

Applicability of Point Load Approximation to Traffic Load Estimation

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ABSTRACT: In this study, the applicability of the point load approximation (PLA) to vehicle load identification, such as wheel loads and gross vehicle weight (GVW), is investigated. Bridge responses from simulation and in-house experiments were used to examine the applicability. Focusing on the use of accelerometers, which are relatively economical to implement, the rotation angles calculated from the accelerometers were used in vehicle load identification by means of the PLA. Comparisons were also made with the results identified using displacements. Observations showed that utilizing filtering and regularization enabled higher accuracy of vehicle weight identification than the result from the bridge-weigh-in-motion (B-WIM). The validity of B-WIM using rotation angles calculated from acceleration data was also observed.

1. INTRODUCTION

Vehicle weight data from weigh-in-motion (WIM) and bridge-weigh-in-motion (B-WIM) provides information on the wheel load of vehicles, gross vehicle weight, speeds, and axle spacings, and it can be used to calculate load effects due to passing vehicles or combinations of vehicles. B-WIM can measure actual bridge performance under traffic loading and provide information on the load sharing between girders and on the dynamic amplification (ISWIM 2022). It is important to understand traffic loads because the fatigue damage of steel bridges is mainly caused by stresses and displacement behavior of members due to active loads (Tateishi et al. 1995).

Since Moses (1979) proposed the B-WIM method to measure traffic loads by using the influence lines, B-WIM has been actively discussed (e.g., Cardini et al. 2009, Ojio et al. 2016). Most of the B-WIMs use the strain response of the bridge and compare the signal with the influence lines to estimate the axle weight by inverse analysis. However, although the strain gauges used in B-WIM provide relatively high-

precision measurements, they are expensive to implement.

In recent years, approaches of B-WIM using accelerometers, which are relatively economical to implement, have been discussed (Nagayama et al. 2017, O'Brien et al. 2020). Utilizing wireless accelerometers for the B-WIM is also investigated as it is portable, does not require a power supply, and can be used for low-cost measurement. Previous studies using accelerometers have proposed methods for obtaining displacement influence lines by integrating the acceleration response twice (Sekiya et al. 2018) and for calculating the displacement response by converting acceleration to rotation angle and using the Kalman filter (Kato et al. 2020). Although research using accelerometers is increasing, the process of converting acceleration to displacement is needed.

Influence lines in existing B-WIM are often calculated from moving vehicle experiments and are affected by the dynamic actions of the vehicles. This has been raised as one of the major sources

of uncertainty in B-WIM systems (Carraro et al. 2020). Under these circumstances, a novel method for estimating axle weight by B-WIM based on various load effects (displacement, velocity, acceleration, rotation angle, and bending moment), called the Point Load Approximation method ("PLA theory"), was suggested (Cantero 2021). PLA theory is an extension of classical beam theory by introducing the finite difference method, which enables axle weight estimation using accelerometers theoretically.

In this study, the applicability of accelerometers in B-WIM is examined by means of PLA using not only acceleration but also rotation angle. B-WIM using displacement measurement is also examined to verify the validity of PLA B-WIM.

2. THEORY OVERVIEW

2.1. Theoretical background

PLA theory is an extension of classical beam theory that approximates external loads from discretely measured load effects using the finite difference method (Cantero 2021). In contrast, in the traditional B-WIM, if the load effect is denoted as LE , the influence line is denoted as δ_{LE} , and the axle weight is denoted as P , the axle weight of the vehicle is estimated solving Eq. (1) for P as an inverse analysis.

$$LE = P \cdot \delta_{LE} \quad (1)$$

Figure 1 shows the relationship between each bridge response (y , a , θ) and external load (q) in the classical beam theory. Classical beam theory consists of continuous functions in space, but it is extremely difficult to prepare continuous functions. Classical beam theory relates external loads to continuous functions by differentiating them in space, whereas in PLA, discrete data are differentiated by finite difference methods and related to external loads.

If X is the number of times of differentiation, n is the number of sensors, and α denotes the finite difference coefficient, PLA theory estimates the axle weight so that the sum of squares of the differences of both sides in Eq. (2) is minimized.

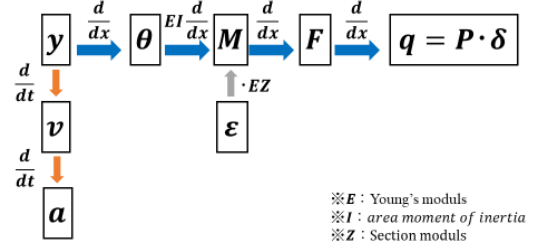


Figure 1: Relationship between each bridge response, etc. and external loads in classical beam theory.

$$\sum_i^n \alpha_{X,i} LE(x_i) = P \cdot \delta_{LE} \quad (2)$$

Eq. (2) is linear and thus superposition holds, and it can be extended to the time domain for the vehicle with N axles. Eq. (3) is a general expression of PLA theory that extends Eq. (2).

$$\sum_i^n \alpha_{X,i} \sum_j^N \frac{d^m}{dt^m} (LE_{i,j}(t)) = \sum_j^N P_j \cdot \frac{d^m}{dt^m} (\delta_{LE}(x_{P,j}(t))) \quad (3)$$

where $x_{P,j}(t)$ represents the position of the j -th axis at time t .

As an example, when the load effect LE is displacement, the finite difference coefficients and influence lines are defined in Eq. (4) and Eq. (5), respectively.

$$\left\{ \begin{array}{l} \alpha_{4,1} = \frac{24}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)(x_1 - x_5)} \\ \alpha_{4,2} = \frac{-24}{(x_1 - x_2)(x_2 - x_3)(x_2 - x_4)(x_2 - x_5)} \\ \alpha_{4,3} = \frac{24}{(x_1 - x_3)(x_2 - x_3)(x_3 - x_4)(x_3 - x_5)} \\ \alpha_{4,4} = \frac{-24}{(x_1 - x_4)(x_2 - x_4)(x_3 - x_4)(x_4 - x_5)} \\ \alpha_{4,5} = \frac{24}{(x_1 - x_5)(x_2 - x_5)(x_3 - x_5)(x_4 - x_5)} \end{array} \right. \quad (4)$$

where, x_n ($n=1, 2, \dots, 5$) is the distance from the entrance of the bridge.

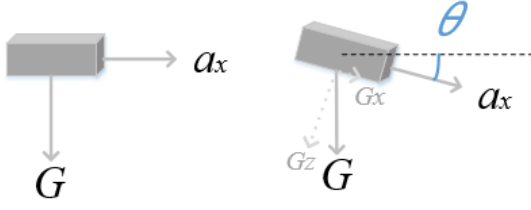


Figure 2: Relationship between accelerometer inclination and gravity.

$$\delta_y(x_p) = \frac{1}{6EIh^4} \begin{cases} 0 & , x_p \leq x_1 \\ x_p^3 & , x_1 < x_p \leq x_2 \\ 4h^3 - 12h^2x_p + 12hx_p^2 - 3x_p^3 & , x_2 < x_p \leq x_3 \\ -44h^3 + 60h^2x_p - 24hx_p^2 + 3x_p^3 & , x_3 < x_p \leq x_4 \\ (4h - x_p)^3 & , x_4 < x_p \leq x_5 \\ 0 & , x_5 < x_p \end{cases} \quad (5)$$

where, h denotes the distance between each sensor when they are implemented with equal distance, and EI indicates the stiffness of the bridge, i.e., E is Young's modulus and I is the second moment of area.

The advantage of using the finite difference method is that not only can multiple sensor results be taken into account in the calculation, but also the response of the bridge can be related to the external loads in a manner independent of the boundary conditions (support conditions and structure). The independence of the PLA method from boundary conditions is due to the fact that the application of second-order differences to bending moments to obtain external loads eliminates the terms

associated with boundary conditions (Cantero 2021, He et al. 2017, Chen et al. 2018).

In summary, PLA can analytically obtain influence lines independent of boundary conditions, and it is possible to obtain the local response of bridges along the bridge length using accelerations as well as other physical responses such as displacement, strain, etc.

2.2. Utilization of Rotation Angle

This study uses not only the acceleration data but also the rotation angle estimated from the acceleration responses as a physical response for PLA. The rotation angle is estimated from triaxial acceleration data installed on bridges following Figure 2 and Eq. (6) (Nagayama et al. 2017). Figure 2 shows the acceleration and gravity components in the direction of the bridge axis.

$$\theta = \arcsin\left(\frac{a_x}{G}\right) \quad (6)$$

where a_x indicates the acceleration in the longitudinal direction of the bridge and G denotes the gravitational acceleration.

3. SIMULATION-BASED INVESTIGATION

To examine the validity of PLA, the accuracy of axle weight identification using PLA is first checked on simulation.

3.1. Numerical models for simulation

A vehicle-bridge interaction analysis is carried out to simulate bridge responses under a moving vehicle. The vehicle is modeled as a half-car model, and the bridge is modeled as a simply supported beam. The vehicle speed was assumed 0.9 m/s, the axle distance of the vehicle model is

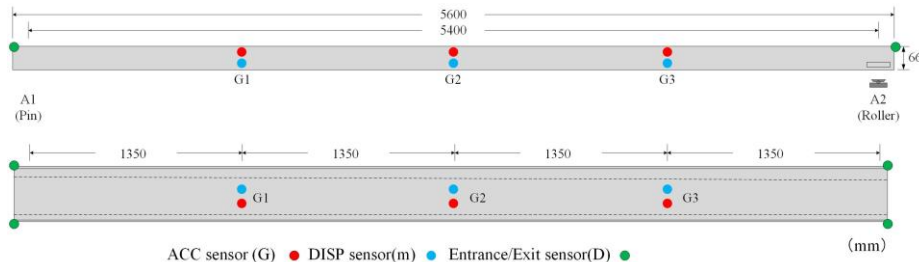


Figure 3: Displacement and acceleration sensor placement in the simulation.

Table 1 : Error in weight estimation accuracy

	DISP	ANGLE	ACC
Axle Error	3.1%	3.4%	1.2%
GVW Error	0.15%	0.004%	-0.09%

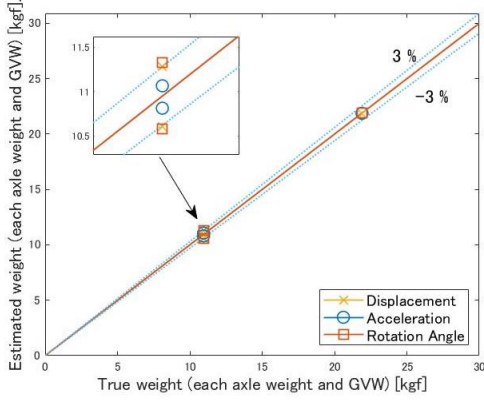


Figure 4: Accuracy of axle weight estimation for each load effect in simulation.

0.4 m, the gross vehicle weight (GVW) is 21.9 kgf (about 215 N) and the axle load of each axle is 10.95 kgf (about 107 N).

The span length of the bridge model is 5.4m, Young's modulus is $2.1 \times 10^{11} N/m$, the cross-sectional area is $6.6 \times 10^{-3} m^2$, and second moment of area is $5.5 \times 10^{-7} m^4$. Bridge responses at five points equally spaced between the supports are observed. However, since the vertical displacement on the support is zero, the finite difference method is applied assuming that there is a virtual sensor that does not actually measure the bridge response. Figure 3 shows the general layout of the bridge and the sensor locations in simulation.

3.2. B-WIM by PLA using simulation data

A second-order Butterworth filter with a cutoff frequency of 1 Hz was applied to each data from the simulation before estimating axle loads of the vehicle by means of PLA. The results of the application of PLA theory are shown in Figure 4. The estimation error of the axle weight by PLA is summarized in Table 1. Figure 4 and Table 1 show that PLA is valid and can estimate axle weight and GVW with high accuracy when there is no noise in the response. In order to investigate the

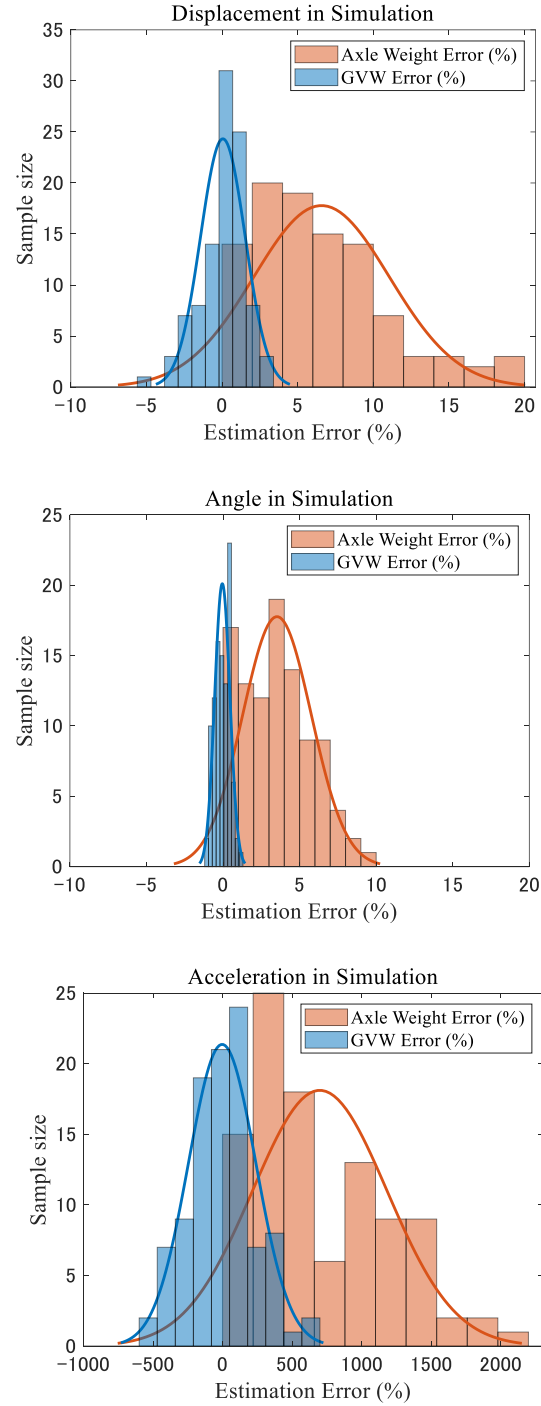


Figure 5: Axle and GVW errors for each load effect at SNR=30. (top) displacement, (middle) rotation angle, (bottom) acceleration. Axle weights are the average of the absolute values of the errors for the front and rear axles.

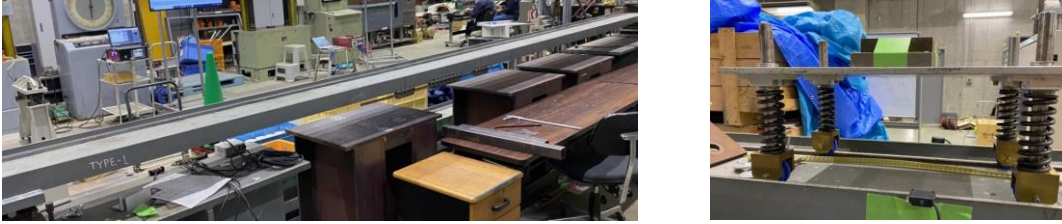


Figure 6: Overview of the model bridge experiment. Left) model bridge, Right) the model vehicle.

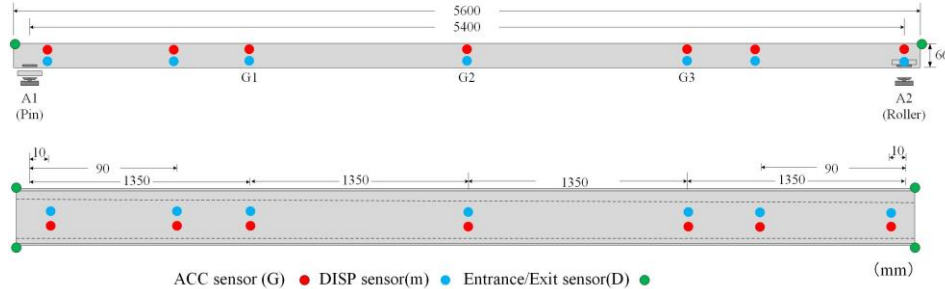


Figure 7: Sensor deployment on model bridge.

identification performance of PLA under noise, simulation data from 100 runs with $SNR=30$ is examined. Histograms of the identification error of PLA considering noise are summarized in Figure 5. Here, the axle weight error is expressed as the average of the absolute errors both for the front and rear axles.

Focusing on the estimation errors in Figure 5, it can be seen that the acceleration data, in particular, has remarkably low robustness to noise. On the other hand, the rotation angle shows little influence against noise, and the average error is almost the same as the error without considering the noise.

The reason for the low estimation accuracy in acceleration can be attributed to the fact that acceleration is mainly a dynamic component, while B-WIM is derived from a smooth influence line. A low-pass filter (LPF) was applied to the acceleration, but the LPF may have difficulty extracting the static component of the bridge in the acceleration. Observations through the simulation showed that using acceleration data is still a challenge in B-WIM even using PLA, while rotation angles demonstrated high robustness against noise.

4. IN-HOUSE EXPERIMENT AND VALIDATION

4.1. Model Bridge

An in-house experiment on a model bridge is carried out to examine the applicability of PLA in the real world. The model bridge and the model vehicle are shown in Figure 6. The properties of the model vehicle and model bridge are as follows. A two-axle vehicle with an axle distance of 0.2 m, and GVW of 23.2 kgf (about 228 N), and an axle load of each axle is 11.6 kgf (about 114 N). The model bridge has a span length of 5.4 m, two rails are allocated on the bridge surface as the path for the vehicle. The width of the two rails is 199 mm and width of bridge is 287 mm. The bearings are pin bearings on the entrance side and roller bearings on the exit side.

15 runs were conducted, five of which were used to estimate the stiffness of the bridge, and ten of which were used to confirm the accuracy of the weight estimation. The stiffness was estimated from the displacement and rotation angle responses using the PLA. Seven accelerometers and displacement transducers were installed as shown in Figure 7.

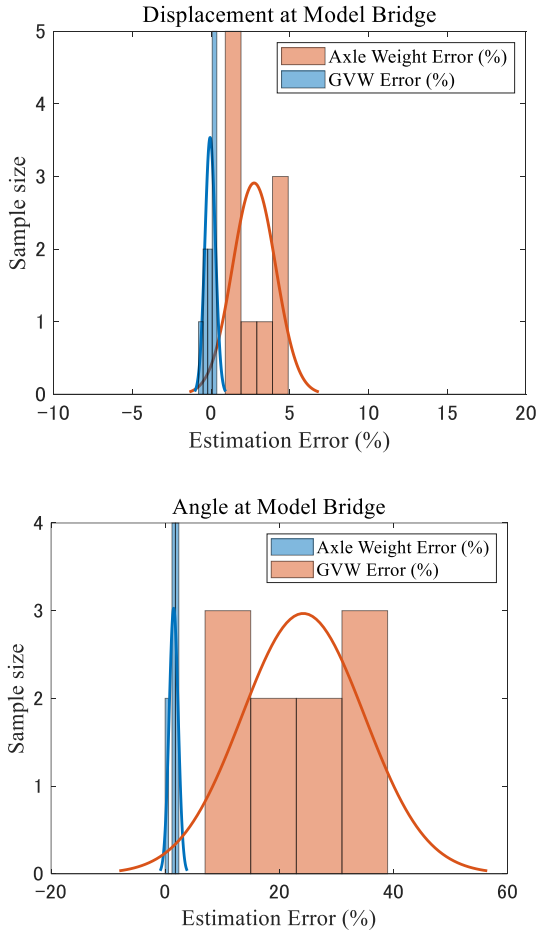


Figure 8: Axle weight estimation from a model bridge experiment. (top) displacement, (bottom) rotation angle.

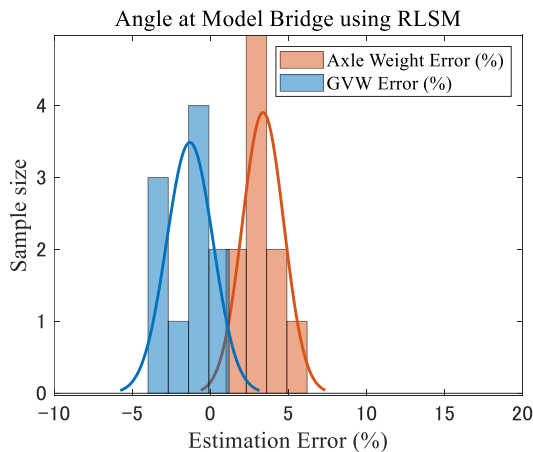


Figure 9: Accuracy of axle weight and GVW estimation from rotation angle data using the Regularized Least Squares Method.

4.2. B-WIM by PLA using experimental data

PLA is applied to data from the in-house experiments. Displacement and rotation angle are data from the bridge. LPF was applied to each measured data with a cutoff frequency of 1 Hz same as the simulation-based investigation. Identification errors for the axle load and GVW are shown in Figure 8. Figure 8 shows that the identification accuracy of the axle load from displacement data is higher than that from the rotation angle, while the identification accuracy for GVW is comparable with each other.

4.3. B-WIM by PLA from rotation angle with the regularized least-squares method

To improve the accuracy of axle weight estimation using the rotation angle, Tikhonov's L2 regularization is introduced. The L-curve method is used to determine the hyperparameters in the Tikhonov regularization. The results of axle weight identification by the regularized least-squares method are shown in Figure 9. Compared with Figure 8, Figure 9 shows a drastic improvement in the axle load identification using the rotation angle. On the other hand, the accuracy of GVW estimation was slightly lower than without regularization.

4.4. Comparison with conventional B-WIM

The identification results of the axle load and GVW from PLA are compared with those obtained by the conventional B-WIM method using the influence line. The displacement at the span center and the rotation angles measured at the entrance of the bridge are examined. The identification results are summarized in Figure 10. It is noted that for axle weight identification regularization was used while only a low-pass filter was applied for the identification of the GVW. This is because, in a preliminary study, the applying LPF alone was highly accurate in identifying the GVW, whereas regularization was effective in improving the accuracy of axle weight identification, especially at rotational angles.

Observations from the comparison demonstrated that the PLA was more accurate in

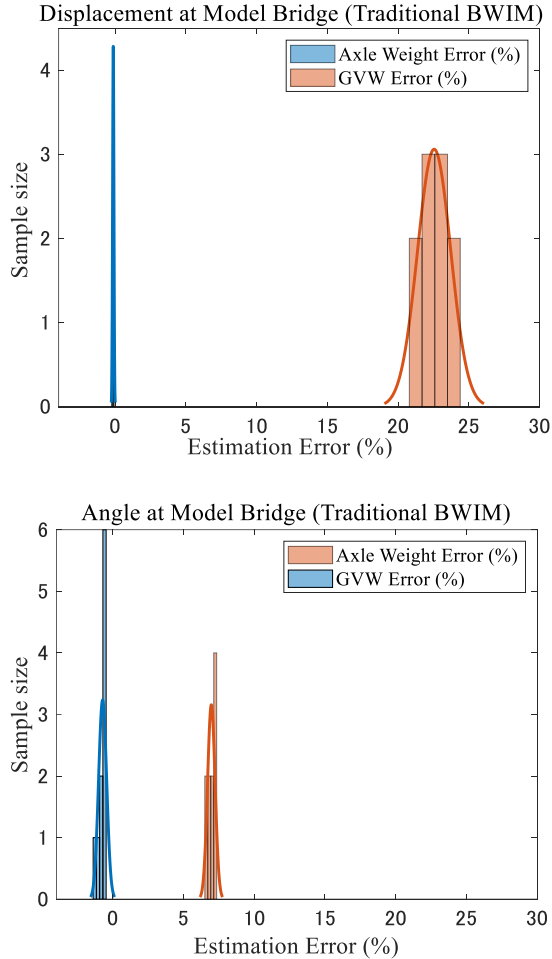


Figure 10: Accuracy of traditional BWIM estimates of axle weight and GVW using displacement and rotation angle data. (top) displacement, (bottom) rotation angle.

axle weight estimation than the conventional B-WIM.

5. CONCLUSIONS

In this study, simulations and in-house experiments on the model bridge were conducted to investigate the applicability of PLA for axle weight and GVW identifications using displacement, acceleration, and rotation angle.

Investigations using simulation data showed that PLA was effective for all loading effects in the absence of noise. However, the accuracy of axle weight identification using acceleration data was affected by the noise. Investigations based on experimental data demonstrated that the

displacement data were more effective than the rotation angle data. Considering regularization in axle weight and GVW identification by PLA resulted in drastic improvement in axle weight identification.

Observations from the comparison demonstrated that the PLA was more accurate in axle weight estimation than the conventional B-WIM.

Future studies include the application of PLA to real bridges and axle weight and GVW identification under multiple vehicles on multiple lanes.

6. ACKNOWLEDGMENTS

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