Optimal risk-based design of a RC frame under column loss scenario

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ABSTRACT: Structural systems may be subject to abnormal loads with small probability of occurrence but great intensity, which may lead to local damage, and even loss of loadbearing elements. In this context, it is important to design structural systems to avoid disproportionate progressive collapse. In this paper, we address the optimal risk-based design of a simple RC frame subjected to sudden loss of an internal column. The optimization targets compressive arch, Vierendeel and catenary actions under column loss condition, also considering both serviceability and ultimate failure modes under normal loading conditions. Since initial damage is associated with large uncertainties, the column loss probability is treated as an independent parameter in the optimization. The numerical model employed in the analysis combines physical and geometrical non-linearities. Steel rebar behavior is represented by an elastoplastic model with isotropic hardening, whereas confined concrete is simulated via a combination of Mazars μ model with the modified Park-Kent model. Failure probabilities are evaluated by weighted average simulation, and the risk optimization is performed using the Firefly algorithm. Kriging metamodels of limit states and reliability indexes are employed to cope with the high computational burden. Results show two quite different optimal configurations for beams and columns of the frame: a more conventional design is obtained under small column loss probability, whereas the optimal solutions clearly benefit from catenary action under large column loss probability since enough ductility is provided in terms of ultimate steel strain. These results confirm, in a more practical setting, a previous outcome by the authors that the column loss probability is the main parameter controlling optimal design of the frames. The transition between optimal solutions mentioned above can be associated with a threshold column loss probability, above which designing or strengthening the structure to withstand column loss becomes cost-effective.

1. INTRODUCTION

Structural disasters due to extreme events raised awareness about the need for robust design against disproportionate failure due to abnormal loads. Progressive collapse happens when an initial member failure triggers the failure of its adjacent elements, in resemblance to a cascade effect, resulting in a final collapse much more severe in comparison to the initial event. The probability of structural collapse $P[C]$ due to multiple hazards is given as (Ellingwood 2006; 2007):

$$P[C] = \sum_H \sum_{LD} P[C|LD, H]\, P[LD|H]\, P[H] \quad (1)$$

with $P[H]$ being the probability of hazard occurrence; $P[LD|H]$ is the conditional probability of local damage for a hazard H; and $P[C|LD, H]$ is the conditional probability of collapse for a local damage LD and hazard H.

Beck et al. (2020; 2022) uses this formulation considering $P[LD|H]\ P[H]$ as the probability of local damage P_{LD}, combining column loss and intact structure scenarios in a single objective function in order to study the cost-benefit of considering column loss situations for usual civil engineering structures.

This framework is adopted by Ribeiro et al. (2022) in the risk-based optimization of a reinforced concrete (RC) frame subjected to the sudden removal of its middle column at its first story. Since small ductility is considered in terms of ultimate steel strain, optimal design against progressive collapse consists in enhancing the compressive arch action (CAA) mechanism of the beams with a slightly increase in the column's robustness. Hence, with beam's rebar rupture occurring either at the instability stage after CAA (snap-through) or right at the beginning of catenary action (CA), the optimal reinforcement decision consists in ensuring that the beam spans adjacent to the removed column does not surpasses its CAA peak strength, otherwise rebar rupture can barely be prevented.

However, several authors have shown that CA can increase the ultimate structural resistance for load and displacement values significantly greater than those for CAA capacity, being a resourceful last line of defense against progressive collapse (Adam et al. 2018; Yi et al. 2008; Lew et al. 2014; Yu and Tan 2013; Sasani et al. 2007; Parisi and Scalvenzi 2020). In order to investigate how the benefits of every main resisting mechanism affects the optimal risk-based design of a RC frame, an increased ultimate steel strain is herein adopted to also ensure rebar failure for later stages of CA.

2. METHODOLOGY

The RC planar frame considered in this study is shown in Figure 1.

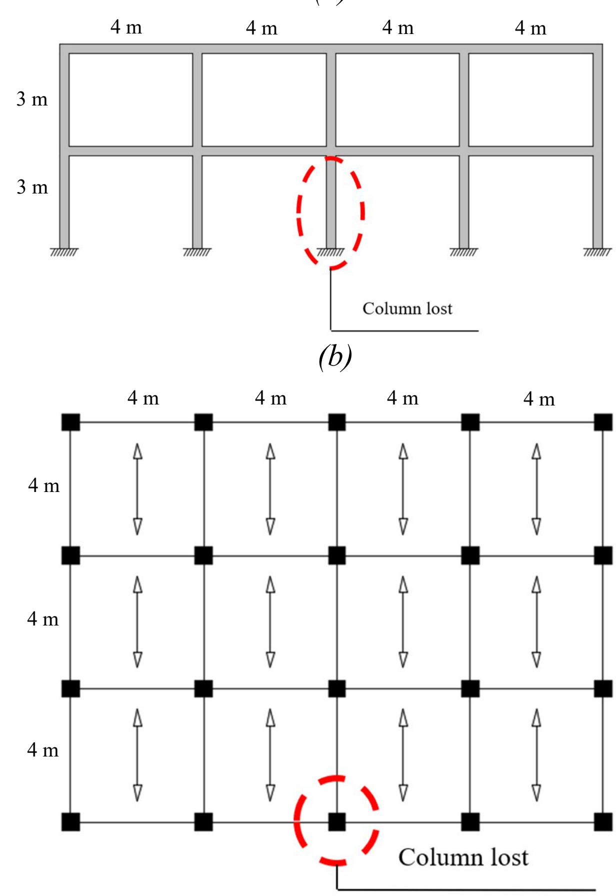

Figure 1: RC frame front view (a) and plan view (b).

Span length for beams and columns are 4 m and 3 m, respectively. All structural elements have cross section width of 200 mm, concrete cover of 25 mm, stirrup diameter of 6 mm, concrete compressive strength f_c' of 45 MPa, modulus of elasticity E_c of 35.5 GPa and concrete tensile strength f_{ctm} of 3.33 MPa. Each member has longitudinal rebars with yielding strength f_y of 511 MPa and modulus of elasticity E_S equal to 212 GPa. Columns have a spacing between stirrups of 100 mm along its entire length.

It is considered 7.0 kN/m² for both dead load and live load, with an additional 2.0 kN/m due to non-structural components over the beams. Since floors are one-directional, this leads to a nominal dead load D_n of 16 kN/m and live load L_n of 14 kN/m.

The 6 design parameters to be optimized are: beam depth (h_B); area of top and bottom beam reinforcements (A_T and A_B); stirrup spacing at the beams (s_t); reinforcement area at the columns (A_P) and column cross section depth (h_P). These parameters are considered as random variables, with only their mean value being optimized. No discontinuities are considered along the length of the elements, and the same optimal design for beam and column is attributed to every beam and column, respectively.

2.1. Risk optimization

The risk optimization problem follows the framework proposed by Beck et al. (2020; 2022), but adapted to the RC frame studied herein:

$$\begin{aligned} C_{TE} = C_M + C_{ef\ I,SE} + C_{ef\ I,BE} + \\ + C_{ef\ I,SH} + C_{ef\ I,FA} + C_{ef\ CL,SR} + \\ + C_{ef\ CL,SH} + C_{ef\ CL,FA} \end{aligned} \quad (2)$$

where C_{TE} is the frame total expected cost; C_M is the manufacture cost; C_{ef} refers to expected cost of failure; and subscripts I and CL stands for intact and column loss scenarios, respectively.

Besides several failure modes being able to occur for both scenarios, a handful of the most representative ones are chosen. For intact structure it is considered serviceability failure in terms of maximum displacement of the beams (SE); bending failure at the midspan of the beams (BE); shear failure at the beams (SH); and flexo-axial compression failure for the columns (FA). Missing column scenario considers the ultimate failure modes of tensile rebar fracture in catenary action (SR), shear failure at beams (SH) and flexo-axial compression of columns (FA).

SINAPI (2022) database is adopted to estimate C_M in Reais (R$), which is then converted to Euros (€) considering 1.00 € equal to R$ 5.50. Construction cost C_M is composed by cost of formwork, obtainment of concrete, pouring of concrete, obtainment of steel rebars, and placing of steel rebars.

For a given failure mode, the expected cost of failure C_{ef} is given by the product of a cost multiplier k times C_M times the probability P_f that the considered failure mode occurs. Thus, for column removal scenarios, the probability of local damage P_{LD} also multiplies $k \times C_M \times P_f$. The multipliers k are chosen according to the order of severity of each failure mode and regarding the real-life ratio between the cost of the building and the cost of reconstruction after failure (Beck et al. 2020; 2022). Therefore, k is assumed equal to 5 for SE, 20 for BE, 30 for SR, and 60 for the brittle and severe failure modes SH and FA.

Considering $\boldsymbol{d} = \{h_B, A_B, A_T, s_t, A_P, h_P\}$ as a vector of random design variables within a design domain $\mathcal{D}$, the cost-benefit analysis is done by solving the optimization problem given by:

$$\begin{aligned} &\text{find } \boldsymbol{d}^* \\ &\text{which minimizes } C_{TE}(\boldsymbol{d}) \\ &\text{subject to } \boldsymbol{d} \in \mathcal{D} \end{aligned} \quad (3)$$

The meta-heuristic firefly algorithm (FA) is used for the risk optimization process due to its efficiency for highly nonlinear problems and its simple formulation (Yang 2008; Yuan-Bin et al. 2013). Initially, a total of 9000 fireflies are generated over the entire design domain, and only the 100 brightest are kept in further 100 iterations. The light absorption coefficient γ_{FA} is kept equal to zero, ensuring that all fireflies are able to see each other and thus favoring the convergence to the global optima. The attractiveness coefficient β_0^{FA} is set proportional to the brightest firefly, and the randomization coefficient α_{FA} is set to 0.2 over all iterations.

2.2. Reliability analysis

The weighted average simulation method (WASM), proposed by Rashki et al. (2012), is used to estimate the failure probabilities P_f. This is a very appropriate technique for optimizing random design variables since estimating P_f depends only on the indicator function $I(\boldsymbol{x})$ and the weight index $W(\boldsymbol{x})$ for n_{sp} sample points, with $\boldsymbol{x}$ being the random variable vector. Therefore, changing the mean value of the candidate for optimal design only requires the reevaluation of the weight index $W(\boldsymbol{x})$, thus allowing the usage of the same sample over the entire process.

$$P_f = \frac{\sum_{k=1}^{n_{sp}} I(x_k)\, W(x_k)}{\sum_{k=1}^{n_{sp}} W(x_k)} \quad (4)$$

Table 1 shows details for all random variables (RVs) herein considered. A sample with 8 million points is used to estimate every P_f for 2000 optimal candidates, which are generated via Latin Hypercube Sampling over $\mathcal{D}$. These optimal candidates are used to elaborate a metamodel for every $\beta = -\Phi^{-1}(P_f)$, reducing even further the computational burden to compute C_{TE}.

Table 1: Uncertainty modelling.

RV	*Distr.*	*Mean*	*CoV*
Beam depth (h_B)	*Normal*	*To be optimized*	*0.01*
Bottom beam reinf. (A_B)	*Normal*	*To be optimized*	*0.05*
Top beam reinf. (A_T)	*Normal*	*To be optimized*	*0.05*
Stirrup spacing at beams (s_t)	*Normal*	*To be optimized*	*0.05*
Column depth (h_P)	*Normal*	*To be optimized*	*0.01*
Column reinf. (A_P)	*Normal*	*To be optimized*	*0.05*
Concrete strength	*Lognormal*	*45 MPa*	*0.12*
Yielding strength	*Normal*	*511 MPa*	*0.05*
Concrete self-weight	*Normal*	*25 kN/m³*	*0.05*
Ultimate steel strain	*Normal*	*0.20*	*0.14*
Dead load	*Normal*	*1.05 D_n*	*0.10*
50-year live load	*Gumbel*	*1.00 L_n*	*0.25*
Arbitrary point in time live load	*Gamma*	*0.25 L_n*	*0.55*
Model error	*Lognormal*	*1.107*	*0.229*

2.3. Structural analysis

The position-based finite elements (FE) of 2D layered beams proposed by Coda and Paccola (2014) are used herein. Each beam span is discretized into 15 FE with a fifth-degree of approximation, and each column into 3 FE with the same degree. A total of 15 layers with 1 integration point each is used to discretize the cross-sections, being 13 for concrete and one for each rebar reinforcement.

Steel rebar behavior is represented by a uniaxial model of isotropic hardening. As for concrete, μ-Model (Mazars et al. 2015) is used to represent its damage evolution and unilateral behavior. Stirrups cannot be explicitly considered, but its influence over concrete ductility is regarded by considering the resulting uniaxial curve from the Modified Park-Kent Model (Park et al. 1982) to automatically calibrate the μ-Model parameters for each sample point.

Two structural analyses are performed for every sample point: one for the intact structure, where an increasing uniform load is applied over each beam span; and one for the column loss scenario, where the uniform load is increased only over the beam spans directly affected by the sudden column removal.

For intact structure it is obtained: the uniform load corresponding to a maximum mid-span displacement of 7 mm; the uniform load that leads to its ultimate beam bending moment; the greatest observed shear force at the beams; and the maximum acting axial force and bending moment at the columns. As for column loss scenario it is obtained: the uniform load that leads to the tensile rebar rupture; the uniform load correspondent to the CAA peak; the greatest observed shear force at the beams until CAA; and the maximum axial force and bending moment at the columns. Due to symmetry in the structural geometry and loading conditions, only half of the structure is modelled for both scenarios.

Dynamic effects due to the sudden column removal are regarded conservatively by a dynamic amplification factor of 2.0 multiplying the applied load when evaluating the limit states.

2.4. Metamodeling

Kriging is used to create simplified, yet accurate, models for the limit states evaluation and reliability indexes in order to perform the risk optimization with a significantly reduced computational cost. This technique is used due to its known efficiency and robustness for structural reliability problems and its great performance at multi-dimensional level (Kroetz et al. 2017; Kaymaz 2005).

A sufficient number of support points n_S is required to make the metamodel accurate in comparison to the original. The base of functions chosen to generate the simplified model is a cubic polynomial with all possible crossed terms. Also, the non-isotropic hyperparameters $\boldsymbol{\theta}$ are calibrated by the minimization of the reduced likelihood function proposed by Dubourg (2011) via Firefly Algorithm:

$$\boldsymbol{\theta} = \arg \min_{\boldsymbol{\theta} \in n_\theta} \mathcal{L}(\boldsymbol{\theta}) \qquad (5)$$

with $\mathcal{L}(\boldsymbol{\theta}) = \sigma^2(\boldsymbol{\theta})\ |R(\boldsymbol{\theta})|^{1/n_s}$; n_θ is the number of hyperparameters coordinates to be evaluated; $\sigma^2(\boldsymbol{\theta})$ is the metamodel variance; and $R(\boldsymbol{\theta})$ is a matrix containing the correlation between pairs of support points.

3. RESULTS

Table 2 shows the optimal risk-based design for a scenario of intact structure ($P_{LD} = 5 \times 10^{-6}$) and for progressive collapse ($P_{LD} = 1$).

Table 2: Optimal risk-based design solutions.

Parameter	*Scenario*	
	Conventional	*Column loss*
h_B *(mm)*	*350*	*320*
A_B *(mm²)*	*263* ($\sim 2\phi 10 + 1\phi 12$)	*653* ($\sim 2\phi 20$)
A_T *(mm²)*	*220* ($\sim 2\phi 12$)	*266* ($\sim 2\phi 12$)
s_t *(mm)*	*275*	*165*
A_P *(mm²)*	*440* ($\sim 4\phi 12$)	*590* ($\sim 4\phi 14$)
h_P *(mm)*	*280*	*285*
C_M (€)	*3461.96*	*4136.75*
C_{TE} (€)	*3488.25*	*4181.01*

For intact structure, serviceability failure is dealt with considering a great h_B with small A_B and A_T, ensuring an optimal moment of inertia in terms of the individual material costs, expected cost of failure, and safety. Since beam bending is considered only at the midspan, only A_B has an optimal value above lower bound adopted for rebar reinforcement ($\sim 2\phi 12$). Shear failure is dealt with considering a combination of great h_B and large spacing s_t between stirrups. Even though a reduced h_B with more stirrups (smaller s_t) can lead to a similar performance against shear failure, greater A_B and A_T would be necessary to avoid serviceability and bending failures, leading to a greater cost for similar safety levels. Hence, an increased h_B is chosen due to it being advantageous for all adopted beam failure modes, allowing smaller longitudinal and transversal reinforcement ratios for the beams and leading to a minimal C_{TE}. Columns have an optimal depth h_P of 280 mm with the minimal rebar area A_P considered for such members ($\sim 4\phi 12$). Similar safety against column failure is also possible with minimal h_P and greater A_P, but this combination is not efficient in terms of C_M due to the relative costs of concrete and steel.

Progressive collapse has a slightly reduced h_B, increased rebar reinforcement (mostly A_B), a significantly reduced s_t for the beams, and an overall increase in the column's robustness. Reducing h_B serves two purposes: it gets bottom and top rebars closer, which reduces their stress differences at CA and enhances its ultimate capacity; and reduces de CAA peak, leading to a smaller shear force acting over the beam. Yet, both longitudinal and transversal ratios have to be increased in order to ensure safety against steel rupture and shear failure. As for the columns, they have to get slightly more robust in order to sustain the increased axial forces and bending moments due to beams being able to reach greater ultimate capacity at CA, which can stress the columns beyond the capacity of their usual design. Hence, weak columns may compromise progressive collapse mitigation even if CA is able to be fully developed over the double-span.

Table 2 also shows that C_{TE} is slightly greater than C_M for both intact and progressive collapse scenarios, meaning that all failure modes are satisfactorily mitigated in terms of expected cost of failure C_{ef}. In contrast, this is not observed in Ribeiro et al. (2022) for progressive collapse failure modes, which happened due to a lack of ductility for steel reinforcement being considered. Hence, ensuring that ductility is provided to also allow a full development of CA is better, in terms of C_{TE}, than relying only on flexural, Vierendeel and CAA mechanisms.

Figures 2 and 3 show the reliability index β^* and expected cost of failure C^*_{ef}, respectively, at the optimal design for each failure mode as P_{LD} increases. This covers scenarios where occurrence of an initial local damage due to a triggering event is very unlikely and also those of certain event. It should be noticed that for column loss scenario β refers to the conditional probability of the failure mode occurrence multiplied by P_{LD}.

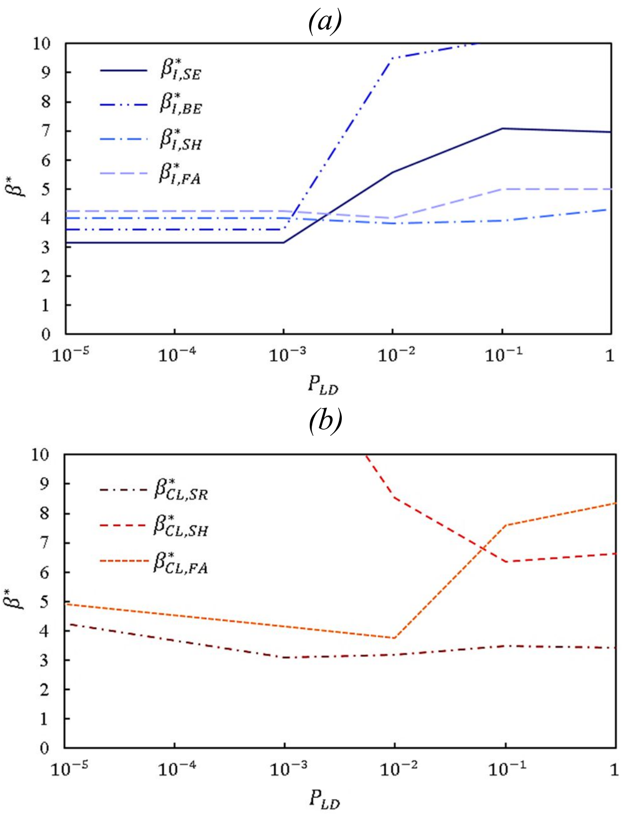

Figure 2: Optimal reliability indexes for failure modes of intact structure (a) and column loss scenario (b).

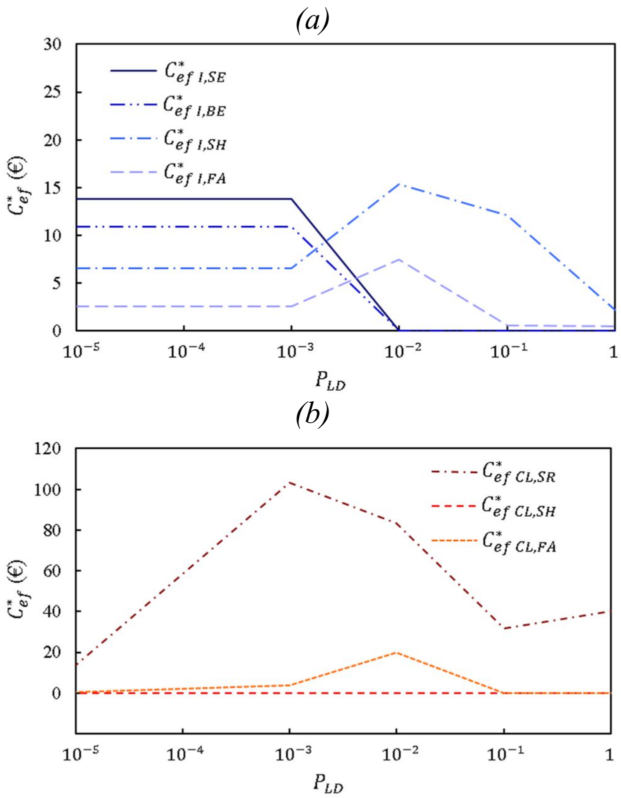

Figure 3: Optimal expected cost of failure for intact structure (a) and column loss scenario (b).

For conventional design, Figure 2a shows that optimal reliability indexes are 3.20 for serviceability, 3.70 for beam bending failure, 4.10 for shear failure and 4.70 for column failure. Since greater cost multipliers k are adopted to represent the increased severity of shear and column failures, the optimization algorithm increases the safety margin against these severe modes in order to ensure economy. Hence, the best balance between safety (increasing C_M) and economy (reducing C_{ef}) is found. This is also true for progressive collapse design, with only enough safety margin ($\beta \approx 3.10$) against steel rupture being kept and greater safety being imposed for shear and column failures (Figure 2b).

As shown in Figure 3, the sudden change in optimal design after $P_{LD} = 10^{-3}$ happens due to increasing expected costs of rebar rupture as P_{LD} grows. This coincides with a reducing reliability index for this failure mode as P_{LD} increases, reaching 3.10 at $P_{LD} = 10^{-3}$ (Figure 2b).

Following Beck et al. (2022), $P_{LD} = 10^{-3}$ is a threshold local damage probability P_{LD}^{th}, after which it is better, in terms of the safety x economy balance, to actually consider a column loss scenario when designing. Adding reinforcement to mitigate steel fracture increases C_M, but using a conventional design for $P_{LD} > 10^{-3}$ leads to a probability of steel rupture so great that the resulting C_{ef} becomes much superior than the necessary cost of reinforcement to avoid it.

In addition, optimal design for progressive collapse also leads to greater safety margins for the intact structure failures modes as well. Figure 2a shows that for $P_{LD} > P_{LD}^{th}$ there is a significant increase in safety for beam bending and serviceability failures due to an increasing rebar reinforcement. Column safety at intact scenario increases since for greater P_{LD} they are designed to withstand greater axial forces and bending moments. Shear failure for the intact structure initially has a decrease in safety (Figure 2a) and a increase in its C_{ef} (Figure 3a), which is due to the reduction of h_B (to enhance CA capacity) being greater than the immediate reduction of s_t (to avoid shear failure). However, as P_{LD} gets close to 1, s_t reduces faster, keeping C_{ef} for shear failure close to zero for both scenarios.

It should be noticed that at P_{LD}^{th} only the beam optimal design shows a significant change in order to mitigate progressive collapse. Increase in column robustness happens later, at $P_{LD} = 10^{-2}$, with reliability index for column failure reaching a minimum of 3.90 (Figure 2b). As P_{LD} increases and CA capacity is enhanced, greater axial compressive forces and bending moments are expected to be imposed over the remaining columns of the damaged structure, mostly at the ones adjacent to the removed column.

Therefore, as the optimization leads to beams with greater ultimate capacity at CA for increasing P_{LD}, starting at $P_{LD}^{th} = 10^{-3}$, the usual optimal design for columns is appropriate until $P_{LD} = 10^{-2}$. At later stages, columns have to be also increasingly reinforced in order to effectively sustain the increased loads due to column removal and allow the full development of CA capacity.

4. CONCLUSIONS

This study shows how the optimal design of an RC frame with enough ductility changes after a column loss scenario starts to be more relevant.

- The risk-based optimization framework is able to find the best compromise between safety and economy for both conventional and column loss scenarios. To that aim, the procedures accounts for the relative costs of concrete and steel, the amount of each material over the frame, and how the resources have to be allocated, to provide safety against each failure mode.
- Optimal design under conventional scenarios considers a large beam depth to ensure safety against all intact structure failure modes, thus leading to a smaller amount of both rebars and stirrups.
- Column loss scenario has an optimal design fully benefiting from the enhancement of ultimate capacity at catenary action, with a slightly reduced beam depth and increased rebar reinforcement. Columns are also reinforced to avoid its premature failure before ultimate capacity of catenary action. Besides, shear capacity is increased by reducing the spacing between stirrups.
- Optimal design against column loss also provides more safety to the intact structure.
- In terms of risk-based optimal design, providing ductility is essential to allow that catenary action actually helps to mitigate progressive collapse. Flexural, Vierendeel and compressive arch action are not enough to avoid rebar rupture during or right after snap-through instability of double-span beams.
- According to past studies, a threshold local damage probability can be defined to consider a column loss scenario in design. However, while beam reinforcement is immediately addressed above this threshold, column reinforcement is only needed under greater probabilities to avoid premature failure before ultimate capacity at catenary action.

5. ACKNOWLEDGMENTS

Funding by FAPESP grants 2019/13080-9 and 2019/23531-8 is cheerfully acknowledged.

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