Stable Model of Outcrossing Rate for Time-dependent Reliability

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ABSTRACT: The problem of time-dependent reliability arises in engineering practice due to the degradation of structural material properties over time and the involvement of time-dependent loads. For time-dependent reliability analysis (TRA), outcrossing rate method is one of the main methods with the key issue of solving the outcrossing rate. Recently, EPHI2 method has been proposed as an advanced outcrossing rate method that can efficiently computes outcrossing rate by means of a two-component parallel system model. However, EPHI2 method will lead to unstable results, due to its sensitivity to time intervals. Therefore, a new method called EPHI2+ is proposed in this paper, which has a more stable expression for the outcrossing rate. To show the corresponding improvement of EPHI2+ method, the comparison of the outcrossing rate results obtained by EPHI2 and EPHI2+ methods was shown. The application of EPHI2+ method for TRA is investigated by two numerical examples and it is found that EPHI2+ method can be applied effectively with sufficient stability and accuracy.

1. INTRODUCTION

The purpose of structural reliability analysis is to evaluate the probability that an engineering structure completes the expected function during its service life. Since the material properties and external loads of structures are significantly time-dependent and stochastic, it is necessary to conduct time-dependent reliability analysis (TRA) on actual engineering structures.

The computational approaches of TRA can be generally classified into two categories, which are sample-based methods and analytical methods. Monte Carlo simulation (MCS) is a widely used sample-based method with high accuracy, but also has a high computational cost, especially when dealing with highly reliability structures. In order to alleviate the huge computational cost of MCS, an extreme value-based approach is proposed (Wang and Chen 2017), in which the time-dependent reliability problem is transformed into a time-independent one using the substitution of the limit state function (LSF), and the extreme value of LSF.

For analytical methods, TRA can be related to the outcrossing rate of the response crossing a specified threshold to reach the failure domain (Rice 1944). The solution of outcrossing rate is an essential problem for this method. Due to the difficulty of calculation, outcrossing rate method can only be applied to several special cases (Breitung and Rackwitz 1982). Thus, PHI2 method was proposed and widely applied as an alternative method (Andrieu-Renaud et al. 2004), which defines the outcrossing rate through the classical two-component parallel system reliability model. In most cases, the time-independent reliability calculations in PHI2 method are based on FORM or SORM, but when finite elements are involved, the moment method has a higher efficiency (Zhang et al. 2021). However, PHI2 method is sensitive to time intervals and computationally inefficient (Andrieu-Renaud et al. 2004), which are due to the limitations of finite differences as well as numerical integration, respectively. To avoid these two drawbacks, PHI2+ (Sudret 2008) and EPHI2 (Li et al. 2022) methods have been proposed. PHI2+ method replaces the finite difference with a derivative method to derive an analytic expression for the outcrossing rate, which has better stability than PHI2 method, but inaccurate values may occur in some cases (Li et al. 2022). In order to avoid the numerical integration in PHI2 method, EPHI2 method is proposed, which is a new outcrossing approach with explicit model. Compared with PHI2 method, EPHI2 method has higher computational efficiency, but the derivation also involves finite differences, which leads to EPHI2 method being sensitive to time intervals. That is, too large value of time interval will make the finite difference inaccurate, while too small value will lead to instabilities. Therefore, there is no such outcrossing rate method that is both accurate and stable.

In the present paper, a new method with a more stable and accurate expression of the outcrossing rate is proposed, which solved the problem of sensitivity to time intervals and is referred to as EPHI2+ method. The remainder of this paper is organized as follows. First, a brief review of the time-dependent reliability statement is presented. Then, the proposed EPHI2+ method is described, with the stable expression for the outcrossing rate derived and procedure for TRA summarized. Third, two numerical examples are introduced to investigate the applicability of EPHI2+ method in TRA. Finally, the findings are summarized. The results show that EPHI2+ method is sufficiently accurate and has a high stability with insensitivity to time intervals.

2. PROPOSED EPHI2+ METHOD

2.1. Time-dependent reliability statement and notation

The time-dependent LSF is normally expressed as $G(\mathbf{X}, \mathbf{Y}(t), t)$, where t denotes time; $\mathbf{X} = [X_1, X_2,..., X_n]$ is an n-dimensional time-dependent random vectors; $\mathbf{Y}(t) =[Y_1(t), Y_2(t),..., Y_m(t)]$ represents an m-dimensional time-dependent random process vector. In the outcrossing rate method, failure of a structure during the forecast time interval $[0, T]$ is interpreted as initial moment failure or outcrossing event occurring at least once over the forecast time interval. And the outcrossing event is defined as $G(\mathbf{X}, \mathbf{Y}(t), t)$ crossing the limit state from the safe domain to the failure domain. The corresponding cumulative failure probability $P_{f,c}(0, T)$ is defined as (Andrieu-Renaud et al. 2004):

$$P_{f,c}(0,T) = \text{Prob}[\{G(\mathbf{X},\mathbf{Y}(0),0) \le 0\} \cup \{N^+(0,T) > 0\}] \tag{1}$$

where $N^+(0, T)$ denotes the number of outcrossing. The upper bound of $P_{f,c}(0, T)$ is approximated by (Ditlevsen and Madsen 2007):

$$P_{f,c}(0,T) \le P_{f,i}(0) + \int_0^T \nu^+(t)dt \tag{2}$$

where $P_{f,i}(0)$ is the failure probability at initial moment; $\nu^+(t)$ denotes the outcrossing rate, which can be calculated by (Hagen and Tvedt 1991):

$$\nu^+(t) = \lim_{\Delta t \to 0, \Delta t > 0} \frac{\text{Prob}[G(\mathbf{X},\mathbf{Y}(t),t) > 0 \cap G(\mathbf{X},\mathbf{Y}(t+\Delta t), t+\Delta t) \le 0]}{\Delta t} \tag{3}$$

For convenience, $G(\mathbf{X}, \mathbf{Y}(t), t)$ and $G(\mathbf{X}, \mathbf{Y}(t+\Delta t), t+\Delta t)$ are referred to as G_1 and G_2 hereafter.

2.2. Derivation of EPHI2+ method

Firstly, according to the defining equation of the derivative, the following equation can be constructed:

$$\nu^{+}(t)=\lim_{h\to 0}\frac{f_t(h)-f_t(0)}{h}=f_t'(0) \tag{4}$$

where h indicates the time interval, $f'_t(.)$ is the derivative of $f_t(h)$ with respect to h, and the equation for $f_t(h)$ is introduced as:

$$f_t(h)=P(\{G(\mathbf{X},\mathbf{Y}(t),t)>0\}\cap\{G(\mathbf{X},\mathbf{Y}(t+h),t+h)\leq 0\}) \tag{5}$$

Following the results of EPHI2 (Li et al. 2022), $f_t(h)$ can be expressed in two ranges: $\beta(t+h)/\beta(t)\geq -\rho_G$ and $\beta(t+h)/\beta(t)<-\rho_G$. Where $\beta(t)$ and $\beta(t+h)$ are the time-independent reliability index of G_1 and G_2, normally obtained with FORM or SORM. ρ_G represents the correlation coefficient of G_1 and G_2, which is obtained by following equation:

$$\rho_G=-\alpha(t).\alpha(t+h) \tag{6}$$

where $\boldsymbol{\alpha}(t)$ and $\boldsymbol{\alpha}(t+h)$ are unit normal vectors corresponding to the linearized margin at fixed instants t and $t+h$, which are represented as $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ in the following. Also for convenience, $\beta(t)$ and $\beta(t+h)$ are referred to as β_1 and β_2, respectively, and ρ_G is indicated by ρ.

For $\beta_2/\beta_1\geq -\rho$, the expression for $f_t(h)$ is given by:

$$f_t(h)=\begin{cases}\Phi(-\beta_2)-P_1-P_0(1-2\dfrac{\varphi}{\pi}) & \varphi_2\geq\dfrac{\pi}{4}\\ \Phi(-\beta_2)-P_1-P_2+P_0\cdot 2\dfrac{\varphi}{\pi} & \varphi_2<\dfrac{\pi}{4}\end{cases} \tag{7}$$

And for $\beta_2/\beta_1<-\rho$, $f_t(h)$ is expressed by the following equation:

$$f_t(h)=\begin{cases}\Phi(-\beta_2)-\Phi(-\beta_1)+P_0\cdot 2\dfrac{\varphi}{\pi} & \varphi_2\geq\dfrac{\pi}{4},\varphi_1\geq\dfrac{\pi}{4}\\ \Phi(-\beta_2)-\Phi(-\beta_1)+P_1-P_0(1-2\dfrac{\varphi}{\pi}) & \varphi_2\geq\dfrac{\pi}{4},\varphi_1<\dfrac{\pi}{4}\\ \Phi(-\beta_2)-\Phi(-\beta_1)+P_1-P_2+P_0\cdot 2\dfrac{\varphi}{\pi} & \varphi_2<\dfrac{\pi}{4}\end{cases} \tag{8}$$

where

$$P_0=\Phi^2\left(-\frac{\beta_0}{\sqrt{2}}\right) \tag{9}$$

$$P_1=\Phi(-\beta_1)\cdot\Phi(-\sqrt{\beta^2{}_0-\beta^2{}_1}) \tag{10}$$

$$P_2=\Phi(-\beta_2)\cdot\Phi(-\sqrt{\beta^2{}_0-\beta^2{}_2}) \tag{11}$$

$$\beta_0=\sqrt{\frac{\beta^2{}_1+2\rho\beta_1\beta_2+\beta^2{}_2}{1-\rho^2}} \tag{12}$$

$$\varphi=arccos(-\rho) \tag{13}$$

$$\varphi_1=arccos\frac{\beta_1}{\beta_0} \tag{14}$$

$$\varphi_2=arccos\frac{\beta_2}{\beta_0} \tag{15}$$

Since the limit values corresponding to $h\rightarrow 0$ in the above expressions are of significance, the following series expansion is used:

$$\alpha_2=\alpha_1+h\alpha_1'+\frac{h^2}{2}\alpha_1''+o\left(h^2\right) \tag{16}$$

$$\alpha_2'=\alpha_1'+h\alpha_1''+o(h) \tag{17}$$

Since $\boldsymbol{\alpha}$ is a unit vector, the following series of equations can be derived:

$$\|\alpha_1\|^2=1\quad \|\alpha_1\|^{2\prime}=\alpha_1\cdot\alpha_1'=0 \tag{18}$$

$$\left[\alpha_1\cdot\alpha_1'\right]'=\|\alpha_1'\|^2+\alpha_1\cdot\alpha_1''=0 \tag{19}$$

Based on Eqs. (6) and (16)- (19), the following equations are obtained:

$$\rho=-1+\frac{h^2}{2}\|\alpha_1'\|^2+o\left(h^2\right) \tag{20}$$

$$\sqrt{1-\rho^2}=h\|\alpha_1'\|+o(h) \tag{21}$$

In order to obtain $f'_t(0)$, the derivatives of P_0, P_1 and P_2 need to be calculated first. Let us define D_0, D_1 and D_2 as the derivatives of P_0, P_1 and P_2, respectively. They can be derived as follows:

$$D_0=-\sqrt{2}\Phi\left(-\frac{\beta_0}{\sqrt{2}}\right)\cdot\varphi\left(-\frac{\beta_0}{\sqrt{2}}\right)\beta_0' \tag{22}$$

$$D_1=-\Phi(-\beta_1)\cdot\varphi(-\sqrt{\beta^2{}_0-\beta^2{}_1})\frac{\beta_0}{\sqrt{\beta^2{}_0-\beta^2{}_1}}\beta_0' \tag{23}$$

$$D_2 = -\varphi(-\beta_2)\cdot\beta_2'\cdot\Phi(-\sqrt{\beta^2{}_0-\beta^2{}_2}) \\ -\Phi(-\beta_2)\cdot\varphi(-\sqrt{\beta^2{}_0-\beta^2{}_2})\cdot\frac{1}{\sqrt{\beta^2{}_0-\beta^2{}_2}}(\beta_0\beta_0'-\beta_2\beta_2') \tag{24}$$

According to Eq. (4), the total derivation is based on the premise that $h \to 0$, and thus β_0 and β'_0 can be regarded as β_1 and β'_1, respectively. With this premise, the later equation can also be easily obtained by using the finite difference form of β'_1:

$$\rho\beta_1+\beta_2 = h\beta_1'+o(h) \tag{25}$$

Now, based on Eqs. (12), (21) and (25), $\sqrt{\beta^2{}_0-\beta^2{}_1}$ can be transformed into the following simple equation. Note that in order to get rid of the sign of the absolute value in the derivation process, β'_1 is assumed to be greater than 0 here for the time being:

$$\sqrt{\beta^2{}_0-\beta^2{}_1} = \frac{|\rho\beta_1+\beta_2|}{\sqrt{1-\rho^2}} = \frac{h|\beta_1'|}{h\|\alpha_1'\|} = \frac{\beta_1'}{\|\alpha_1'\|} \tag{26}$$

Similarly, $\sqrt{\beta^2{}_0-\beta^2{}_2}$ can be approximated to obtain a simplified equation:

$$\sqrt{\beta^2{}_0-\beta^2{}_2} = \frac{|\beta_1+\rho\beta_2|}{\sqrt{1-\rho^2}} = \frac{h|\beta_1'|}{h\|\alpha_1'\|} = \frac{\beta_1'}{\|\alpha_1'\|} \tag{27}$$

Substituting Eqs. (26), (27)into (23), (24), the following equations are acquired:

$$D_0 = -\sqrt{2}\Phi\left(-\frac{\beta_1}{\sqrt{2}}\right)\cdot\varphi\left(-\frac{\beta_1}{\sqrt{2}}\right)\beta_1' \tag{28}$$

$$D_1 = -\Phi(-\beta_1)\cdot\varphi\left(\frac{\beta_1'}{\|\alpha_1'\|}\right)\beta_1\|\alpha_1'\| \tag{29}$$

$$D_2 = -\varphi(-\beta_2)\cdot\beta_1'\cdot\Phi\left(-\frac{|\beta_1'|}{\|\alpha_1'\|}\right) \tag{30}$$

Meanwhile, the derivative of φ, defined as D_φ, can be got as:

$$D_\varphi = \|\alpha_1'\| \tag{31}$$

So far, the derivation of all intermediate parameters has been completed. Combining Eqs. (7) and (8) with Eqs. (28)- (31), the derivatives of $f_t(h)$ in all cases can be obtained, which are defined as D_{f1}, D_{f2}, D_{f3}, D_{f4}, D_{f5}, respectively:

$$D_{f_1} = -\varphi(-\beta_2)\beta_1'-D_1-D_0+P_0\cdot 2\frac{D_\varphi}{\pi} \tag{32}$$

$$D_{f_2} = -\varphi(-\beta_2)\cdot\beta_1'-D_1-D_2+P_0\cdot 2\frac{D_\varphi}{\pi} \tag{33}$$

$$D_{f_3} = -\varphi(-\beta_2)\beta_1'+P_0\cdot 2\frac{D_\varphi}{\pi} \tag{34}$$

$$D_{f_4} = -\varphi(-\beta_2)\beta_1'-D_1-D_0+P_0\cdot 2\frac{D_\varphi}{\pi} \tag{35}$$

$$D_{f_5} = -\varphi(-\beta_2)\beta_1'-D_1-D_2+P_0\cdot 2\frac{D_\varphi}{\pi} \tag{36}$$

The derivation of the ranges of Eqs. (32)-(36) follows. Based on Eq (25), $\beta_2/\beta_1 \geq -\rho$ can be translated into inequality as $\beta'_1 \geq 0$. Similarly, $\beta_2/\beta_1 < -\rho$ can be transformed into $\beta'_1 < 0$. A more detailed classification involves the derivation of β_0 as follows:

$$\beta_0 = \sqrt{\frac{\beta^2{}_1+2\rho\beta_1\beta_2+\beta^2{}_2}{1-\rho^2}} = \frac{|\beta_1-\beta_2|}{\sqrt{1-\rho^2}} = \frac{|\beta_1'|}{\|\alpha_1'\|} \tag{37}$$

Thus, the comparison of the value of φ_1 and φ_2 for $\pi/4$ becomes a comparing of $|\beta_1'|$ and $\sqrt{2}\|\alpha_1'\|\cdot\beta_2$. Hence, the complete expressions of EPHI2+ method are illustrated below, for $\beta'_1 \geq 0$:

$$\nu^+(t) = \begin{cases} -\varphi(-\beta_2)\beta_1'-D_1-D_0+P_0\cdot 2\frac{D_\varphi}{\pi} & \beta' \geq \sqrt{2}\|\alpha_1'\|\cdot\beta_2 \\ -\varphi(-\beta_2)\cdot\beta_1'-D_1-D_2+P_0\cdot 2\frac{D_\varphi}{\pi} & \beta' < \sqrt{2}\|\alpha_1'\|\cdot\beta_2 \end{cases} \tag{38}$$

For $\beta'_1 < 0$:

$$\nu^+(t) = \begin{cases} -\varphi(-\beta_2)\beta_1'+P_0\cdot 2\frac{D_\varphi}{\pi} & \beta_1' \leq -\sqrt{2}\|\alpha_1'\|\cdot\beta_1 \\ -\varphi(-\beta_2)\beta_1'-D_1-D_0+P_0\cdot 2\frac{D_\varphi}{\pi} & -\sqrt{2}\|\alpha_1'\|\cdot\beta_1 < \beta_1' \leq -\sqrt{2}\|\alpha_1'\|\cdot\beta_2 \\ -\varphi(-\beta_2)\beta_1'-D_1-D_2+P_0\cdot 2\frac{D_\varphi}{\pi} & \beta_1' > -\sqrt{2}\|\alpha_1'\|\cdot\beta_2 \end{cases} \tag{39}$$

2.3. Time-Dependent Reliability Analysis Procedure Based on the Proposed Model

With a defined outcrossing rate $v^+(t)$, TRA can be easily conducted through EPHI2+ method. The procedure of EPHI2+ method is presented in Figure 1, which includes the following six steps:

1. Construct time-independent LSF G_1 and G_2 at t and $t+\Delta t$, respectively. And replacing $\mathbf{Y}(t)$ with the associated random variable.
2. Calculate the instantaneous failure probability $P_{f,i}(0)$ at the initial moment $t = 0$.
3. Calculate unit vectors $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ and reliability indexes β_1, β_2 corresponding to G_1 and G_2, respectively. In Steps 2 and 3, FORM or SORM is applied.
4. Determine $v^+(t)$ according to the proposed EPHI2+ method, as given in Eqs. (38) and (39)
5. Repeat steps 1- 4 with a new moment t and $t+\Delta t$, until $v^+(t)$ is calculated for all time moments.
6. Calculate $P_{f,c}(0, T)$ based on Eq. (2).

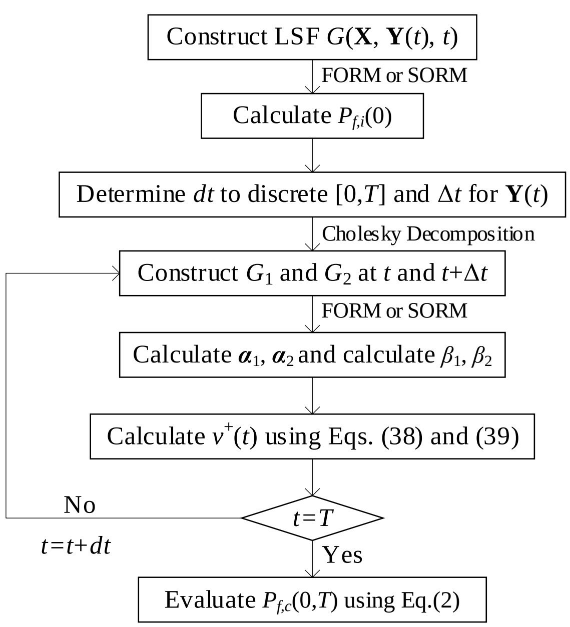

Figure 1: Flowchart of EPHI2+ method

3. ILLUSTRATIVE EXAMPLES

3.1. Example 1: Steel bending beam

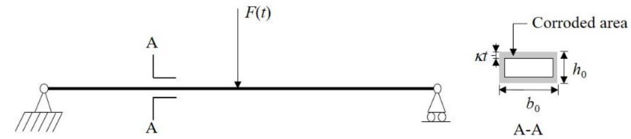

Figure 2: Corroded bending beam in Example 1

In the first example, a steel bending beam is investigated, as shown in Figure 2. The steel bending beam is subjected to a time-dependent pinpoint load $F(t)$ at mid-span and a time-independent self-weight distributed load, while the cross-section decreases with time due to corrosion. The limit state considered is dominated by the ultimate bending capacity of the section at the mid-span, and the corresponding LSF as $G(\mathbf{X}, \mathbf{Y}(t), t)$, is given by (Andrieu-Renaud et al. 2004):

$$G(\mathbf{X},\mathbf{Y}(t),t)=\frac{(b_0-2\kappa t)(h_0-2\kappa t)^2}{4}f_y-\frac{F(t)L}{4}-\frac{\rho_{st}b_0h_0L^2}{8} \tag{40}$$

where b_0 and h_0 are the initial width and height of the beam, respectively; f_y is the yield stress of the steel; L =5m is the length of the bending beam; ρ_{st} =78.5kN/m^3 is the weight density of the steel; and κ=0.03mm/a is the corrosion rate.

Table 1: Random variables and random processes for Example 1

Parameters	*Distributions*	*Mean*	*COV*	*Autocorrelation*
f_y *(MPa)*	*Lognormal*	*240*	*0.1*	*N/A*
b_0 *(m)*	*Lognormal*	*0.20*	*0.05*	*N/A*
h_0 *(m)*	*Lognormal*	*0.04*	*0.1*	*N/A*
F(t) (kN)	*Gaussian process*	*3.5*	*0.2*	*Eq.(41)*

The statistics of the random quantities in this example are listed in Table 1, where $F(t)$ is a stationary Gaussian process with an autocorrelation function given as:

$$\rho_F(\Delta t)=\exp\left[-\left(\frac{\Delta t}{\lambda}\right)^2\right] \tag{41}$$

where λ =1 month is the correlation length.

The time-dependent reliability of the beam is evaluated over a period of 20 years. As a comparison, the precise values of the time-dependent reliability of steel bending beam is investigated using PHI2, PHI2+, EPHI2, and EPHI2+ methods. the MCS method (with 10^6 samples for each moment) is also applied as a reference for accuracy.

For PHI2 and EPHI2 methods, Δt is chosen to be 0.008 years, aiming to satisfy $\rho_F(\Delta t)$ between 0.99 and 0.995. For PHI2+ method, the value of Δt is suggested to be 0.01λ (Sudret 2008). Since the expression of EPHI2+ method is stable, the smaller value of Δt is much better, so Δt is taken to be the same value as in PHI2+ method.

In EPHI2+ method, the calculation of $v^+(t)$ is based on the estimation of the time instant reliability indices β_1, β_2 and each unit vector $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$. Firstly, the limit state functions G_1 and G_2 are constructed at Δt and $t+\Delta t$. Since G_1 and G_2 in this example are not strongly nonlinear, FORM is used. After $P_{f,i}(0)$ is easily obtained, a decomposition of the time-dependent random process $F(t)$, which is represented by two random variables $\{F_{x1}, F_{x2}\}$ with correlation coefficients, is required. Then, $\{F_{x1}, F_{x2}\}$ is represented by two independent normal variables $\{u_1, u_2\}$ based on the Cholesky decomposition as follows:

$$\begin{pmatrix} F_{x1} \\ F_{x2} \end{pmatrix} = \left(700\left[\rho_F(\Delta t)u_1 + \sqrt{1-\rho_F^2(\Delta t)}u_2\right] + 3500\right) = \begin{pmatrix} 700u_1 + 3500 \\ 693.034u_1 + 98.5012u_2 + 3500 \end{pmatrix} \tag{42}$$

Based on Eq. (42), G_1 and G_2 can be calculated as:

$$G_1 = \frac{(b_0 - 2\kappa t)(h_0 - 2\kappa t)^2}{4} f_y - \frac{(700u_1 + 3500)L}{4} - \frac{\rho_{st} b_0 h_0 L^2}{8} \tag{43}$$

$$G_2 = \frac{[b_0 - 2\kappa(t+\Delta t)][h_0 - 2\kappa(t+\Delta t)]^2}{4} f_y - \frac{(693.034u_1 + 98.5012u_2 + 3500)L}{4} - \frac{\rho_{st} b_0 h_0 L^2}{8} \tag{44}$$

Based on FORM, $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, and β_1, β_2 can be easily obtained. ρ_G for PHI2 and EPHI2 methods can be acquired by Eq. (6). Then the outcrossing rate $v^+(t)$ can be calculated by using PHI2, PHI2+, EPHI2 and EPHI2+ methods, respectively. The results of the time-dependent failure probability are presented in Figure 3.

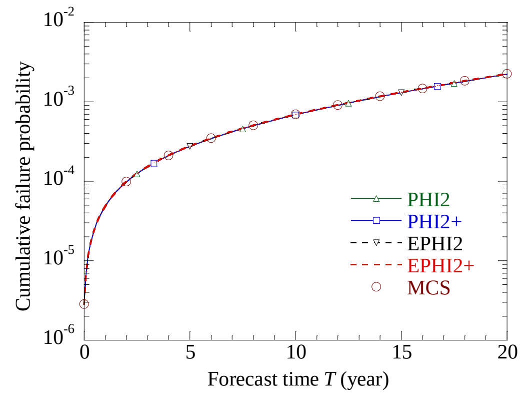

Figure 3: Changes of time-dependent cumulative failure probability for Example 1

To compare the stability of each method, Δt is chosen as other values, from 8×10^{-5} to 8×10^{-2} years. Then, by repeating the above steps for different values of the time intervals, the outcrossing rate can be calculated for the initial instant $t = 0$. The comparison results of each method are shown in Figure 4.

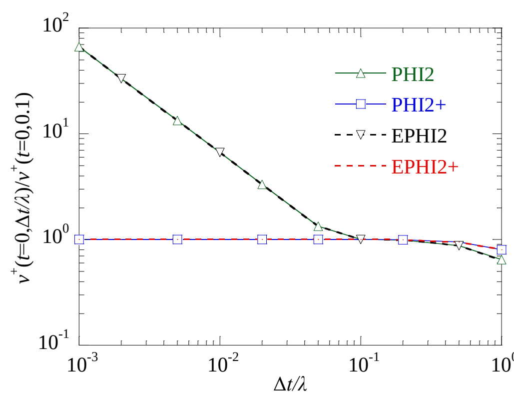

Figure 4: Sensitivity of the outcrossing rate with respect to time interval for Example 1

From the results represented in Figure. 3 and Figure. 4, it can be found that:

1. The cumulative failure probabilities obtained with EPHI2+ method are always close to those of PHI2, PHI2+, and EPHI2 methods during

the considered prediction intervals. The relative error between EPHI2+ method and MCS is within 1%, which demonstrates the accuracy of EPHI2+ method.

2. For all values of the time interval, the outcrossing rate calculated by EPHI2+ method remains almost the same, while PHI2 method as well as EPHI2 method shows sensitivity to changes in the time interval, such as the instability of the numerical values that occur when the interval Δt decreases. This indicates that EPHI2+ method has sufficient stability and is insensitive to time intervals.

3.2. Example 2: Corroded pipeline joint

The second example considers the time-dependent reliability of a corroded pipeline joint with operating pressure (Gong and Frangopol 2019). The diameter (D) of the pipe joint is 324 mm, the nominal wall thickness (wt_n) is 4.32 mm. Besides the pipeline joint has a nominal specified minimum yielding strength (SMYS) of 359 MPa, and a nominal maximum operating pressure (MOP) of 5 MPa. It is assumed that the joint contains a single corrosion defect. The time-dependent LSF is as follows:

$$G(t) = p_b(t) - p(t) \tag{45}$$

$$p_b(t) = \xi \frac{2wt\left(\sigma_y + 68.95\right)}{D} \left[\frac{1 - \frac{0.85d(t)}{wt}}{1 - \frac{0.85d(t)}{M(t)wt}} \right], \frac{d(t)}{wt} \le 0.8 \tag{46}$$

$$M(t) = \begin{cases} \sqrt{1 + 0.6275\frac{l(t)^2}{Dwt} - 0.003375\frac{l(t)^4}{(Dwt)^2}}, & \frac{l^2(t)}{Dwt} \le 50 \\ 3.3 + 0.032\frac{l(t)^2}{Dwt}, & \frac{l^2(t)}{Dwt} > 50 \end{cases} \tag{47}$$

where $p_b(t)$ and $p(t)$ represent the bursting capacity pressure and operating pressure at time t; wt and σ_y are the wall thickness, and yield strength, respectively; ξ is a model error; $M(t)$ is the Folias factor at time t; $l(t)$ and $d(t)$ are the length and depth of the corrosion defect at time t, which are assumed as follows (Gong and Frangopol 2019):

$$l(t) = l_0 + g_l t \tag{48}$$

$$d(t) = d_0 + d_g(t) \tag{49}$$

where l_0 and d_0 are the initial length and initial defect depth, respectively; g_l is the length growth rate; and $d_g(t)$ is a gamma process, which has the probability density function as:

$$F\left(d_g(t) \mid at, \beta\right) = \beta^{at}\left(d_g(t)\right)^{at-1} \exp\left(-\beta d_g(t)\right) / \Gamma(at) \tag{50}$$

where $\Gamma(.)$ is the gamma function; a and β are the shape and rate parameters, which are assumed to be 4.0×10^2 and 4.0×10^3, respectively. $d_g(t)$ follows equation as:

$$d_g(t+\Delta t) = d_g(t) + d_g(\Delta t) \tag{51}$$

where $d_g(\Delta t)$ obeys the gamma distribution $F(d_g(\Delta t)|a\Delta t, \beta)$. The correlation coefficient between $d_g(t)$ and $d_g(t+\Delta t)$ as $\rho_d(\Delta t, t)$, is $\sqrt{\frac{t}{t+\Delta t}}$ (Gong and Zhou 2017).

The operating pressure is assumed as a Gaussian process with the autocorrelation function $\rho_p(\Delta t) = \exp(-\Delta t^2/4)$. The probabilistic properties of all the parameters included in this analysis are summarized in Table 3.

Table 3: Random variables and random processes for Example 2.

Parameters	*Distributions*	*Mean*	*COV*
wt	*Normal*	wt_n	*0.015*
σ_y	*Normal*	*1.1SMYS*	*0.035*
ξ	*Gumbel*	*1.297*	*0.258*
l_0	*Normal*	*50(mm)*	*0.15*
d_0	*Normal*	*0.30* wt_n	*0.15*
g_l	*Weibull*	*3.0(mm/a)*	*0.15*
p	*Gaussian process*	*1.05MOP*	*0.03*

The results of the cumulative failure probability for each method are shown in Figure 5.

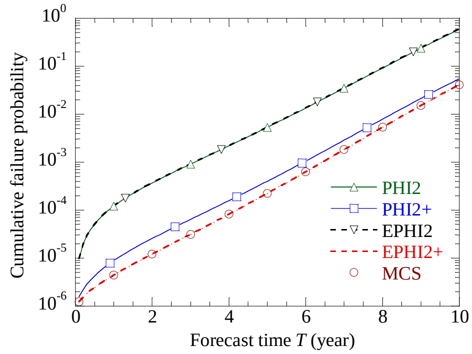

Figure 5: Changes of time-dependent cumulative failure probability for Example 2

From the results represented in Figure 5, it can be found that:

The results obtained by EPHI2+ method and MCS are essentially the same. When dealing with several different types of non-stationary random processes, the results obtained by PHI2 and EPHI2 methods differ significantly from the values of MCS, which is due to the fact that the time intervals chosen by PHI2 and EPHI2 methods cannot satisfy the condition for $\rho_d(\Delta t, t)$ and $\rho_p(\Delta t)$ both in 0.99-0.995. And due to the non-stationary random processes involved, the results of PHI2+ method are larger than those of MCS. Therefore, EPHI2+ method can be applied with accurate and stable results when faced with several different types of random processes.

4. CONCLUSIONS

A time-dependent reliability method called EPHI2+ is proposed, and a stable outcrossing rate model is developed. The accuracy and stability of EPHI2+ method are investigated by two examples. It was found that:

The time-dependent reliability can be easily and accurately calculated with the help of the proposed stable outcrossing rate model. Examples show that EPHI2+ method can obtain results close to those of MCS, which indicates that EPHI2+ method has sufficient accuracy. The proposed EPHI2+ method is insensitive to changes in the time interval, and the examples show that EPHI2+ method can obtain more stable results.

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