Multi-objective evaluation of target reliabilities for structural fire design

Ranjit Kumar Chaudhary
Ph.D. Student, Dept. of Structural Engineering and Building Materials, Ghent University, Ghent, Belgium

Thomas Gernay
Assistant Professor, Dept. of Civil and Systems Engineering, Johns Hopkins University, Baltimore, United States

Ruben Van Coile
Associate Professor, Dept. of Structural Engineering and Building Materials, Ghent University, Ghent, Belgium

ABSTRACT: Target reliabilities for structural design are mostly informed by risk-based optimization. Such optimization is often computationally challenging, and especially in structural fire engineering involves numerous design parameters, objectives, and constraints. Usually, a single-objective optimization-based approach is adopted, but it is mostly inefficient and could result in uneconomical design solutions. Multi-objective optimization-based approach can address these issues and is therefore considered in the current study for evaluating target reliabilities for the structural fire design. A reinforced concrete slab exposed to natural fires is considered as a case study. The target reliabilities for the slab vary between 1.0 and 4.0 for the structural fire design. The consideration of the environmental cost does not influence the result of cost optimization in RC slab except in the case of a higher trade-off of CO_2 *emission with dollars, while post-fire repairability considerations result in a significantly increased target reliability level.*

1. INTRODUCTION

Design target reliabilities are typically based on calibration from risk-based design optimizations from the perspective of societal stakeholders (Rackwitz, 2000; JCSS, 2001). Economic optimization is performed by balancing the cost and consequences of implementing safety measures. The optimizations are mostly single-objective based, and if multiple objectives exist they are generally transformed into a common utility function (Hopkin et al., 2021), whereas for multiple failure modes an interaction of the failure modes is assumed. However, real-world design problems involve multiple objectives and design parameters, while design constraints may be non-linear and there may be various utility functions. Additionally, single-objective optimizations assume the objective function space as convex which is often not accurate (Deb, 2011). Multi-objective optimization (MOO) based decision-making has the potential to address these limitations. Therefore, in this study, MOO is adopted to evaluate target reliabilities for the design of fire-exposed structures.

A reinforced concrete slab exposed to natural fire is considered as a case study. The target reliabilities are initially developed from a cost-benefit perspective. Then, the target reliabilities are evaluated considering other objectives such as resilience and sustainability. The benefits of the MOO are highlighted through comparison with the single-objective optimization. This study, therefore, recommends MOO as a rational approach to evaluate target reliabilities for a reliable, economical, and sustainable structural design.

2. ECONOMIC OPTIMIZATION AND CODE CALIBRATION

In this study, the target reliabilities are derived through lifetime cost optimization. Rackwitz's approach is commonly adopted for optimization, where the lifetime utility, Y is given by (Rackwitz, 2000):

$$Y = B - C - A - D = B - K \qquad (1)$$

In eq. (1), B refers to the benefit derived from the structure's existence, C is the sum of the cost of construction (C_0) and the safety investment cost (C_1), A is the obsolescence cost, and D is the damage cost resulting from structural failure. When the reconstruction time is considered negligible in comparison to the time between structural failures, B can be considered constant and independent of cost optimization. This simplifies maximizing Y to minimizing K (termed as the lifetime cost). The components of K are elaborated in eqs. (2) - (3), where ω is the yearly obsolescence rate, λ the yearly probability of occurrence of adverse event, P_f is the probability of structural failure subjected to the occurence of adverse event (and is associated with a limit state, Z) and ξC_0 is the expected damage cost in case of structural failure (sum of the direct and indirect costs), presented as a factor (ξ) of the construction cost. For cost optimization, the safety investment cost (C_1) is the up-front (present time) value, while A and D refer to future entities (evaluated over the structure's lifetime). These are therefore discounted to obtain a net present value. For this, a continuous discounting rate (γ) is adopted.

$$A = C\frac{\omega}{\gamma} = (C_0 + C_1)\frac{\omega}{\gamma} \qquad (2)$$

$$D = \frac{\lambda P_f}{\gamma}\,\xi C_0 \qquad (3)$$

On substituting the cost components in eq. (1), the lifetime cost optimization can be written as:

$$\min_p \left[K = (C_0 + C_1(p))\left(1 + \frac{\omega}{\gamma}\right) + \frac{\lambda P_f(p)}{\gamma}\,\xi C_0 \right] \qquad (4)$$

where p is the structural design parameter. The minimization in eq. (4) yields an optimum value of p, which thus guides design. The corresponding structural failure probability is the optimum failure probability ($P_{f,opt}$). The optimum reliability index (β_{opt}) is obtained as the inverse cumulative density function (Φ) of $P_{f,opt}$. The optimum reliability obtained through the cost optimization approach outlined here agrees with the standard normal design target reliabilities in international guidelines (Chaudhary et al., 2023) for $\lambda = 1/year$. Here, the approach is extended for deriving target reliabilities for structural fire design considering multiple design variables, objectives, and constraints.

3. MULTI-OBJECTIVE OPTIMIZATION

Unlike traditional optimization, which finds the best solution for a single objective, MOO involves finding the best solutions for multiple objectives simultaneously. A general MOO problem is formulated as (Deb, 2011):

$$\begin{Bmatrix} Maximize/minimize \\ f_m(x), \;\; m = 1,2,\ldots,M \\ Subjected\ to \\ g_j(x) \ge 0, \;\; j = 1,2,\ldots,J; \\ h_k(x) = 0, \;\; k = 1,2,\ldots,K; \\ x_i^L \le x_i \le x_i^U, \;\; i = 1,2,\ldots,n \end{Bmatrix} \qquad (5)$$

where, $x = [x_1, x_2 \ldots\ldots x_n]^T$ is a vector of design variables. The design variables themselves are subjected to the constraints, i.e., lower L and upper U as the bound. This constitutes the design space D for variables. $g_j(x)$, and $h_k(x)$ are sets of inequality and equality constraints, while $f_m(x)$ represent the objective set and form objective space Z.

The set of solutions obtained from MOO is called Pareto-optimal solutions. These solutions dominate all other possible solutions in the objective space and the boundary they form is called the Pareto-optimal front. MOO has been efficient for design optimization for a wide variety of problems, such as the seismic design of a structure, bridge maintenance, and planning, construction project management, etc (Alothaimeen and Arditi, 2019). Evolutionary algorithms are commonly adopted for MOO (NSGA-II here).

4. CASE STUDY: FIRE-EXPOSED REINFORCED (RC) CONCRETE SLAB

A one-way RC slab exposed to natural fire is considered as a case study. The slab is 0.2 m thick, with a tensile reinforcement of 0.000785 mm^2/m. The reinforcement has a clear concrete cover of 15 mm (i.e., a reinforcement axis distance of 20 mm from the bottom face). The characteristic strength of the concrete is 30 MPa and the characteristic yield strength of reinforcing bars is 500 MPa. The slab is from a compartment of size 6 m × 6 m × 3.5 m. The compartment belongs to a 3-story multi-family dwelling with floor area of 5 times the compartment size at each story. The cost of the entire building is therefore 15 times the cost of the considered compartment. The construction cost is 215 \$/ft^2 (national US average based on Gordian, 2022). The demolition and reconstruction costs (C_{tot}) is considered 1.05 times the construction cost (i.e., 2430 \$/m^2).

The slab is exposed to the fire at the bottom face. The fire exposure is a Eurocode parametric fire (EN1991-1-2:2002). The same slab has been investigated by Chaudhary et al. (2021) for probabilistic assessment of fire-exposed structures. The bending moment capacity (for both normal and fire exposure scenarios) of the slab is evaluated as:

$$M_R = A_s k_{f_y(T)} f_{y,20°C}\left(h - c - \frac{\phi}{2}\right) - 0.5\frac{\left(A_s k_{f_y(T)} f_{y,20°C}\right)^2}{b f_{c,20°C}} \tag{6}$$

where, A_s refers to the reinforcement area of the slab, h the depth of the slab, b the width, c the clear concrete cover, and ϕ the diameter of reinforcing bars. $k_{f_y(T)}$ is the reinforcement yield strength retention factor at temperature T and $f_{y,20°C}$ and $f_{c,20°C}$ are the ambient yield and characteristic strength for reinforcement and the concrete, respectively.

For the evaluation of the ambient bending moment capacity, the reinforcement temperature is considered as 20°C, while for the fire exposure, a 1-D heat transfer analysis is done. The design capacity of the slab is evaluated as 59 kNm (with 1.15 and 1.5 as the safety factor for steel and concrete strength). The evaluation of probabilistic distribution for bending moment capacity takes into account the stochastic parameters listed in Table 1; for specifics please refer to (Chaudhary et al., 2023). Figure 1 shows the probabilistic distribution for the moment capacity evaluated for both ambient and fire design conditions. As expected, the mean bending moment capacity of the slab decreases with the increase in fire load density.

Table 1 Stochastic parameters for failure probability evaluation of the RC slab

Variables	**Mean, μ**	**CoV (distribution)**
Slab thickness, *h*	200 [m]	0.025 (normal)
Concrete strength, f_c	42.9 [MPa]	0.15 (lognormal)
Reinforcement yield strength retention factor, $k_{fy,T}$	Temperature dependent logistic model	
Reinforcement axis, *a*	a_{nom} + 5 [mm]	σ = 5 (beta; 4,4)
Reinforcement area, A_s	1.02 $A_{s,nom}$ [mm^2/m]	0.02 (normal)
Fire load density, q_f	(300-1500) [MJ/m^2]	0.3 (gumbel)
Opening factor, *O*	0.04 [m$^{1/2}$]	-
Permanent load, M_G	M_G [kNm]	0.1 (normal)
Imposed load, M_Q	0.2× M_Q [kNm]	0.95 (gamma)
Capacity estimation, K_R	1.1 [-]	0.11(lognormal)
Load estimation, K_E	1.0 [-]	0.1 (lognormal)

Considering the exceedance of bending moment capacity as a limit state, a target reliability index (β_t) of 3.8 (for moderate consequence and a 50-year reference period) needs to be verified for normal design (EN 1990: 2002). In this study, the target reliability is developed for the RC slab considering fire exposure. For this, a risk-based optimization is carried out. Two parameters (reinforcement area A_s and its axis distance a) are identified as design variables for the fire design.

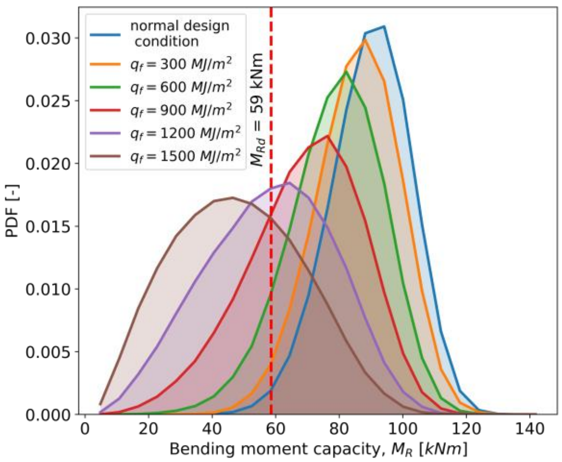

Figure 1 Probabilistic distribution for bending moment capacity of the slab at ambient temperature and in fire (constant opening factor of 0.04 m$^{1/2}$; fire load density described by a Gumbel distribution with cov of 0.3 and listed mean value).

5. MULTI-OBJECTIVE OPTIMIZATION FOR RC SLAB

5.1. Input data

The minimization of the lifetime cost in eq. (4) involves determining the investment cost (including its obsolescence value) and the (lifetime) failure cost. The investment cost for the slab only depends on the reinforcement area and not the axis distance and is thus evaluated as the cost of added reinforcement (ΔA) over a reference area. A minimum area for the slab is considered 0.000393 mm^2 (10 ϕ bars spaced at 200 mm center-to-center) and the axis position is 12 mm from the bottom surface. The cost of reinforcement (C_s) is 1.31 \$/lb (in Gordian, 2022) i.e., 2.89 \$/kg. A ω of 0.022/year is adopted to evaluate the obsolesce cost.

The evaluation of structural failure cost involves defining λ, ξ, γ, and determining P_f. For the considered compartment, λ of 0.0001/year is adopted. Note that λ is for total fires, while the parametric fires are flashover fires. In the absence of an active fire protection system and a fire-fighting service, the flashover fire probability is (as a conservative approximation) considered equal to the reported fire frequency. γ is considered as 0.03/year (for societal stakeholders). The failure cost factor ξ is a variable in this study and is discussed further in section 5.3.

5.2. Evaluation of structural failure probability

The P_f for the slab is evaluated considering the exceedance of the bending moment capacity of the slab as the limit state and is given by:

$$P_f \;=\; P[Z < 0] \qquad (7)$$

where $Z = K_R M_{R,fi} - K_E\left(M_G + M_Q\right)$. $M_{R,fi}$ refers to the bending moment capacity for fire exposure and M_G and M_Q are the permanent and imposed load moments, respectively. K_R and K_E refer to the capacity and load model uncertainties. The M_G and M_Q for the slab are evaluated from the ambient design capacity, considering load safety factors (details in Chaudhary et al., 2023) and $M_{R,fi}$ is evaluated based on eq. (6). Figure 2 shows the failure probability calculated for the RC slab for fire exposure with a nominal fire load density of 780 MJ/m^2 (corresponds to mean fire load density in residential buildings based on EN 1991-1-2: 2002) and an opening factor of 0.04 m$^{1/2}$ (deterministic value). The P_f has been evaluated for entire range of A_s and a. The evaluation is carried out through the Monte-Carlo approach.

5.3. Single objective optimization and multi-objective constraints

The main goal for the fire design of the considered slab is the minimization of the lifetime cost. The total lifetime cost (K_p) for the slab ($\$/m^2$) for the compartment can be obtained by rewriting eq. (4) as eq. (8). The term independent of the design variable is not considered in the optimization. In eq. (8), the initial construction cost (C_0) is modified to the cost of replacement (C_{tot}).

$$K_p = C_s \times \Delta A(p) \times \rho_{steel}(1+\frac{\omega}{\gamma}) + \frac{\lambda P_f(p)}{\gamma}\,\xi \times C_{tot} \tag{8}$$

where, $\Delta A(p)$ stands for the added reinforcement over the minimum area (0.000393 m^2) and determines the safety investment cost C_1.

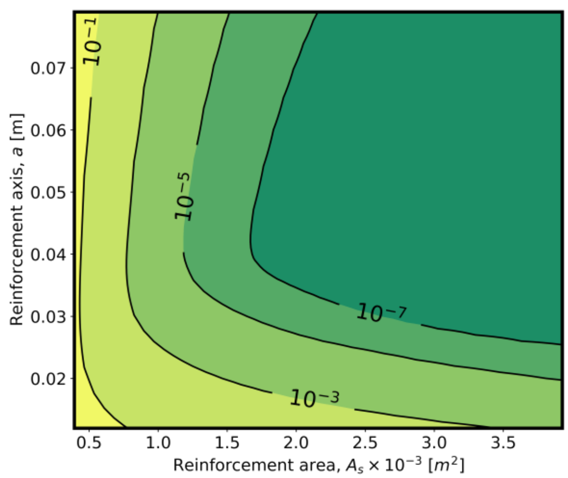

Figure 2 Failure probability of RC slab for fire exposure with nominal fire load density of 780 MJ/m^2 at absolute values of reinforcement area A_s and axis distance a.

The investment cost associated with the increased reinforcement area in the slab is shown in Figure 3. The costs in the figure are shown after multiplying with the compartment area (i.e., 6 m × 6 m × K_p). For a reinforcement area of 0.00393 m^2 (highest value considered), the investment cost (including lifetime obsolescence) is 5000\$. The figure also displays the net present value (NPV) of lifetime failure cost for ξ of 100. This damage is 100 times the compartment cost and is 6.67 (= 100/ 15) times the cost of replacing the entire building (costs of similar scale observed by Kanda and Shah, 1997). At the minimum area, the failure cost is 6850 \$ and as expected reduces to around 0 \$ for higher reinforcement areas. The total lifetime cost (sum of the investment and the failure cost) is minimum (940 \$) at a reinforcement area ($A_{s,opt}$) of 0.000825 m^2 (reinforcement area increase of 40 mm^2), reducing the total lifetime cost by about 5 % compared to the reference situation. This minimum value of the lifetime cost is for a constant reinforcement axis (reference axis *a* of 20 mm), while a further reduction is possible at different *a*. Evaluating the minimum lifetime cost for the entire range of *a* is computationally expensive.

Optimizing A_s and *a* for fire exposure only however impacts the ambient design reliability. As visualized in Figure 4, the ambient reliability indices (β_{amb}) of the slab varies non-linearly for different A_s and *a*. Therefore, the ambient design reliability needs to be implemented as a boundary condition. This further adds to the computational burden of cost optimization. Note that the ambient reliability indices have been evaluated considering probabilistic load models with a 50-year reference period, while the fire exposure evaluations consider load models for an arbitrary point in time load (intended for deriving target reliabilities conditional on fire exposure).

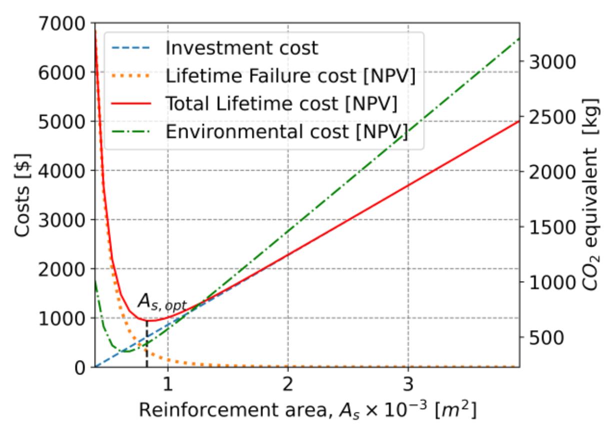

Figure 3 Total lifetime cost (sum of investment and failure cost) and lifetime environmental cost for RC slab at absolute values of reinforcement area. Constant axis distance of 20 mm and nominal fire load density of 780 MJ/m^2.

There are however also other objectives that should be considered as part of the optimization. First of all, also the environmental cost is being prioritized recently following increased global warming concerns (Gervasio and Dimova, 2018). The environmental cost increases with the increased reinforcement area of the slab, while at the same time, it decreases for a reduced risk of failure. The total lifetime environmental cost (K_e) for the RC slab of the compartment is evaluated based on eq. (9). Figure 3 presents the NPV of the lifetime environmental cost ($kgCO_2/m^2$) at different A_s and a fire exposure with a nominal fire load density of 780 MJ/m^2. Here, the environmental cost has been considered in CO_2 emissions (being the most common indicator). The global warming potential for the increased reinforcement area (GWP_{steel}) is 3.21 $kgCO_2$ per kg of steel (Gervasio and Dimova, 2018). The global warming potential resulting from structural failure $\mathrm{GWP_{tot}}$ amounts to 168 $kgCO_2$ per square meter of building area for a multi-family dwelling (Gervasio and Dimova, 2018). The environmental cost in Figure 3 could go as high as 1500 kg CO_2. A continuous discounting rate (γ_e) of 0.014/year is considered for the net present evaluation of the environmental cost resulting from structural failure based on (Kula and evans, 2011) In eq. (9), ξ_e represents the ratio of total area damaged by fire to the compartment area. Considering damage to the entire building, ξ_e of 15 is considered).

$$K_e = GWP_{steel} \times \Delta A(p) \times \rho_{steel} + \frac{\lambda P_f(p)}{\gamma_e} \xi_e \times \mathrm{GWP_{tot}} \tag{9}$$

A third objective acknowledges there is an increased interest among societal stakeholders toward rapid recovery and limited functionality loss of structure after a fire event (Van Coile et al., 2017). This can be ensured by applying post-fire repairability as a constraint for the structural fire design. For this, in the case of the RC slab, the maximum allowable temperature of reinforcing bars is considered as 600°C (Neves, 1996). The reinforcement loses strength permanently following heating to a temperature higher than this. A probabilistic evaluation is carried out and the reliability target for the serviceability limit (yearly) is considered a design constraint ($\beta_s =$ 2.9 based on EN1990: 2002). This reliability is evaluated as part of the Monte Carlo assessment by checking the rebar temperature.

It is practically inconvenient to consider all the design objectives and constraints listed above as part of a single-objective optimization. Thus, the multi-objective optimization approach is adopted hereafter.

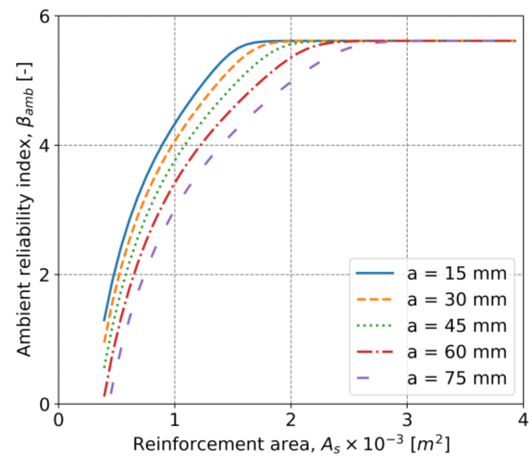

Figure 4 Ambient reliability indices for the RC slab at absolute values of reinforcement areas and axis distances, considering a 50-year reference period.

5.4. Multi-objective evaluation of target reliabilities

The optimization for RC slab involves minimizing three costs: (i) investment cost (ii) failure cost, and (iii) environmental cost, and considering two design constraints: (i) ambient design target reliability and (ii) post-fire repairability. The minimization of the costs in eqs. (8) and (9) involve optimizing only for ΔA and P_f, while other variables are not a function of the vector of design parameters, p . It therefore becomes computationally efficient to evaluate MOO-based Pareto-fronts and Pareto-front solutions first considering the minimization of ΔA and P_f as objectives. Substituting the Pareto-front objectives in eq. (8), we get a vector of K_p. The Pareto-front solution corresponding to the minimum K_p is the optimum solution ($A_{s,opt}$ and a_{opt}). The evaluated objective space and the design space for the RC slab for different nominal fires are displayed in Figure 5. For this evaluation, only β_t of 3.8 is considered a constraint. The Pareto-fronts for the RC slab with both design constraints (β_t and β_s) have also been evaluated. The evaluated Pareto-fronts considering β_s also as constraint is not shown.

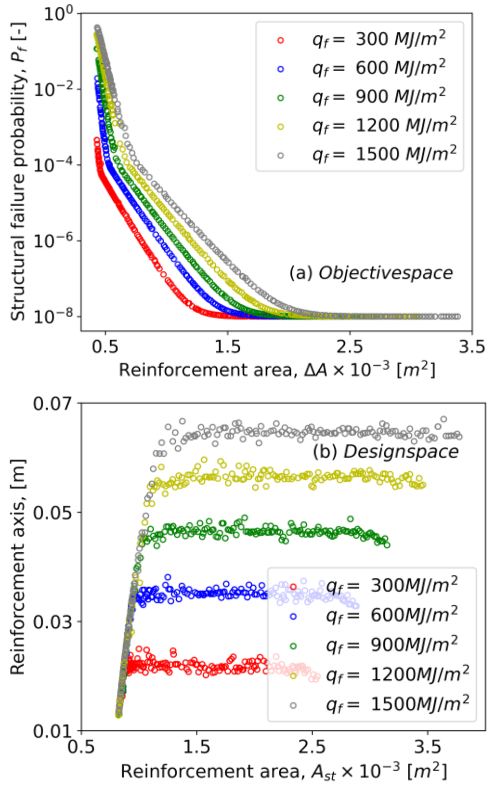

Figure 5 (a) Objective space and (b) design space for MOO-based lifetime cost minimization for RC slab, considering different nominal fires.

With $A_{s,opt}$, and a_{opt} evaluated, the optimum failure probability ($P_{f,opt}$) and the reliability index (β_{opt}) can readily be assessed. The cost factor ξ is considered a variable in this study, with values from 1 to 10,000 as shown in Figure 6. The Figure shows β_{opt} evaluated for fire exposure for different nominal q_f and ξ. For ξ of 1.0 (when the damage is equal to the loss of a compartment), the optimum reliability reduces steeply up to the fire load density of 900 MJ/m^2 in comparison to the higher fire load. This is because the optimum design parameters are governed by $\beta_{t,amb}$ for qf below 900 MJ/m^2, whereas for higher q_f, the fire-induced failure cost becomes significant and so also influences the optimum design. Note that the same level of investment results in an increase of P_f as the fire load density increases. Thus, the decrease in optimum reliability observed for increasing fire load density does not imply a reduced optimum safety investment level. This is observed in Figure 6 for ξ equal and above 10. These cases always have increased investment costs with the increase in q_f while optimum reliability mostly reduces.

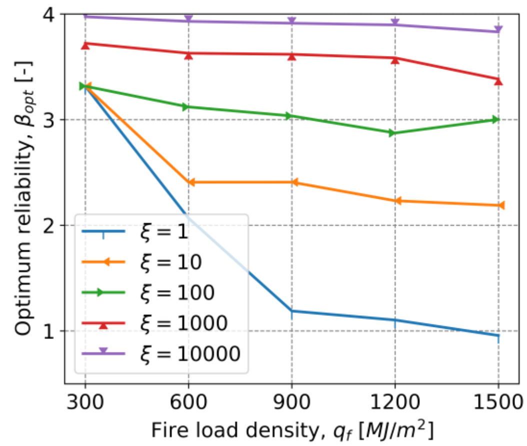

Figure 6 Optimum reliabilities evaluated based on lifetime cost optimization of RC slab for different nominal fires.

Minimizing environmental cost has CO_2 emissions as a utility, while other objectives are in monetary value and their trade-off is still unclear. A trade-off factor (τ) of 0.1 and 2 \$ per kg of CO_2 is considered in the following. The optimum design is obtained by calculating K_p and K_e for the Pareto-front objectives based on eq. (8) and (9), respectively. The minimum of $(K_p + \tau\, K_e)$ is then specified as the optimum solution. The results for β_{opt} considering the environmental cost are shown in Figure 7. The environmental cost does not influence β_{opt} at τ of 0.1, while it decreases at τ of 2.0. The decrease of β_{opt} is the result of the investment cost being increased. The consideration of the post-fire repairability constraint leads to an increased β_{opt} for the fire limit state of Eq. (7) in comparison to the results presented above where the ambient target reliability was the as only constraint. This is because the requirement of post-fire performance results in an increase of the optimum axis distance *a*, and (indirectly) an increase in optimum reinforcement area.

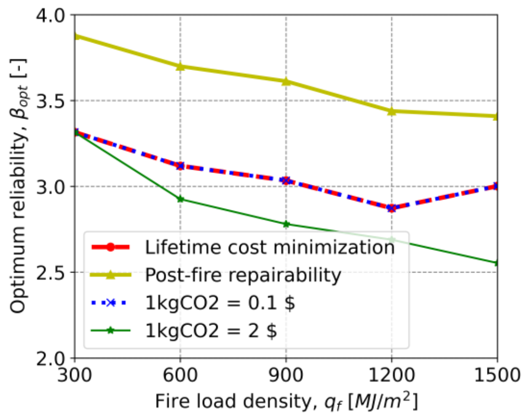

Figure 7 Comparison of the optimum reliabilities based on lifetime cost optimization derived with and without environmental cost and the post-fire repairability. ξ of 100 is considered.

6. CONCLUSIONS

Cost optimization-based target reliabilities are evaluated in this study for structural fire design. The single objective-based approach is found computationally inefficient when multiple design variables, objectives, and constraints exist, while MOO is a flexible approach and allows the evaluation of target reliability efficiently. Further, in the MOO approach, design objectives can have different utilities (dollars and CO_2 emissions here). Depending on the fire and the failure consequences, the target reliabilities for the RC slab vary between 1.0 and 4.0. The environmental cost has no influence on the cost optimization at a lower trade-off of CO_2 emissions with dollars, while a higher trade-off could result in reduced target reliability. The consideration of post-fire repairability leads to a significantly increased target reliability level in comparison to the normal cost-optimization target for fire design.

7. REFERENCES

Alothaimeen, I. and Arditi, D., (2019). "Overview of multi-objective optimization approaches in construction project management". Multicriteria Optimization-Pareto-Optimality and Threshold-Optimality, IntechOpen, 4, 88185.

Chaudhary, R.K., Van Coile, R. and Gernay, T., (2023). "Cost-optimization based target reliabilities for design of structures exposed to fire". (Unpublished).

Chaudhary, R.K., Van Coile, R. and Gernay, T., 2021. "Potential of surrogate modelling for probabilistic fire analysis of structures". Fire Technology, 57(6), 3151-3177.

Deb, K., (2011). "Multi-objective optimisation using evolutionary algorithms: an introduction". In Multi-objective evolutionary optimisation for product design and manufacturing, Springer , London, 3-34.

Der Kiureghian, A., and Ditlevsen, O. (2009). "Aleatory or epistemic? Does it matter?" *Structural Safety*, 31(2), 105–112.

EN 1990: 2002. Eurocode - Basis of structural design.

EN 1991-1-2: 2002. "Actions on structures. Part 1–2: General actions—Actions on structures exposed to fire".

Gervasio, H. and Dimova, S., (2018). "Environmental benchmarks for buildings". Publications Office of the European Union: Luxembourg.

Gordian (2022). Assemblies Costs Book, RSMeans.

Hopkin, D., Fu, I. and Van Coile, R., (2021). Adequate fire safety for structural steel elements based upon life-time cost optimization. Fire Safety Journal, 120, 103095.

JCSS (2001). "Probabilistic Model Code, Part 1—Basis of Design". Joint Committee on Structural Safety.

Kanda, J. and Shah, H., (1997). "Engineering role in failure cost evaluation for buildings". Structural Safety, 19(1), 79-90.

Kula, E. and Evans, D., (2011). "Dual discounting in cost-benefit analysis for environmental impacts". Environmental impact assessment review, 31(3), 180-186.

Neves, I.C., Rodrigues, J.P.C. and Loureiro, A.D.P., (1996). "Mechanical properties of reinforcing and prestressing steels after heating. Journal of Materials in Civil Engineering", 8(4), 189-194.

Rackwitz, R., (2000). "Optimization—the basis of code-making and reliability verification. Structural safety", 22(1), 27-60.

Van Coile, R., Bisby, L., Rush, D. and Manes, M., (2017). "Design for post-fire use: a case study in fire resilience design". In 2nd International Conference on Structural Safety under Fire and Blast Loading, London.