Nonlinear Model-Data Fusion with Minimal Sensing for Performance-Based Seismic Monitoring of Instrumented Buildings

Milad Roohi, Ph.D.
Assistant Professor, Durham School of Architectural Engineering and Construction, University of Nebraska–Lincoln, Omaha, NE, 68182 USA

Eric M. Hernandez, Ph.D.
Associate Professor, Department of Civil and Environmental Engineering, University of Vermont, Burlington, VT, 05405 USA

David V. Rosowsky, Ph.D.
Vice President for Research, Kansas State University, Manhattan, KS, 66506 USA

ABSTRACT:

This paper presents a new concept for performance-based monitoring (PBM) of instrumented buildings subject to earthquakes. This concept is achieved by simultaneously combining and advancing existing knowledge from structural mechanics, signal processing, and performance-based earthquake engineering paradigms. The PBM concept consists of 1) measurement, 2) dynamic response reconstruction, 3) damage analysis, and 4) loss analysis and decision making. The main theoretical contribution of the proposed concept is the derivation of a nonlinear model-data fusion algorithm called nonlinear model-based observer (NMBO) for state estimation in nonlinear structural systems with minimal sensing (i.e., a limited number of global response measurements). The NMBO employs an efficient iterative algorithm to combine a nonlinear model and limited noise-contaminated response measurements to estimate the complete nonlinear dynamic response of the structural system of interest. The main advantage of the proposed observer over existing nonlinear recursive state estimators is that it is specifically designed to be physically realizable as a nonlinear structural model. This results in many desirable properties, such as improved stability and efficiency. The proposed methodology is validated using three case studies of experimental and real-world large-scale instrumented buildings. The first case study is a 6-story steel moment-resisting frame building in Burbank, CA, using the recorded acceleration data from the 1994 Northridge earthquakes. The second case study is an extensively instrumented six-story wood frame building tested in a series of full-scale seismic tests in the final phase of the NEESWood project at the E-Defense facility in Japan. The third case is a seven-story reinforced concrete structure in Van Nuys, CA, severely damaged during the 1994 Northridge earthquake.

1. INTRODUCTION

This paper proposes a new concept for Performance-Based Monitoring (PBM) of instrumented buildings subjected to earthquakes. This concept combines existing knowledge from structural mechanics, signal processing, and performance-based earthquake engineering paradigms. Specifically, the proposed PBM concept aims to assess the state of cumulative mechanics-based damage and its uncertainty throughout the structure during an earthquake and use this information for performance-based monitoring, damage detection, and localization of instrumented buildings. This approach deviates from the traditional approach used in structural monitoring and damage identification, which seeks changes in the structural parameters before and after an earthquake.

Since the PBM concept is developed on a probabilistic basis, the outcome can be used to obtain the probability of various losses based on the defined decision variable and be integrated into a decision-making process by city officials, building owners, emergency managers, or other officials. However, there are technical difficulties in implementing the proposed structural monitoring approach, primarily due to the challenges associated with a) reconstructing (or estimating) the seismic response (or full state) time histories of the instrumented building from the incomplete and noise-contaminated measured seismic response, and b) obtaining stress and strain fields as functions of the full state and perform damage quantification based on the damage sensitive response quantities.

The main challenge of the response reconstruction process is developing signal processing methods that provide a robust and accurate estimate of the seismic response and require a minimum number of seismic sensors. The main challenge of the damage quantification process is to select damage measures that a) can be reconstructed from the estimated seismic response, b) are physically meaningful and correlate well with the level of damage experienced during loading cycles, c) can account for cumulative damage during short or long period weak, moderate and strong ground motions, and d) can be interpreted by engineers to make rapid and reliable decisions regarding post-earthquake reoccupancy of the building.

This paper aims to address the challenges by proposing a new concept for performance-based seismic monitoring (PBM) of instrumented buildings. The PBM concept is capable of reconstructing (or estimating) the seismic response (or full state) time histories of a minimally instrumented building from the incomplete and noise-contaminated measured seismic response, and obtaining stress and strain fields as functions of the full state and performing damage quantification based on the damage sensitive response quantities including geometric damage features, element-by-element demand-to-capacity ratios, and damage indices.

2. BUILDING AND MEASUREMENT MODELS OF INTEREST

This paper focuses on seismic monitoring of instrumented buildings whose floor diaphragms can be assumed to be rigid for in-plane deformations. For this type of structure, the response to seismically induced ground acceleration can be accurately modeled by the following simultaneous set of nonlinear differential equations given by

$$\mathbf{M}\ddot{q}(t)+\mathbf{C}_D\dot{q}(t)+f_r(x(t))=\mathbf{M}\mathbf{b}\ddot{u}_g(t)+w(t) \quad (1)$$

where the vector $q(t)\in\mathbb{R}^n$ represents the relative displacement (with respect to the ground) of all stories. The vector $z(t)\in\mathbb{R}^h$ is an auxiliary variable that models nonlinear hysteretic structural behavior. $x(t)=\begin{bmatrix} q(t)^T & \dot{q}(t)^T & z(t)^T \end{bmatrix}^T$ is the state of the system. $f_r(x(t))=f_r(q(t),\dot{q}(t),z(t))$ is the resultant restoring force vector. The time history $\ddot{u}_g(t)$ is the measured ground motion time history, the matrix $\mathbf{b}$ is the influence matrix. The vector $w(t)$ accounts for the effects of unmeasured excitations and model error. $\mathbf{M},\mathbf{C}_D$ and $\mathbf{K}$ are the mass, damping and stiffness matrices. This research considers the acceleration response measurements of an instrumented building can be modeled in discrete time as

$$y(t_k)=h[x(t_k)]+v(t_k) \quad (2)$$

where y_k is a vector of m acceleration measurements recorded at time t_k and $h(x_k)$ is given by

$$\begin{aligned}h[x(t_k)]=-\mathbf{c}_2\mathbf{M}^{-1}[&\mathbf{C}_D\dot{q}(t_k)\\&+f_r(q(t_k),\dot{q}(t_k),z(t_k))]+v(t)\end{aligned} \quad (3)$$

and $\mathbf{c}_2\in\mathbb{R}^{m\times n}$ is a Boolean matrix that maps the DoFs to the measurements, and $v(t)\in\mathbb{R}^{m\times 1}$ is the measurement noise.

3. DEVELOPMENT OF THE PBM CONCEPT

The proposed PBM is achieved by simultaneously combining and advancing existing knowl-

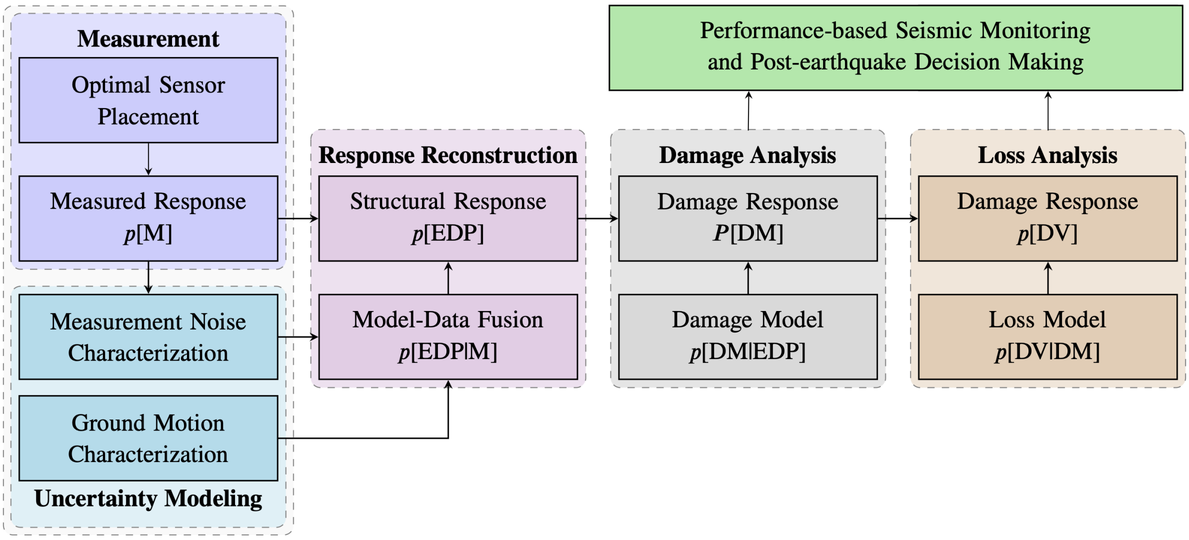

Figure 1: Summary of the proposed nonlinear model-data fusion with minimal Sensing for performance-Based Seismic Monitoring of Instrumented Buildings

edge from structural mechanics, signal processing, and performance-based earthquake engineering paradigms and consists of (1) measurement, (2) dynamic response reconstruction, (3) damage analysis and (4) loss analysis. Figure 1 presents a summary of the proposed PBM concept. The outcome of every step of the proposed concept is characterized by one of four generalized variables, including Response Measurement (M), Engineering Demand Parameter (EDP), Damage Measure (DM), and Decision Variable (DV). Using the Total Probability Theorem, the proposed framework equation is expressed by

$$p[\mathrm{DV}] = \iiint p[\mathrm{DV}|\mathrm{DM}]\, p[\mathrm{DM}|\mathrm{EDP}]\, p[\mathrm{EDP}|\mathrm{IM}] \ldots \quad p[\mathrm{IM}]\, d\mathrm{IM} \,.\, d\mathrm{EDP} \,.\, d\mathrm{DM} \tag{4}$$

where $p[\mathrm{M}]$ is the probability density of the measurement set, and $p[\mathrm{EDP}|\mathrm{M}]$ is the conditional probability of experiencing a level of response parameter given measurement set M. Except for a few special cases, solving the multidimensional integrals in Equation 4 is a very complex and challenging task as it requires the complete probability distribution of each of three generalized variables (M, EDP, and DM) to be estimated. For instance, to estimate $p[\mathrm{EDP}|\mathrm{M}]$ in the special case of linear structural systems (which can be described by linear models), the densities $p[\mathrm{EDP}|\mathrm{M}]$ are Gaussian. This means they can be characterized by mean vectors and covariance matrices; thus, the mathematical solution becomes trackable. This is important because, in real-world applications, there are many cases that can be addressed using this special case. However, in the case of more complicated systems, where there is a need to solve the nonlinear filtering problem, there does not exist a finite set of parameters that can characterize the densities $p[\mathrm{EDP}|\mathrm{M}]$.

3.1. *Measurement*

The first step of the proposed PBM concept consist of optimum sensor placement to determine the type, number, and location of the sensors and quantification of the uncertainty of unmeasured excitation w and measurement noise v.

3.1.1. *Optimal Sensor Placement*

In practice, this process begins with sensor type selection considering technical, logistical, and economic constraints. This paper is restricted to accelerometers due to their popularity in typical seismic instrumentation. Thus, this step requires only determining the number and locations of the sensors, which is typically known as the *optimal sensor placement* problem. Here, the meaning of the term "optimal" depends on the objective of sensor placement, which can be identification, damage detection, or response reconstruction. The PBM concept aims to place accelerometers in locations containing maximum information for response reconstruction, i.e., select the number and locations of sensors to minimize the uncertainty of response reconstruction. This minimization can be achieved by selecting an optimality criterion based on the variance of a user-defined objective function related to the state of the system, such as displacement, internal forces, and stresses. The proposed criterion is the sum of the inter-story drift estimation variances. Therefore, the optimal sensor placement can be achieved by solving an optimization problem to select the optimal measurement matrix, $(\mathbf{c}_2)_{opt}$, subject to maximum inter-story drift (ISD) estimation variance being bounded by a maximum allowable variance of $\sigma^2_{\max}$, which can be specified based on the expected accuracy to determine performance-based post-earthquake reoccupation category of the building of interest. This optimization problem can be formulated as follows

$$\begin{aligned} (\mathbf{c}_2)_{opt} &= \underset{\mathbf{c}_2}{\operatorname{argmin}} \operatorname{tr}(\mathbf{P}_{ISD}) \\ \text{s.t. } & \left[\sigma^2_{ISD}(k,k)\right]_{k=1:n} < \sigma^2_{\max} \end{aligned} \tag{5}$$

where $\operatorname{tr}(\mathbf{P}_{ISD}(k,k))$ is the inter-story drift estimation error covariance matrix given by

$$\mathbf{P}_{ISD}(k,k) = \begin{cases} \mathbf{P}(1,1), & \text{if } k = 1 \\ \mathbf{P}(k,k) + \mathbf{P}(k-1,k-1) - 2\mathbf{P}(k,k-1), & \text{if } k > 1 \end{cases} \tag{6}$$

and $\mathbf{P}$ is the displacement estimation error covariance matrix, is given by

$$\mathbf{P} = \mathbb{E}\left[[q(t) - \hat{q}(t)][q(t) - \hat{q}(t)]^T\right] \tag{7}$$

$\sigma^2_{ISD}(k,k)$ is the k th diagonal element of interstory drift estimation error covariance matrix, k is story number, and n is total number of stories.

3.1.2. *Uncertainty Modeling*

The first step continues with modeling the uncertainty in the unmeasured ground motion excitations and the measurement noise. The uncertainty modeling will be explicitly performed using power spectral density (PSD). The uncertainty of unmeasured seismic ground motions can be modeled using a Kanai-Tajimi power spectral density $\phi_{ww}(\omega)$ corrected by an amplitude function to obtain a non-stationary ground motion acceleration. The measurement noise $\phi_{vv}(\omega)$ will be modeled using a zero-mean Gaussian sequence with a noise-to-signal RMS (root mean-square) selected based on the expected accuracy of measurements.

$$\phi_{vv}(\omega) = \phi_0$$

3.2. *Response Reconstruction*

Once data becomes available from a seismic event, given by $p[\mathrm{M}]$, response reconstruction is the second step of the proposed framework. Response reconstruction refers to the estimation of unmeasured response quantities of interest or engineering demand parameters (EDP) from a limited number of global response measurements, described as $p[\mathrm{EDP}|\mathrm{M}]$. An accurate response reconstruction in the step is vital to prevent underestimation or overestimation of the actual response of the building. Further, the estimated uncertainty bound helps to develop a set of maximum, mean, and minimum seismic demands to consider the best and worst-case scenarios in assessing the performance of the instrumented building.

In a Bayesian framework, response reconstruction is called filtering, which estimates the current conditional probability density function of the full state given the past and presents noisy and incomplete measurements. Thus, the solution to the response reconstruction problem provides a distribution that incorporates all the statistical information of the state obtained from available measurements and the initial condition. Given a sequence of measurements $\mathbf{y}_k = \{y_1, \ldots, y_k\}^T$, the filtering problem consists of determining an estimate of the system state x_k based on $\mathbf{y}_k$. This implies that the filtering problem is solved to find the complete solution, which is provided by the probability density function $p(x_k|\mathbf{y}_k)$. To solve the filtering problem, it is required to determine how probability density function $p(x_t|\mathbf{y}_{k-1})$ changes once observation y_t becomes available and then obtain $p(x_t|\mathbf{y}_k)$. It can be shown that an expression can be derived for the one step ahead prediction density as follows

$$p(x_{t+1}|\mathbf{y}_t) = \int p(x_{t+1}|x_t)\, p(x_t|\mathbf{y}_t)\, dx_t \qquad (8)$$

Solving the integral in this equation is a challenging task except for a few special cases. In the case of linear systems, the celebrated Kalman filter (KF) is the optimal and most widely used approach. However, there is no unique and optimal solution in the case of nonlinear systems. Researchers have proposed sub-optimal nonlinear state estimation and filtering algorithms based on simplification or approximation techniques. In the following, this paper presents a nonlinear model-based observer that can provide estimates based on approximations of the probability density functions using the first two statistical moments.

3.2.1. Nonlinear Model-based Observer

The equation of a nonlinear state observer (NMBO) for state estimation in nonlinear systems Roohi et al. (2021a) is given by

$$\begin{aligned} \mathbf{M}\ddot{\hat{q}}(t) &+ \left(\mathbf{C}_D + \mathbf{c}_2^T \mathbf{E} \mathbf{c}_2\right) \dot{\hat{q}}(t) \\ &+ f_r(\hat{q}(t), \dot{\hat{q}}(t), z(t)) = \mathbf{c}_2^T \mathbf{E} \dot{q}_m(t) \end{aligned} \qquad (9)$$

where $\hat{q}(t)$ is the time history of the estimated response at all DOFs of the model and $\dot{q}_m(t)$ is the measured velocity response of the system. To determine $\mathbf{E}$, the objective function to be minimized is the trace of the displacement error covariance matrix given by

$$\begin{aligned} J &= \mathrm{tr}(\mathrm{P}) \\ &= \mathrm{tr}\left(\mathbb{E}\left[(q(t)-\hat{q}(t))(q(t)-\hat{q}(t))^T\right]\right) \end{aligned} \qquad (10)$$

Interested readers are referred to Roohi et al. (2021a) for further details on the derivation and implementation of the NMBO.

3.3. Damage Analysis

The third step of the proposed framework is to estimate the damage measure (DM) from the estimated response and compare the DMs with performance-based acceptance criteria. The outcome of this step is given by $p[\mathrm{DM}|\mathrm{EDP}]$, which is the probability of DM given EDP. Based on the selected damage measure, the $[DM|EDP]$ is calculated at the element or system level. Then, the outcome is evaluated using the acceptance criteria to determine the post-earthquake re-occupancy category of the instrumented building and to detect and localize element-level structural damage. The PBM concept employs three approaches to estimate damage measures, including 1) geometric damage features; 2) element-by-element demand-to-capacity ratios (DCR); and 3) element-by-element damage indices (DI).

3.4. Loss Analysis and Decision Making

Since the PBM concept is developed on a probabilistic basis, the estimated DMs can be used as input to loss models to obtain the probability of various losses based on the commonly used decision DVs, given by $p[\mathrm{DV}|\mathrm{DM}]$. Here, a loss model defines the relationship between a DM and DVs and the $p[\mathrm{DV}|\mathrm{DM}]$ is evaluated depending on the desired expression of loss. The outcome of damage and loss estimation can be integrated into a decision-making process by city officials, building owners, and emergency managers (Roohi and Hernandez (2020)).

4. VALIDATION OF THE PBM CONCEPT USING CASE STUDIES OF REAL-WORLD INSTRUMENTED BUILDINGS

4.1. Case Study of Burbank Steel Building (CSMIP Station 24370)

The first case study is a partially instrumented 6story steel building located in Burbank, CA and uses acceleration data from the California Strong Motion Instrumentation Program (CSMIP) Station 24370 during the 1991 Sierra Madre and 1994 Northridge earthquakes. Of particular interest is to estimate demand-to-capacity ratios (DCR) and their corresponding uncertainty in all members of an engineered and partially instrumented steel moment resisting frame (SMRF) structure. Figure 2 presents a location of building instrumentation (accelerometers) and schematic of measurements used and the model-based observer of the Burbank building. Figure 3 shows the results for bending and axial DCR in the perimeter frame. Readers are kindly

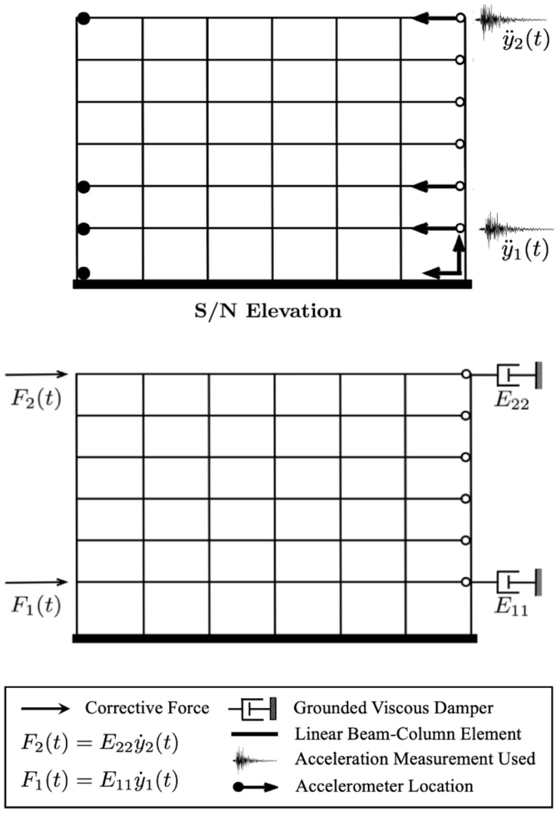

Figure 2: Location of building accelerometers (top) and the OpenSees implementation of the NMBO for the Burbank building (bottom)

referred to Hernandez et al. (2018) for more information about this case study.

4.2. Case Study of NEESWood Full-scale Tests

The second case study is a six-story wood frame Capstone building tested in a series of full-scale seismic tests in the final phase of the NEESWood project at the E-Defense facility in Japan. The building was tested in three intensity levels with over 300 channels to record acceleration, displacement, strain, and optical tracking measurements. To validate the proposed PBM methodology, a nonlinear structural model was developed in OpenSEES and the measured seismic response and photo records of the building were used. Figure 4 presents photographs of the test building with locations of the accelerometers used along with a schematic of the OpenSEES NMBO of the building. The measured data from the instrumented building were used as feedback to the OpenSEES NMBO to compare drift estimates with recorded data using optical tracking light (OTL). Results, as shown in Figure 4, indicate that the building could be categorized as LS after Test 5, and the actual drift fell within the uncertainty bounds provided by the NMBO. Element-by-element damage indices were also computed and Figure 5 illustrates the estimates obtained for 5$^{\text{th}}$ story. For more information about this case study, readers are referred to Roohi et al. (2019).

Figure 3: Estimated bending and axial demand-to-capacity ratios on perimeter frame of CSMIP 24370 during the Northridge earthquake

4.3. Case Study of Van Nuys Hotel Testbed (CSMIP Station 24386)

The third case is a 7-story reinforced concrete structure located in Van Nuys, CA. The building was instrumented by the CSMIP (Station 24386) and was severely damaged during the 1994 Northridge earthquake. The measured data from this building has been widely analyzed in the literature. This case study validates the PBM methodology in the case of a reinforced concrete building that experienced severe localized damage. The seismic response estimation results are obtained by implementing OpenSEES-NMBO on the Van Nuys building using measured data from the 1994 Northridge earthquakes. Figure 4 presents the location of accelerometers along with the OpenSEES NMBO of the Van Nuys building. Figure 7 presents the story-by-story and building-level estimated probability of post-earthquake performance levels of Van Nuys building during the 1994 Northridge earthquakes. Figure 8 depicts reconstructed element-by-element damage indices by

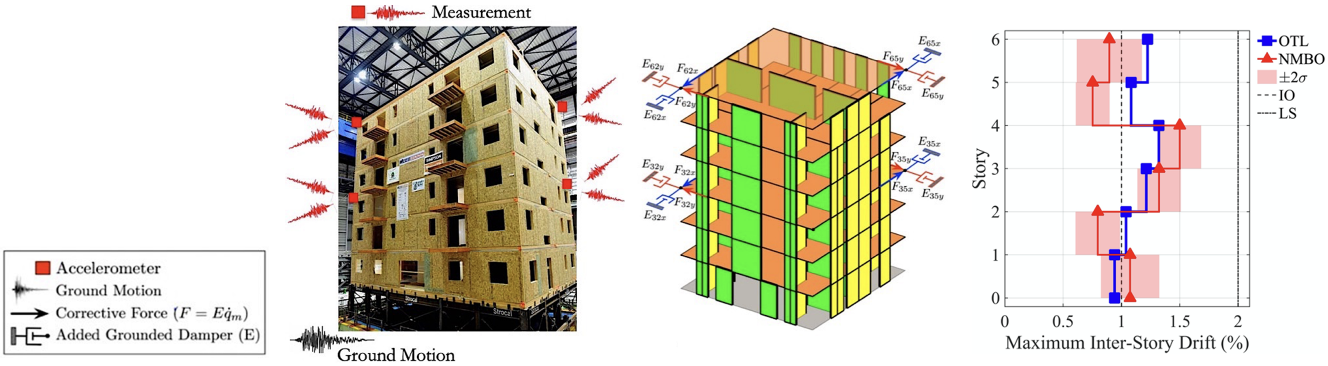

Figure 4: NEESWood capstone building (left), OpenSEES NMBO (middle), and Comparison of NMBO maximum inter-story drift ratio estimates obtained from reconstructed displacements using the NMBO and real measurements using optical tracking lights during the Northridge earthquake (right)

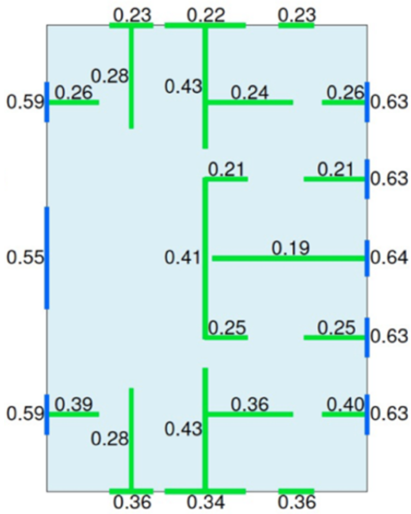

Figure 5: Estimated element-by-element damage indices at story 5 during test 5

OpenSEES-NMBO during 1994 Northridge earthquake. As can be seen, an element-by-element comparison of estimated damage indices with post-earthquake inspection results confirms the accuracy of damage localization using the proposed mechanistic approach. Readers are referred to Roohi et al. (2021b) and Roohi et al. (2020) for further details.

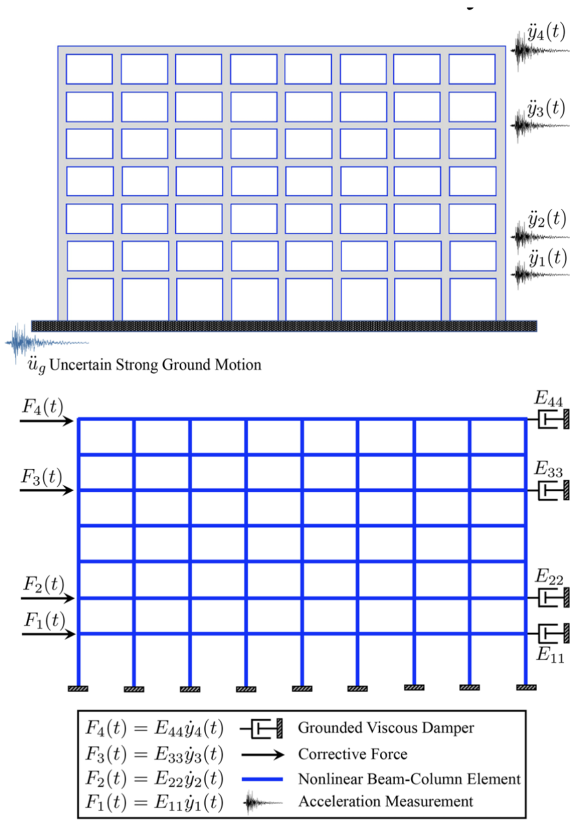

Figure 6: Location of the building accelerometers (top). The OpenSEES NMBO of the Van Nuys building (bottom)

5. CONCLUSIONS

This paper presented the development and practical application of a new concept for performance-based seismic monitoring (PBM) of instrumented buildings. The PBM concept is capable of reconstructing (or estimating) the seismic response (or full state) time histories of a minimally instrumented building from the incomplete and noise-contaminated measured seismic response, and obtaining stress and strain fields as functions of the full state and performing damage quantification based on the damage sensitive response quanti-

	p[ISD<IO]	p[IO≤ISD<LS]	p[LS≤ISD<CP]	p[ISD≥CP]
Story 7	0.99	0.01	0.00	0.00
Story 6	0.57	0.43	0.00	0.00
Story 5	0.44	0.56	0.00	0.00
Story 4	0.00	0.96	0.04	0.00
Story 3	0.00	0.34	0.65	0.01
Story 2	0.00	0.86	0.13	0.00
Story 1	0.01	0.65	0.33	0.00

	p[ISD<IO]	p[IO≤ISD<LS]	p[LS≤ISD<CP]	p[ISD≥CP]
Building	0.00	0.19	0.80	0.01

Figure 7: Story-by-story and building-level estimated probability of post-earthquake performance levels of Van Nuys building during the 1994 Northridge earthquakes

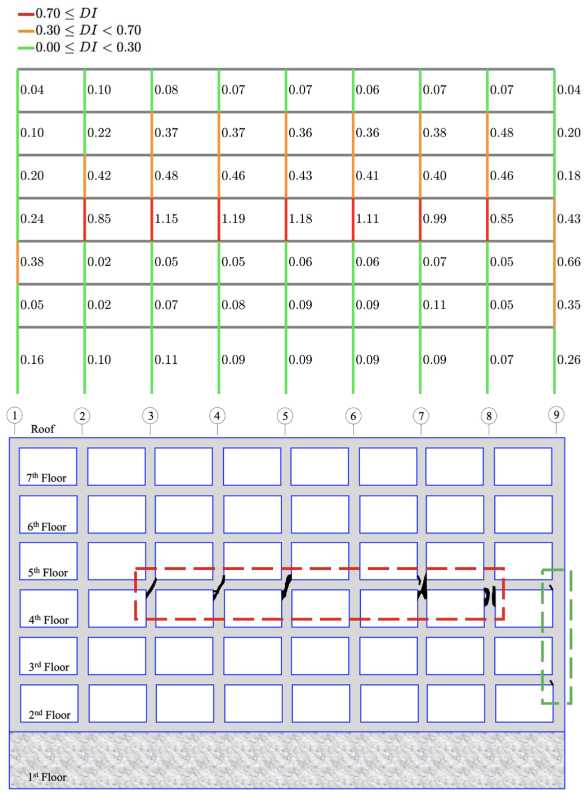

Figure 8: Reconstructed element-by-element damage indices by OpenSEES-NMBO (top) and seismic damage experienced in south view (bottom) during the 1994 Northridge earthquake

ties including geometric damage features, element-by-element demand-capacity ratios and damage indices. The proposed methodology is validated using three case studies of experimental and real-world large-scale instrumented buildings. The results presented in this paper constitute the most accurate and the highest resolution seismic response and damage measure estimates obtained for instrumented buildings. The proposed framework will help officials make more informed and swift decisions regarding post-earthquake assessment of critical instrumented buildings, thus improving the seismic resilience of communities.

6. REFERENCES

Hernandez, E., Roohi, M., and Rosowsky, D. (2018). “Estimation of element-by-element demand-to-capacity ratios in instrumented smrf buildings using measured seismic response.” *Earthquake Engineering & Structural Dynamics*, 47(12), 2561–2578.

Roohi, M., Erazo, K., Rosowsky, D., and Hernandez, E. M. (2021a). “An extended model-based observer for state estimation in nonlinear hysteretic structural systems.” *Mechanical Systems and Signal Processing*, 146, 107015.

Roohi, M. and Hernandez, E. M. (2020). “Performance-based post-earthquake decision making for instrumented buildings.” *Journal of Civil Structural Health Monitoring*, 10, 775–792.

Roohi, M., Hernandez, E. M., and Rosowsky, D. (2019). “Nonlinear seismic response reconstruction and performance assessment of instrumented wood-frame buildings—validation using neeswood capstone full-scale tests.” *Structural Control and Health Monitoring*, 26(9), e2373.

Roohi, M., Hernandez, E. M., and Rosowsky, D. (2021b). “Reconstructing element-by-element dissipated hysteretic energy in instrumented buildings: application to the van nuys hotel testbed.” *Journal of Engineering Mechanics*, 147(1), 04020141.

Roohi, M., van de Lindt, J. W., Rosenheim, N., Hu, Y., and Cutler, H. (2020). “Implication of building inventory accuracy on physical and socio-economic resilience metrics for informed decision-making in natural hazards.” *Structure and infrastructure engineering*, 17(4), 534–554.