Realistic system reliability estimates in design of levee systems

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ABSTRACT: Reliability-based design of levee systems requires proper treatment of spatial variability in reliability analysis. In practice, this is often dealt with by standardized factors that represent the typical spatial variability. Such factors are then used to upscale cross-sectional reliability to system reliability. As for instance choices in schematization and definition of parameter distributions influence the exact meaning of a cross-sectional reliability calculation, these choices also determine what factors should be used. For instance, combination of factors aimed at upscaling 'representative' cross-sectional reliability estimates with a cross-section that represents the worst possible cross-section results in excessively low estimates of system reliability. In this paper we investigate the treatment of uncertainty in the reinforcement design of a riverine levee that is sensitive to inner slope instability and piping erosion. By integrally looking at the schematization of cross-sections and how this translates to system reliability we determine realistic bounds for system reliability.

1. INTRODUCTION

Flood defense systems protect billions of assets from flooding. To maintain these systems effectively, investments can be prioritized based on cost and risk reduction on a system level (Klerk et al., 2021). For a risk-based approach to investments in flood defences, it is thus important to consider the reliability of the defenses.

In the Netherlands, reliability targets for flood defences are set at the system level. A large reinforcement program (HWBP) is currently in place to improve many of the flood defenses to meet that safety level. Designs for reinforcements are often based on reliability-based analyses at a cross-section level. However, translating system-level targets to specific requirements for individual sections of the flood defense (series) system requires assumptions about the spatial variability and the contribution of different failure modes at different sections to the overall system reliability (Jongejan et al., 2020).

Combining the limit state functions of different cross-sections can be done by integrated probabilistic analysis, or by using approximative methods such as the Equivalent Planes method (Roscoe et al., 2015). Yet, such methods are either time-consuming or not straightforward to apply during a dike reinforcement project.

To this end, standardized factors are typically used to account for spatial variability with respect to the system probability. Such factors can be used to translate the reliability requirement of the system to that of a specific cross-section (and vice versa). However, to obtain realistic reliability estimates, the assumptions in the schematization of a cross-section (e.g. on spatial averaging) need to be consistent with these factors.

In this paper we study two key aspects of this translation from cross-sectional to system reliability:

1. The influence of assumptions in the schematization of cross-sections on the system reliability.
2. How spatial variability in the field (e.g., in geometry or soil parameters) influences the translation of cross-sectional reliability to system reliability.

Section 2 describes the basic principles of system reliability of flood defence systems, Section 3 outlines the approach for the case study. Section 4 presents results and Section 5 discusses conclusions and outlines steps for follow-up research.

2. SYSTEM RELIABILITY OF FLOOD DEFENCE SYSTEMS

In this paper we consider flood defence segments along rivers (hereafter levees), which are series systems of typically ~20 kilometers. In analysis of such systems, these are typically split up into sections, which are relatively homogeneous stretches with a length between 250 to 1500 meters. Such homogeneous sections are then characterized by a cross-section, which is used to compute the reliability for different failure modes.

In this analysis, we need to account for the variability along the section because random variables (e.g., geotechnical parameters) vary spatially. For flood defences this is typically done through a Gaussian autocorrelation function, which consists of a non-ergodic (base) correlation that does not vary spatially, and a spatially variable (ergodic) component. Contributions of different random variables can be obtained by for instance influence coefficients from a first-order reliability method computation, after which limit state functions can be combined using the Equivalent Planes method, or such a case can be solved by an integrated Monte Carlo simulation.

In practice, often semi-probabilistic computations are used to determine the reliability of flood defences. To assess whether such cross-sections meet the system requirements these are assessed using a cross-sectional reliability target $P_{\text{req,cs}}$, which is obtained through:

$P_{\text{req,cs}} = \frac{P_{\text{req,sys}}}{N}$ with $N = 1 + \frac{a \cdot L_{\text{sys}}}{b}$, where N is the number of independent sections in the system that are represented by cross-sections, L_{sys} is the length of the levee system, a is the part of the system that contributes to the system failure probability and b is the equivalent length of an independent system which is determined based on a general analysis of the autocorrelation as shown before. Default length effect factors for piping erosion and inner slope instability are shown in Table 1 (Jongejan et al., 2020). An assumption underlying this approach, is that cross-sections are schematized as representative rather than being the weakest spot, as the probability of a weak spot increasing with length is accounted for with the length-effect factors.

Table 1 Default length effect factors.

	a [-]	b [m]
Piping erosion	0.9 or 0.5	300
Inner slope instability	0.033	50

The system reliability is typically estimated from cross-sectional reliability estimates in a stepwise manner. First, cross-sectional results are translated into estimates for a section, and then into estimates for the entire system. However, the default factor a represents the sensitive fraction of entire systems. Therefore, this factor cannot be applied to translation of reliability estimates from cross-section to section. Therefore we propose to introduce a section-dependant factor a_{sec} for each section.

The failure probability for a section P_{sec} can be determined using the following formula:

$P_{\text{sec}} = P_{\text{cs}} \cdot N_{\text{sec}}$ with $N_{\text{sec}} = \frac{a_{\text{sec}} \cdot L_{\text{sec}}}{b}$, where P_{cs} is the failure probability of the cross section, a_{sec} is the part of the section that contributes to the failure probability, and L_{sec} is the length of the section, which yields the number of independent

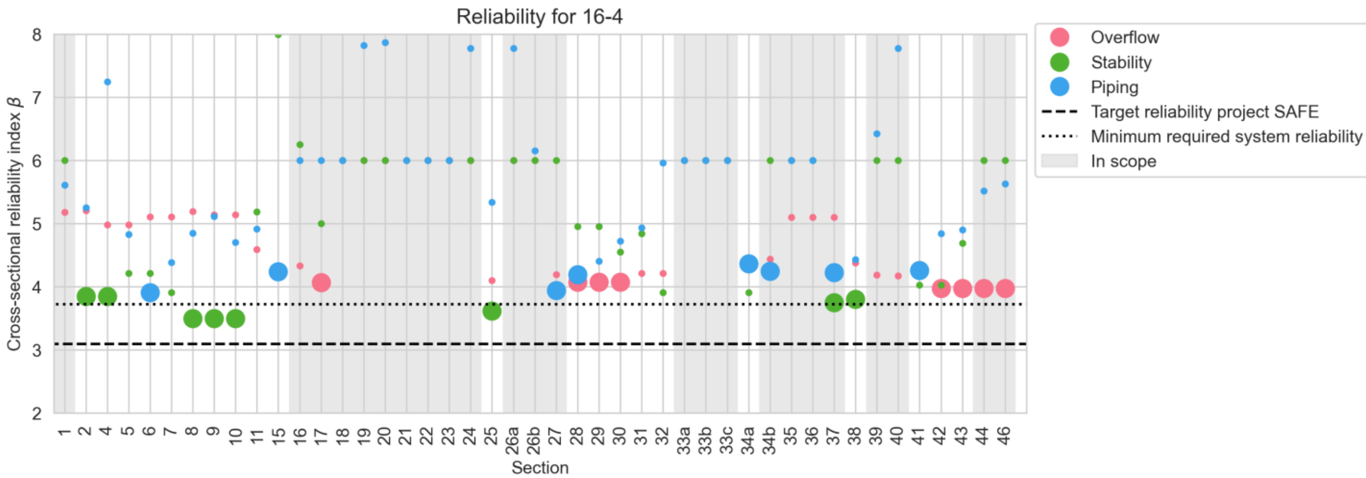

Figure 1 Current reliability indices as available for 16-4, after reinforcement. Reinforced sections are assumed to have a reliability index of 6.0. Larger circles indicate the 8 weakest sections for each failure mode.

parts in the section N_{sec}. For backwards erosion piping and inner slope stability these estimates per section an (under assumption of independence) be translated to a system reliability for a system of n sections using(Klerk, 2022):

$$P_{sys} = 1 - \prod_{1}^{n} (1 - P_n) . \quad (1)$$

An important challenge is to meaningfully derive a value for a_{sec}, and this value strongly depends on the local characteristics of the section and how the underlying schematization to determine the cross-sectional reliability was derived. For instance, if such a schematization is conservative in the sense that it represents the worst cross-section in the section one should not upscale at all, and N_{sec} should be approximately equal to 1. If the cross-section is representative and the section is completely homogeneous as intended in the guidelines, one should use a value a_{sec}=1.

In practice, sections are rarely fully homogeneous. For instance, a section might be homogeneous in terms of the subsoil parameters, but the dominant exit points for backwards erosion piping can be in a few perpendicular ditches, which means that the contributing part of such a section is relatively small. Thus, schematizing it as different (homogeneous) sections is very cumbersome. For inner slope stability on- and offramps to access neighboring residences can cause much higher reliability estimates at parts of a section, meaning that these parts don't contribute as well.

3. APPROACH

The study presented in this paper is part of a dike reinforcement project aimed at reinforcing the levee between the towns of Streefkerk, Ameide and Fort Everdingen (SAFE), along the Dutch Rhine river. This project is special because only a part of the levee is reinforced immediately, while the remainder is to be reinforced in 20 years from now. The levee consists of two separate segments (16-3 and 16-4), which each have a maximum allowable probability of flooding (standard) of 1/10,000 per year. The goal of the water authority is that after the first reinforcement the probability of flooding of both segments is < 1/1,000 (β>3.09), and in the next 20 years it will be further reinforced to meet the standard in 2050.

For each section (semi-)probabilistic analysis for slope instability, backwards erosion piping and overflow/overtopping are available for cross-sections that are considered representative. From these results, we can estimate the system probability using equation 1, with an appropriate assumption for the a_{sec} and/or N_{sec}. Typically, after reinforcement, the system failure probability will be dominated by non-reinforced sections, which is illustrated in Figure 1 for segment 16-4. For these areas typically less data, and less advanced computations are available.

To improve the system probability estimate we consider two main routes of action:

- Revisiting the schematization to improve the estimate of the cross-sectional reliability (β_{cs}). Note that we use reliability index β interchangeably with the failure probability P_f (where $P_f = \Phi(-\beta)$).
- Better estimating a_{sec} or N_{sec} and improving the representativeness of cross-sections by altering the division of the segment in sections.

Each of the approaches can lead to an improvement of the system reliability estimate, substantiating the target of a failure probability <1/1,000 is attained or not. To evaluate this on a system level, we compute the system failure probability for bounds where N_{sec}=1 and N_{sec} is based on the characteristics of the section (default a_{sec}=1, unless refined in the analysis). By selecting the most dominant sections for the system failure probability we iteratively refine these upper and lower bounds.

3.1. Inner slope instability

For slope stability, we start with the failure probability for all cross-sections from semi- or full probabilistic reliability analyses. For semi-probabilistic analyses, the probability is estimated using a relation between the Factor of Safety with design values with the reliability index (Kanning et al., 2017). Full probabilistic calculations are made using FORM within the Probabilistic Toolkit of Deltares, coupled to the D-Stability software. For the cross-sections being reinforced, the reliability is assumed to increase to a target level of 6.0, which means that they will no longer contribute to the overall system failure probability.

For cross-sections that have a significant impact on the system's failure probability, we explore ways to improve their reliability. This may involve adjusting the sensitive fraction (a_{sec}) based on the cross-section's geometry or performing a more detailed probabilistic calculation or refinement of the schematization. The aim is to enhance the reliability estimate for the section, and ultimately the system reliability.

3.2. Backwards Erosion Piping

Backward erosion piping, piping in the rest of the paper, is a failure mode where a head difference over the levee results in groundwater flow below the levee which results in backward erosion, forming a pipe. In the Netherlands, the Sellmeijer model is used to assess the safety with respect to piping (Pol, 2022). The typical analysis of a single cross-section means that weak spots along a longer stretch might remain undetected, which is accounted for by using length-effect factors. Here we use a novel approach (exit point method) based on dense data where many combinations of entrance and exits points of piping paths are assessed. An example is shown in Figure 2.

To compute the system reliability, we apply a moving window of 300 meters. The highest P_f of the points within a window is assumed as representative for the whole window, as points within a window are strongly correlated. The windows are subsequently combined using the assumption of independence. Benchmarks using exact (but computationally very expensive) methods show that this method is sufficiently accurate (Kanning, 2021).

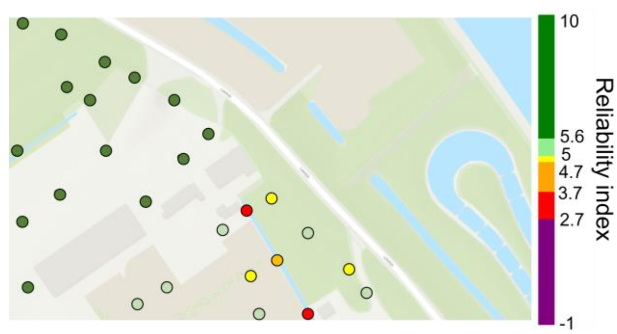

Figure 2 Exit point based piping assessments from the SAFE project. Colors indicate the safety of a point.

4. CASE STUDY

4.1. General description of system

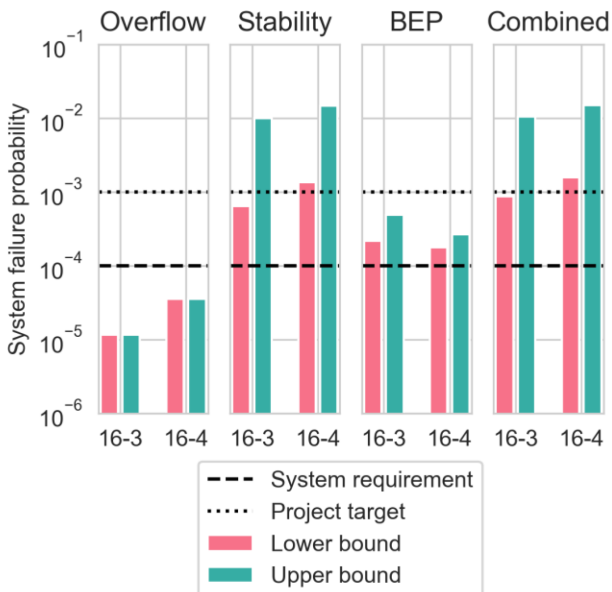

Figure 3 Initial bounds for different failure mechanisms.

Levee segments 16-3 and 16-4 are located along the Lek, a branch of the Dutch Rhine River. Both segments differ in terms of characteristics: 16-3 is densely built, and the largest safety deficits occur for the mechanism of inward slope stability. Segment 16-4 is characterized by a highly variable soil stratification due to its geological history due to former river branches. Typically this the area is more sensitive to backwards erosion piping (BEP), but also has issues with slope stability.

The initial upper and lower bounds for the system failure probability are given in Figure 3. Here we can see that the failure modes slope stability and BEP contribute the most to the system probability for both segments, and we can see that the lower bound (almost) satisfies the project target of a 1/1,000 failure probability.

In the following sections we will illustrate refinements for different sections. These sections were selected based on their contribution in terms of either a low β_{cs} or a high L_{sec}. Refinements to β_{cs} influence both the lower and upper bound, refinements to L_{sec} can reduce the upper bound. In particular we look at sections 47 and 8 to 10 for slope stability. For piping we will consider the implications of using the previously described exit point method.

4.2. Improving reliability estimate for slope stability at section 47 (16-3)

Section 47 has a significant contribution to the system failure probability The initial estimated section reliability is between 2.96 and 3.76 (lower/upper bound). The low reliability is due to two causes. First, the reliability of the considered cross-section is relatively low, and second, the cross-section is assumed to be representative for the entire section with a length 844 meter, which means a length effect of roughly $N_{sec} = 18$.

The latter is certainly a conservative estimate, since the geometry varies considerably along the section, which is a non-ergodic and spatially variable property. As a large part of the possible cross-sections along the dike section has a less inclined slope, these are more stable than the considered cross-section. Moreover, some parts have a much wider dike crest, which generally leads to an order of magnitude lower probability of flooding. Given that a major part of the section has such a favorable geometry, the considered cross-section is only representative for a length of 290 m, which means N_{sec} is 6.7, rather than 18.

Though the above improvement is relatively straightforward, and leads to more realistic failure probably estimates, a much larger influence is retrieved when we (re)consider the reliability index β_{cs}. Initially the reliability index was estimated by a semi-probabilistic slope stability analysis, which can be enhanced by updating the cross-section with improved estimates for the statistics of the soil parameters from regional soil investigation. So, the impact of better estimates of

Table 2: Improvement of the reliability estimates of section 47 in 4 different steps

Step	β_{cs}	N_{sec}	β_{sec}	comments
0	3.76	18	2.96	Base analysis
1	3.76	6.7	3.25	Update sensitive fraction a_{sec}
2	4.13	6.7	3.67	Refined geometry
3	4.42	6.7	3.99	New phreatic lines
4	4.78	6.7	4.38	Updated soil properties
5	6.57	6.7	6.28	Probabilistic analysis

the cross-sectional reliability has a much larger impact on the system probability than reducing the contribution factor a_{sec}.

The cross-sectional reliability estimate has been improved in a stepwise manner for section 47, considering first a new surface geometry of the cross-section, then redefining the phreatic lines (loads) and finally updating the soil properties following more recent site investigations. These results are summarized in Table 2 and show that the combination of the updated N_{sec} and β_{cs} lead to a reliability index of the section of 4.38 instead of 2.96. As such the failure probability is reduced by a factor 260 overall, due to the almost 100 times lower cross-sectional failure probability, and a roughly 3 times lower N_{sec}.

Finally, a full probabilistic calculation was done in step 5, using FORM. The results show that the reliability estimates of step 1-4 β_{cs} (based on semi-probabilistic calculations) can be substantially improved further by doing probabilistic analysis.

4.3. Analysis of sections 8 to 10

A similar analysis is done for the sections 8 to 10, for which a cross-sectional reliability β_{cs} was found to be 3.49 (N_{sec} = 34.3), based on a semi-probabilistic safety assessment. The total length of sections 8 to 10 is 1745 meters, characterized by considerable variation in subsoil stratification and geometry, see Figure 4.

It was decided to first split the long stretch into 5 more homogeneous (and independent) subsections and calculate the reliability for a cross-section in each of these subsections, see Table 3. The subdivision was performed based on examination of the GEOTOP 3D model for the soil stratigraphy (Dabekaussen et al., 2019) and was verified with local CPTs at or close to the crest of the dike. Other elements like the occurrence of ditches or offramps in the hinterland of the dike were also considered (represented by the lines in the top of Figure 4, which indicate the surface elevation at regular distances from the outer crest. Straight lines represent a uniform geometry).

To identify the subsections with the lowest reliability, we first determined the reliability index conditional on the design water level $\beta|h_{des}$. Note that $P(h>h_{des})$ = 1/10,000, such that the actual reliability is then much higher. Based on this analysis we identify subsections A and B as the most critical ones. Subsections C and D are

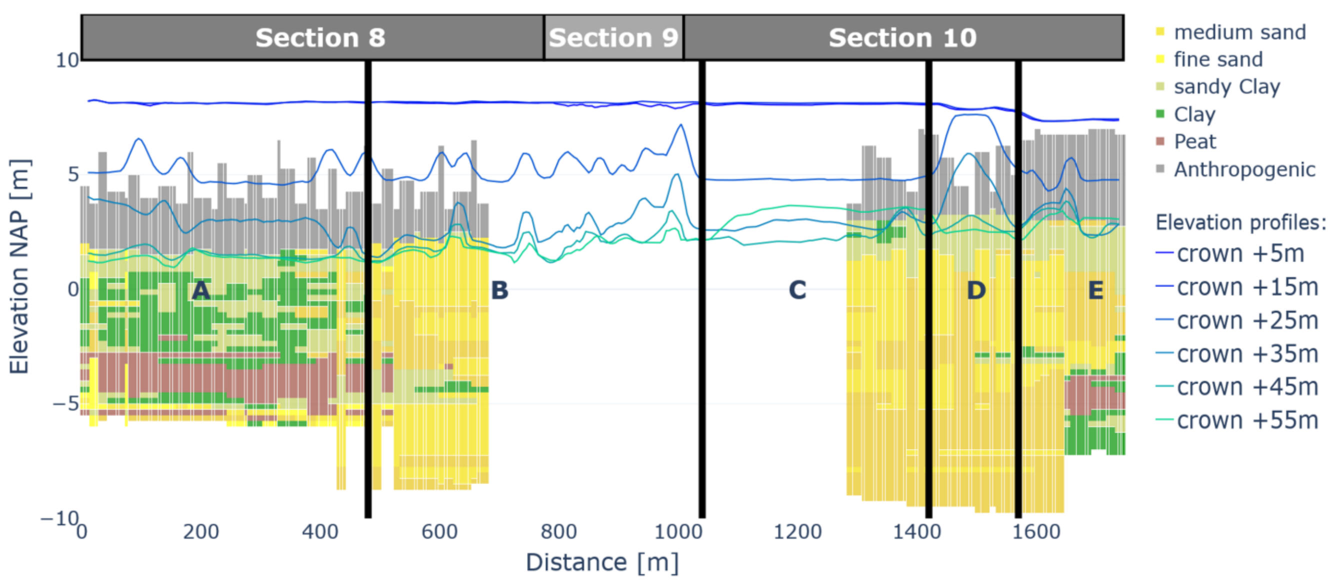

Figure 4 Longitudinal profile of section 8-9-10 with GEOTOP subsurface model and lines representing surface elevation at given distances parallel to the dike crown. Both old and new sections are indicated.

Table 3: Reliability results for the section 8-9-10 split into 5 subsections.

Subsection	Length	p_i	β	N_i
A	480	3.5e-7	4.96	9.6
B	560	6.5e-6	4.36	11.2
C	380	<1.0e-15	>8.0	7.6
D	150	<1.0e-15	>8.0	3.0
E	175	2.0e-4	8.0	3.5

Table 4: Reliability results for the section 8-9-10 into 9 subdivisions

Subsection	Length	p_i	$\beta\|h_{des}$	β_{cs}	N_i
A1	80	2.3e-6	4.58		1.6
A2	100	7.1e-9	5.67		2.0
A3	225	3.5e-7	4.96		4.5
A4	75	4.1e-6	4.46		1.5
B1	250	3.8e-3	2.67	3.91	5.0
B2	310	6.5e-6	4.36		6.2
C	380	<1e-15	8.0		7.6
D	150	<1e-15	8.0		3.0
E	175	<1e-15	8.0		3.5

not expected to contribute at all, subsection E can become unstable, but has a very wide crest such that this will not lead to flooding. Hence the reliability index was set to 8.

The subsections A and B were then split up into respectively 4 and 2 sub-parts to further investigate the influence of the variability of the geometry within these subsections, especially if ramps (access roads for cars to access houses from the crest) are located along the levee. Based on this we find the lowest $\beta|h_{des}$ to be 2.67 at section B1. A full probabilistic analysis for the cross-section of B1 results in $\beta_{cs} = 3.91$, which means the failure probability is a factor 5.2 lower than the initial estimate.

More importantly, this analysis shows that only one subsection of 250 meters contributes significantly to the total probability of failure of the section 8-9-10, instead of the initial 1750 m of the total stretch. Concretely, this reduced the upper bound for the failure probability of sections 8 to 10 by a factor 36, mainly due to the reduction in sensitive length. Note that this reduction of the sensitive length is only made possible by calculating the reliability at more regular intervals along the levee, which requires more engineering effort but can potentially reduce reinforcement or risk costs.

4.4. Improvements to the computations of Backwards Erosion Piping (BEP)

With the large number of computations in the exit point approach (see 3.2), a translation using a_{sec} and b is not required, and one can combine these results directly to determine the failure probability of a section. We consider section 11 which is a long dike section of 1400m, where reliability indices are computed for 228 exit points using a probabilistic approach. This differs from the underlying bounds in Figure 3 for which rough semi-probabilistic computations were performed for a single location. The P_f was estimated to be 4.55e-7. Figure 5 shows that this value is conservative for a large part of the section, but weaker spots between 1200m and 1400m were missed and revealed through the exit point method, resulting in a much higher P_f of 4.04e-3. This example advocates for a denser analysis of cross-sections or exit points to obtain more accurate results.

Next, the system reliability is computed for the entire segment 16-4 with the window method based on all the exit points. A window size of 300m is assumed for consistency reasons with the b factor for BEP. Moreover, the reliability has been overwritten to β_{cs}=6 for all cross-sections in the reinforcement plan, so that they no longer influence the system reliability. Using the window approach the P_f of the system is found to be 9.10e-3. This means the updated failure probability is

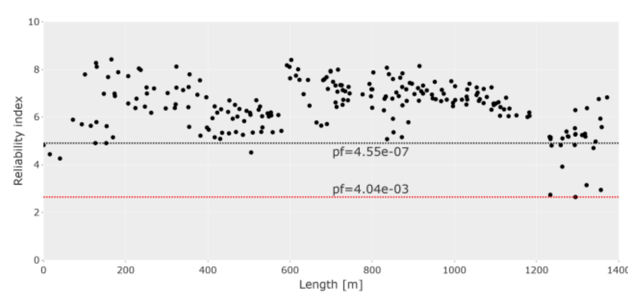

Figure 5 Reliability indices of exit points of section 11. The upper line is the initial reliability β_{sec}=4.91, the lower line is the weakest exit point.

significantly higher than previously thought in Figure 3.

5. CONCLUSIONS

In this paper we have illustrated efforts to improve system reliability estimates for levee segments along a river. This is relevant, as spatial variability and choices in schematization can have a large influence on the translation from cross-sectional estimates to a system level and vice versa.

For slope stability we have illustrated the approach for two parts of the system: one location where the initial cross-sectional reliability estimate was found to be very conservative, and where we reduced the factor accounting for the length that is sensitive to failure.

At another location we redefined the bounds of multiple sections to account for variance in stratification and geometry, and found that only about 15% of the section has a relevant contribution to the system failure probability, while having a higher reliability than originally thought. As such reanalysis of the system failure probability in Figure 3 shows that slope stability is found to be between 2.6e-3 and 2.0e-4 for segment 16-3, and between 6.9e-3 and 6.2e-4 for 16-4. As such, system failure probability was reduced by a factor 2-4 and further analysis of sections that are now dominant can likely result in achieving the target P_f<1/1,000.

For backwards erosion piping we found that the initial schematization was not conservative in all cases, and in some locations the reliability was significantly lower than initially thought. This was revealed through application of the exit point approach, which enables explicitly considering geometric variations in for instance the leakage length. The exit point approach also reveals that in many cases a limited length of the section meaningfully contributes to the failure probability of the section, which is important information when planning reinforcement measures.

Future efforts will focus on a full computational analysis where many cross-sections are evaluated to derive a general guideline for dealing with spatial variability for slope stability. Additionally, existing semi-probabilistic assessment guidance for backwards erosion piping needs to be revisited to ensure that all relevant geometric variations are accounted for in a reliability analysis.

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