A simple formula for time-dependent seismic reliability assessment of aging structures

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ABSTRACT: The capacity deterioration of structures over time may impair their seismic structural reliability significantly, and thus should be taken into account when assessing their seismic performance. In this paper, a simple formula is proposed for time-dependent seismic reliability analysis of deteriorating structures. The capacity deterioration on the temporal scale is captured by the generalized capacity and deterioration function, and the Fréchet distribution is used to describe the probabilistic behaviour of the peak ground acceleration. The proposed formula is based on some minor assumptions on the probability models of the deterioration process and of the seismic hazard, and does not involve the computation of integrals, which is beneficial for use in practical engineering. A numerical example is presented to demonstrate the accuracy and applicability of the proposed reliability analysis method.

1. INTRODUCTION

Earthquakes are among the significant natural hazards that threaten structural serviceability and safety of civil engineering structures. Engineered structures are expected to withstand the impact of earthquakes during their service life with acceptable seismic performance (Cornell et al., 2002; Anbazhagan et al., 2009; Wang et al., 2019). However, these structures often suffer from environmental or operational conditions such as chloride-induced corrosion, leading to degraded seismic performance that may fall below an acceptable level as assumed for new ones (Ghosh and Padgett, 2010; Wang et al., 2021). Taking into account the uncertainties associated with both the earthquake actions and the structural properties, a reliability analysis should be employed to estimate the seismic structural performance quantitatively.

The concept of time-dependent fragility curve has been widely used in the literature to represent the time-dependent variation of seismic structural performance (Choe et al., 2010; Song et al., 2019; Granello et al., 2020). However, the time-dependent fragility curves are representative of the seismic structural capacity and demand at a specific time point, and thus cannot reflect the accumulation of risks over a reference period of time (note that the structural damage/failure may occur at any time and thus the risk increases with the duration of considered service period). To this end, time-dependent seismic reliability assessment is a powerful tool to estimate the structural ability of withstanding future earthquake loads within a given service period.

The work by Mori and Ellingwood (1993) was among the early attempts to estimate time-dependent structural reliability, which modeled the occurrence of load sequence as a Poisson point process. However, a three-fold integral is included in the estimate of structural reliability when taking into account the uncertainties associated with initial structural resistance, the resistance deterioration and the external load process (Mori and Ellingwood,

1993), which could hinder its application in practical engineering. Akiyama et al. (2011) evaluated the displacement ductility capacity and life-cycle seismic reliability of corroded reinforced concrete bridge piers by integrating the probabilistic hazard of airborne chlorides. In their model, it was assumed that the events of structural failure at different times are statistically independent, and thus the impact of the temporal correlation associated with the seismic structural capacity was not considered. Kurtz et al. (2016) developed a method for multi-scale network reliability analysis of bridge networks, using the time-variant fragility models to represent the time-variation of bridge performance under earthquake excitations. However, while their method offers snapshots for the reliability of bridge network at different time points, it cannot account for the accumulation of earthquake risk over time.

In this paper, a simple formula is developed for time-dependent seismic structural reliability assessment. The concept of generalized seismic capacity is adopted to represent the seismic fragility, which has a cumulative distribution function (CDF) that is exactly the same as the fragility curve. The proposed method takes into account the uncertainties associated with the generalized capacity, the capacity deterioration process and the earthquake loads. A linear deterioration function is used to model the time-variation of seismic structural capacity. The accuracy and applicability of the proposed method are verified through an example.

2. Deterioration model for seismic structural capacity

The fragility curves have been extensively used to represent the vulnerability of a structure subjected to earthquake excitations, providing the probability of a certain damage state of the post-hazard structure (Li and Ellingwood, 2006; Guo et al., 2016; Cui et al., 2018). A fragility curve can be obtained by computing the probability that the demand exceeds the structural capacity conditioned on a specific hazard intensity. A practical approach to understanding the fragility curve is to adopt the concept of generalized seismic capacity, denoted by R, whose CDF is exactly the same as the fragility curve (Baker, 2008; Wang and Zhang, 2020; Wang et al., 2020).

The generalized capacity R has the same unit as the seismic intensity measure, and is a measure of the performance of a structure in an earthquake event which, if exceeded by the ground motion intensity, would result in a damage state. Mathematically,

$$\Pr(DS|x) = \Pr(R \leq x) \tag{1}$$

in which $\Pr(\ )$ denotes the probability of the event in the brackets, DS is the damage state, and x is the intensity measure. A fragility curve is typically assumed to have a lognormal distribution shape in practice, so the generalized capacity R is also lognormally distributed. With this regard, the probability density function (PDF) of R, $f_R(r)$, takes a form of

$$f_R(r) = \frac{1}{\sqrt{2\pi}r\nu} \exp\left[-\frac{1}{2}\left(\frac{\ln r - \kappa}{\nu}\right)^2\right], \quad r \geq 0 \tag{2}$$

where the two parameters κ and ν are the mean value and the standard deviation of $\ln R$, respectively. Note that Eq. (1) has been conditioned on a specific value of x. If the uncertainty associated with the intensity measure is also taken into account, using the law of total probability, it follows that

$$\Pr(DS) = \int_0^\infty \Pr(DS|x)f_X(x)dx = \Pr(R \leq X) \tag{3}$$

where X, written in the capital form, denotes the intensity measure as a random variable, and $f_X(x)$ is the PDF of X conditional on the occurrence of one earthquake event. Considering the impact of structural capacity deterioration, as well as the time-variation of seismic demand, on the temporal domain, Eq. (3) is rewritten as follows,

$$\Pr(DS(t)) = \Pr(R(t) \leq X) \tag{4}$$

where $DS(t)$ and $R(t)$ are the damage state and the generalized capacity at time t respectively. Based on Eq. (4), a deterioration function, denoted by $G(t)$, is introduced to describe the deterioration process of the generalized capacity $R(t)$ (Wang and Zhang, 2020), which is defined as follows,

$$R(t) = R_0 \cdot G(t) \tag{5}$$

where R_0 is the initial generalized capacity. Clearly, $G(0) \equiv 1$ deterministically. At any time $t > 0$, $G(t)$ is a random variable varying within an interval of $[0, 1]$. There are many candidate distribution types for $G(t)$ with a support of $[0, 1]$, e.g., Beta distribution, truncated distribution, among others (Wang et al., 2016). Specifically, the Beta distribution can be used to derive a simple expression for seismic structural reliability, as will be shown later (see Eq. (19)). Another important feature for Beta distribution is that, a Beta-distributed random variable can be interpreted as the ratio of two Gamma distributed variables (with numerator not exceeding denominator). This is similar to the definition of $G(t)$, being equal to the ratio of $R(t)$ to R_0 (see Eq. (5)). When $G(t)$ is modeled as a Beta random variable for $t > 0$, the PDF of $G(t)$ is written as follows for $g \in [0, 1]$,

$$f_{G(t)}(g) = \frac{\Gamma(p(t)+q(t))}{\Gamma(p(t))\Gamma(q(t))} g^{p(t)-1}(1-g)^{q(t)-1} \quad (6)$$

where Γ is the Gamma function, $\Gamma(z) = \int_0^\infty x^{z-1}\exp(-x)dx$, $p(t) > 0$ and $q(t) > 0$ are two time-variant shape parameters.

In Eq. (5), the deterioration process of generalized capacity, $R(t)$, is modeled as follows in this paper (Wang and Zhang, 2020),

$$G(t) = 1 - \alpha_0 \cdot t \quad (7)$$

in which $\alpha_0 > 0$ is a rate parameter (random variable). With Eq. (7), $G(t)$ can be expressed in terms of T and $G(T)$ as follows,

$$G(t) = 1 - [1 - G(T)] \cdot \frac{t}{T} \quad (8)$$

The deterioration function $G(t)$ in Eq. (8) enables that the time-variation of structural generalized capacity (fragility curve) on the temporal scale can be described, which is by nature a continuous stochastic process. Such a model is an essential ingredient in structural time-dependent reliability assessment (as will be detailed in the following, see Eq. (13)), and its role cannot be simply achieved through individual fragility curves.

3. PROBABILISTIC MODEL OF SEISMIC LOAD

Frequently-used intensity measures for earthquake loads include peak ground acceleration (PGA), peak ground velocity, and others (Elnashai and Di Sarno, 2015). In this paper, the PGA will be used to represent the earthquake load, with uncertainties arising from the earthquake events in terms of frequency and magnitude. For a reference period of $[0, \Delta t]$ (in years), the Extreme Type II distribution (also known as Fréchet distribution) can be used to reasonably model the probabilistic behavior of PGA, denoted by A (Cornell, 1968). Mathematically, the CDF of A takes a form of

$$F_A(x) = \exp\left[-\left(\frac{x}{\epsilon}\right)^{-k}\right] \quad (9)$$

in which ϵ is a scale parameter and k is a shape parameter. The assignment of a Fréchet distribution for the PGA is based on the assumption that the occurrence of earthquakes follows a Poisson point process.

According to Eq. (9), the mean value and the variance of A are determined as follows, respectively,

$$\mu_A = \epsilon\Gamma\left(1 - \frac{1}{k}\right), \quad \text{if } k > 1$$
$$\sigma_A^2 = \epsilon^2\left[\Gamma\left(1 - \frac{2}{k}\right) - \Gamma^2\left(1 - \frac{1}{k}\right)\right], \quad \text{if } k > 2 \quad (10)$$

It is assumed in this paper that $k > 1$. Note that in Eq. (9), the parameter ϵ is dependent on the duration of the time interval Δt, while k only depends on the characteristics of the site seismicity. In fact, it can be shown that ϵ^k is proportional to Δt (Cornell, 1968; Wang, 2021). Based on this fact, let ϵ_y denote the value of ϵ associated with a period of one year, and it follows that

$$\frac{\epsilon^k}{\Delta t} = \epsilon_y^k \quad (11)$$

With Eq. (9), Eq. (4) can be rewritten as $\Pr(DS(t)) = \Pr(R(t) \leq A) = F_A(R(t))$. Since the structural capacity $R(t)$ is typically far greater than A, at the upper tail, F_A can be approximated as follows,

$$F_A(x) \approx 1 - \left(\frac{x}{\epsilon}\right)^{-k} = 1 - \epsilon^k x^{-k} = 1 - H(x) \quad (12)$$

where $H(x) = \epsilon^k x^{-k}$ is the hazard curve. The approximation in Eq. (12) was used by Cornell et al. (2002) to derive a closed-form solution for structural seismic performance. The parameter k in Eqs. (9) and (12) has a typical value of $1 \sim 4$ for most engineering cases in the US (Yun, 2000; Cornell et al., 2002).

4. TIME-DEPENDENT SEISMIC RELIABILITY ASSESSMENT

The occurrence of unsatisfactory structural performance, measured by the damage state, could be at any time during service life, reflecting the accumulation of seismic risks over time. In this paper, the structural seismic reliability over a time period of $[0, T]$ is defined as the probability that the damage state does not occur over $[0, T]$. The structural reliability would be essentially dependent on the specific damage state of interest.

Let $\mathrm{Rel}(T)$ denote the time-dependent seismic reliability of a structure over $[0, T]$ (in years). First, the interval $[0, T]$ is subdivided into n identical sections, namely $[t_0 = 0, t_1], (t_1, t_2] \dots (t_{n-1}, t_n = T]$, where n is large enough. Let A_i denote the maximum PGA within the ith interval for $i = 1, 2, \dots n$. Modeling the capacity deterioration using Eq. (5), it follows that,

$$\mathrm{Rel}(T) = \Pr\left(R_0 \cdot G(t_i) > A_i, \forall i = 1, 2, \dots n\right) \quad (13)$$

Correspondingly, the structural failure probability, denoted by $P_f(T)$, is simply $1 - \mathrm{Rel}(T)$.

The model in Eq. (8) is used to describe the deterioration process $G(t)$. If the uncertainties associated with both R_0 and $G(T)$ are taken into account, Eq. (13) becomes

$$\mathrm{Rel}(T) = \int_{R_0} \int_{G(T)} \prod_{i=1}^{n} F_{A_n}\left[r_0 \cdot \left(1 - (1-g) \cdot \frac{t_i}{T}\right)\right] \\ \cdot f_{R_0, G(T)}(r_0, g) dg dr_0 \quad (14)$$

in which $f_{R_0, G(T)}(r_0, g)$ is the joint PDF of R_0 and $G(T)$, and A_n is the maximum PGA within each time interval (with a duration of T/n). Assuming that R_0 is statistically independent of $G(T)$, Eq. (14)

is rewritten as follows,

$$\mathrm{Rel}(T) = \int_{R_0} \int_{G(T)} \exp\left(-\frac{r_0^{-k} \epsilon_y^k T}{k-1} \cdot \frac{g^{1-k} - 1}{1 - g}\right) \\ \cdot f_{R_0}(r_0) f_{G(T)}(g) dg dr_0 \quad (15)$$

in which $f_{G(T)}(g)$ is the PDF of $G(T)$, and $f_{R_0}(r_0)$ is the PDF of R_0.

Note that for most engineering structures guided by structural design standards, the seismic reliability in Eq. (15) is close to 1. With this context, one can use the first-order Taylor expansion to approximate the core of Eq. (15). Note that for a real number $x \to 0$, $\exp(x) \approx 1 + x$ (an example of this approximation is in Eq. (12)). Thus, Eq. (15) can be approximated by

$$\mathrm{Rel}(T) \approx \int_{R_0} \int_{G(T)} \left[1 - r_0^{-k} \epsilon_y^k T \frac{g^{1-k} - 1}{(1-g)(k-1)}\right] \\ \cdot f_{R_0}(r_0) f_{G(T)}(g) dg dr_0 \\ = 1 - \frac{\epsilon_y^k T}{k-1} \int_{R_0} r_0^{-k} f_{R_0}(r_0) dr_0 \int_{G(T)} \frac{g^{1-k} - 1}{1-g} f_{G(T)}(g) dg \quad (16)$$

In Eq. (16), R_0 is modeled as a lognormal variable as discussed before, whose PDF is as in Eq. (2), with which

$$\int_{R_0} r_0^{-k} f_{R_0}(r_0) dr_0 = \exp\left(\frac{1}{2} k^2 \nu^2 - k\kappa\right) \\ = \mu_{R_0}^{-k} \left(1 + \frac{\sigma_{R_0}^2}{\mu_{R_0}^2}\right)^{\frac{1}{2}(k^2 + k)} \quad (17)$$

where μ_{R_0} and $\sigma_{R_0}^2$ are the mean value and the variance of R_0 respectively. Furthermore, the item $G(T)$ in Eq. (16) is assumed to follow a Beta distribution (see Eq. (6) with $t = T$). With a requirement that $p(T) > k - 1$ and $q(T) > 1$, one has

$$\mathcal{G}(p(T), q(T), k) = \int_{G(T)} \frac{g^{1-k} - 1}{1 - g} f_{G(T)}(g) dg \\ = \frac{\Gamma(p(T) + q(T))\Gamma(1 - k + p(T))}{\Gamma(p(T) + q(T) - k)\Gamma(p(T))(q(T) - 1)} + \frac{1 - p(T) - q(T)}{q(T) - 1} \quad (18)$$

Thus, Eq. (16) becomes

$$\mathrm{Rel}(T) = 1 - \frac{\epsilon_y^k T}{k-1} \mu_{R_0}^{-k} \left(1 + \frac{\sigma_{R_0}^2}{\mu_{R_0}^2}\right)^{\frac{1}{2}(k^2 + k)} \mathcal{G}(p(T), q(T), k) \quad (19)$$

It can be seen from Eq. (19) that the seismic reliability analysis method does not involve the calculation of integrals, which is beneficial for its application in practical engineering. It was mentioned earlier that a closed-form solution for seismic structural reliability was developed in Cornell et al. (2002) without considering the impact of seismic capacity deterioration.

5. NUMERICAL EXAMPLE

In this section, a numerical example will be used to demonstrate the accuracy and applicability of the proposed reliability method. Consider the reliability of a bridge for reference periods up to 50 years (that is, T is up to 50 years). The fragility curve at the initial time, with respect to PGA, has a median value of $1.03g$ and a dispersion of 0.7 (and correspondingly, a coefficient of variation (COV) of 0.8 for the generalized seismic capacity), where g is the acceleration of gravity. This condition is consistent with the *complete collapse* damage state of a typical multispan continuous concrete bridge in the central and southeastern US, as reported in Nielson and DesRoches (2007). The determination of fragility curve for structures subjected to earthquake loads has been extensively discussed in the literature, which typically requires finite element based modeling of the structure. Since the aim of this section is to examine the applicability of the proposed reliability method, details on obtaining the fragility curve will not be repeated herein. Let $A_{10/50}$ denote the characteristic value of PGA with an exceedance probability of 10% over 50 years, which may be accessible via the regional seismic maps. One can then determine ϵ_y as follows,

$$\epsilon_y = \left(-\frac{\ln 0.9}{50}\right)^{\frac{1}{k}} A_{10/50} \qquad (20)$$

Suppose that the generalized capacity, which has the same CDF as the fragility curve at the initial time, degrades by 20% on average over a service life of 50 years, with which $G(50)$ has a mean value of 0.8. The COV of $G(50)$ is assumed to be 0.12 unless otherwise stated. Suppose that R_0 is statistically independent of the deterioration process, with which Eq. (19) applies. The resulting PDF of

$G(50)$ is plotted in Fig. 1(a). According to Eq. (5), the CDF of $R(50)$ can be determined uniquely, and is presented in Fig. 1(b).
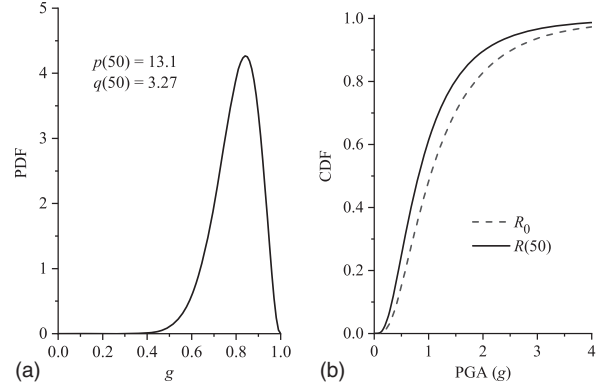

Figure 1: Probability distributions of R_0, $G(50)$ and $R(50)$. (a) PDF of $G(50)$. (b) CDFs of R_0 and $R(50)$.

The bridge's failure probabilities for reference periods of 10, 30 and 50 years, obtained by the proposed method (Eq. (19)), are presented in Table 1 for different values of k. It is assumed that $A_{10/50} = 0.1g$ in Table 1. The failure probability increases with a longer reference period due to the accumulation or failure risks over time. With a fixed $A_{10/50}$, a greater value of k leads to a smaller failure probability. This can be explained by observing the behaviour of F_A (CDF of A on a yearly basis) at the upper tail, as shown in Fig. 2. With the same annual exceedance probability (0.0021) for $0.1g$, a greater k results in F_A being closer to 1, and thus a smaller probability that the PGA exceeds structural capacity.
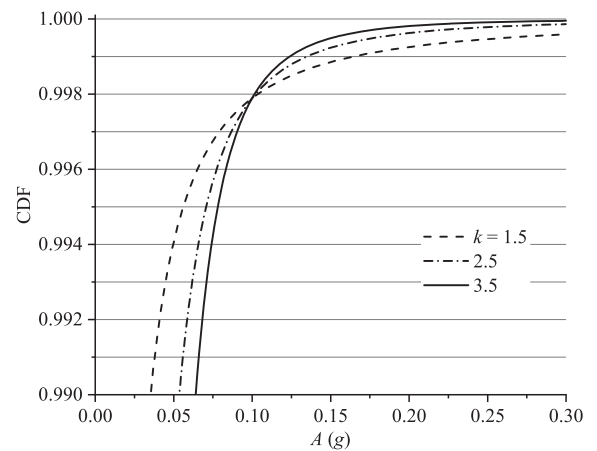

Figure 2: CDFs of PGA associated with different values of k for $A_{10/50} = 0.1g$.

In order to verify the accuracy of Eq. (19), a Monte Carlo simulation (MCS)-based method is used to approximate the *true* solution to the seismic reliability. The basic idea is to introduce a counter c_{sim}, having an initial value of 0, which increases by 1 if the sampled structural performance is satisfactory within each simulation run. With m replications of simulation, the seismic structural reliability can be approximated by c_{sim}/m when m is sufficiently large.

With 10^6 simulation runs and $n = 500$, the relative errors of the analytical results with respect to the simulated reliabilities are presented in Table 1. It can be seen that the results of Eq. (19) agree well with the simulated results. The computation efficiency of the reliability assessment based on the proposed equations is significantly improved compared with the MCS. For example, with $k = 1.5$ and $T = 50$ years in Table 1, using the commercial software Matlab 2019a on a computer with Intel(R) Core(TM) i7-8700 CPU@3.2GHz, the computational times for Eq. (19) and MCS are 0.01s and 420s, respectively. This is because the reliability analysis method in Eq. (19) only involves some simple algebra without the computation of integrals.

In Eq. (19), the impact of structural generalized capacity deterioration has been taken into account. If one simply uses the initial capacity to represent the case for the whole service life without considering the time-variation of $R(t)$, the seismic reliability, denoted by $\text{Rel}_{nd}(T)$, can be computed as follows,

$$\text{Rel}_{nd}(T) = \int_0^\infty F_{A_y}^T(r_0) f_{R_0}(r_0) dr_0 \qquad (21)$$

where A_y is the maximum PGA on a yearly basis. For comparison purposes, the reliabilities with a non-degrading generalized capacity are presented in Table 1. The difference between Eqs. (19) and (21) is slight when $T = 10$ years, where the role of capacity deterioration in seismic structural reliability is negligible. With a relatively long reference period, the seismic structural reliability is overestimated if the deterioration of structural capacity is ignored. This impact is further amplified with a severer capacity deterioration or greater uncertainty associated with the deterioration process.

The effects of the initial generalized capacity, the capacity deterioration and the magnitude of earthquake loads (measured by $A_{10/50}$) on seismic structural reliability are examined in Fig. 3. First, Fig. 3(a) presents the time-dependent failure probabilities for reference periods up to 50 years with different COVs of R_0, assuming that $k = 3.5$ and $A_{10/50} = 0.1g$. The case of $\text{COV}[R_0] = 0.8$ corresponds to that used in Table 1 (with the dispersion of R_0 being 0.7). A greater COV of R_0 leads to a greater failure probability due to the increased structural fragility, and this effect is amplified in the presence of a smaller reference period. For example, when the COV of R_0 increases from 0.1 to 1.0, $P_f(50)$ increases from 5.01×10^{-5} to 2.37×10^{-3} (by 46 times), while $P_f(10)$ increases from 6.78×10^{-6} to 4.19×10^{-4} (by 61 times). It should be noted that the sensitivity of $P_f(T)$ to the COV of R_0 would be weakened in the presence of more severe seismic hazard. For example, if $A_{10/50}$ becomes $0.2g$ in Fig. 3(a), $P_f(50)$ and $P_f(10)$ increase by 6 and 7 times respectively as the COV of R_0 varies from 0.1 to 1.0. This is because, with a greater value of $A_{10/50}$, the role of seismic hazard becomes increasingly dominant in the structural failure probability compared with that of the generalized seismic capacity. The failure probabilities for reference periods up to 50 years are presented in Fig. 3(b) with different COVs of $G(50)$, where $A_{10/50} = 0.1g$ and $k = 3.5$. The mean value of $G(50)$ equals 0.8 for all cases. When the COV of $G(50)$ varies from 0.1 to 0.25, $P_f(50)$ increases from 5.22×10^{-4} by 80%. However, the sensitivity of $P_f(T)$ to the COV of $G(50)$ is weakened in the presence of a shorter reference period. In Fig. 3(c), the impacts of $A_{10/50}$ (taking a value of $0.1g$ or $0.2g$) and the mean value of $G(50)$ (0.8 or 0.6) on the time-dependent failure probabilities are presented, assuming that $k = 3.5$. A greater value of $A_{10/50}$ or a severer deterioration of the generalized capacity leads to a greater failure probability due to the increased seismic risks. For example, with the mean value of $G(50)$ being 0.8, $P_f(50)$ increases from 5.31×10^{-4} to 5.33×10^{-3} as $A_{10/50}$ increases from $0.1g$ to $0.2g$. This effect is amplified with a shorter reference period, where the seismic hazard plays a greater role in structural failure probability.

Table 1: Failure probabilities for reference periods up to 50 years with different values of k ($A_{10/50} = 0.1g$).

| T	Method	$k = 1.5$		$k = 2.5$		$k = 3.5$	
		$P_f(T)$	Error (%)	$P_f(T)$	Error (%)	$P_f(T)$	Error (%)
10 years	Eq. (19)	1.14×10^{-3}	−3.32	3.01×10^{-4}	1.47	1.30×10^{-4}	2.34
	Simulation-based	1.18×10^{-3}	/	2.97×10^{-4}	/	1.27×10^{-4}	/
	Non-degrading	1.10×10^{-3}	−6.37	2.86×10^{-4}	−3.80	1.19×10^{-4}	−5.99
30 years	Eq. (19)	3.66×10^{-3}	1.78	1.02×10^{-3}	6.40	4.61×10^{-4}	7.96
	Simulation-based	3.60×10^{-3}	/	9.54×10^{-4}	/	4.27×10^{-4}	/
	Non-degrading	3.30×10^{-3}	−8.18	8.52×10^{-4}	−10.73	3.50×10^{-4}	−17.99
50 years	Eq. (19)	6.59×10^{-3}	2.18	1.94×10^{-3}	3.37	9.49×10^{-4}	6.98
	Simulation-based	6.45×10^{-3}	/	1.88×10^{-3}	/	8.86×10^{-4}	/
	Non-degrading	5.49×10^{-3}	−14.97	1.41×10^{-3}	−24.94	5.74×10^{-4}	−35.25
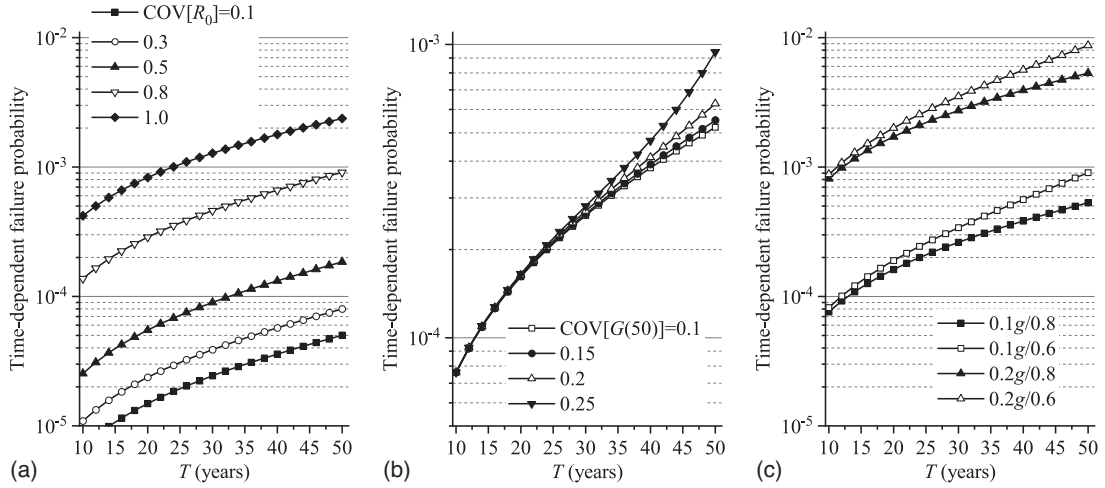
Figure 3: Time-dependent failure probability for reference periods up to 50 years. (a) Impact of COV of R_0. (b) Impact of COV of $G(50)$. (c) Impacts of PGA and deterioration rate.

6. Concluding remarks

In this paper, a simple formula for time-dependent seismic reliability analysis of aging structures is developed, taking into account the uncertainties associated with the structural initial capacity, the capacity deterioration and the stochastic seismic loads. The formula is based on some minor assumptions, and only involves simple algebra without the computation of integrals.

The accuracy and applicability of the proposed method are demonstrated through a numerical example. The computation efficiency is improved significantly by the proposed analytical method compared with the MCS.

Sensitivity analysis shows that with an increasing COV of R_0 and/or $G(T)$, the structural failure probability increases. Furthermore, the probability distribution of the maximum PGA affects the seismic reliability significantly. With a fixed exceedance probability, a greater value of k (shape parameter) results in a smaller failure probability, as it reduces the probability that the PGA exceeds the structural generalized capacity. This result shows the importance of properly identifying the local seismic load characteristics for seismic structural reliability assessment.

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