regarded as made up of pN errors from a Normal distribution with standard deviation σ_1 and qN errors from a normal distribution with standard deviation σ_2, the two distributions having the same mean m. Thus, if the observed mean deviation, standard deviation and fourth moment are d, σ and m_4 respectively, the parameters will be related to the expected values of the statistics by the following three equations:

(1) $$p\sigma_1 + q\sigma_2 = \sqrt{\frac{\pi}{2}}\,d$$

(2) $$p\sigma_1^2 + q\sigma_2^2 = \sigma^2$$

(3) $$p\sigma_1^4 + q\sigma_2^4 = \frac{1}{3}m_4$$

The mean of the distribution is taken equal to the sample mean and the bias in the sample moments caused by the use of an estimated mean is neglected.

Solving (1) and (2) for σ_1 and σ_2 we obtain:-

(4) $$\sigma_1 = \sqrt{\frac{\pi}{2}}\,d\left\{1 + \sqrt{\frac{q}{p}\left(\frac{2\sigma^2}{\pi d^2} - 1\right)}\right\}$$

(5) $$\sigma_2 = \sqrt{\frac{\pi}{2}}\,d\left\{1 - \sqrt{\frac{p}{q}\left(\frac{2\sigma^2}{\pi d^2} - 1\right)}\right\}$$

together with a second solution in which σ_1 and σ_2 are interchanged. If we specify that σ_1 shall be the larger standard deviation, the second solution can be rejected.

Writing r for $\frac{2\sigma^2}{\pi d^2} - 1$, substituting in (3) from (4) and (5) and re-arranging (remembering that $q = 1-p$):-

$$1 + 6r - 3r^2 + 4r^{3/2}\left(\frac{1-2p}{\sqrt{p(1-p)}}\right) + \frac{r^2}{p(1-p)} = \frac{4m_4}{3\pi^2 d^4}$$

Putting $x = 1-2p$ and writing Q for $\frac{4m_4}{3\pi^2 d^4} - 1$, this becomes:-

$$Q - 6r + 3r^2 = \frac{8r^{3/2}x}{\sqrt{1-x^2}} + \frac{4r^2}{1-x^2}$$

This last equation can easily be rationalized into a quadratic in x^2. The quadratic will give two roots in x^2 and therefore four roots in x, but it can be seen that two of these are extraneous roots introduced by the process of rationalisation, and that the extraneous roots are of equal magnitude and opposite sign to those which satisfy the original equation.

The two roots in x^2 are:-

$$x^2 = 1 - 4r^2\left\{\frac{Q + 2r + 3r^2 \pm 4\sqrt{r(Q - 2r - r^2)}}{(Q - 6r + 3r^2)^2 + 64r^3}\right\}$$

These were calculated for each of the four samples, and from each, a pair of values of p were calculated, using:-

$$p = \tfrac{1}{2}(1 - x)$$

The two extraneous values of p were then identified by numerical substitution in (4), (5) and (3). In each case it was found that one of the two remaining values of p corresponded to negative values of σ_2 and could, therefore, be rejected, leaving only one value which was physically acceptable. The one remaining value of p is given in Table 7, together with the values of σ_1 and σ_2 calculated from (4) and (5).

Appendix 2. Adjustment of mean square errors of 0800 forecasts for interpolation bias.

Let x and y be the values of the actual and forecast components, respectively, at time t (in the present case, 0800Z), both being measured from the mean of the actual component. It will be assumed that this is also the mean of the forecast component, - the degree of error involved will be that implied in neglecting the square of the mean value of $x-y$ in comparison with the mean square value of $x-y$.

The true value of x is unknown, the known values of x being those for times $t = 0, 1, 2 \ldots\ldots$ where the unit of time is 12 hours. If $t = 1+p = 2-q$, p being less than unity, the estimate of x given by linear interpolation is:-

(1) $$z = qx_1 + px_2$$

x_1 and x_2 being the values of x for $t=1$ and $t=2$.

The problem is, given σ^2_{y-z} , the mean square value of $y-z$ to estimate σ^2_{x-y} the mean square value of the unknown quantity $x-y$.

(For the data with which we are concerned, $t=0$ refers to 1500Z and p = 5/12, q = 7/12).

The form in which the problem will be approached is that of estimating the correlation coefficient r_{xy} between x and y in terms of the observable correlation coefficient r_{yz} between y and z .

Two separate causes may be expected to contribute towards giving r_{yz} a positive value. Firstly, y and z will be positively correlated with x , as both are estimates of x . Secondly, both y and z will be positively correlated with a the latest actual component known when the forecast was made. In the present case, a is the value of x at $t=0$.

It will be assumed that the magnitude of the actual correlation coefficient between y and z is exclusively the result of the two causes just mentioned.

This assumption leads us to equate to zero the second-order partial correlation coefficient, $r_{yz.ax}$.

Expressed in terms of the total correlation coefficients involved, this gives

$$\frac{\begin{vmatrix} 1 & r_{xy} & r_{ax} \\ r_{zx} & r_{yz} & r_{az} \\ r_{ax} & r_{ay} & 1 \end{vmatrix}}{\left\{ \begin{vmatrix} 1 & r_{zx} & r_{ax} \\ r_{zx} & 1 & r_{az} \\ r_{ax} & r_{az} & 1 \end{vmatrix} \begin{vmatrix} 1 & r_{xy} & r_{ax} \\ r_{xy} & 1 & r_{ay} \\ r_{ax} & r_{ay} & 1 \end{vmatrix} \right\}^{1/2}} = 0$$

as the denominator cannot become infinite, we may equate the numerator to zero. Solving for r_{xy}, this gives:-

(2) $$r_{xy} = \frac{r_{yz}(1 - r_{ax}^2) - r_{ay}(r_{az} - r_{ax} r_{zx})}{r_{zx} - r_{az} r_{ax}}$$

The required correlation coefficient r_{xy} has now been expressed in terms of the observed correlation coefficient r_{yz} and the quantities r_{zx}, r_{ax}, r_{az} and r_{ay}.

In order to obtain estimates of the first two of these four quantities, we must make use of the assumption referred to earlier that the correlation coefficient r_t between successive values of the component separated by a short time-interval t is approximately a linear function of t. It will first be proved that, neglecting any diurnal variation in the variance σ_x^2 of the actual component, this assumption is equivalent to taking z to be the "least-squares" estimate of x based on x_1 and x_2. Neglecting the diurnal variation of σ_x^2, the standard deviations of x, x_1 and x_2 are all equal, and the regression equation of x on x_1 and x_2 reduces to:

(3) $$x' = \frac{r_p - r_1 r_q}{1 - r_1^2} x_1 + \frac{r_q - r_1 r_p}{1 - r_1^2} x_2$$

(r_1, is the correlation coefficient between successive values of the actual component 12 hours apart).

Then if $x' = z$, we have, by equating coefficients in (1) and (3):-

$$\frac{r_p - r_1 r_q}{1 - r_1^2} = q \qquad \frac{r_q - r_1 r_p}{1 - r_1^2} = p$$

which, together with $p + q = 1$ are equivalent to:

$$r_p = 1 - p(1 - r_1), \quad r_q = 1 - q(1 - r_1)$$

that is to say, are equivalent to giving to r_p and r_q the values obtained by linear interpolation between 1 and r_1, which proves the proposition.

The value of r_{zx} now follows immediately. For, since z is a least-square estimate of x, the regression coefficient of x on z is unity, so that

$$r_{zx} = \frac{\sigma_z}{\sigma_x}$$

We may calculate σ_z from (1) by taking the standard deviation of both x_1 and x_2 as equal to σ_x.

This gives:-
$$\sigma_z^2 = \sigma_x^2(p^2 + q^2 + 2pqr_1)$$
$$= \sigma_x^2\{1 - 2pq(1 - r_1)\}$$
$$= \sigma_x^2 - pq\sigma_1^2$$

so that:

(4)
$$r_{zx} = \sqrt{1 - pq\frac{\sigma_1^2}{\sigma_x^2}}$$

The quantity r_{ax} is simply the value of r_t for $t = 1 + p$. Applying linear interpolation again, we find:-

(5)
$$r_{ax} = qr_1 + pr_2$$
$$= \frac{q\sigma_1^2 + p\sigma_2^2}{2\sigma_x^2}$$

The third unknown r_{az} may be expressed in terms of the first two, using only the assumptions already made. For, expanding the co-variance of z and a according to the terms in the right-hand side of (1), we find:-

$$r_{az} = \frac{q\sigma_x^2 r_1 + p\sigma_x^2 r_2}{\sigma_z \sigma_x}$$
$$= \frac{\sigma_x}{\sigma_z}(qr_1 + pr_2)$$

(6)
$$= \frac{r_{ax}}{r_{zx}}$$

Now substituting from (6) in (2):-

(7)
$$r_{xy} = \frac{r_{yz} r_{zx}(1 - r_{ax}^2) - r_{ax} r_{ay}(1 - r_{zx}^2)}{r_{zx}^2 - r_{ax}^2}$$

The fourth unknown, r_{ay} may be determined directly from the values of the 1500Z actual and 0800Z forecast components for the month in question. The observed value of r_{yz} may then be used to estimate r_{xy} by means of equations (4), (5) and (7), and σ_{x-y}^2 may be calculated from

$$\sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 - 2r_{xy}\sigma_x\sigma_y$$

using for σ_x the value calculated from all the actuals for the same period.

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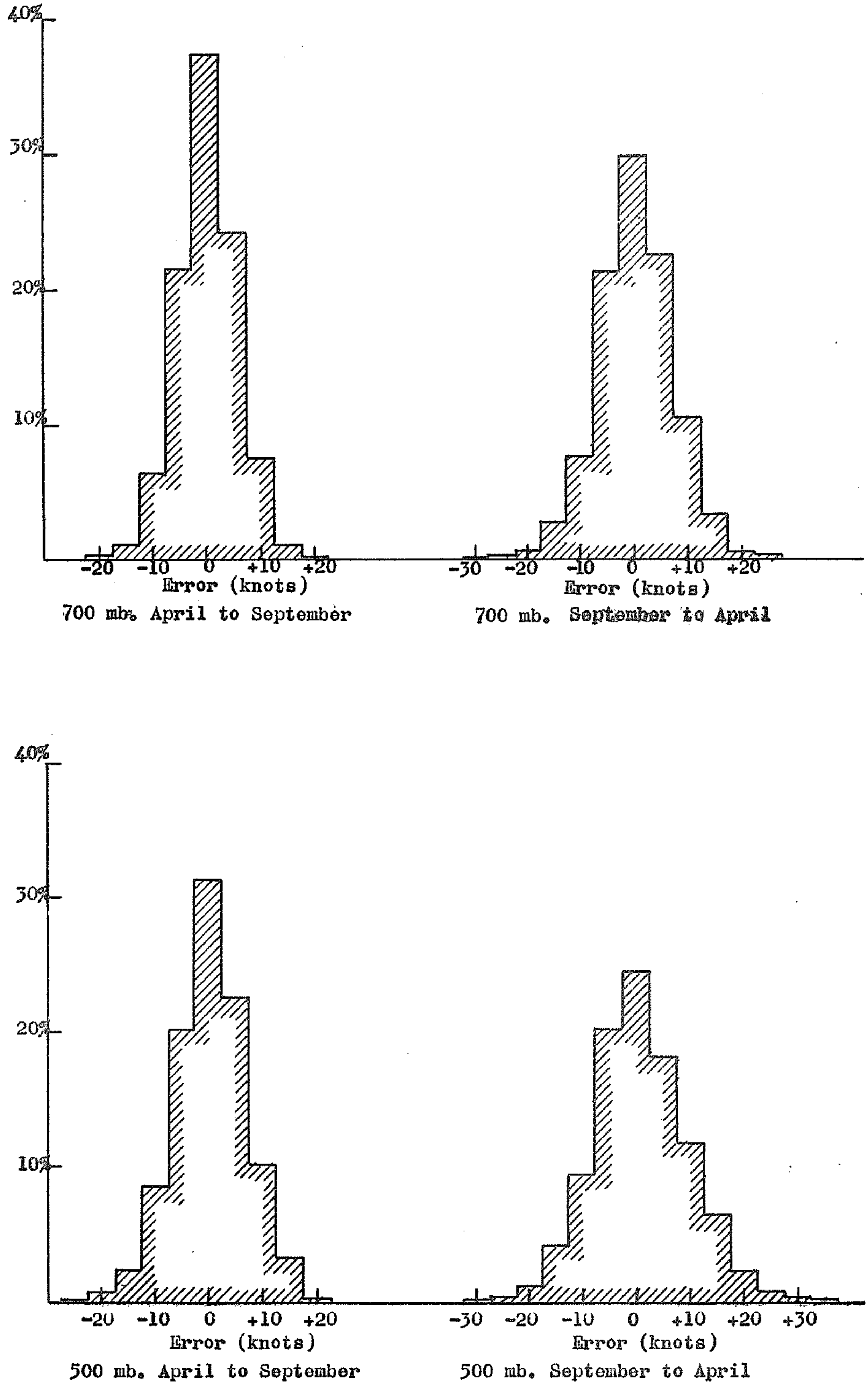

Fig. 1. Percentage frequency distribution of errors in forecast components on the Shannon-Gander great circle route for the period April 1952 to March 1954.

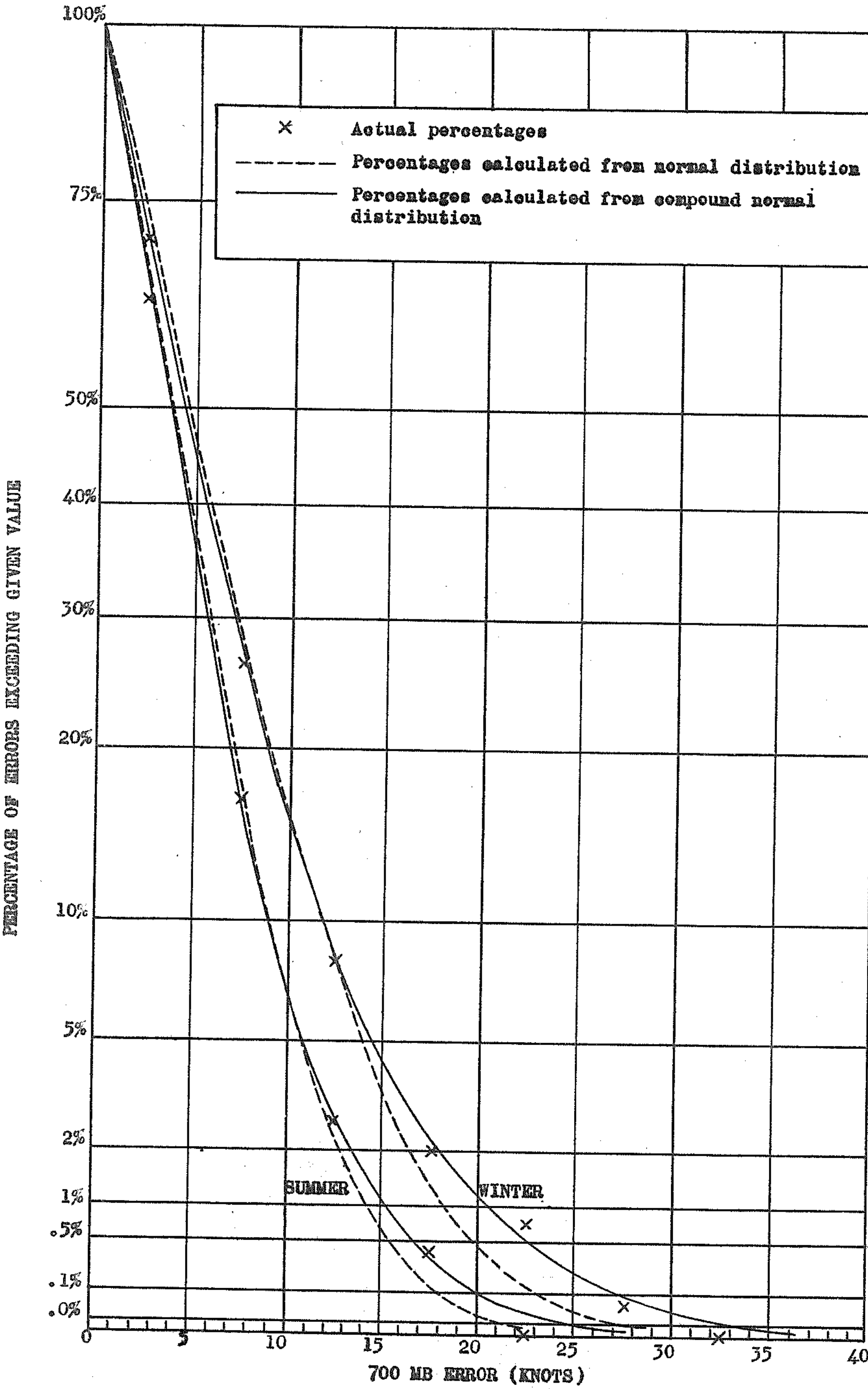

Fig. 2. Actual and calculated percentages of errors in the forecast 700 mb Shannon-Gander great-circle component numerically exceeding stated values. The curves marked "Summer" refer to the periods April to September, inclusive, 1952 and 1953, and the curves marked "Winter" refer to the periods October 1952 to March 1953 and October 1953 to March 1954.

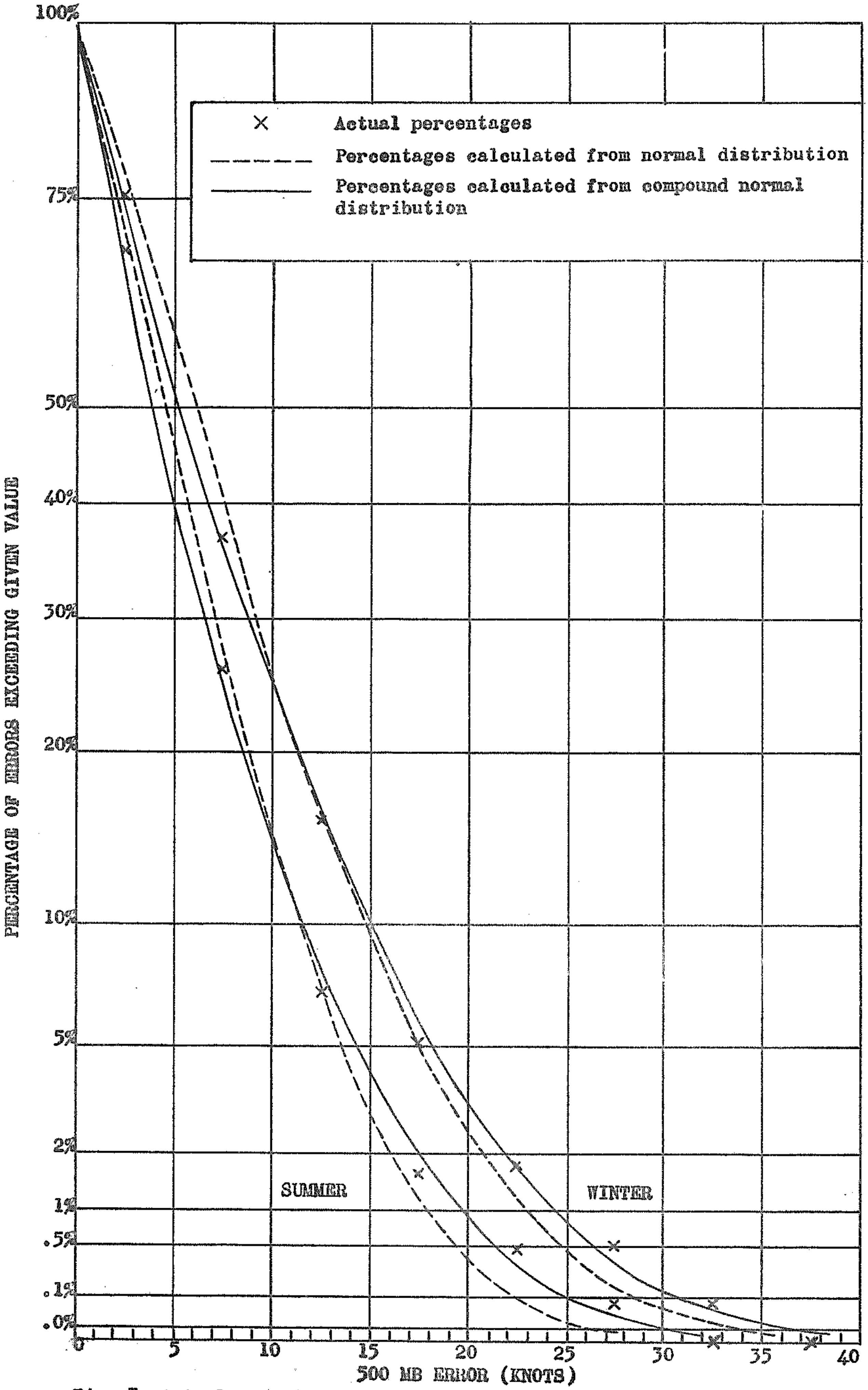

Fig. 3. Actual and calculated percentages of errors in the forecast 500 mb. Shannon-Gander great-circle component numerically exceeding stated values. The curves marked "Summer" refer to the periods April to September, inclusive, 1952 and 1953, and the curves marked "Winter" refer to the periods October 1952 to March 1953 and October 1953 to March 1954.

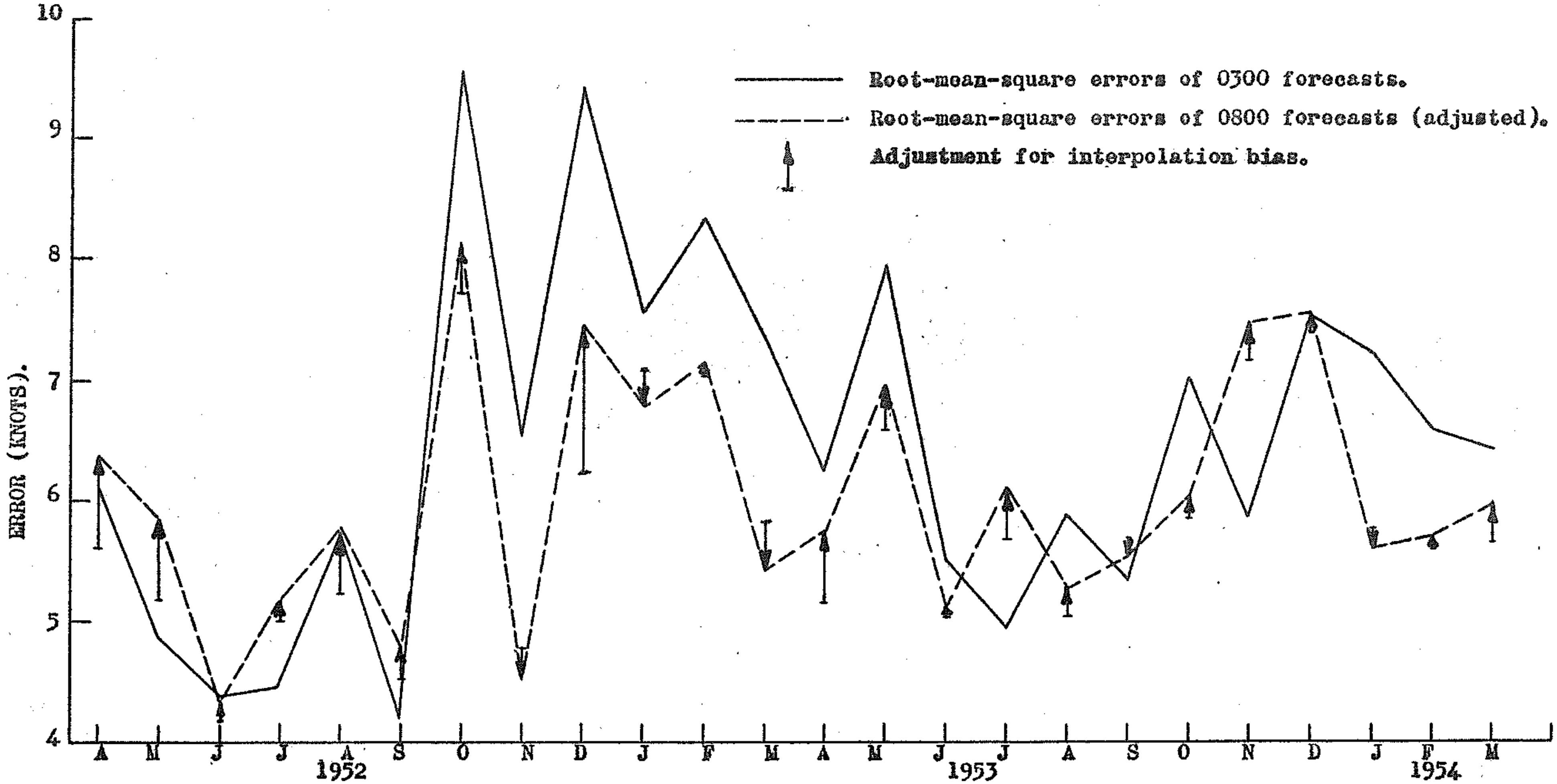

Fig. 4. Monthly root-mean square errors of forecast components on the Shannon-Gander great circle at 700 MB.