METEOROLOGICAL SERVICE

15489

TECHNICAL NOTE No. 39

OCEAN WAVES

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DUBLIN
MARCH 1975

Price: 30p

OCEAN WAVES

1. GENERAL DESCRIPTION

It is a matter of common observation that the surface of the sea rarely presents a smooth mirror-like appearance even on a calm day, and that as a wind develops the surface quickly takes on an undulating appearance, ripples form and these enlarge into waves as the wind increases. Friction between the moving air and the flat sea surface induces turbulent eddies in the air. As a result some of the air particles previously moving parallel to the surface now strike against it and distort it, producing ripples. Once this occurs the ripples are acted on by the horizontally moving air particles as well. These create excess pressure on the windward slope of the ripple and also diminished pressure on the lee side. This causes the ripples to move downwind and to increase in size as long as the wind is moving faster than the wave which it is producing. Acting against the wind force and tending to restore the water surface to its undisturbed position are the forces of surface tension and gravity. For waves of more than a few centimetres length the effect of surface tension is unimportant.

The details of the physical processes which transfer energy from the wind to the water surface - leading to the production of waves - are still not completely understood. Observation shows that as the wind increases, so does the height of the waves. This height increases with the length of time for which the wind has been blowing (the duration), and also with the distance across the water over which the wind has been blowing unimpeded (the fetch), but for any steady wind speed the waves eventually reach a maximum height. Such waves are said to be "fully developed", and the sea surface is described as a fully developed sea.

2. BASIC THEORY

Elementary wave theory shows that, to a first approximation, small waves on the boundary surface between two fluids (e.g. water and air) will be waves having a vertical cross-section in the form of a sine-curve. However, observation of any sea surface affected by winds of Beaufort Force 3 or more shows that the resulting waves are far removed from simple sine-waves. Instead of producing a simple sine-wave with a definite period and height, the wind may be regarded instead as producing a family of sine-waves of differing periods and heights which combine to form the observed shape of the sea surface. In theory an instantaneous picture of a vertical cross-section of this surface can be reduced by Fourier analysis to its component sine-curves but the result is of little value in practical forecasting of waves. Analysis does show however that for any particular wind speed sine-waves of all periods up to a certain maximum period are produced.

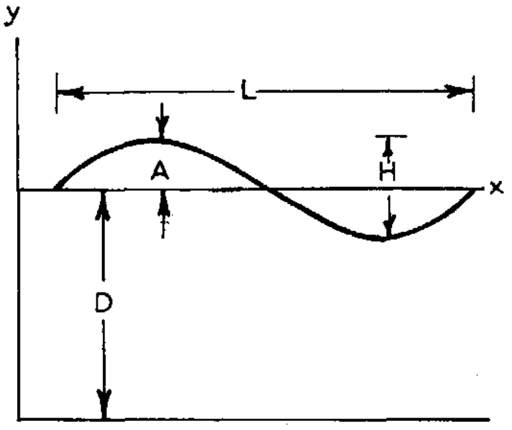

Fig. 1

Figure 1 shows a simple sine-wave, with amplitude, height and length of A, H and L, moving with a velocity V along the surface of a liquid whose depth is D. The period of this wave is T, so that

$$V = L/T$$

The equation of this wave is

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{L}\right)$$

and theory shows that the velocity of the wave is given by

$$V = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi D}{L}}$$

For $D/L \geq \frac{1}{2}$ $\tanh \frac{2\pi D}{L} \doteqdot 1$ so that $V = \sqrt{\frac{gL}{2\pi}}$

or, using $V = L/T$, $L = \frac{gT^2}{2\pi}$, and $V = \frac{gT}{2\pi}$.

For g in feet/sec^2 this gives approximately

$$V \text{ (in knots)} = 3\,T \text{ (in seconds)}$$

$$L = 5\,T^2 = 1\tfrac{1}{3}\sqrt{L} \text{ (in feet).}$$

The above formulae are valid where the half-wave length is less than the depth of the water. Such waves are described as <u>surface waves</u>, and are the type normally encountered over the ocean, where the precise depth is irrelevant so long as it exceeds the half wave-length.

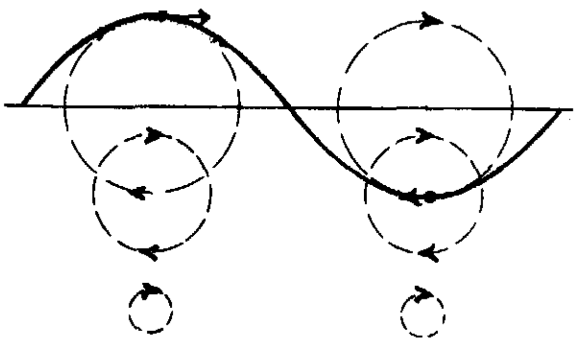

Fig. 2

In a wave (see Fig. 2) the water particles at the sea surface move in almost circular orbits in a vertical plane, forwards when at the wave crest and backwards when in the trough. The diameter of this orbit is the same as the height of the wave. Particles below the surface also move in circular orbits but the diameter of these orbits decreases geometrically with the depth.

At a depth of one-ninth the wavelength the diameter of the orbit is reduced to one-half that at the surface, while at a depth of one-half the wavelength it is reduced to $^1/23$ the surface value. Thus for particles below this depth the amount of movement is negligible, and if the sea bed is below this depth it presents no appreciable impediment to the motion of the wave.

As waves increase in height, the wave profile departs from a simple sine curve. The circular motion of the water particles tends to heighten and sharpen the crests, and reduces and flattens the troughs, so that the curve become more like a trochoid (the curve which is formed by the motion of a point on a disc as the disc rolls on a flat surface).

Another theoretical result is that the maximum wave height possible without the wave crest breaking is one-seventh of the wavelength. In practice, however, this ratio seldom exceeds one-tenth. (The Marine Observer's Handbook states one-thirteenth.)

3. SIGNIFICANT AND MAXIMUM WAVE-HEIGHTS

A typical sea wave can be regarded as being made up of a series of sine waves of differing amplitudes and periods, A_i and T_i. From A_i we can define a number

$$E = \sum A_i^2$$

This number is a theoretical measure of the energy of the wave and it is also related to the wave heights in the sea. The average wave height is given by $1.77\sqrt{E}$. A height more frequently used in theory is the significant height (usually denoted by H_s or $H_{\frac{1}{3}}$). This is defined as the average of the highest one-third of the waves, and is of significance because it has been found to agree fairly closely with the wave height obtained by experienced observers aboard ship who are attempting to estimate the average height of the higher well-defined waves. This height H_s is given by $2.83\sqrt{E}$.

Another height which is sometimes used is the maximum wave height, H_{max}. This is obtained by taking the highest wave occurring during each of a series of intervals, and finding its average value. There are differences of opinion as to how H_{max} is related to H_s. Darbyshire (see later) used $H_{max} = 1.6\ H_s$, based on typical wave records of about 10 minutes duration and containing about 100 waves, but other workers have obtained slightly different values, due perhaps to having used records of different length. For practical purposes a value $H_{max} = 1.5\ H_s$ can be taken, but it must be remembered that extreme waves will occur in isolated cases which will be higher than this derived value. In the above case where records each containing about 100 waves are used if the observation is repeated a large number of times then 5% of the values obtained will be higher than $1.94\ H_s$. If longer intervals are used, giving about 1000 waves per record, then 5% of the highest values observed will exceed $2.22\ H_s$. These figures, which are based on theory, are due to the individual components which make up the wave, each reaching a maximum simultaneously, a phenomenon which can occur only infrequently. Such waves are occasionally observed in practice but as they exist only for a very short time they will often go unobserved. These outsized waves are destroyed by their own height. As described earlier, the water particles on the sea surface rotate in vertical circles whose diameter equals the height of the wave. As this height increases the particles acquire greater speeds, so that eventually - when at the crest of the wave - they are moving forward faster than the crest itself is moving. Hence the wave breaks as the crest topples forward into the trough ahead of it.

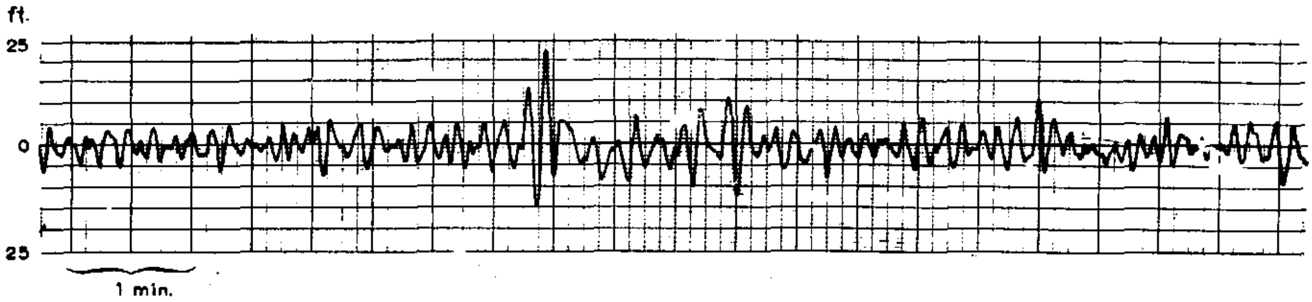

Fig. 3. Portion of Wave Record from Daunt Light Vessel, 12 January 1969

The highest wave recorded in Irish waters occurred at the Daunt Light Vessel off the coast of Co. Cork on 12 January 1969. Figure 3 (taken from "Wave Data for the Kish, Barrels and Daunt Rock Light Vessels", by A.D.H. Martin of The Irish Lights Office), shows a portion of the wave record, and indicates how much this solitary wave exceeded the normal waves which occurred during the ten minute period. Its true height, after corrections had been applied to allow for the limitations of the recording equipment, was 42½ feet.

The highest wave recorded in the North Atlantic was almost 67 feet (20.4 metres) at Ocean Weather Station Juliett (52°N, 19°W) on 12 September 1961.

4. WAVE SPEED

Waves move in the same direction as the wind. As shown earlier, individual sine waves move with a speed given by V = 3T. Actual waves produced by the wind are composed of individual sine waves of different wavelengths and periods, which move with different speeds. The effect of this is to produce alternate areas of fairly large and well defined waves (where the major sine-components are roughly in phase with each other) and areas of smaller ill-defined waves (where the same components are mostly out of step). The areas of larger waves are known as wave groups, in them the individual components appear to form at the rear of the group and move forward through it, finally disappearing as they move out through the forward edge. Such wave groups move with a velocity V = 1.5 T, where T is an average period for the waves in the group. For waves of periods 7 to 10 seconds, which are fairly typical of ocean waves, this gives speeds of 10 to 15 knots so that in 12 hours they will move downwind distances of 120 to 180 nml, equal to two to three degrees of latitude.

5. WIND-WAVE DIAGRAMS

Much of the difficulty in deriving theoretical relationships lies in trying to measure accurately corresponding values of wind speed and wave height. Wind speed is normally measured at a height of 10 metres or more above the sea surface, but this value can differ from that of the wind actually in contact with the sea, the difference depending on the stability of the air and also on the degree of turbulent mixing already present as a result of interaction between air and sea. In addition, wind reports over the sea are generally subjective estimates rather than instrumental measurements. Wave heights also are more often than not visual estimates on the part of the observer. Even with moderate winds, waves are usually irregular in shape and of varying height, with smaller wavelets superimposed on the larger waves. The observer is instructed to estimate an average height and period for the higher, well-defined waves, ignoring these smaller wavelets. Naturally between one observer and another there may be different opinions as to what constitutes a well-defined wave. Anemometers and wave recorders have been in use in a limited number of ships during the past twenty years or so, and their observations have helped to improve the accuracy of the wind/wave relationships. However, even with instrumental measurements there is still uncertainty in the exact values of wind and wave upstream from the ship, and these factors must also be taken into consideration when deriving any relationships.

A number of scientists have derived theoretical relationships connecting wind speed, duration and fetch with the resulting wave height. For practical purposes they expressed their results, not in mathematical formulae, but in the form of nomograms relating some or all of these parameters. Figures 4 to 8 show some of these nomograms, all of which have been used by different meteorological or oceanic institutes. Since their initial hypotheses differed, and since each worker introduced empirical constants, based on different sets of observed waves and winds (the latter measured by anemometers of different heights), it is not surprising that there is some dissimilarity in their results.

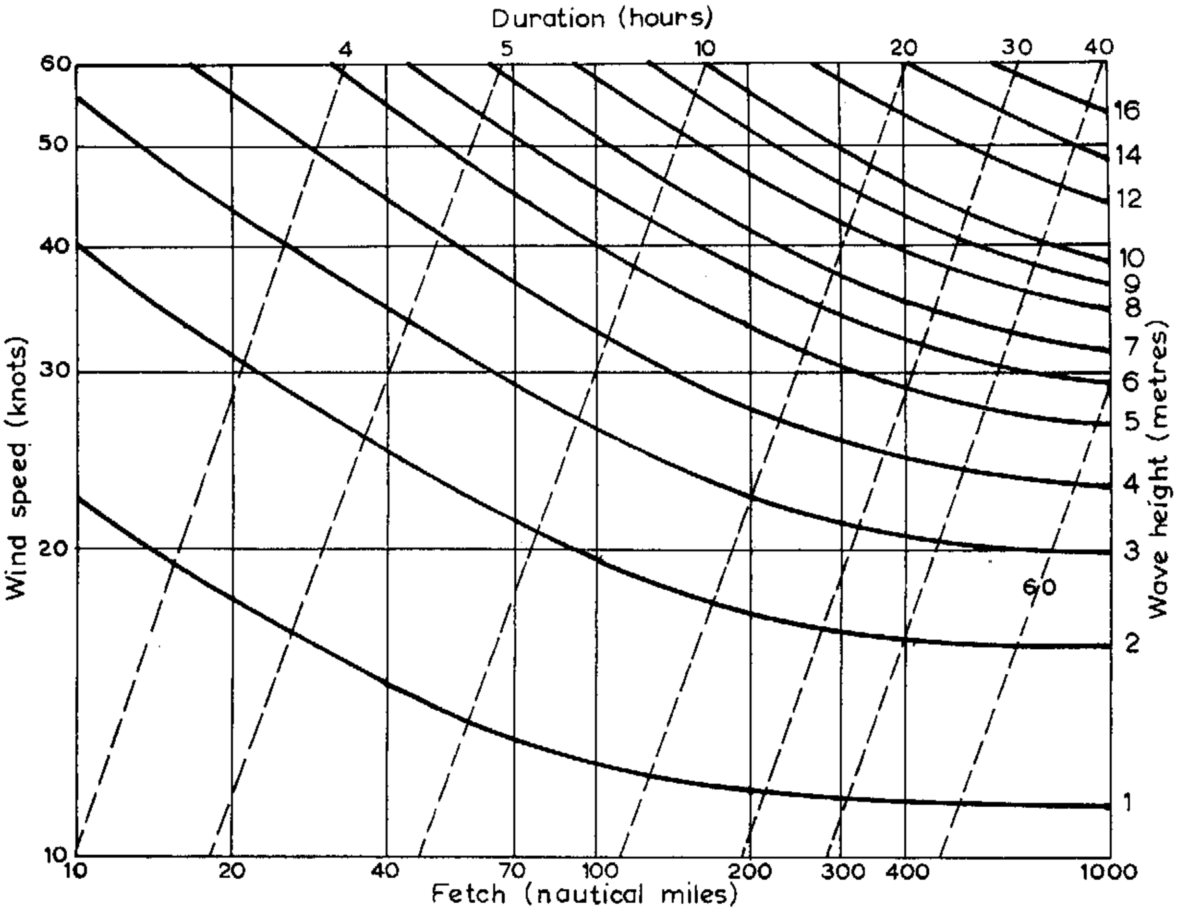

Fig. 4. Bretschneider's nomogram for significant wave heights

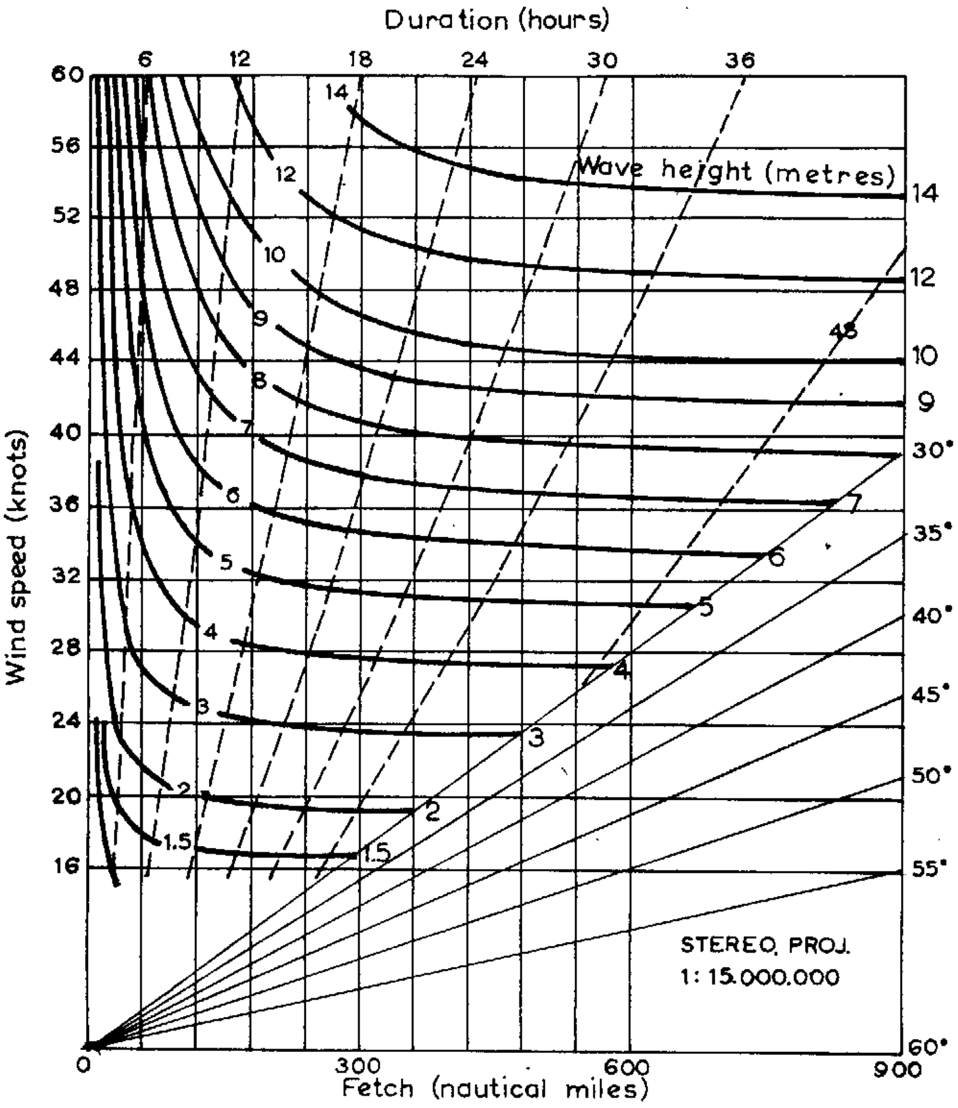

Fig. 5. Dorrestein's nomogram for significant wave heights

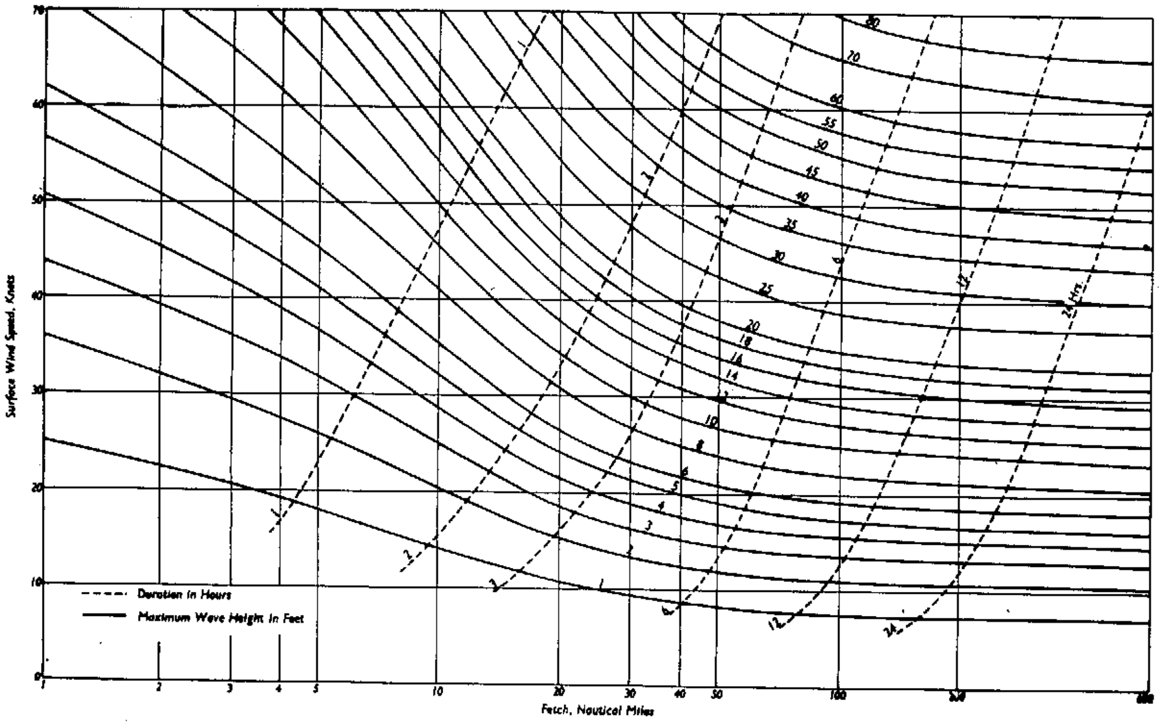

Fig. 6a. Darbyshire and Draper's nomogram for maximum wave heights, oceanic waters

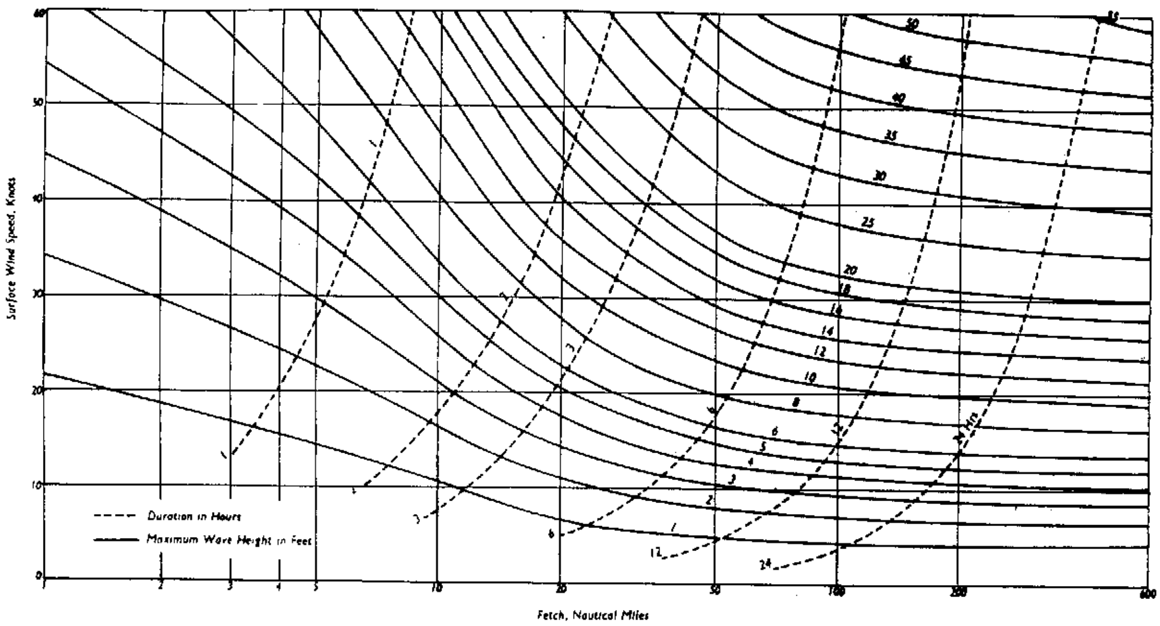

Fig. 6b. Darbyshire and Draper's nomogram for maximum wave heights, coastal waters

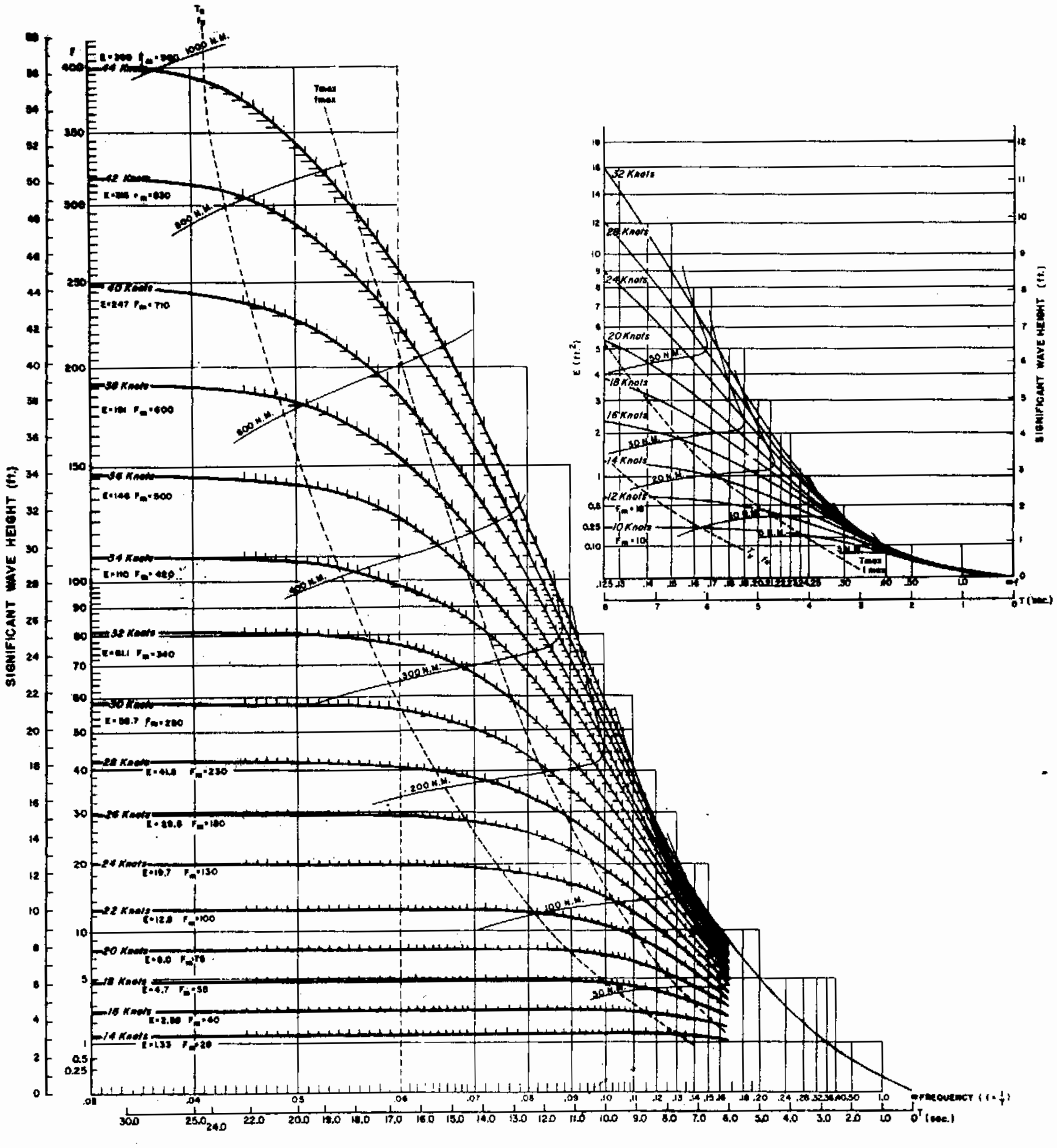

Fig. 7a Pierson, Neumann and James' nomogram for significant wave heights, with different fetches and winds of 10 to 44 kt.

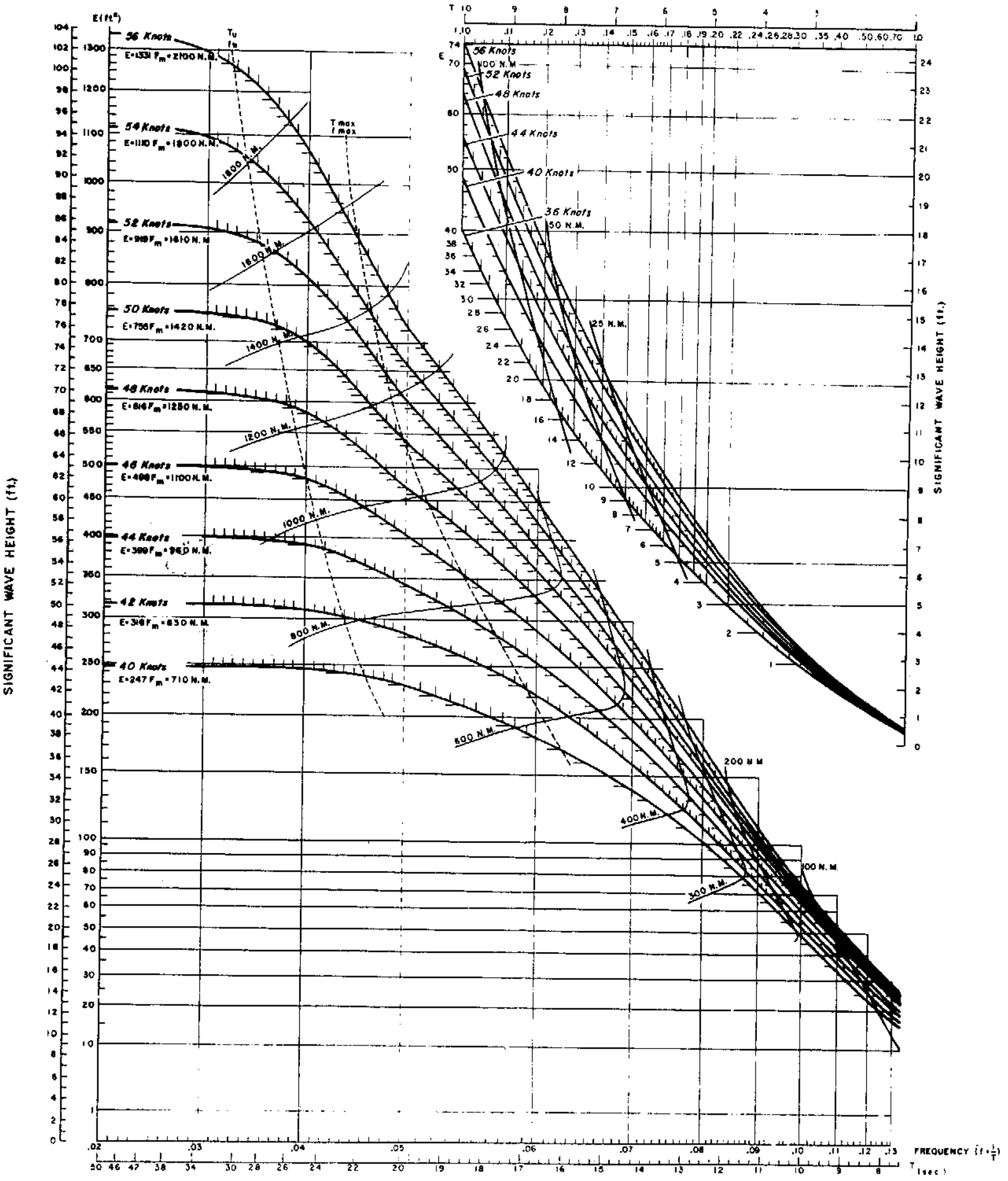

Fig. 7b Pierson, Neumann and James' nomogram for significant wave heights, with different fetches and winds of 36 to 56 kt.

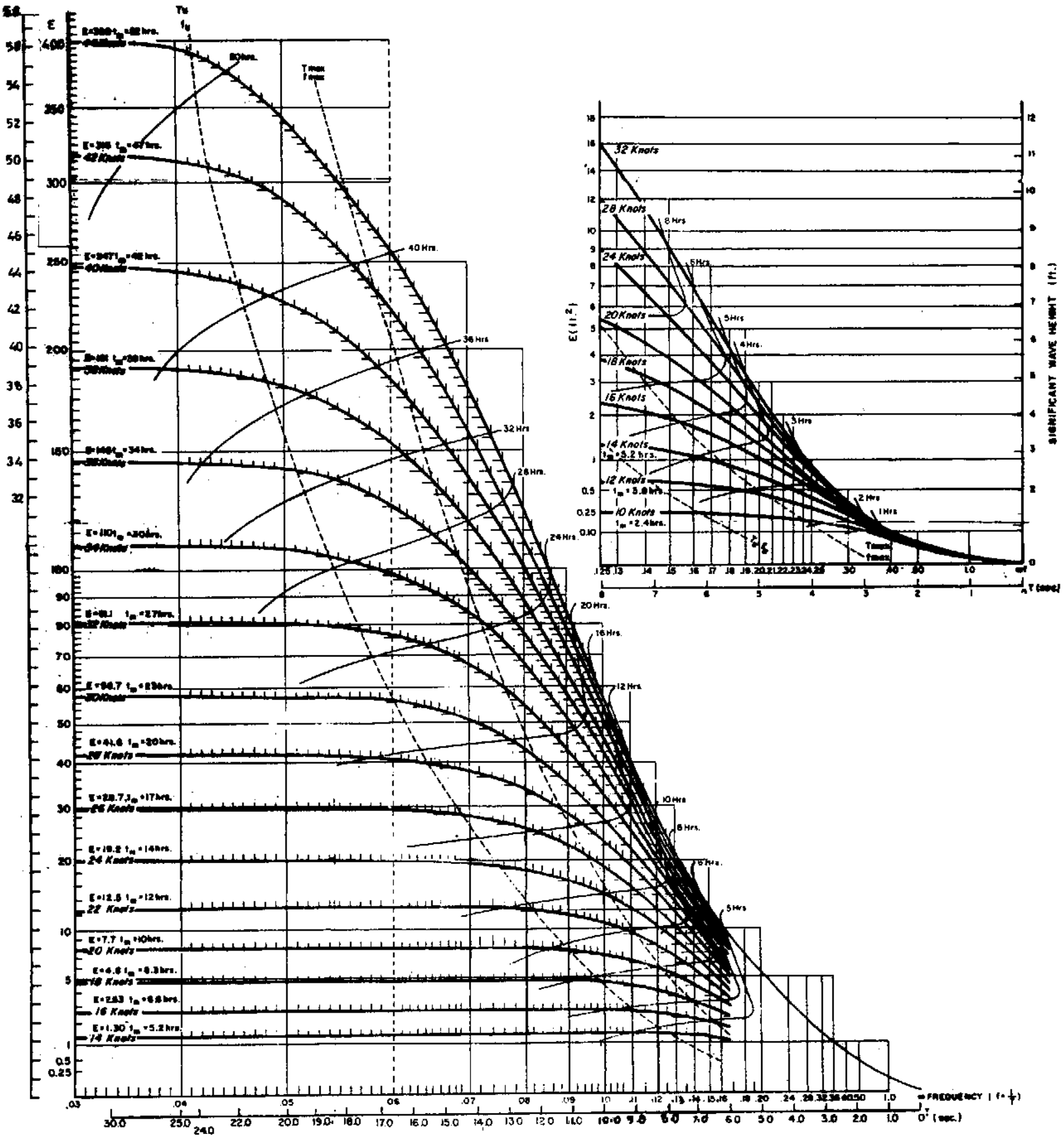

Fig. 7c Pierson, Neumann and James' nomogram for significant wave heights, with different durations and winds of 10 to 44 kt.

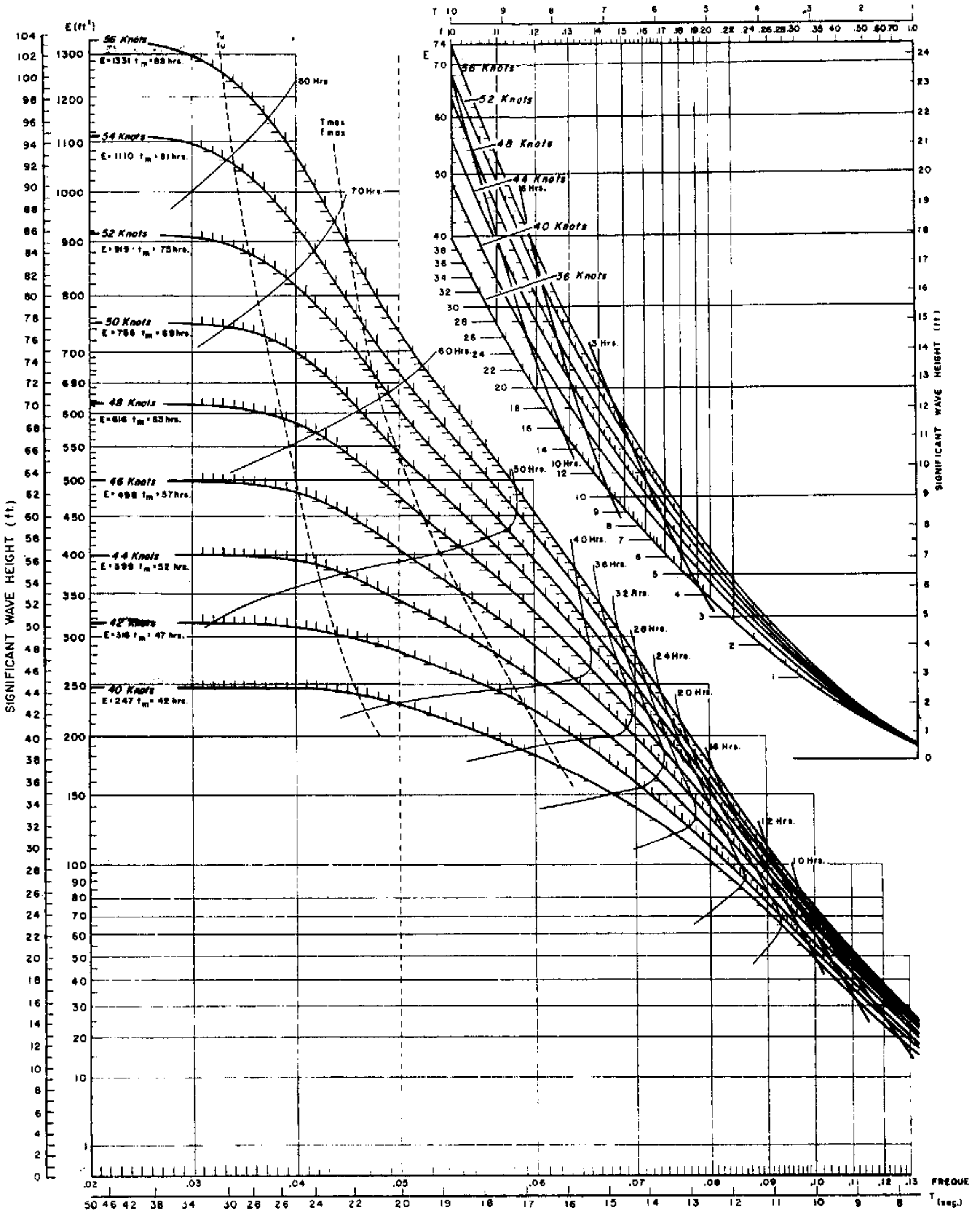

Fig. 7d Pierson, Neumann and James' nomogram for significant wave heights, with different durations and winds of 36 to 56 kt.

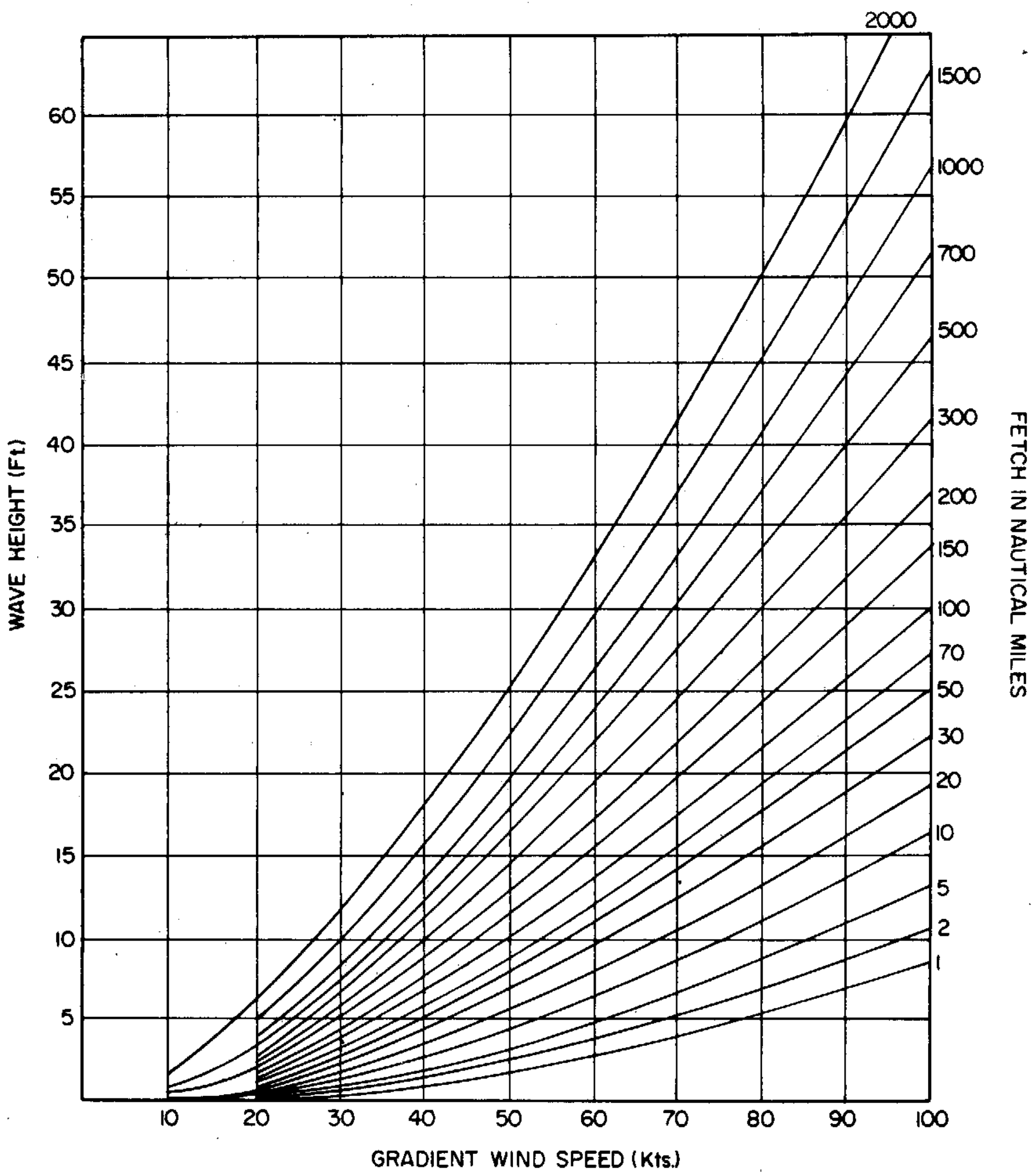

Fig. 8a. Suthons' nomogram for wave heights, with <u>limited fetch</u>

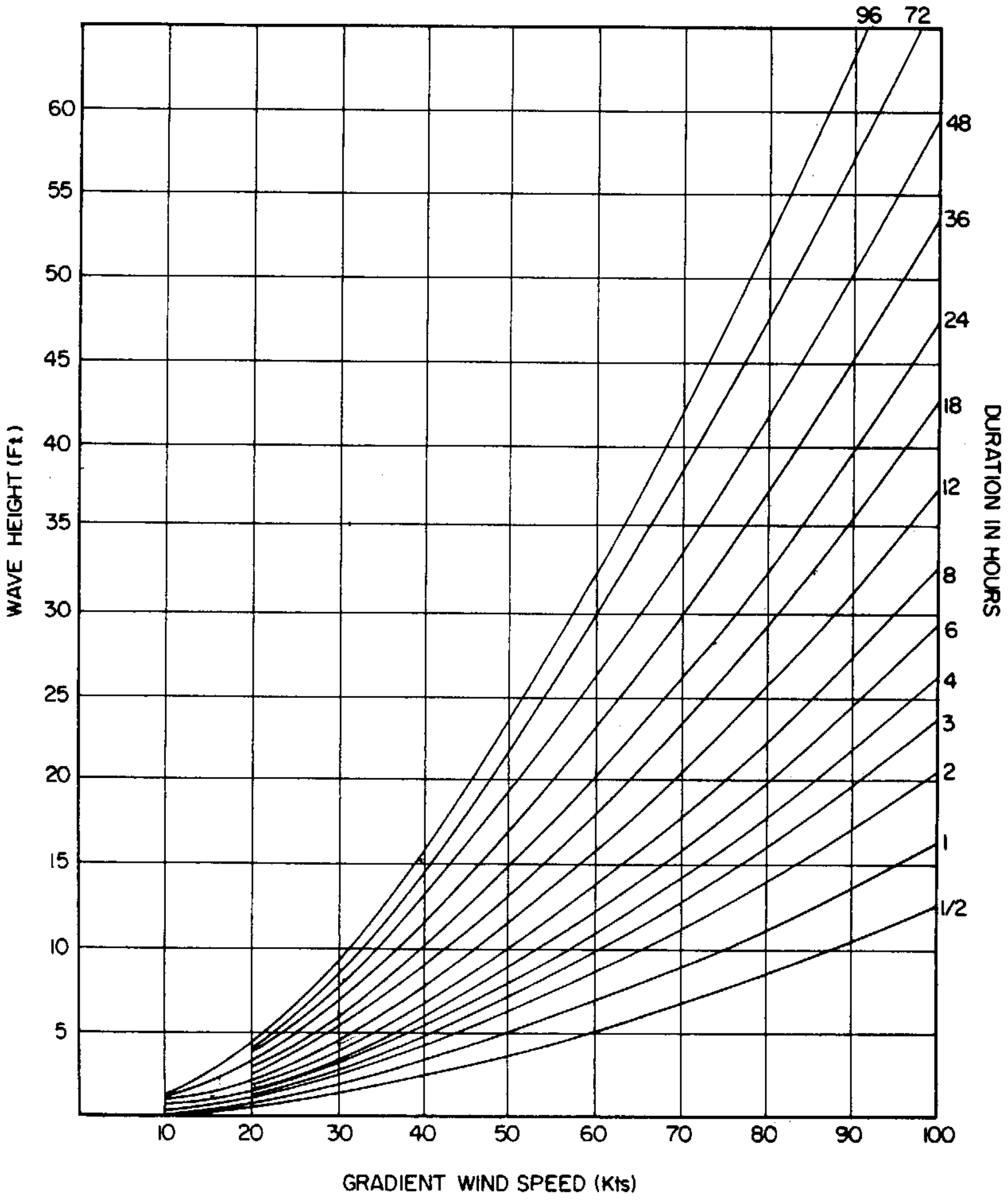

Fig. 8b. Suthons' nomogram for wave heights, with limited duration

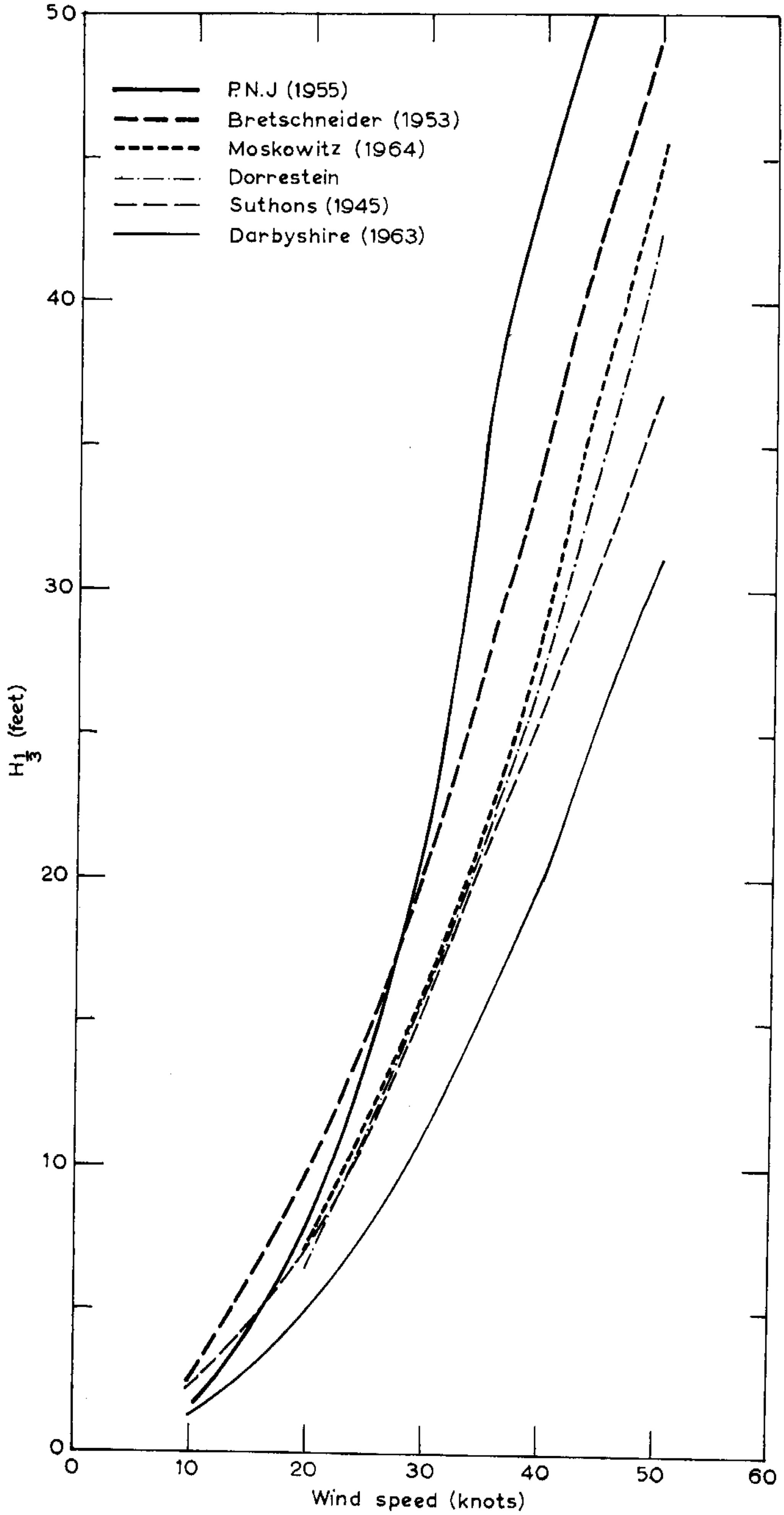

Fig. 9. Comparison of significant wave heights with fetch 1000 nml and duration 50 hours

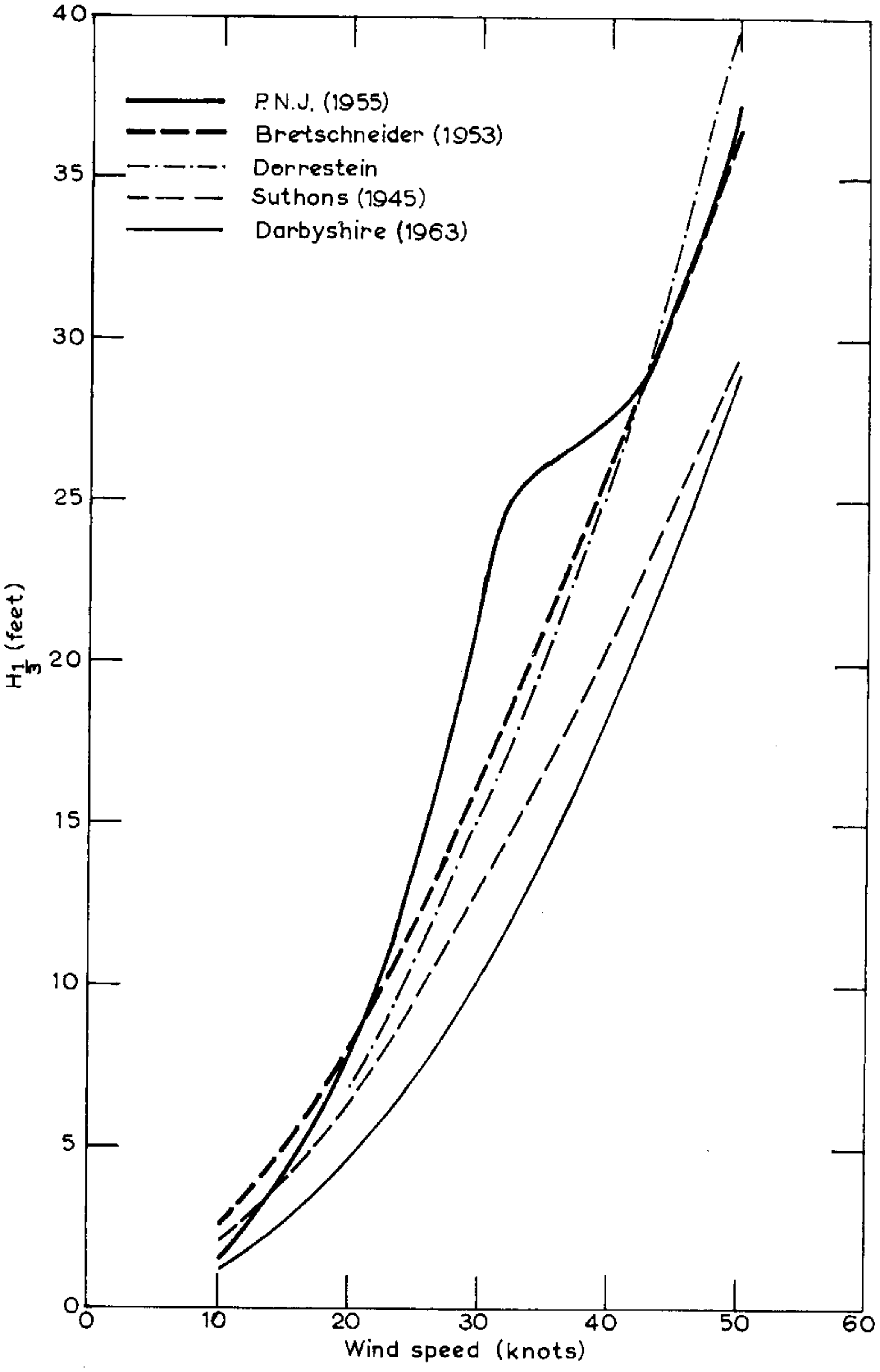

Fig. 10. Comparison of significant wave heights with fetch 500 nml and duration 25 hours

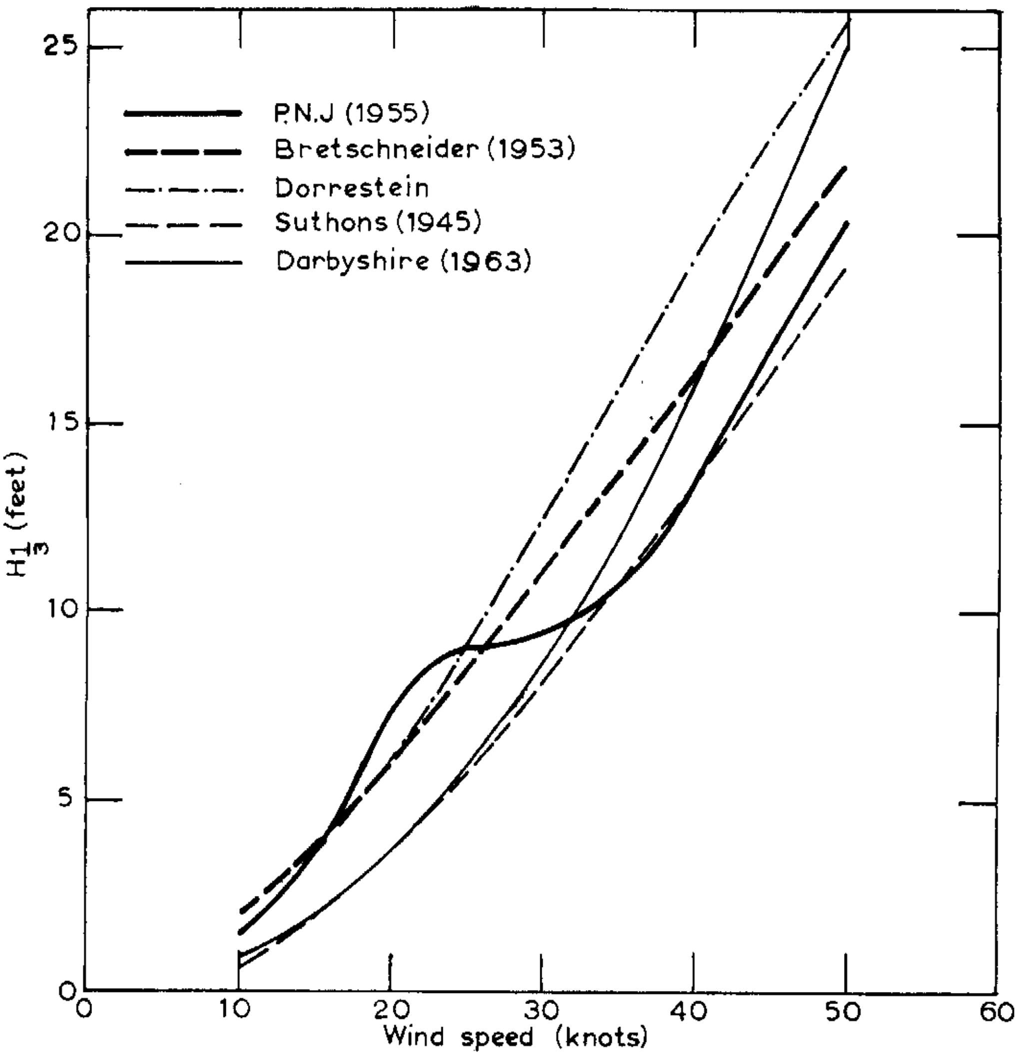

Fig. 11. Comparison of significant wave heights with fetch 100 nml and duration 10 hours

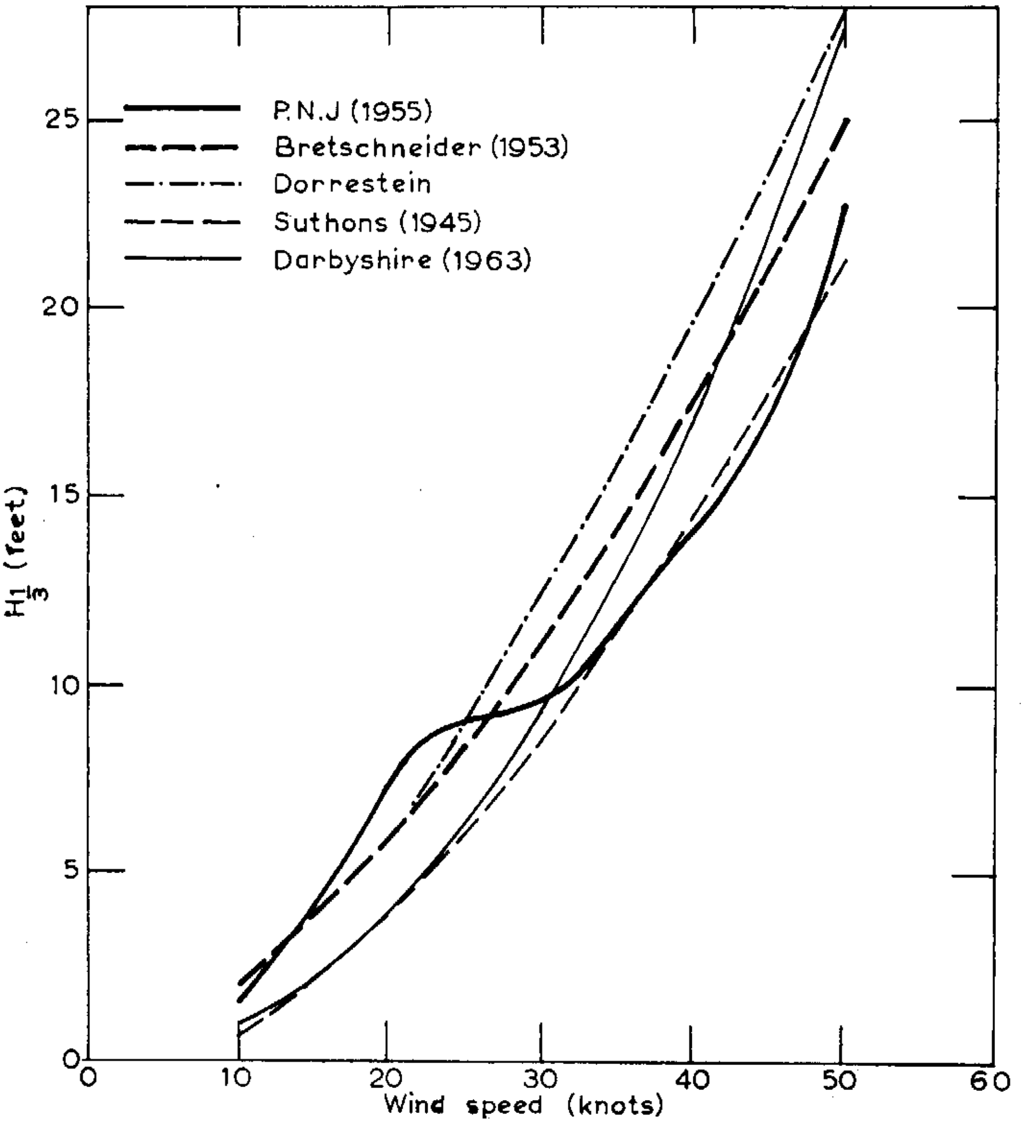

Fig. 12. Comparison of significant wave heights with fetch 300 nml and duration 10 hours

Figures 9 to 12 compare the significant wave heights obtained from these nomograms for a range of wind speeds under different conditions of fetch and duration. (Note: no allowance has been made for the different anemometer heights, which ranged from 7½ to 14 metres, but Suthons' "gradient" winds have been converted to "surface" winds by multiplying by a factor of ⅔, and the "maximum" wave heights in Darbyshire & Draper's nomograms have been reduced to "significant" wave heights by dividing by 1.60). Figure 9 shows also a relation produced by Moskowitz in the case where fetch and duration are unlimited and $H_s = .0182\ V^2$, (in units of feet and knots). On examining these graphs perhaps the surprising thing is that there is not more dissimilarity in the results obtained by the different workers.

6. FORECASTING OF SEA WAVES

From nomograms such as these the wave height produced by a particular wind blowing for a certain time over a known fetch can be read off directly. If either the fetch or the duration is insufficient to produce a fully developed wave it is the lower of the two heights obtained by using the given values of fetch and duration separately which must be taken.

By this means a forecast can be made of the wave height at a selected point for some future time, assuming the wind remains unchanged in the interim. Such a wave height is based on the assumption that no waves are present initially. In practice, waves usually do exist, but can be allowed for in the following way. From the nomogram can be found the time for which the existing wind would have had to blow to produce this size of wave. This time is added to the required forecast interval to give a total duration, which is then used with the wind to obtain a final overall height from the nomogram.

In the case where the wind speed is not steady but is increasing (or is expected to increase) during the period the simplest procedure is to assume a mean wind over the period. The average of the initial and final winds could be used, but it has been found by experience that when these values differ by more than about five knots a better result is obtained by taking a value equal to the final wind less one-quarter of the difference between the initial and final values. Using this wind and the duration or fetch the wave height at the end of the period is obtained as before.

Note: Strictly speaking, in the above operations it is not the initial wind and wave height at the selected point which should be used, but those values which were occurring at the same time at a point just far enough up-wind for the wave there to reach the selected point by the end of the period.

(The opposite case - where the wind decreases during the period - will be dealt with later under SWELL).

As well as changing in speed, winds can also change in direction during the period being considered. If this change is less than 30 degrees it can be ignored without any appreciable loss of accuracy. If the change is 30 degrees or more two things will happen: (i) The new wind will start producing waves which will grow with time. Their height can be obtained from the diagram for wind-speed and duration. These waves will propagate downwind through the already existing waves at an angle to them, (ii) the original waves (formed by the wind which existed earlier) will die away, unless the new wind has a component acting in the same direction as before, strong enough to maintain them. If this component is not able to maintain them at their existing height they will soon decrease to a size which it can support.

7. FETCH

A fetch can be defined as an area of the sea surface, or a distance across the sea surface, over which the wind is blowing with a constant speed and direction. When a steady wind is blowing from off the land the fetch at any selected point is simply the distance from the shore, measured along the track of the wind, and the fetch is described as being limited.

However cases of limited fetch also occur in mid ocean, far from land, where the fetch is limited - not by a coastline - but by a front or by any other appreciable change in the direction or spacing of the isobars. In such cases the fetch at a selected point may alter as the fronts or pressure systems move. A problem may then arise in deciding the most appropriate distance to use for the fetch. We will consider two examples:

Figure 13a on the next page shows the positions of a cold front and the isobars behind it at two different times (the details for the earlier occasion being indicated by dashed lines). Initially the fetch is the area A'B'C'D', which moves downwind to ABCD during the interval. Since the waves formed in A'B'C'D' move downwind into ABCD the effective fetch at the later time can be taken as A'B'CD and the distance A'B' to CD used in calculations. As the front passes a particular point the waves there will increase rapidly to the value produced by the post-frontal wind over the total fetch.

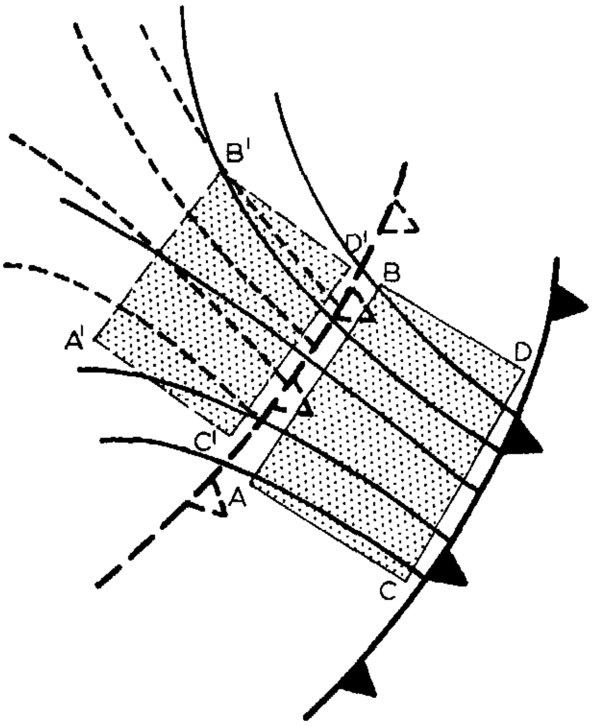

Fig. 13a

Figure 13b shows an open frontal wave moving eastwards and the fetch on the northern side of it (again the dashed lines refer to the earlier position and corresponding isobars). This fetch moves against the wind from A'B'C'D' to ABCD, but the waves increase and move downwind from A'B' to C'D'. Hence the waves formed in A'B'C'D' are of no assistance in developing waves in ABCD. The effective fetch in this case at the end of the interval is ABCD, the highest waves being along CD. To the west of this line winds are now decreasing and so waves are also beginning to decrease.

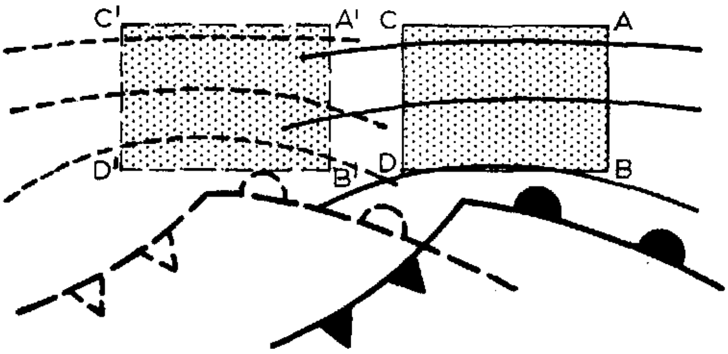

Fig. 13b

8. SWELL

So far we have considered only the case where the wind is either increasing or steady. While the wind continues to increase energy is transferred from it to the sea. This energy goes into increasing the height of the longer components in the waves, as these components travel more rapidly than those of shorter wavelength and have speeds nearer to that of the wind. When the wind becomes steady a state is reached finally where there is no further transfer of energy from wind to wave, and the dominant wave component is travelling at the same speed as the wind. (This means that the wave group - as described earlier - moves with half the speed of the wind).

If the wind decreases in strength or changes direction, those wave components with longer wavelengths will now be moving faster than the wind or its component at right angles to the waves, so the forward sides of their crests push the wind along, effectively transferring energy from the sea to the air. The result will be a gradual decrease in the height of the wave. If the wind disappears completely all the wave components will suffer attenuation in the same way, but the shorter components will disappear more rapidly because they are smaller initially and so contain less energy than the longer components. In addition, internal friction of the water has a damping effect on all the undulations of the sea, and since the shorter waves undulate more rapidly (they have shorter periods), they are more affected than the longer waves. As the shorter components disappear the waves gradually take on a smoother and more sinusoidal (or trochoidal) appearance and the crests - measured across the direction of motion - become longer. This type of wave is called swell.

Swell is also produced when waves move out of the region where they have been created by the wind, or when the pressure system which produced the waves moves away. For example, a depression may have strong northwesterly winds on its rear side. These will produce waves which will move southeastwards as swell into regions which never experienced strong winds at all. Depressions moving northeastwards across the Atlantic and passing between Ireland and Iceland can produce waves which move southeastwards as swell and arrive several days later on the coast of Morocco where they can disrupt loading operations in the port of Casablanca.

So swell may be defined as waves produced by winds which no longer exist or which exist elsewhere. Waves produced by winds still existing in an area are described as "sea" and such an area is called a "generating area".

It may be appropriate to summarize here the differences in appearance between sea and swell.

In SEA the individual waves are lumpy in appearance, sometimes with sharp crests. Individual crests can be followed by eye for only a few wavelengths. Looking across the general wave direction any particular crest will extend only a distance equal to two or three times the distance between crests and the individual crests will not all be aligned in the same direction, short portions of separate crests will appear at angles of up to 20 or 30 degrees to each other. The waves will not all be of the same size - small waves will be superimposed on the larger ones - and even with a strong wind - there will be areas where the general waveheight is momentarily quite low.

SWELL waves are much more rounded, and consecutive crests are nearly always of similar height. Individual crests are much more extensive (up to 6 or 7 times the wavelength) and persist for much longer.

From the theoretical viewpoint sea is made up of components whose periods and wavelengths cover a wide range of values and whose directions of motion vary over 20 or 30 degrees. In swell the components cover only a narrow range of periods and directions.

In practice, both sea and swell exist together and provision is made for both to be reported in ships' observations. Generally with sea and swell arriving from well separated directions both wave systems can be fairly easily distinguished. If both systems arrive from approximately the same direction there is more difficulty in separating them by eye,and there is a strong possibility that some of the waves which an observer interprets as swell may in fact be older and more regular waves formed by the existing wind farther away from the ship. For this reason when long-crested rounded waves (typical of swell) are observed arriving from within 20 degrees of the wind direction the observer is instructed to regard such waves as forming a separate system only when their period is at least four seconds greater than that of the larger waves in the existing "sea". (In analysing a wave chart the analyst must keep in mind that this instruction is not always adhered to, and that ship reports may be received giving sea and swell coming from the same direction with closely similar periods. In such cases both waves will be regarded as sea, and its height taken to be the greater of the two reported values.)

9. HEIGHT OF SWELL

Once waves have left the generating area internal friction removes the components with the shortest periods fairly quickly, but it reduces the long-period swell only comparatively slowly. The main factor in reducing the height of this swell is its angular dispersion. As mentioned earlier wave crests produced by the influence of the wind do not lie in exactly parallel lines, mainly because turbulent eddies in the wind cause it to fluctuate about a mean direction. As a result the waves move in directions which vary from this mean direction, and will continue to do so even after the wind has stopped.

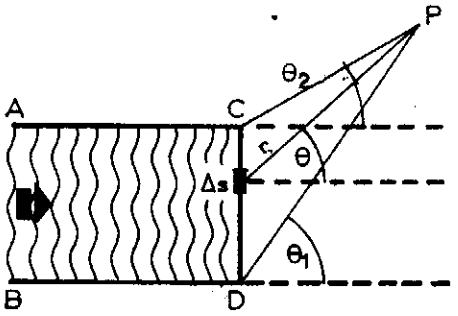

Fig. 14

Figure 14 shows a fetch ABCD with the wind moving in the direction A to C. When the wind ceases the waves continue to move in their original slightly divergent directions so that some of the swell will reach a point such as P, which is not directly "downwind" from CD. Since the energy of the wave is being spread out over a wider front as the wave moves on, and no further energy is available from the wind, the wave height decreases as it moves away from CD.

Observation and theory suggest that the energy arriving at P (outside the wave-generating area) from a small portion of the wave-front Δs, is proportional to $\cos^2\theta$, where θ is as shown in Fig. 14, as well as being inversely proportional to the square of the distance (r) from Δs to P.

Figure 15 (reproduced from Pierson, Neumann and James' book) is a graph from which the percentage energy arriving at P from the total wavefront CD can be obtained by subtracting the percentages corresponding to the two angles θ_1 and θ_2 in Fig. 14. θ_1 and θ_2 are both measured in same direction. (The derivation of this graph is given in Appendix 1). This provides a method of calculating the height (H) of the swell at a point P in terms of the wave height (H_o) which existed along CD:

$$H^2/H_o^2 = E/E_o$$

$$\text{hence } H = H_o\ (E/E_o)^{\frac{1}{2}}\ .$$

Figures 16 and 17 show values calculated for H as a percentage of H_o at varying distances and in varying directions from fetches of two different widths D_1 and D_2. It can be seen that to the left and right of the fetch the swell dies away quickly. "Downwind" along the centre line of the fetch the wave-height decreases to three-quarters of its original value

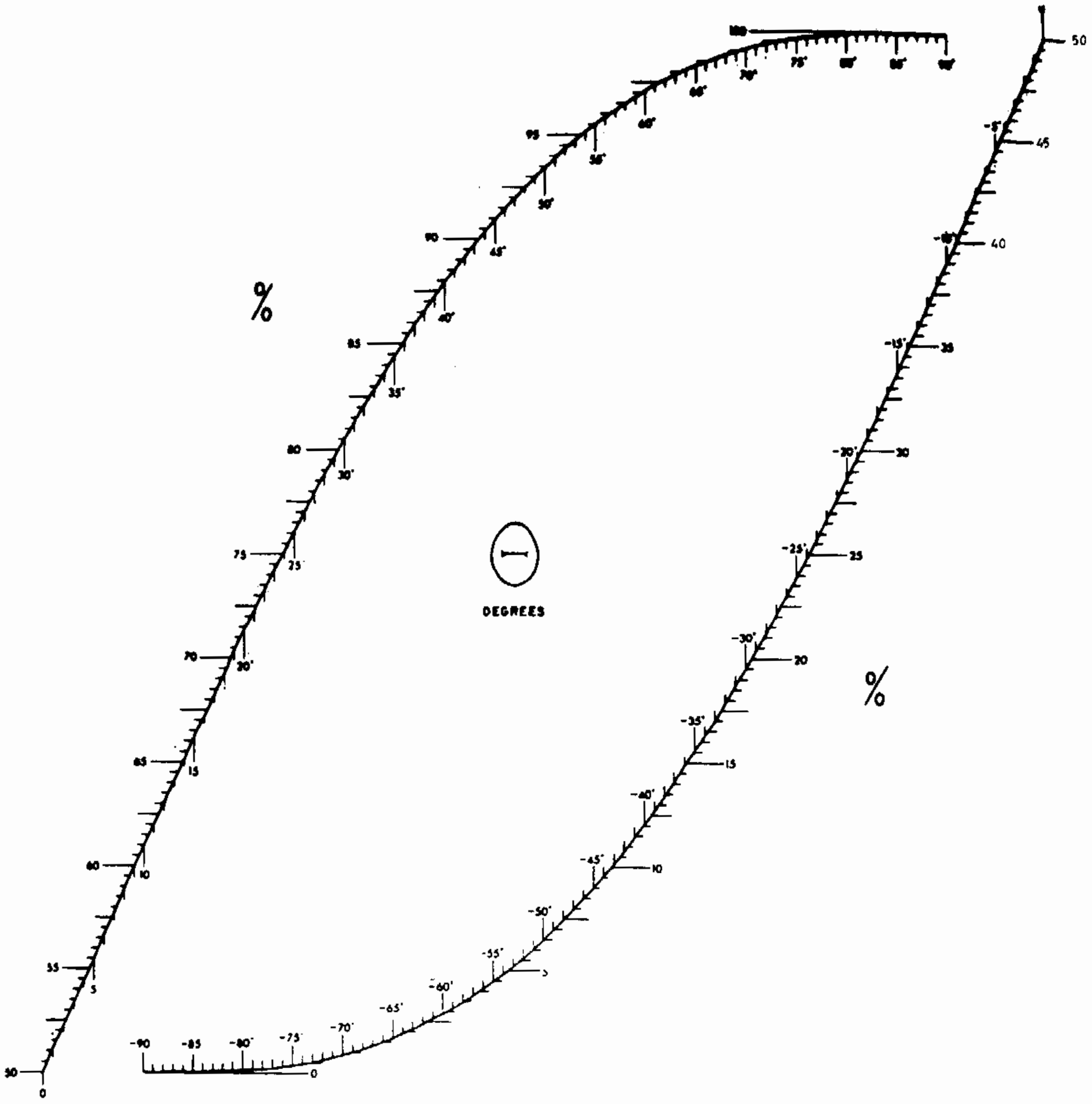

Fig.15. Angular dispension of wave energy

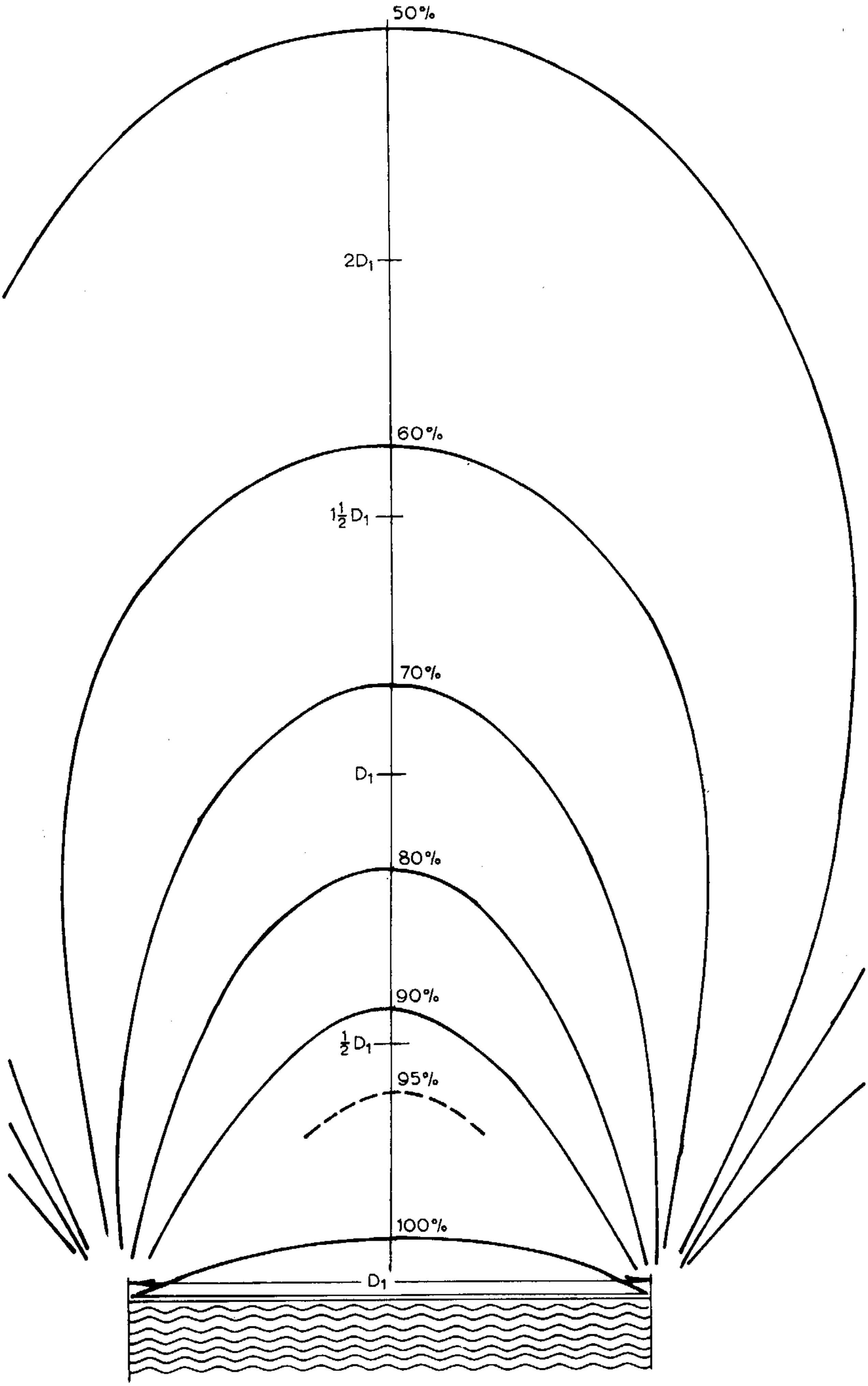

Fig. 16. Decrease of swell height outside generating area, expressed as percentage of original height

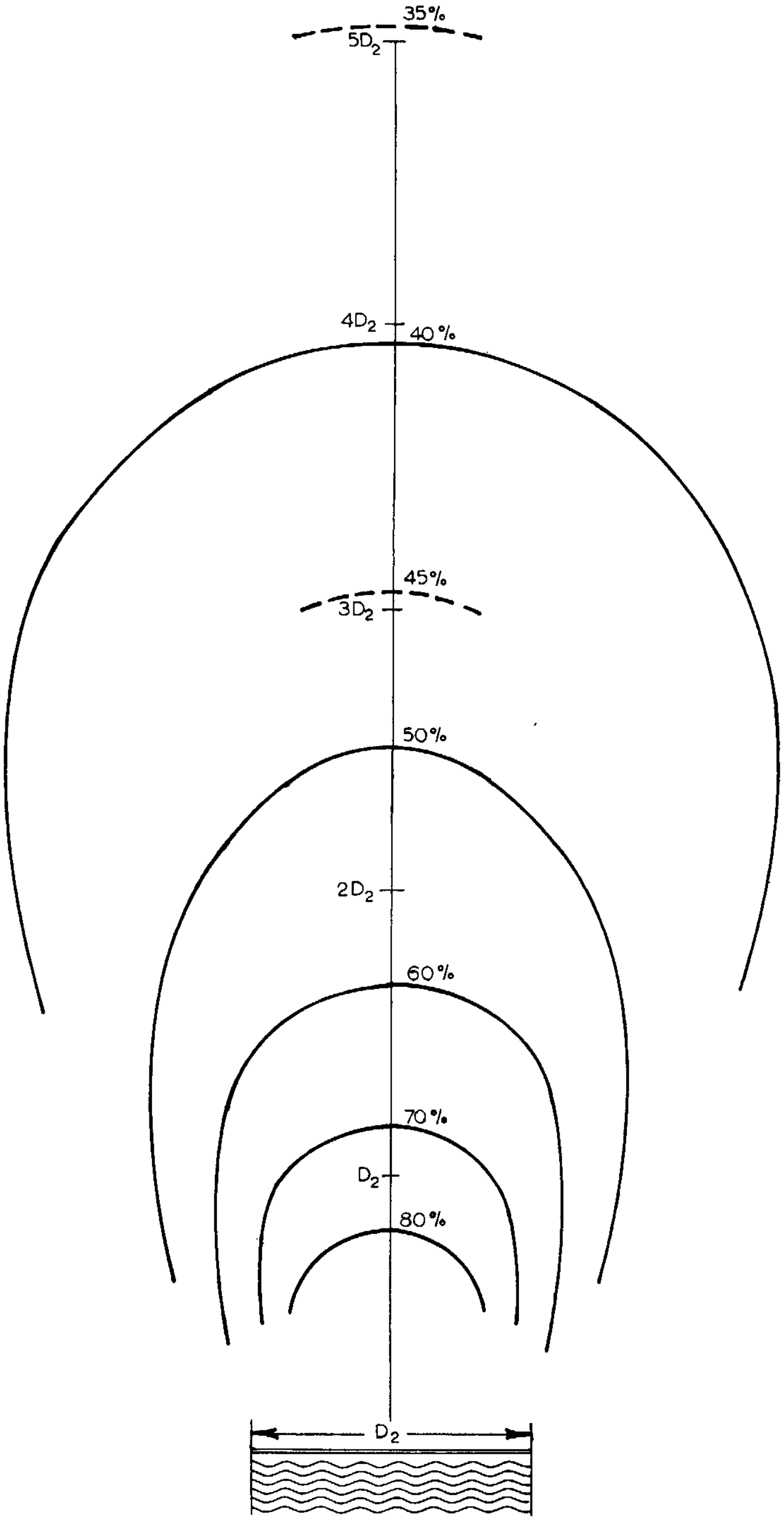

Fig. 17. Decrease of swell height outside generating area, expressed as percentage of original height

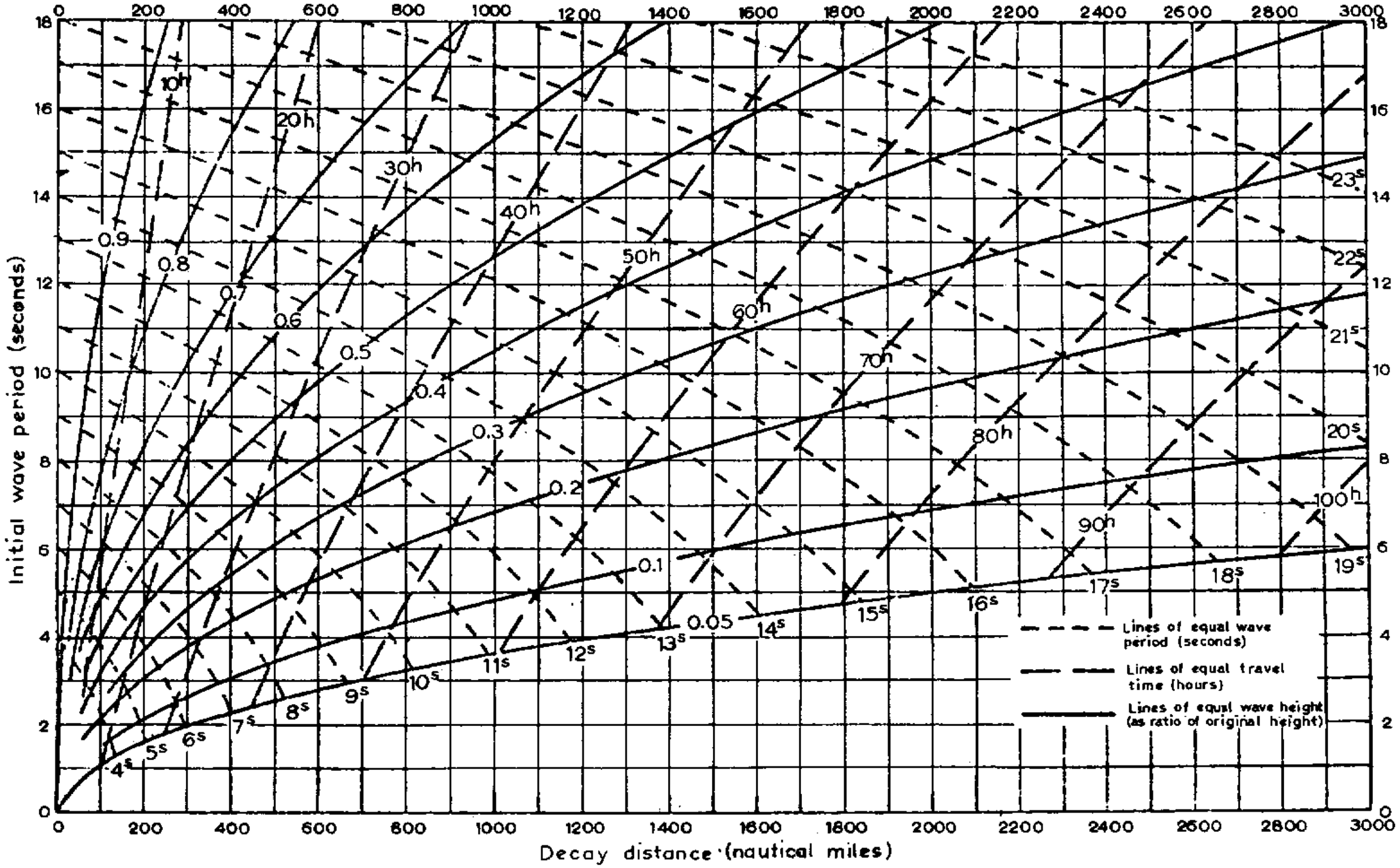

Fig. 18. Variation in swell height and period with time and distance of travel (over calm water)

after a distance equal to the width of the fetch, but thereafter the height decreases more slowly and is still more than one-third its original value after a distance equal to five times the width of the fetch.

It should be noted that figures 16 and 17 are based solely on geometrical considerations, the particular scales are irrelevant. However the actual width of the fetch will be relevant if it is necessary to estimate the time at which the swell will arrive at a given point.

10. SPEED OF MOVEMENT OF SWELL

The dominant period of the sea depends on the speed of the wind which is producing it, and also on the duration and fetch of the wind, if either of these is limited. It therefore behaves in a manner similar to the height of the sea. Table 1 gives an approximate relationship between the significant wave height (in metres) and the wave period (in seconds). The derivation of this Table may be found in Appendix 2.

Height	1	2	3	4	5	6	7	8	9	10	11	12	metres
Period	5½	7	8½	9½	10	11	11½	12	12½	13	13½	14	seconds

Table 1: Significant Wave Height and corresponding Wave Period

<u>The speed of movement</u> (V) of the swell depends on its period (T), (V = 1.5 T where V is in knots and T is in seconds). The period of the swell is initially the period of the dominant component in the sea, produced by the wind. As the shorter components die away an increase is produced in the effective period of the swell. There is also some transfer of energy from waves of shorter wavelength to those of greater length which produces a similar result. Both processes lead to a greater speed of movement of the swell.

Figure 18 is a nomogram prepared by the U.S. Hydrographic Office in 1951. This shows how the period and height of a wave vary with distance when the wave is travelling over a sea surface unaffected by wind. For example (1) a wave initially of period 10 seconds and height H takes 46 hours to travel 1000 nml and after this distance its period has increased to 14½ seconds while its height has decreased to 0.37 H. If the width of the fetch is taken into account (as shown in figure 17) the same reduction in height after the same distance is obtained where the width of the fetch is about 225 nml. (on the vertical axis in fig. 17 37% corresponds to a distance of about 4.45 x D_2, hence D_2 = 225 nml).
(2) a wave initially of period 8 seconds and height H travels 180 nml in 12 hours and by the end of this time and distance the period and height are 9 seconds and 0.71 H. (In this case 71% on the vertical axis of figure 17

corresponds to 1.1 x D_2, hence 1.1 x D_2 = 180 nml, so here the appropriate width of the fetch is about 164 nml). A fetch of width 225 nml, as in the first example, would produce a height of 0.80 H after a distance of 180 nml. (If D_2 = 225 nml then 180 nml = 0.8 x D_2, which corresponds to 80% on the vertical axis in figure 17). While therefore there is not complete agreement between the results obtained by the two methods of determining swell decay, these results are still of the same order of magnitude, and greater accuracy is probably unnecessary in view of the imprecision which is associated with the width of fetch, wave periods, etc. in any real synoptic situation.

In practice the Netherlands' Meteorological Institute in preparing its North Atlantic wave charts at 12 hourly intervals uses a rule of thumb which moves a swell-wave 180 nml (3 degrees of latitude) in the 12 hours and reduces the height by multiplying it by a factor of 0.75.

Other simple factors for swell height and movement which have been proposed and used are:

1. Marine Observer's Handbook (due apparently to Darbyshire and Draper): At R nml from the point of generation $H_R = H_o x \sqrt{\frac{300}{R}}$. (This obviously does not hold for small values of R).
2. Ogden (London Weather Centre): Height is reduced by one-half after 1200 nml.
3. Burgess (The Marine Society, London): Swell travels at nearly half the speed of the wind in the generating area.

The above methods are all based on the assumption that there is no wind in the area across which the swell is travelling. If a wind is present this will have some effect on the rate of decay of the swell, but so far attempts to deal with this on a quantitative basis have had limited success. A head wind blowing against the swell will reduce its height more quickly while a following wind will tend to sustain it. Cross-winds also generally reduce swell height, but experience has shown that moderate or fresh cross-winds having a slightly favourable component to the swell have only a small effect on swell which has a period greater than 12 seconds.

Principal references:

A.D.H. MARTIN, Wave Data for the Kish, Barrels and Daunt Light Vessels. (Irish Lights' Office, 1971)

W.J. PIERSON, G. NEUMANN, R.W.JAMES, Practical Methods for Observing and Forecasting Ocean Waves. (U.S. Navy Hydrographic Office, 1955)

M. DARBYSHIRE & L. DRAPER, Forecasting Wind-generated Sea Waves. (Engineering, April 1963)

M.R. MORGAN, The Analysis and Forecasting of Sea and Swell in Deep water. (Atmospheric Environment Service, Department of the Environment, Canada)

C.G. KOREVAAR, Methods employed in Wave Analysis. (W.M.O. Regional Training Seminar on Meteorological Services to Marine and Coastal Activities, ROME April 1974)

L. MOSKOWITZ, Estimates of Power Spectrums for Fully Developed Seas for Wind Speeds of 20 to 40 Knots. (Journal of Geophysical Research, December 1964)

N. HOGBEN & F.E. LUMB, Ocean Wave Statistics. (H.M.S.O., 1967).

APPENDIX 1. Angular dispersion of energy

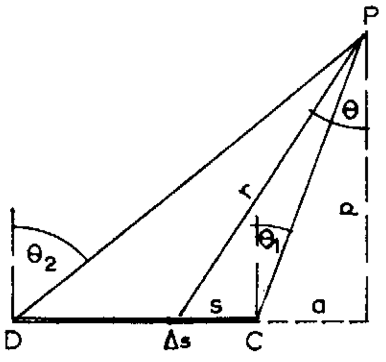

Fig. 19

Using the notation in the adjoining Figure 19 the energy arriving at P from Δs is proportional to $\frac{\cos^2\theta.\Delta s}{r^2}$, and the total energy arriving from the whole wavefront CD will be proportional to

$$\sum_{\theta=\theta_1}^{\theta=\theta_2} \frac{\cos^2\theta.\Delta s}{r^2}\text{, leading to } \int_{\theta=\theta_1}^{\theta=\theta_2} \frac{\cos^2\theta}{r^2}\,ds.$$

We have $r = p\sec\theta$

$s + a = p\tan\theta$

on differentiating $ds = p\sec^2\theta.d\theta$

Hence the integral becomes

$$\frac{1}{p}\int_{\theta_1}^{\theta_2}\cos^2\theta.d\theta = \frac{1}{2p}\left[\theta + \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

Therefore the energy $E = K\left[\theta + \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$

where K is an unknown constant.

If P lies on CD the energy $E_o = K\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} = K\pi$

Hence $K = \frac{E_o}{\pi}$

$$\text{Therefore } \frac{E}{E_o} = \frac{1}{\pi}\left[\theta + \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{\pi}\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\frac{\pi}{2}}^{\theta_2} - \frac{1}{\pi}\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\frac{\pi}{2}}^{\theta_1}$$

The function $\frac{1}{\pi}\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\frac{\pi}{2}}^{\theta}$ can be evaluated for different values of θ.

It is the same as $\frac{1}{\pi}\left(\theta + \frac{\sin 2\theta}{2} + \frac{\pi}{2}\right)$ for θ in radians

or $\frac{\Theta}{180} + \frac{\sin 2\Theta}{2\pi} + \frac{1}{2}$ for Θ in degrees

Figure 15 in the text shows this last function expressed as a percentage for Θ ranging from -90 to +90°.

APPENDIX 2. Relationship between significant wave height and corresponding wave period

Four of the authors mentioned in the text, Darbyshire, Bretschneider, Suthons and Pierson-Neumann-James, devised nomograms which show wave-period as well as significant wave height as a function of the wind, duration and fetch. These nomograms were used to obtain corresponding values of height (H) and period (T) for wind speeds from 10 to 50 knots with the same conditions of duration and fetch as were used in Figures 9 to 12. Graphs of log H (x-coordinate) against log T (y-coordinate) were plotted. It was found that for each author the different conditions of duration and fetch gave - not a single straight line - but a set of closely spaced almost parallel lines. Taking these to be of the form $\log T = m.\log H + c$ the adjoining Table shows for each author the greatest and least values obtained for c and the corresponding value of m.

	c	m
Darbyshire	0.66 to 0.68	0.30 to 0.31
Bretschneider	0.49 " 0.66	0.40 " 0.38
Suthons	0.54 " 0.56	0.40 " 0.43
Pierson, etc.	0.40 " 0.41	0.40 " 0.34

Taking the average for c and m in each case and using $T = H^{m}.10^{c}$ values were found for T over a range of values of H. The Table below shows the results for each author, and also values for T obtained by using an overall average for c and M. In this Table the heights have been expressed in metres, since all reported wave heights are now given in these units; periods are expressed in seconds.

Height	1	2	3	4	5	6	7	8	9	10	11	12
Darbysh.	6.7	8.3	9.4	10.2	11.0	11.6	12.2	12.7	13.1	13.6	14.0	14.3
Bretsch.	6.0	8.2	9.8	11.2	12.3	13.4	14.4	15.3	16.1	16.9	17.6	18.3
Suthons	5.8	7.7	9.2	10.3	11.3	12.2	13.0	13.8	14.5	15.1	15.7	16.3
Pier. etc.	3.9	5.1	5.9	6.6	7.2	7.7	8.1	8.5	8.9	9.2	9.6	9.9
Average	5.6	7.2	8.4	9.4	10.2	10.9	11.5	12.1	12.6	13.1	13.6	14.0

It can be seen that the values obtained from the first three authors are in reasonable agreement, while Pierson's method yields periods which are considerably lower.

"Ocean Wave Statistics" by Hogben and Lumb gives wave data for the period 1953-1961 for the North Atlantic (as well as other oceanic areas). The data for the eastern half of the Atlantic north of 50 N, including the Irish Sea, are given as Areas 2 and 3 on page 6 therein, where the observations are divided according to wave height (in intervals of half-metres) and period (in intervals of two seconds). As might be expected, the waves of any particular height are distributed over several intervals of period. The data for both these Areas were combined and a simplification was made by apportioning the observations for the "half" metres of wave height to the next higher and lower integral values of the height. The Table below shows for each wave height (in whole metres) the periodic interval in which occurred the maximum number of observations. There is fairly good agreement between these observed periods and these average values given in the final row of the above Table which were obtained by calculation. The latter values (slightly approximated) appear again in the final row of the Table below, and the same figures have been used in Table 1 of the text.
(Note: In "Ocean Wave Statistics" no wave heights above 9 metres were observed in the Areas concerned).

Wave Height (metres)	1	2	3	4	5	6	7	8	9
Wave Period, observed (seconds)	≤5	6 or 7	8 or 9	8 or 9	8 or 9	10 or 11	10 or 11	12 or 13	12 or 13
Wave Period, calculated (seconds)	5½	7	8½	9½	10	11	11½	12	12½