Decision-making tool for structural integrity assessment of buried water-transmission mains using a geomechanical approach

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ABSTRACT: Uncertainties and spatial variability of the soil are considered in a geomechanical approach of water-transmission pipelines. This enables to go further than the standard cross-sectional design of the pipe. A hybrid model, which is a mixture of a cross-sectional model and a longitudinal model, was developed. This model assesses the failure modes of the pipe and provides the critical indicators of its current state of performance as well as a prognosis at a longer time.

The management of drinking water distribution networks in big cities involves very important issues according to Mian et al. (2023). One of these challenges is to always guarantee a sustainable water service of satisfactory quality. The optimization of the resources allocated to guarantee this service must therefore be rethought as soon as it is necessary to renew or replace certain portions of the network. This is the case for main water-transmission pipelines, called feeders, which are generally large-diameter pipes (with dimensions between 0.3 m and 4 m in diameter). They consist of a steel cylinder positioned between two layers of concrete and are also known in France as Bonna pipes (Yáñez-Godoy et al. 2017).

The standard design of these pipelines is based on the cross-section of the pipe and on the strong assumption of homogeneous soil characteristics (Afnor 1995). The mechanisms of geomechanical behavior are however difficult to learn because the pipes are placed in an uncertain environment. Indeed, the interaction of the soil with the pipe has an important influence on its stress and strain distribution. A numerical tool is an important support for the treatment of input data, the inclusion of uncertainties and aspects of soil variability, the mechanical calculations, and the post-processing of the results to generate decision-making elements. The paper is structured in three parts. The first part presents the used methodology to implement the geomechanical approach through a hybrid mechanical model. In the second part, the performance criteria that allow the planning of pipeline renewal strategies are presented. Finally, an application case illustrates the approach adopted throughout the document.

1. GEOMECHANICAL APPROACH USING A 2.5D HYBRID MODEL IN AN UNCERTAIN CONTEXT

1.1. Methodology

The model used to represent the studied soil-pipe system was based on 1D (Euler-Bernoulli beam model). The 1D model allows to obtain the settlement profile of the pipe, as well as the stresses coming from the bending effects. The main limitation of the 1D model is that it is unable to take into account the effects in the cross-section, such as the effect of the internal hydraulic pressure and the effect of the lateral soil, hence the interest in thinking about the coupling of longitudinal and transverse effects. Indeed, a 2D model (plane stress model) allows the evaluation of the various circumferential stresses developing in the cross-section of the pipe. The 2D model being very expensive in computation time in an uncertain context, which implies several simulations, a response surface of this model is determined to minimize the computational cost. Both models, 1D and 2D, have been translated into a hybrid model, that has been named "2.5D hybrid model" according to Darwich (2019). The following sections describe the different steps in the development of the geomechanical approach.

1.2. Soil-structure interaction

In global simplified models, the local behavior of soil is replaced by a simplified mechanical model. The most used common model to represent soil-structure interaction is the unidimensional Winkler model where the non-linear aspects of the soil behavior can be modelled, according to Elachachi et al. (2004), by considering types of law as $k_s(t)$ and $k_s(w)$ (where k_s, represents the coefficient of subgrade reaction). The first one describes the progressive degradation of the soil properties and the second one the non-linear feature of the soil. The soil reaction coefficient, k_s is not a measurable physical quantity and depends on the properties of the pipe, in addition to those of the soil it is supposed to represent. The determination of its value has been the subject of many studies and several semi-empirical relationships have been proposed (Elachachi et al. 2004). These formulas involve geometric and mechanical characteristics of the pipes as well as mechanical properties of the soil, including the soil modulus, E_s and the soil Poisson's ratio, υ_s. The values derived from these models are not only quite different but also result in values that are widely dispersed. Here, k_s is related to the mean soil modulus, E_s by Vesic model

$$k_s = \frac{0.65}{d} \sqrt[12]{\frac{E_s d^4}{E_p I_p}} \cdot \frac{E_s}{1-\upsilon^2} \tag{1}$$

with E_p the modulus of elasticity of the Bonna pipe, I_p the moment of inertia of the pipe, ν the soil Poisson coefficient and d the external diameter of the pipe.

1.3. 1D model

The 1D model consists of a set of pipe segments. Each pipe segment of finite length is decomposed into a number of beams connected to each other by nodes (Figure 1). At the ends of pipe segments, a pair of two independent nodes is used to represent the joints. Each beam element is subjected to a uniformly distributed loading, q, and rests on a soil modeled, according to the Winkler model, by a set of independent springs with a coefficient of subgrade reaction or soil reaction coefficient, k_{s1D} (N/m^2), in order to take into account the soil-structure interaction, which is defined as follows

$$k_{s1D} = k_s \times d \times l_e \tag{2}$$

where l_e is the length of a 1D model element for a single spring (Figure 1). A computer program, developed on Matlab, using the finite element method based on beam theory allowed us to build the 1D model. This program constructs the finite element stiffness matrix in an elastic soil (Elachachi et al. 2012). The soil-pipe system is represented as follows

$$\frac{d^2}{dx^2}\left(E_p I_p \frac{d^2 w(x)}{dx^2}\right) + p(x) = q(x) \tag{3}$$

where $w(x)$ is the vertical displacement of the pipe, $q(x)$ is the external load and $p(x)$ is the soil reaction (N/m) on the concrete pipe wall. The soil reaction used here is expressed by the modified Vlasov model (Girija Vallabhan and Das 1991)

$$p(x) = k_{s1D} \cdot w(x) - k_{sh} \frac{d^2 w}{dx^2} \tag{4}$$

where k_{sh} is the shear coefficient (N). These parameters are not soil specific and depend on the properties of the soil and pipe materials, the pipe geometry, the thickness of the soil layer and the load distribution on the pipe. The elementary stiffness matrix of a finite element $[K^e]$ can be written as follows

$$[K^e] = [K_b^e] + [K_s^e] + [K_{sh}^e] \tag{5}$$

where $[K_b^e]$ is the elementary stiffness matrix of the beam, $[K_s^e]$ is the soil stiffness matrix related to the Winkler model, and $[K_{sh}^e]$ is the soil stiffness matrix related to the shear contribution according to Zhaohua and Cook (1983). A discontinuity of the rotation between the ends of two neighboring pipes is considered. The rotation governed by the pipe-joint stiffness ratio r_{joint} (without units) is defined as follows

$$r_{joint} = \frac{R_j L}{E_p I_p} \tag{6}$$

where R_j is the joint stiffness (N.m) and L is the length of a segmented pipe. This joint stiffness is added to the elementary stiffness matrix, Eq. (5).

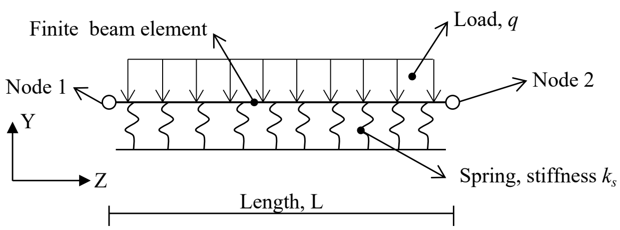


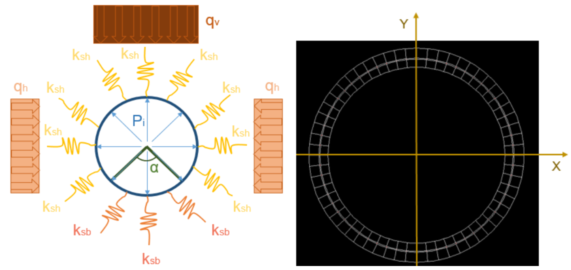
Figure 1: 1D model of a finite beam element lying on an elastic soil and subjected to a uniform load.

1.4. 2D model

The 2D model developed on the Cast3M finite element code (http://www-cast3m.cea.fr) is a model that represents a cross-section of a Bonna pipe (Figure 2a), subjected to vertical loads, q_v, from the self-weight of the soil and surface loads (moving or static), horizontal loads, q_h, from lateral soil effects, and internal hydraulic pressure, P_i. The interaction between the soil and the pipe is taken into account by springs positioned perpendicular to the outer concrete core of the pipe and spaced at a 30° arc. k_{sh} is the stiffness for springs not belonging to the support arc of angle α of the pipe and k_{sb} is the stiffness for springs belonging to it given by

$$k_{sb} = \frac{\pi k_{s1D}}{12 l_e} \tag{7}$$

k_{sb} parameter is associated with the pipe laying bed supposed to be compacted. The model mesh, shown in Figure 2b, consists of QUA8 type elements in a plane stress state. Three types of materials are considered for this model: the concrete of the inner core, the reinforced concrete of the outer core (due to the presence of the steel wires), and the steel cylinder.



(a) (b)

Figure 2: (a) Soil-pipe interactions and applied loads q_v and q_h; (b) Full mesh of the 2D model.

1.5. Surface response model for circumferential stresses

The Response Surface Method (RSM) (Myers et al. 2016) is used to identify an analytical (closed-form) model, based on the results of the 2D model presented in the previous section, which would allow to obtain directly a circumferential and radial bending stress from the input data (loads, pipe laying angle, soil reaction coefficients, mechanical and geometrical characteristics of the soil and the pipe, etc.). The construction of the response surfaces is done following the fitting of polynomials (usually quadratic) and goes through three steps: the construction of a design of experiments, the modeling of the response and finally the graphical representation. This model includes linear, interaction and quadratic effects. The coefficient of determination allow to estimate the quality of the representation.

1.5.1. Design of experiments

A design of experiments (DOE) was carried out using the method of central composite designs (Myers et al. 2016). This method has the advantage of not requiring too many experiments. Five levels were selected for the different factors, κ, shown in Table 1. The total number of points, N_T, for an orthogonality case, of the response surface is given by

$$N_T = n_f + n_\alpha + n_0 \tag{8}$$

where $n_f = 2^{\kappa-1}$ ($\kappa = 6$ factors) is the factorial portion, $n_\alpha = 2\kappa$ is the number of star points (axial points), and $n_0 = 15$ is the number of center points. For 6 factors, the response surface was thus constructed with N_T = 59 points. Figure 3 shows an example of a two-factor, circumscribed-type central composite design. The star points represent the new extreme values (low and high) for each factor in the design. These points are at some distance $\alpha = \sqrt[4]{n_f}$ from the center. The DOE matrix is used as input data matrix to the 2D model to numerically calculate the circumferential stress (59 calculations).

Table 1: Factor levels.

Factor	*Level -2*	*Level -1*	*Level 0*	*Level 1*	*Level 2*
q_v (N/m^2)	67×10^3	75.25×10^3	83.50×10^3	91.75×10^3	100×10^3
a_q	0	0.5	1	2	3
P_i (Pa)	0	250×10^3	500×10^3	750×10^3	100×10^4
E_p (Pa)	3.0×10^{10}	3.5×10^{10}	4.0×10^{10}	4.5×10^{10}	5.0×10^{10}
k_{sb} (N/m)	7.96×10^5	2.39×10^6	3.98×10^6	9.95×10^6	1.59×10^7
b_k	0.3	0.5	0.7	0.8	1.0

a_q is the ratio of horizontal, q_h, to vertical loads, q_v, in the pipe; b_k is the ratio of the stiffness k_{sh} and k_{sb}, i.e. $k_{sh} = b_k \times k_{sb}$.

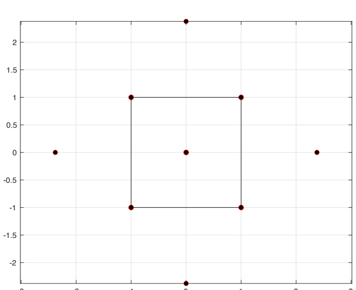


Figure 3: Example of a two-factor, circumscribed-type central composite design.

1.5.2. Modeling of the response

The resulting vector of circumferential and radial bending stresses, computed using the 2D model, was used to determine an analytical model for calculating the stress, σ_{2D}. The impact of factors (linear effects, quadratic effects, orthogonality i.e. interaction of parameters, and residuals) on the variability of the response of the model is therefore integrated. A multilinear regression is required to fit the model response with the following response surface model

$$\sigma_{2D} = a_0 + \sum_{j=1}^{k} a_j x_{ij} + \sum_{j=1}^{k} a_{jj} x_{ij}^2 + \sum_{j=1}^{k} \sum_{l=j+1}^{k} a_{jl} x_{ij} x_{il} + \varepsilon_i \quad (9)$$

where a are the regression coefficients, x are factors (Table 1) and ε_i are the residual terms.

1.5.3. Graphical representation

Figure 4 shows an example comparing the results between the two models, 2D and the response surface, for an isolated point in the concrete outer core. This is the circumferential stress, $\sigma_{\theta\theta}$, at the BE3 point (Figure 5). It is noted here that a positive value of $\sigma_{\theta\theta}$ corresponds to a tensile stress, while a negative value represents a compressive stress in the pipe. The coefficient of determination, R^2, which describes the good fit between the results of the two models is close to 1.

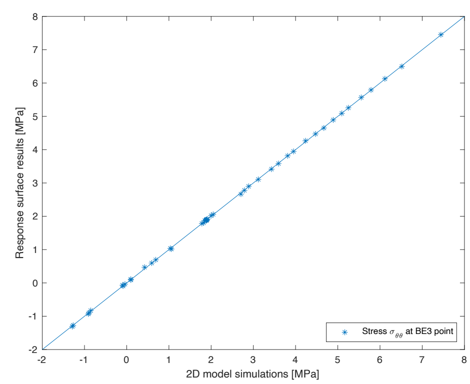


Figure 4: Comparison of 2D model and response surface model results for $\sigma_{\theta\theta}$ at BE3 point.

1.6. 2.5D hybrid model

The combination of the two models, 1D and 2D, has been named “2.5D hybrid model” as a coupling is taken into account. Placing the pipeline in a frame of reference where the x-axis is the axis passing through the left and right ends of the pipe, the y-axis is the vertical axis, and the z-axis is the longitudinal axis of the pipe (Figure 5), the assumptions made for this 2.5D modelling result in the combination of two stress systems, based on different assumptions in a plane stress sate. In the 1D model, the stresses in the cross-section, σ_{xx}, σ_{yy}, and σ_{xy} are assumed to be zero and the axial stress σ_{zz} can vary longitudinally. Whereas in the 2D model, non-zero stresses σ_{rr}, $\sigma_{\theta\theta}$, and $\sigma_{r\theta}$ are sought to be calculated, with the axial stress σ_{zz} assumed to be constant along the pipe. Three points in the thickness of the pipe cross-section are considered at four critical locations: left and right ends and top and bottom ends. As the left and right ends are symmetrical, only one end is retained. The combination of stresses for the proposed 2.5D model is based on the following assumption

$$\sigma_{2.5D} = \sigma_{1D} + \sigma_{2D} \tag{10}$$

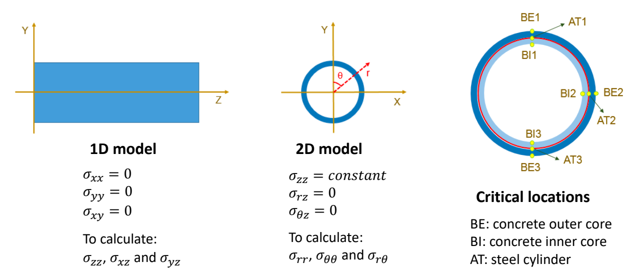


Figure 5: Assumptions for stresses in 1D and 2D models and critical locations in the pipe cross-section.

1.7. Variability of soil properties

The role of longitudinal variability can be studied by considering random fields. Random fields let to model the spatial variability characteristics of soil reaction coefficient, k_s , through some parameters as the mean value m_{ks}, the variance σ_{ks}^2 and the correlation length l_c. This last parameter is linked to an autocorrelation function, $\rho(\tau)$, where τ indicates the distance between two points and describes the spatial structure of the correlation of soil properties. A single exponential model to relate these two scales to an autocorrelation function $\rho(\tau)$ was chosen as follows

$$\rho(\tau) = exp\left(-\frac{|\tau|}{l_c}\right) \text{ for } \tau \leq l_c \tag{11}$$

The experimental acquisition of quantities of interest such as the correlation length, l_c, of soil properties was presented in (Yáñez-Godoy et al. 2019). Results in Yáñez-Godoy et al. (2017) showed that for horizontal l_c values between 6 and 18 m, the increase in the probability of failure of the pipeline (i.e., the reliability index decreases) is greater than for smaller values. A forthcoming study by the authors suggest that the horizontal l_c of the soil below the pipeline could have a more important effect on the structural integrity of the pipe. The spatial variability below the pipeline was assumed to correspond to the soil modulus parameter, E_s, in the model. Indeed, the variability and/or uncertainty related to soil characteristics and therefore to the soil response coefficient comes from the soil, which is in fact a material with spatial heterogeneity resulting from its deposition and aggregation process, the inaccuracy of measurements, and the model's uncertainty. E_s is an important factor for understanding the spatial behaviour of the soil–pipe system.

2. PERFORMANCE CRITERIA TO OPTIMIZE PIPELINE RENEWAL STRATEGIES

Three main potential failure modes for a buried pipe are identified: 1) an excessive displacement of the pipe; 2) an excessively high stress on the pipe, thus compromising the structural integrity of the pipe; and 3) an excessively high joint opening (which could compromise the tightness of the pipes and result in a drop in pressure, unserved users, etc.). Structural integrity, for example, is expressed in terms of circumferential stresses with a limit state function quantifying the state of cracking, for a reinforced concrete pipe, as follows

$$g_\sigma = \sigma_R - \sigma_S \leq 0 \tag{12}$$

where σ_R is the ultimate acceptable stress of the concrete and σ_S is the maximum loading stress. For the points in the thickness of the pipe cross-section that are located in concrete (BE and BI, Figure 5), the results of the stresses obtained, σ_S, are compared to the value of the tensile strength of concrete, σ_R, calculated according to the following equation

$$\sigma_R = E_p \times \varepsilon_{d0} \tag{13}$$

where $\varepsilon_{d0} = 10^{-4}$ is the tensile strain threshold.

The failure modes are linked through three different criticality indicators, I_{cr}, which will help to understand which phenomenon or potential event has the highest occurrence. Indeed, the models developed are used in a probabilistic context and allow the formation of a set of performance criteria that are defined from the definition of limit states. The criticality indicators help to optimize pipeline renewal strategies, e.g., through a proactive approach, by identifying alert thresholds, etc. The probabilistic approach makes it possible to identify the areas that are likely to deteriorate in a pipe and to forecast the evolution of the indicators in the long term.

For each of the failure modes, two reliability indices β are calculated for two distinct limit states:

- The serviceability limit state (SLS), the exceeding of which compromises normal service conditions (induces minor effects).
- The ultimate limit state (ULS), corresponding to the state which, if reached, could lead to damaging consequences at the level of a portion of the pipe or in its totality.

The expression for the reliability index β is given by

$$\beta = \frac{ln\left\{\frac{R}{S}\left[\left(1+CoV_S^2\right)/\left(1+CoV_R^2\right)\right]^{1/2}\right\}}{\left\{ln\left[\left(1+CoV_S^2\right)\left(1+CoV_R^2\right)\right]\right\}^{1/2}} \tag{14}$$

with R the value of the acceptable stress, S the mean maximum stress, CoV_R the coefficient of variation of the acceptable stress (here taken to be zero, as R is supposed to be deterministic) and CoV_S the coefficient of variation of the maximum stress, assuming that both R and S follow a lognormal distribution. The values of β for the ULS and SLS can be specified by the user according to the standards or the desired performance levels. For example, Eurocode 0 recommends a value of 1.5 for the SLS, which corresponds to a probability of failure of 6.7%, and a value of 3.8 for the ULS, which corresponds to a probability of failure of 0.007%. For buried pipes, 3 levels of performance or criticality indicators, I_{cr}, can be considered:

- Safe pipe: no intervention by the manager is necessary; all calculated β values are higher than the value indicated as acceptable, that is to say $I_{cr} = 0$.
- Pipe to be inspected: the manager must carry out a follow-up in terms of inspection for the pipe studied; there is at least one computed β value (not several) that is lower than the value indicated as acceptable, that is to say $I_{cr} = 1$.
- Pipe to be maintained or renewed: there is a high probability that a failure of the pipe would have occurred; maintenance or renewal is then necessary because several calculated β values are lower than the value indicated as acceptable, that is to say $I_{cr} = 2$.

3. APPLICATION OF THE GEOMECHANICAL APPROACH DEVELOPED USING A DECISISON-MAKING TOOL

The case study concerns the renewal of an 800 mm inner diameter Bonna pipe in an urban municipality. The pipe consists of 15 individual sections, each 6 m long, for a total length of 90 m. The pipe is laid at an average depth of 2.5 m in a dense sand and gravel soil; E_s=150 MPa. The quality of the pipe laying is assumed to be good. The operating pressure of the pipe is 8 bar. The static load due to road traffic is assumed to be 67 kN/m^2. In a first step, after the installation and commissioning of the pipeline, the current state of the pipeline is assessed. The spatial variability of the soil is considered by assuming the following values to characterize it and by taking into account a Gaussian random field of E_s : $l_c = 6$ m and $COV = 0.5$. Mean value of k_s (73.99 MN/m^3) is calculated with Eq. (1). The threshold values for the limit states considered are:

- Circumferential pipe stresses: SLS, 5 MPa; ULS, 8 MPa.

- Pipe displacement: SLS, 30 mm; ULS: 60 mm.
- Pipe joint opening: SLS, 0.01°; ULS, 0.02°.

3.1. Failure modes assessment

The number of Monte Carlo simulations is 1×10^4. The maximum mean circumferential tensile stress is 4.44 MPa and is reached at point BE1. The mean value of the maximum displacements is 2.5 mm. The mean value of the maximum joint openings is 0.006°. Figure 6 shows both the empirical cumulative distribution function (CDF) of the maximum circumferential pipe stress at point BE1 and a fit with a lognormal distribution. Confidence intervals are shown in Table 2. As an example, Figure 7 shows for a single simulation the longitudinal profile of soil variability and circumferential stress in BE1.

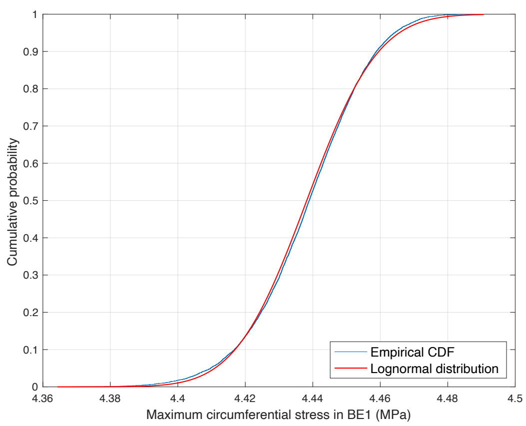


Figure 6: CDF of maximum circumferential pipe stress at point BE1.

Table 2: Confidence intervals of the possible failure modes.

Failure mode	Mean	Std. dev.	5%	95%
Circumferential pipe stress (MPa)	4.44	0.02	4.41	4.47
Pipe displacement (mm)	2.5	0.5	1.8	3.4
Pipe joint opening (°)	0.006	0.002	0.003	0.009

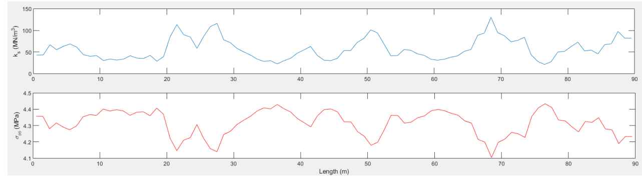


Figure 7: Longitudinal profile of soil variability (above figure) and circumferential stress in BE1 (below figure).

3.2. Current assessment of criticality indicators

The reliability indices computed with Eq. (14) show no signs of failure on the pipeline. In fact, all criticality indicators are "in the green light", giving fairly high indexes as shown in Table 3.

Table 3: Confidence intervals of the possible failure modes at $t = 0$ years.

Failure mode	β_{SLS}	β_{ULS}
Circumferential pipe stress	31.9	157.5
Pipe displacement	11.9	15.2
Pipe joint opening	15.9	18.2

3.3. Temporal assessment of criticality indicators

Although the issue of durability is not addressed in this study, a very simple example is given by considering a time-dependent degradation of the reinforced concrete coating of the pipe. The internal and external degradation of the pipe is expressed globally by a power law with coefficients of degradation α, β as follows:

$$\left(E_p I_p\right)_t = \left(E_p I_p\right)_{t0}\left(1 - \alpha \cdot t^{\beta}\right) \quad (15)$$

where $\left(E_p I_p\right)_{t0}$ and $\left(E_p I_p\right)_t$ are respectively the values of the initial bending stiffness and the bending stiffness at a given time t. Considering values of $\alpha = 0.05$ and $\beta = 0.5$, a computation was made at $t = 70$ years which generally corresponds to the lifetime of the pipeline. This allowed us to obtain the values shown in Table 4. It can be verified by the criticality indicators that in normal operating conditions this renewed pipeline will correctly comply with regulatory standards. One notices a "positive" effect of k_s on soil displacements but in contrast it results in an increase of the pipe stresses. Indeed, the term E_pI_p which is part of Eq. (1), by being reduced in time, increases the value of k_s resulting in a stiffer soil but creating more important circumferential pipe stress.

Table 4: Confidence intervals of the possible failure modes at $t = 70$ years.

Failure mode	β_{SLS}	β_{ULS}
Circumferential pipe stress	*24.5*	*120.6*
Pipe displacement	*12.0*	*15.3*
Pipe joint opening	*15.6*	*18.0*

4. CONCLUSIONS

The 2.5 hybrid model showed its value in assessing the performance of a pipeline by considering simultaneously its cross-section and its length. The large amount of information to be inputted in the performance analysis of a pipe has been simplified by the development of a purpose-designed tool. The values from the analysis allow to improve the setting of some performance criteria, especially concerning the limit values to be taken into account. The versatility of the data management also allows to consider several scenarios by checking the sensitivity of a particular parameter, either for the type of soil, or for its variability effects on the structural behavior. The non-linear behavior of the soil is to be considered for future developments as well as the implementation of a durability module for reinforced concrete.

5. ACKNOWLEDGMENTS

The authors would like to thank Syndicat des eaux d'Ile de France (SEDIF) for its involvement in this project. Created in 1923, SEDIF is a public utility that is responsible for supplying drinking water to subscriber towns and municipalities in the Greater Paris area. The authors also thank G. Darwich and A. Darolles for their contributions to the development of the IT tool.

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