Reliability-based study on the duration of load effect in timber structures under wind loads

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ABSTRACT: This paper presents a reliability-based study on the duration of load (DOL) effect in timber members under wind loads. First, an auto regressive and moving average (ARMA) time-series model was established to simulate 50 years wind speed record in three Canadian cities. The model parameters were calibrated for each city based on historical data from the GEM-LAM platform of Environment Canada. Comparisons on the statistics and probability distribution of hourly wind speeds and yearly maximum wind speeds were conducted to validate the use of ARMA model. Subsequently, reliability analysis was carried out to evaluate the short-term and long-term reliability of timber members under simulated wind loads. The process considered the randomness of wind loads and the damage accumulation behaviour of timber. Finally, recommended values of the load duration factor under wind loads were obtained and compared with the values in the current Canadian design standard.

1. INTRODUCTION

Wood, as a nonlinear viscoelastic material, is one of the few engineering materials where design codes account for its design strength based on the load duration. The strength properties of wood can deteriorate under sustained and/or repeated loading, which is well known as the duration of load (DOL) effect. Various models have been proposed to capture this intrinsic feature of timber material, including empirical models, fracture mechanics models, deformation dynamic models, energy-based models, and damage accumulation models. Amongst these models, in North America calibrated damage accumulation models are recognized in the code development process, where the DOL effect is simulated as a process of irreversible damage accumulation. There are three main types of damage accumulation models, i.e., Gerhards and Link's model, Barrett and Foschi's model and Foschi and Yao's model. The damage accumulation of timber in Gerhards and Link's model was assumed as an exponential function of the stress ratio while in Barrett and Foschi's model and Foschi and Yao's model, the rate of damage accumulation depended on both the stress ratio and the damage state. A threshold stress ratio was also proposed in the Foschi's models, which means that the damage can only accumulate when the applied stress exceeds the threshold value.

In timber structures design standards, a load duration factors are typically used to adjust the design strength of timber under different load durations to account for load duration effects. In Canada a reliability-based approach was adopted (Foschi et al. 1989) to calibrate the load duration factor in the Canadian Code on Engineering Design in Wood CSA O86.1. The key idea of this approach was that the long-term reliability considering DOL effect should be the same as the target reliability under short-term loads. In US Ellingwood and Rosowsky (1991) adopted a similar probabilistic approach to determine the load duration factor in US National Design

Standards (NDS). Later, this reliability-based approach was also used to study the load duration factor in Eurocode. Sørensen et al. (2005) calibrated the load duration factor under different loads in Eurocode with different damage accumulation models. Kohler and Svensson (2011) improved the calibration approach by considering the uncertainty of the damage accumulation model. Recently, Li et al. (2020) calibrated the load duration factor for Chinese standard based on the reliability-based approach, where the effect of load ratios and coefficients of variation of timber strength on the load duration factor were revealed.

Previous research calibrated the load duration factor under different loads including dead load, live load and snow load. However, the duration of load effect under wind loads has not been comprehensively considered. With increasing interests in construction of tall timber buildings, their performance under wind loads can be an important design limit state. Thus, it is of interest to further investigate the load duration factor under wind loads. In CSA O86.1, wind loads are considered as short-term loading for which the continuous or cumulative specified load duration does not exceed 7 days. Here member strengths under typical test conditions (~15 min.) are reduced by a factor of 1.09 to account for short-term duration of load (the corresponding load duration factor is 1.15).

In this study, a reliability-based approach was used to study the duration of load effect in timber structures under wind loads. First, an auto regressive and moving average (ARMA) time-series model was established to simulate 50 years wind speed record in three Canadian cities. The site-specific model parameters were calibrated based on historical data from the GEM-LAM platform of Environment Canada. Comparisons on the statistics and probability distribution of hourly wind speeds and yearly maximum wind speeds were conducted to validate the developed ARMA model. Reliability analysis was then carried out to estimate the short-term and long-term reliability of timber members under the simulated wind loads with consideration of the randomness of wind loads and damage accumulation behaviour of timber. Finally, recommended values of the load duration factor under wind loads were obtained and compared with the values in the current design standard.

2. WIND SPEED MODEL

2.1. Auto regressive and moving average (ARMA) model

Wind load applied on timber structures at a geographic location is highly variable and time-dependent. Establishing a reasonable wind load model requires accurate models to simulate and forecast the variation of wind speeds as well as dynamic effects of wind gusts. In this study the influence of wind gusts is ignored. Time series models, which include auto regressive (AR) model, moving average (MA) model, auto regressive and moving average (ARMA) model, etc., have been developed by some scholars to simulate wind speed records and achieve good agreements. In this study, the site-specific ARMA time series model was established to simulate the time history of wind speed (Karki et al. 2006).

The sequential equation of ARMA (n, m) model is shown in Eq. (1).

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_n y_{t-n} + \alpha_t - \theta_1 \alpha_{t-1} - \theta_2 \alpha_{t-2} - ... - \theta_m \alpha_{t-m} \quad (1)$$

where ϕ_i (i = 1, 2, … , n) and θ_j (j = 1, 2, … , m) are the auto regressive and moving average model parameters, respectively; y_t is the time series value; and $\{\alpha_t\}$ is a normal white noise process with zero mean and a variance of σ_a^2, i.e., $\alpha_t \in \mathrm{NID}(0, \sigma_a^2)$ (NID denotes normally independently distributed).

In this study, the time series value is defined by Eq. (2). The order of ARMA model was chosen as (3, 2).

$$y_t = (OW_t - \mu_t) / \sigma_t \quad (2)$$

where OW_t is the observed wind speed at hour *t*; μ_t and σ_t are the mean and standard deviation of observed wind speed at hour *t*, respectively.

Historical wind speed data in the GEM-LAM platform of Environment Canada were used to calibrate the model parameters. This database has the time series wind speed data with a time step of 10 minutes over three years. The historical data for three typical Canadian cities including Ottawa, Saskatoon and Montreal were collected and analyzed. The specific location for each city was at the international airport (open terrain). The observed data were adjusted to the standard 10 m height in most design standards by employing the power law in National Building Code of Canada (NBCC). The hourly mean and standard deviation of observed wind speed were calculated, and the time series values of wind speed (at a frequency of one reading per hour) were obtained. Then, the nonlinear least-square method was implemented to estimate the parameters of ARMA model for each city. The calibrated ARMA model and corresponding model parameters are shown in Eqs. (3)-(5).

$$y_t = 1.4263 y_{t-1} - 0.4479 y_{t-2} - 0.0180 y_{t-3} + \alpha_t - 1.2313\alpha_{t-1} + 0.2673\alpha_{t-2} \quad (3)$$
$$\alpha_t \in \mathrm{NID}(0, 1.1666^2)$$

$$y_t = 1.4897 y_{t-1} - 0.5024 y_{t-2} - 0.0154 y_{t-3} + \alpha_t - 1.3211\alpha_{t-1} + 0.3434\alpha_{t-2} \quad (4)$$
$$\alpha_t \in \mathrm{NID}(0, 1.1687^2)$$

$$y_t = 1.5526 y_{t-1} - 0.6067 y_{t-2} + 0.0250 y_{t-3} + \alpha_t - 1.3536\alpha_{t-1} + 0.3813\alpha_{t-2} \quad (5)$$
$$\alpha_t \in \mathrm{NID}(0, 1.1726^2)$$

2.2. Model validation

A series of comparisons were conducted to validate the calibrated ARMA time series model for three different cities. The calibrated ARMA models were used to randomly and sequentially simulate the hourly wind speeds over 50 years for three cities. The actual wind speeds were obtained from the HLY01 digital archive of Environment Canada (EC). The archive has been maintained by EC since 1953. The wind speeds in this archive are 1- or 2-minute average wind speed recorded just before the top of the hour, or 10-minute average wind speed recorded just before the top of the hour. The 10 m-height wind data at the international airport of three cities (standard condition in most design standards) were collected as an actual database in this study. The collected duration is 50 years (from 1971 to 2020).

As shown in Figure 1, the cumulative probability of wind speed over 50 years for actual and simulated results agree well, which means the ARMA model can reasonably reflect the probability distribution of wind speeds in these three different cities. Additionally, Table 1 shows the comparisons of statistics of actual and simulated hourly wind speeds over 50 years. The standard deviations were found to be close for actual and simulated wind speeds in three cities. The simulated mean wind speed over 50 years was a bit higher (less than 15%) than the actual mean. The difference is acceptable considering slight overestimation of the mean wind speed is conservative when determining the load duration factor. In summary, the calibrated ARMA models can well simulate the global variation and probabilistic characteristics of wind speeds.

Considering the maximum wind speed is important for the long-term damage accumulation analysis, the statistics of yearly maximum wind speeds obtained from the actual database and ARMA model were also compared. Table 2 summarizes the statistics of actual and simulated yearly maximum wind speeds in three cities. Three random 50 years wind speed records generated by the ARMA model were used for comparison in each city. The results showed that all the simulated wind records have similar average of yearly maximum speeds and coefficient of variation of yearly maximum speeds compared to the actual wind records. Also, the simulated maximum wind speeds over 50 years were at the same level as the actual maximum. The above observations demonstrated that the calibrated ARMA time series model can reasonably capture the maximum wind speed in three cities.

3. DAMAGE ACCUMULATION MODEL

The damage accumulation model proposed by Foschi and Yao (1989) was adopted in this study to consider the duration of load effect of wood. The mathematical expression of the damage accumulation model is shown in Eq. (6).

$$\begin{cases} \dfrac{d\alpha}{dt} = a\left[\dfrac{\tau(t)}{\tau_s} - \sigma_0\right]^b + c\left[\dfrac{\tau(t)}{\tau_s} - \sigma_0\right]^n \alpha; \\ \dfrac{d\alpha}{dt} = 0; \quad \tau(t) - \sigma_0\tau_s \le 0 \end{cases} \tag{6}$$

where α denotes the damage state, $\alpha = 0$ means no damage while $\alpha = 1$ means failure; $\tau(t)$ and τ_s are the applied stress at time *t* and the initial short-term strength, respectively; σ_0 is the threshold ratio; and *a*, *b*, *c* and *n* are model parameters.

In this damage accumulation model, τ_s is determined by short-term ramp tests. The product of σ_0 and τ_s is defined as a threshold where the damage accumulation is initiated. τ_s, *a, b, c* and *n* are constants for a specified structural member but vary randomly for different members. Thus, the four parameters including τ_s, *b, c* and *n* were modelled as independent lognormal variables. The parameter of *a* can be obtained by the other four parameters. The calibrated model parameters for No. 2 and better visually graded Western Hemlock timber in bending (Foschi et al. 1989) were adopted in this study.

Table 1. Comparisons of statistics of actual and simulated hourly wind speeds over 50 years (Unit: km/h).

City	*Mean of actual data*	*Mean of simulated results*	*Standard deviation of actual data*	*Standard deviation of simulated results*
Ottawa	*13.4*	*15.4*	*8.2*	*7.0*
Saskatoon	*15.7*	*17.3*	*8.9*	*8.2*
Montreal	*14.6*	*17.0*	*9.0*	*7.8*

Table 2. Comparisons of statistics of actual and simulated yearly maximum wind speeds (Unit: km/h).

City	*Actual or simulated*	*Maximum speed over 50 years*	*Average of yearly maximum speeds*	*Coefficient of variation of yearly maximum speeds*
Ottawa	*Actual*	*74*	*56.3*	*10.4%*
	ARMA1	*75.5*	*56.2*	*12.5%*
	ARMA2	*82.8*	*56.3*	*12.9%*
	ARMA3	*72.4*	*55.1*	*11.8%*
Saskatoon	*Actual*	*87*	*63.9*	*13.8%*
	ARMA1	*96.8*	*64.5*	*15.3%*
	ARMA2	*87.6*	*62.1*	*14.8%*
	ARMA3	*82.2*	*63.8*	*14.6%*
Montreal	*Actual*	*83*	*62.8*	*14.9%*

City	*Actual or simulated*	*Maximum speed over 50 years*	*Average of yearly maximum speeds*	*Coefficient of variation of yearly maximum speeds*
	ARMA1	*81.5*	*58.1*	*12.6%*
	ARMA2	*80.5*	*59.5*	*11.8%*
	ARMA3	*75.3*	*57.2*	*10.8%*

Note: Actual denotes the actual wind speed obtained from HLY01 digital archive of Environment Canada; ARMA1-3 denote the randomly simulated wind speed using ARMA time series model.

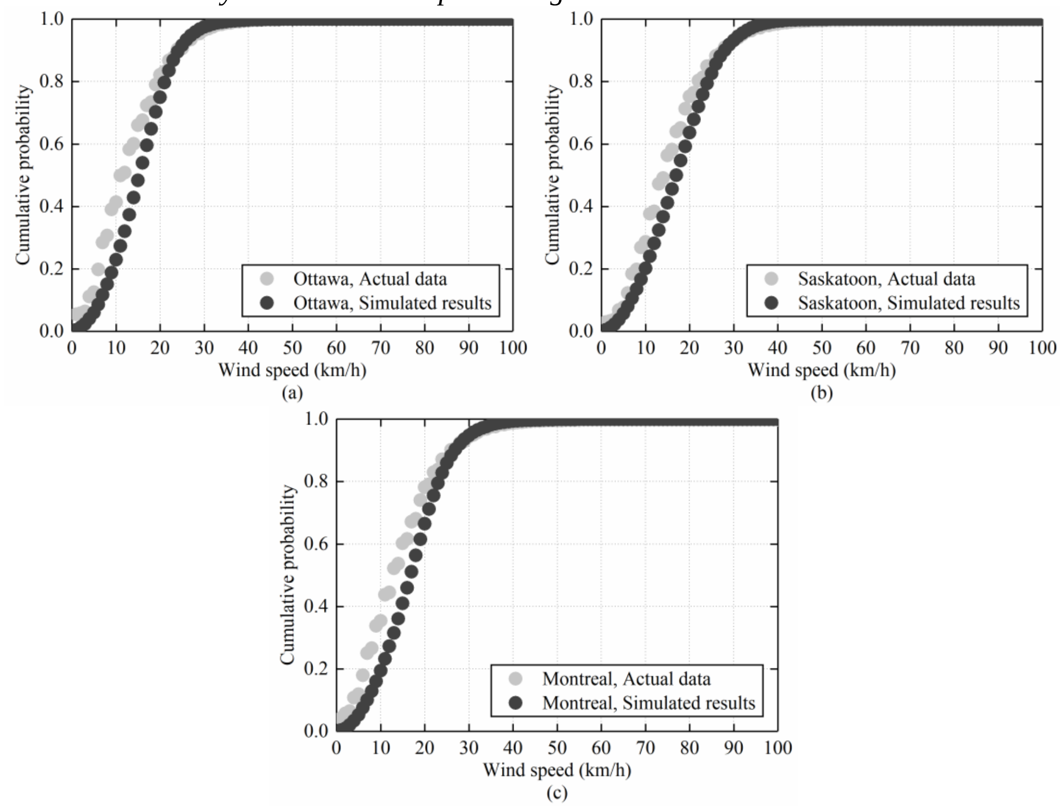

Figure 1: Comparisons between actual and simulated cumulative probability of wind speeds over 50 years in (a) Ottawa, (b) Saskatoon, and (c) Montreal.

Because the closed-form integration of Eq. (6) is difficult for an arbitrary load history, a recurrence relationship was developed to simplify the complex calculation, as shown in Eq. (7). For the simplified method, the random load was divided into different time intervals, where the applied stress keeps constant within each interval. And the wood damage at any time in the load history can be determined step by step using the recurrence relationship.

$$\alpha_i = \alpha_{i-1} K_i + L_i \tag{7}$$

$$K_i = exp\left[c\left(\tau_i - \sigma_0 \tau_s\right)^n \Delta t\right] \tag{8}$$

$$L_i = \frac{a}{c}\left(\tau_i - \sigma_0 \tau_s\right)^{b-n}\left(K_i - 1\right) \tag{9}$$

4. RELIABILITY-BASED CALIBRATION OF LOAD DURATION FACTOR UNDER WIND LOADS

4.1. Benchmark model

A benchmark model was proposed for the reliability-based calibration of load duration factor under wind loads, as shown in Figure 2. Similar to the calibration of load duration factor of beams under vertical loads, single timber member was considered in the benchmark model. The wood species and grade were Western Hemlock and No. 2 and better (visually graded), respectively. The sectional size of member was 38 mm × 140 mm. The mean and CoV of short-term bending strength of the wood were 47.83 MPa and 0.41, respectively (Foschi et al. 1989). The characteristic bending strength was 23.14 MPa. A 1 m × 1 m wind plate was assumed on the top of timber members to catch wind. The center of wind plate was set at the 10 m height above the ground. The boundary condition of timber members was chosen as fixed end. For this benchmark model, the maximum stress of timber members was at the edge of the fixed end section. The member length can be determined by the ultimate limit state design equation with different resistance factors.

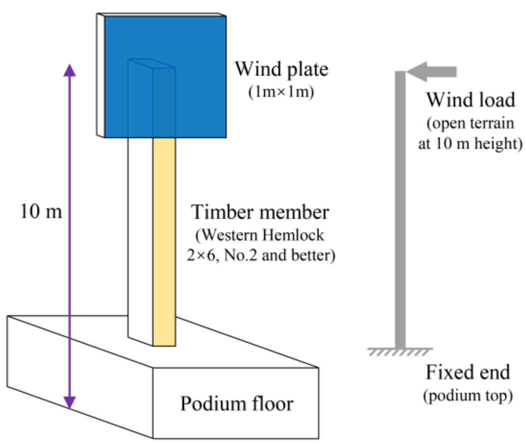

Figure 2: Benchmark model for the calibration of load duration factor under wind loads.

4.2. Short-term reliability analysis

The ultimate limit state design equation for wind loads according to CSA O86 is shown in Eq. (10).

$$1.4E(W_n) = \phi R_{0.05} \tag{10}$$

where $E(W_n)$ is the design wind load effects ignoring wind gust; ϕ is the resistance factor; $R_{0.05}$ is the specified strength; 1.4 is the wind load factor.

The performance function G of short-term reliability analysis can be written as Eq. (11).

$$G = R - \frac{\phi R_{0.05}}{1.4} \times \frac{E(W)}{E(W_n)} \tag{11}$$

where R and $E(W)$ are the random variables related to the bending strength and wind load effects, respectively.

Altogether four main steps were included in the short-term reliability analysis. STEP 1 was conducted to define key parameters including service location, timber species, resistance factor, etc. The resistance factor was chosen from 0.6 to 1.2 with an increment of 0.1. STEP 2 was about the wind speed calculation. First, the design wind speeds in three cities were taken from NBCC, which correspond to loads with a 1/50 probability of being exceeded (50 years return). The specific design values for Ottawa, Saskatoon and Montreal were 90.7 km/h, 96.1 km/h and 93.9 km/h, respectively. Subsequently, the calibrated ARMA time series models for three cities were adopted to randomly generate the hourly wind speeds over 50 years and obtain the maximum wind speed over 50 years.

In STEP 3 the short-term damage of timber members under wind loads was calculated. The short-term bending resistance was randomly chosen based on the lognormal distribution of short-term strength. Then the performance function was calculated, and the short-term damage state can be determined. STEP 4 was performed to calculate the short-term reliability index using Monte Carlo approach. For a certain city and a certain resistance factor, STEP 2 and STEP 3 were repeated 10^7 times to determine the probability that the performance function be negative (failure) and the corresponding reliability index.

4.3. Long-term reliability analysis

The performance function G for long-term reliability analysis is shown in Eq. (12).

$$G = 1 - \alpha \tag{12}$$

where α is the long-term damage state based on the damage accumulation model of timber. $G > 0$ implies survival, $G = 0$ implies limit state and $G < 0$ implies failure.

Five main steps were conducted during the long-term reliability analysis. STEP 1 was the same as that in the short-term reliability analysis. In STEP 2 the length of benchmark timber members was determined. First, the design wind speed was obtained for a certain city. Then the reference wind velocity (speed) pressure q can be calculated. The length of timber member can be determined with a chosen resistance factor by Eq. (13).

$$1.4 \times \frac{pAl}{bh^2/6} = \phi R_{0.05} \tag{13}$$

where A is the area of wind plate; b, h and l are the width, depth and length of timber members, respectively.

STEP 3 considered the simulation of wind speed time history. The calibrated ARMA time series models were conducted to randomly generate the hourly wind speed record over 50 years for a certain city. Then the wind speed record can be transformed as the time history of reference wind velocity (speed) pressure.

STEP 4 was conducted to calculate the long-term damage accumulation of timber. First, the time history of maximum stress in timber members was obtained. The model parameters of damage accumulation model and timber resistance of each timber replicate were generated randomly. Then, the long-term damage state can be analyzed sequentially based on the recurrence relationship shown in Eq. (7). Lastly, the long-term performance function of Eq. (10) was calculated to check if G was negative.

Finally in STEP 5 the long-term reliability index was estimated considering the duration of load effect. The reliability evaluation was performed using Monte Carlo approach. STEP 3 and STEP 4 were repeated 10^6 times to create a large number of samples. The number of failures was counted to calculate the probability of failure and the corresponding reliability index for a given resistance factor.

4.4. Calibration of load duration factor

The load duration factor K_D is a modification of the resistance factor which was related to a target reliability in the short-term. Based on the previous short-term and long-term reliability analysis, the relationship between reliability index and resistance factor can be captured for short-term and long-term. For a given target reliability, different resistance factors for long-term and short term were obtained. Then the load duration factor can be determined by Eq. (14).

$$K_D = \phi_L / \phi_S \tag{14}$$

where ϕ_S and ϕ_L are the short-term and long-term resistance factors, respectively.

5. RESULTS

Figure 3 shows the relationships between the reliability index and the resistance factor under wind loads. Comparing the reliability index versus resistance factor relationships for wind pulse durations of 1 hour and 10 minutes shows minor difference.

For a target reliability index of 3.0 for bending members, the calibrated load duration factors under wind loads in three cities are shown in Table 3. The results showed that the load duration factors were similar for different durations of wind pulse (1 hr. vs. 10 min.) in all cases. Moreover, the load duration factors for Ottawa and Montreal were similar. The load duration factor for Saskatoon was less than that in other two cities, which was about 0.8. The possible reason for this difference amongst cities was the different wind speed (load) ratios. The wind speed (load) ratio was defined as the ratio of the average of 1000 50 years maximum wind speeds to the design wind speed. The wind speed ratios for Ottawa and Montreal were both 0.81 while that for Saskatoon was 0.95. Here the NBCC design wind load for Saskatoon seems to be less conservative compared to that in the other two Canadian cities when compared to the simulated wind speed data. The K_D factor in CSA O86.1 is slightly conservative for Ottawa and Montreal but unconservative for Saskatoon.

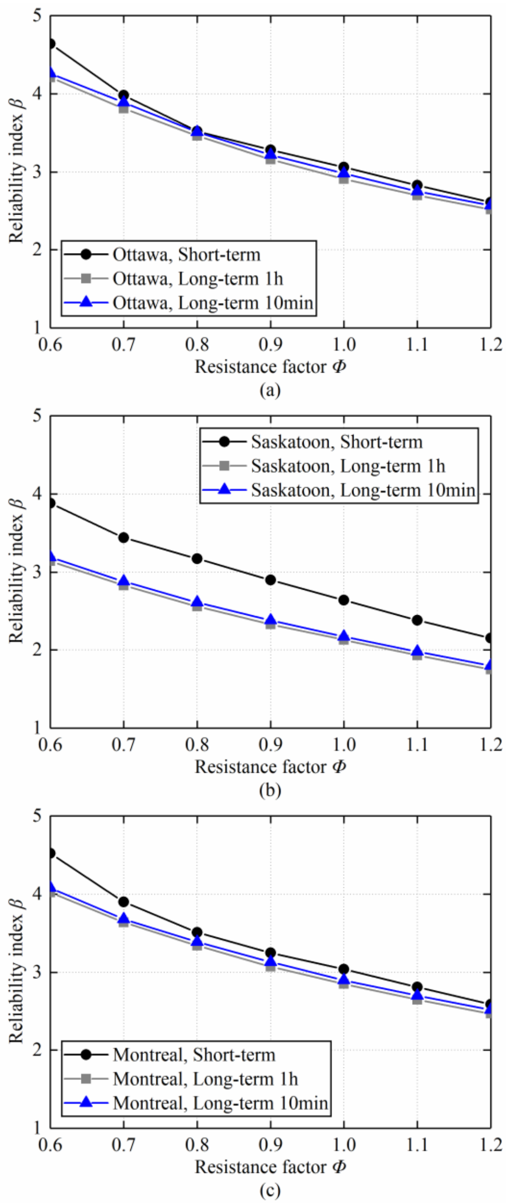

Figure 3: Relationship between the reliability index and the resistance factor under wind loads in (a) Ottawa, (b) Saskatoon, and (c) Montreal.

Table 3: Load duration factors for wind loads.

Pulse duration	*Ottawa*	*Saskatoon*	*Montreal*
1 h	*0.94*	*0.75*	*0.92*
10 min	*0.97*	*0.77*	*0.94*

6. CONCLUSIONS

This paper presents a reliability-based study on the duration of load effect in timber structures under wind loads. The ARMA time series model was calibrated and validated to simulate 50 years wind speed record in three Canadian cities. A series of comparisons demonstrated that the ARMA model can well simulate the global variation and probabilistic characteristics of wind speeds. Based on the reliability-based calibration method, the load duration factors for wind loads in three cities were determined. The reliability results show that wind pulse duration between 1 hour and 10 minutes lengths had little effect on the load duration factor. The load duration factor in Saskatoon was different from that in Ottawa and Montreal, which was mainly due to the different wind speed (load) ratios.

7. REFERENCES

Ellingwood, B., and Rosowsky, D. V. (1991). “Duration of load effects in LRFD for wood construction.” *Journal of Structural Engineering*, 117(2), 584-599.

Foschi, R. O., Folz, B. R., and Yao, F. Z. (1989). *Reliability-based design of wood structures.* Structural research series, Report No. 34, Department of Civil Engineering, Canada: University of British Columbia.

Karki, R., Hu, P., and Billinton, R. (2006). “A simplified wind power generation model for reliability evaluation.” *IEEE Transactions on Energy Conversion*, 21(2), 533-540.

Kohler, J., and Svensson, S. (2011). “Probabilistic representation of duration of load effects in timber structures.” *Engineering Structures*, 33(2), 462-467.

Li, Z., Zheng, X., Ni, C., Tao, D., and He, M. (2020). “Reliability-based investigation into the duration of load effect for the design of timber structures based on Chinese standard.” *Structural Safety*, 87, 102001.

Sørensen, J. D., Svensson, S., and Stang, B. D. (2005). “Reliability-based calibration of load duration factors for timber structures.” *Structural Safety*, 27(2), 153-169.