Physically-driven GE-GDEE and its application to stochastic seismic response and dynamic reliability analyses of practical engineering structures

Meng-Ze Lyu
Postdoc, College of Civil Engineering, Tongji University, Shanghai, China

Jian-Bing Chen*
Professor, College of Civil Engineering & State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China

ABSTRACT: Stochastic seismic response and dynamic reliability analyses of large-scale high-rise building structures under earthquake actions is one of the most challenging problems in engineering field. Both the randomness from structural parameters and external excitations has significant effects on the stochastic dynamic behaviors of structures with complex nonlinearity. In such a case, it is difficult to evaluate the first-passage reliability, especially the small probability of failure under rare events. A feasible approach for this problem is developed via the physically-driven globally-evolving-based generalized density evolution equation (GE-GDEE), where the double randomness is characterized in a one- or two-dimensional partial differential equation (PDE) governing the probability density function (PDF). In the developed unified formalism of GE-GDEE, an absorbing boundary process (ABP) of the critical response quantity indicating structural failure or not can be constructed. Its transient PDF satisfies the GE-GDEE, a two-dimensional PDE. The effective drift coefficient in the GE-GDEE essentially represents the physically driving force for the evolution of transient PDF, and can be identified via the observed data given by some representative deterministic dynamic analyses. Then, the GE-GDEE with the determined effective drift coefficient can be solved numerically to capture the transient PDF of the ABP, and then obtain the time-variant reliability by integration further. In the present paper, the stochastic seismic response and dynamic reliability analyses of a 24-story reinforced concrete (RC) shear wall structure with nearly 280,000 degrees of freedom are performed. There are fourteen concrete material parameters considered as probabilistically dependent random variables characterized by vine copulas. The ground motion is modelled by the non-stationary Clough-Penzien spectrum. The time-variant reliability curves of the structure under different thresholds are given, which cannot be achieved by the general Monte Carlo simulation (MCS) due to the limitation of computational cost.

Keywords: high-rise RC shear wall structure; seismic reliability analysis; GE-GDEE; Physically-driven; Rare event.

The probabilistic performance and dynamic reliability evaluation of RC structures under disastrous random dynamic actions, such as strong earthquakes, are of paramount importance in engineering practices (Li & Chen 2009, Zio 2009, Der Kiureghian 2022). In contrast to generic nonlinear dynamic problems, seismic nonlinear dynamic response analysis of real-world RC structures under earthquake is more challenging due to the occurrence of dynamic damage, energy dissipation, and plasticity (Li et al. 2018). There is still a gap between the existing classical stochastic dynamics theory and the refined quantification of dynamic reliability in practical engineering (Li 2021). In summary, the gap mainly arises from three aspects.

First, the existing engineering reliability analyses mostly take one component, such as a beam or a column, (Stewart & Al-Harthy 2008) or a multi-story frame structure (Iervolino et al. 2018) as examples. There have been many attempts to analyze the refined random response and reliability of frame structures (Feng et al. 2022, Cao et al. 2023), but the investigation on more complex engineering structures is rarely involved. Recent experimental studies show that high-rise shear wall structures have more complex dynamic damage characteristics and failure modes under disastrous dynamic action (He et al. 2020, Tao et al. 2021). The numerical evaluation of its random nonlinear response and reliability under stochastic ground motion is of great significance.

Second, several independent random variables were just employed to characterize the uncertainty of structural parameters in the existing reliability evaluation of practical structures (Petryna et al. 2002, Amirkardoust et al. 2020). In fact, the uncertainty involved in structural parameters is exhibited on three different levels: the probability distribution of each parameter, the probabilistic dependence configurations of multiple parameters, and the spatial variability of parameters. For instance, for concrete material, it is found that the multiple constitutive parameters are probabilistic dependent, and it is suggested to capture the probabilistic dependence by vine-copulas (Tao et al. 2020) or hierarchical model (Tao & Chen 2022).

Third, though widely applied in engineering practice, difficulties still exist in various reliability methods for high accuracy and effectiveness to achieve high-dimensional complex nonlinear problems, such as the first-order second-moment method (FOSM) (Cervenka 2013), which is difficult to obtain accurate results during strong nonlinear stage via only the first two moments, and the level-crossing process method (Ma & Wang 2015), where even the modified Markov assumptions cannot guarantee the accuracy. In addition, MCS was also applied (Stewart & Al-Harthy 2008, Akiyama et al. 2011), but their calculation costs are unacceptable for refined reliability analysis of practical large structures.

In the past decades, the deterministic dynamic analysis of large-scale engineering structures has become increasingly sophisticated and complete with the rapid development of modern supercomputing techniques and advanced mechanics basis (Liu et al. 2022). However, the accuracy and efficiency of their stochastic dynamic analysis are still insufficient (Li 2021). In fact, the difficulty of reliability analysis in practical engineering is that the complicated nonlinear dynamic characteristics mask the propagation mechanism of randomness. It is necessary to tackle the solution of constitutive-structural differential equations in two levels (Li et al. 2014, Ren & Li 2018). Therefore, it is hard to capture the propagation of uncertainty in such dynamic systems exhibiting complex nonlinearity, which is not explicit as shown in Duffing-like polynomials or even cannot be expressed in Bouc-Wen hysteresis differential form.

The probability density evolution method (PDEM) provided a new perspective to this problem (Li & Chen 2004). It is revealed that the propagation of uncertainties is driven by the evolution of the physical mechanism of the system (Li & Chen 2008, Chen & Li 2009). Adhering to the global-evolution path of PDEM (Li et al. 2012), the GE-GDEE was established for generic path-continuous process (Lyu & Chen 2022a). The intrinsic drift coefficient in the GE-GDEE is the physically driving “force” for the propagation of uncertainties, and can be identified analytically (Sun & Chen 2022, Luo et al. 2023) or numerically based on data from representative dynamic analyses of the system (Lyu & Chen 2021a, b, Chen & Lyu 2022a, Luo et al. 2022a). In this sense, the method for probabilistic response determination (Chen & Lyu 2022a, Luo et al. 2022b) and reliability analysis (Lyu & Chen 2022a, Sun et al. 2023) via “identifying intrinsic drift coefficients - solving GE-GDEE” was called as the “physically-driven GE-GDEE”. In recent studies, this method has shown high accuracy and efficiency for high-dimensional nonlinear stochastic dynamic problems (Luo et al. 2020, 2022c, Lyu & Chen 2020, 2021c, 2022b-d, Chen

& Lyu 2022b), but it has not yet been applied to stochastic nonlinear analysis of real-world concrete structures, which is one of the most challenging problems.

In the present paper, a comprehensive methodology for stochastic response and dynamic reliability analysis is developed by combining the physically-driven GE-GDEE with the refined nonlinear finite element (FE) dynamic analysis of concrete structures. Sec. 1 elaborates the refined FE dynamic model and uncertainty quantification formalism of practical concrete structures. Sec. 2 introduces the framework of physically-driven GE-GDEE for response determination and reliability analysis. Sec. 3 shows the applications to a real-world RC structure. Concluding remarks are provided in Sec. 4, finally.

1. FORMULATION OF SEISMIC ANALYSIS FOR RC STRUCTURES

1.1. Governing equations for refined seismic nonlinear dynamic analysis

For an engineering concrete structure under earthquakes, the fundamental physical equation of stress for the material element is expressed by the equation of motion (Li et al. 2014), namely,

$$\nabla\cdot\boldsymbol{\sigma}(\boldsymbol{x},t)+\rho\boldsymbol{i}_{\mathrm{L}}\ddot{x}_{\mathrm{g}}(t)=\rho\ddot{\boldsymbol{u}}(\boldsymbol{x},t)+c\dot{\boldsymbol{u}}(\boldsymbol{x},t) \quad (1)$$

with appropriate stress and displacement boundary conditions, where $\boldsymbol{\sigma}(\boldsymbol{x},t)$ is the second-order stress tensor at the position $\boldsymbol{x}$ and the time instant t ; $\dot{\boldsymbol{u}}(\boldsymbol{x},t)$ and $\ddot{\boldsymbol{u}}(\boldsymbol{x},t)$ are, respectively, the velocity and acceleration vector corresponding to the displacement vector $\boldsymbol{u}(\boldsymbol{x},t)$ relative to the ground; ρ and c are the mass density and viscous damping coefficient of material, respectively; $\boldsymbol{i}_{\mathrm{L}}$ is the direction vector of ground motion, e.g., $\boldsymbol{i}_{\mathrm{L}}=(1,0,0)^{\mathrm{T}}$ for ground motion in *x*-direction; and $\ddot{x}_{\mathrm{g}}(t)$ is the nonstationary ground motion acceleration process modelled by Clough-Penzien spectrum (Clough & Penzien 2003).

Meanwhile, the strain of the material element is expressed by the compatibility equation (Li et al. 2014), namely,

$$\boldsymbol{\varepsilon}(\boldsymbol{x},t)=\frac{1}{2}[\nabla\boldsymbol{u}(\boldsymbol{x},t)+\boldsymbol{u}(\boldsymbol{x},t)\nabla], \quad (2)$$

where $\boldsymbol{\varepsilon}(\boldsymbol{x},t)$ is the second-order strain tensor at $\boldsymbol{x}$ and t.

In the elastoplastic damage constitutive model of concrete, the relationship between the stress and strain for the material element can be modelled as (Li et al. 2014)

$$\begin{aligned}\boldsymbol{\sigma}(\boldsymbol{\Theta},\boldsymbol{x},t)&=[\mathbb{I}-\mathbb{D}(\boldsymbol{\Theta},\boldsymbol{x},t)]:\mathbb{E}_0(\boldsymbol{\Theta})\\&\quad:[\boldsymbol{\varepsilon}(\boldsymbol{x},t)-\boldsymbol{\varepsilon}_{\mathrm{p}}(\boldsymbol{\Theta},\boldsymbol{x},t)],\end{aligned} \quad (3)$$

where $\mathbb{D}(\boldsymbol{\Theta},\boldsymbol{x},t)$ is the fourth-order damage tensor; $\mathbb{I}$ is the fourth-order unit tensor; $\mathbb{E}_0(\boldsymbol{\Theta})$ is the fourth-order initial elastic modulus tensor; and $\boldsymbol{\varepsilon}_{\mathrm{p}}(\boldsymbol{\Theta},\boldsymbol{x},t)$ is the second-order plastic strain tensor. Here $\mathbb{D}(\boldsymbol{\Theta},\boldsymbol{x},t)$ and $\boldsymbol{\varepsilon}_{\mathrm{p}}(\boldsymbol{\Theta},\boldsymbol{x},t)$ can be determined iteratively at each analysis step (Wu et al. 2006; Li & Ren 2009) ; and $\boldsymbol{\Theta}=(E_{\mathrm{c}},f_{\mathrm{c}},\varepsilon_{\mathrm{c}},\alpha_{\mathrm{c}},f_{\mathrm{t}},\varepsilon_{\mathrm{t}},\alpha_{\mathrm{t}})^{\mathrm{T}}$ is the basic random vector of the concrete material, including the initial Young's modulus E_{c}, the tensile /compressive strengths $f_{\mathrm{t/c}}$, the tensile /compressive peak strains $\varepsilon_{\mathrm{t/c}}$, and the tensile /compressive shape parameters $\alpha_{\mathrm{t/c}}$, and will be characterized in Subsec. 1.2.

The initial-boundary value problem constructed by Eqs. (1) - (3) can be solved by nonlinear FE analysis if $\boldsymbol{\Theta}$ takes deterministic value.

1.2. Uncertainty characterization of concrete parameters

The uncertainties of the structural materials and external excitations cannot be ignored for real-world engineering systems.

A pragmatic technique based on vine-copulas is introduced to model the probabilistic dependence configurations of the concrete parameters $\boldsymbol{\Theta}=(E_{\mathrm{c}},f_{\mathrm{c}},\varepsilon_{\mathrm{c}},\alpha_{\mathrm{c}},f_{\mathrm{t}},\varepsilon_{\mathrm{t}},\alpha_{\mathrm{t}})^{\mathrm{T}}$. Here, the marginal PDF of E_{c}, f_{c}, ε_{c}, and α_{c} can be determined by test data (Tao et al. 2020). The parameters between different grades are independent. The dependence among f_{c}, ε_{c}, and α_{c} for the same grade can be characterized based on vine-copulas (Tao et al. 2020), namely, their joint PDF reads

$$
\begin{aligned}
p_{f_c\varepsilon_c\alpha_c}(f,\varepsilon,\alpha) = c_{f_c\varepsilon_c|\alpha_c}[F_{f_c|\alpha_c}(f|\alpha), F_{\varepsilon_c|\alpha_c}(\varepsilon|\alpha)] \\
\cdot c_{f_c\alpha_c}[F_{f_c}(f), F_{\alpha_c}(\alpha)]\ c_{\varepsilon_c\alpha_c}[F_{\varepsilon_c}(\varepsilon), F_{\alpha_c}(\alpha)] \\
\cdot p_{f_c}(f)\, p_{\varepsilon_c}(\varepsilon)\, p_{\alpha_c}(\alpha),
\end{aligned} \tag{4}
$$

where $p_{\langle\cdot\rangle}(\cdot)$ and $F_{\langle\cdot\rangle}(\cdot)$ are the PDF and cumulative distribution function (CDF) of random variable $\langle\cdot\rangle$, respectively; $F_{\langle\cdot\rangle|[\cdot]}(\cdot|\cdot)$ is the conditional CDF of random variable $\langle\cdot\rangle$ under the condition of random variable $[\cdot]$, i.e., the CDF of conditional random variable $\langle\cdot\rangle|[\cdot]$; $c_{\langle\cdot\rangle[\cdot]}(\cdot,\cdot)$ is the bivariate copula density function of random variables $\langle\cdot\rangle$ and $[\cdot]$; and $c_{\langle\cdot\rangle[\cdot]|\{\cdot\}}(\cdot,\cdot)$ is the bivariate copula density function of conditional random variables $\langle\cdot\rangle|\{\cdot\}$ and $[\cdot]|\{\cdot\}$. Then, E_c is independent to f_c, ε_c, and α_c but should satisfy Drucker's postulate, and f_t, ε_t, and α_t are assumed to depend on f_c, ε_c, and α_c, and can be determined by the empirical formulas (Guo et al. 1982).

2. PHYSICALLY DRIVEN GE-GDEE

For an RC structure involving random material parameters and subjected to stochastic ground motion, the dynamic reliability in terms of a quantity of interest at a critical position $\boldsymbol{x}_{\text{int}}$, denoted as $u_{\text{int}}(t)$, under a specific safe domain can be defined via the first-passage failure criteria (Lin 1967), namely,

$$
R(t) = \Pr\{u_{\text{int}}(\tau) \in \Omega_s,\ \text{for}\ 0 \leqslant \tau \leqslant t\}, \tag{5}
$$

where $u_{\text{int}}(t)$ is a component or the modulus of $\boldsymbol{u}(\boldsymbol{x}_{\text{int}}, t)$; $\Pr\{\cdot\}$ denotes the probability of the bracketed random event; and Ω_s is the safety domain defined as an open set on the real number field with the boundary $\partial\Omega_s$.

Meanwhile, note that $\ddot{x}_g(t)$ modelled by nonstationary Clough-Penzien spectrum can be expressed as a nonstationary linear function of the outputs of an r-dimensional linear filter system with Gaussian white noise input, e.g.,

$$
\begin{cases}
\ddot{x}_g(t) = \eta(t)\boldsymbol{b}_f^{\mathrm{T}}\boldsymbol{v}_f(t), \\
\dot{\boldsymbol{v}}_f(t) = \mathbf{K}_f\boldsymbol{v}_f(t) + \boldsymbol{g}_f\xi(t),
\end{cases} \tag{6}
$$

where $\eta(t)$ is the non-stationary modulation envelope function (Amin & Ang 1968); $\boldsymbol{b}_f$ and $\boldsymbol{g}_f$ are r-dimensional vectors and $\mathbf{K}_f$ is an $r \times r$ matrix, and they can be determined by the Clough-Penzien spectrum; and $\xi(t)$ is a Gaussian white noise process with intensity D.

Introduce $v_{f,k}(t)$, for $k = 1, \ldots, r$, as an auxiliary process satisfying $g_{f,k} \neq 0$. For the purpose of first-passage reliability analysis defined in Eq. (5), ABPs corresponding to $u_{\text{int}}(t)$ and $v_{f,k}(t)$, denoted as $\breve{u}_{\text{int}}(t)$ and $\breve{v}_{f,k}(t)$, can be constructed as

$$
\begin{cases}
\breve{u}_{\text{int}}(t) = \begin{cases} u_{\text{int}}(t), & \text{if } t < T, \\ u_{\text{int}}(T), & \text{if } t \geqslant T, \end{cases} \\
\breve{v}_{f,k}(t) = \begin{cases} v_{f,k}(t), & \text{if } t < T, \\ v_{f,k}(T), & \text{if } t \geqslant T, \end{cases}
\end{cases} \tag{7}
$$

respectively, where $T = \inf\{t|\ u_{\text{int}}(t) \notin \Omega_s,\ \text{for}\ t \geqslant 0\}$ is the first-passage time of $u_{\text{int}}(t)$ with respect to Ω_s (Redner 2001). Generally, both $\breve{u}_{\text{int}}(t)$ and $\breve{v}_{f,k}(t)$ are path-continuous processes, i.e., their sample paths are continuous functions of t with probability one. According to the unified formulation of GE-GDEE for generic path-continuous processes (Lyu & Chen 2022a), the transient joint PDF of $\breve{u}_{\text{int}}(t)$ and $\breve{v}_{f,k}(t)$, denoted as $p_{\breve{u}_{\text{int}}\breve{v}_{f,k}}(u,v,t)$, satisfies the following PDE:

$$
\begin{aligned}
\frac{\partial p_{\breve{u}_{\text{int}}\breve{v}_{f,k}}(u,v,t)}{\partial t} = -\frac{\partial[\breve{a}_1^{(\text{eff})}(u,v,t)\, p_{\breve{u}_{\text{int}}\breve{v}_{f,k}}(u,v,t)]}{\partial u} \\
-\frac{\partial[\breve{a}_2^{(\text{eff})}(u,v,t)\, p_{\breve{u}_{\text{int}}\breve{v}_{f,k}}(u,v,t)]}{\partial v} \\
+\frac{D g_{f,k}{}^2}{2}\frac{\partial^2 p_{\breve{u}_{\text{int}}\breve{v}_{f,k}}(u,v,t)}{\partial v^2}\mathrm{I}\{u \in \Omega_s\},
\end{aligned} \tag{8}
$$

where $\mathrm{I}\{\cdot\}$ is the indicator functional; and $\breve{a}_1^{(\text{eff})}(u,v,t)$ and $\breve{a}_2^{(\text{eff})}(u,v,t)$ are the intrinsic drift coefficients given as

$$
\breve{a}_1^{(\text{eff})}(u,v,t) =
\begin{cases}
\mathrm{E}[\dot{u}_{\text{int}}(t)\ |\ \breve{u}_{\text{int}}(t) = u;\ \breve{v}_{f,k}(t) = v], & \text{if } u \in \Omega_s, \\
0, & \text{if } u \in \partial\Omega_s,
\end{cases} \tag{9}
$$

and

$$\breve{a}_2^{(\mathrm{eff})}(u,v,t)=$$
$$\begin{cases} \mathrm{E}[\boldsymbol{k}_{\mathrm{f},(k,\cdot)}\boldsymbol{v}_{\mathrm{f}}(t) \mid \breve{u}_{\mathrm{int}}(t)=u;\ \breve{v}_{\mathrm{f},k}(t)=v], & \text{if } u\in\Omega_{\mathrm{s}}, \\ 0, & \text{if } u\in\partial\Omega_{\mathrm{s}}, \end{cases} \tag{10}$$

respectively, in which $\dot{u}_{\mathrm{int}}(t)$ is the velocity response corresponding to $u_{\mathrm{int}}(t)$; and $\boldsymbol{k}_{\mathrm{f},(k,\cdot)}$ is the k-th row vector of $\mathbf{K}_{\mathrm{f}}$.

The intrinsic drift coefficients are the physical driving force for the evolution of PDF but cannot be expressed analytically usually. They should be identified numerically according to the data from representative dynamic FE analyses. Then solving GE-GDEE (8) with the identified intrinsic drift coefficients will yield the transient joint PDF $p_{\breve{u}_{\mathrm{int}}\breve{v}_{\mathrm{f},k}}(u,v,t)$. According to the definition of the ABPs in Eq. (7), the first-passage reliability can be then calculated by

$$R(t)=\Pr\{\breve{u}_{\mathrm{int}}(t)\in\Omega_{\mathrm{s}}\} = \int_{-\infty}^{\infty}\int_{\Omega_{\mathrm{s}}} p_{\breve{u}_{\mathrm{int}}\breve{v}_{\mathrm{f},k}}(u,v,t)\,\mathrm{d}u\,\mathrm{d}v. \tag{11}$$

3. PRACTICAL ENGINEERING CASE

The case investigated in this paper is the No. 6 apartment building on a university campus in Shanghai, China. The apartment building is a 24-story RC shear wall structure. In Figure 1 shown is its FE analysis model. The total height of the building is 69.6 m. There are 7,636 elements included in the one-dimensional frame components modelled as fiber beam elements, and 43,962 elements included in the two-dimensional slab and shear wall components modelled as multilayer shell elements. The structural FE model involves 46,234 nodes, totaling 277,404 degrees of freedom (DOFs).

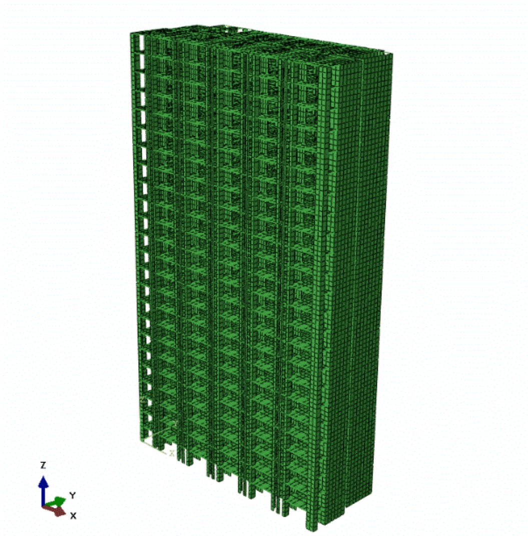

Figure 1: FE model of the building.

The FE model of the structure is modelled by the FE software ABAQUS. The constitutive model of concrete is embedded with a user-defined material subroutine compiled by FORTRAN. The dynamic FE analysis of the structure is computed by Tianhe-2 in the National Supercomputing Center in Guangzhou, China, where a single computing node occupies 24 cores.

Here 400 representative deterministic dynamic FE analyses are conducted, and the top displacement of the structure can be taken as the critical response for the first-passage failure criterion. Further taking the safe domain as $\Omega_{\mathrm{s}}=\{u|\ |u|<b\}$, where b is the threshold of the top displacement, identifying the intrinsic drift coefficients in Eqs. (9) and (10), and solving GE-GDEE (8) can give the time-variant reliability and corresponding probability of failure. In Figure 2 shown are the time-variant probabilities of failure under different thresholds. It can be seen that the probability of failure decreases with the increase of threshold. As the ground motion enters the attenuation section after $t=14$ s, the structural response amplitude gradually decreases, and thus the probability of failure increases slowly until it is stable. In Table 1 shown are the results of probability of failure at terminal time ($t=20$ s) via physically-driven GE-GDEE compared with the failure frequency estimation via 400

representative values directly. It can be seen that for the case with high thresholds, the failure frequency under 400 deterministic analyses is zero, but the physically-driven GE-GDEE can give the results of small probability of failure.

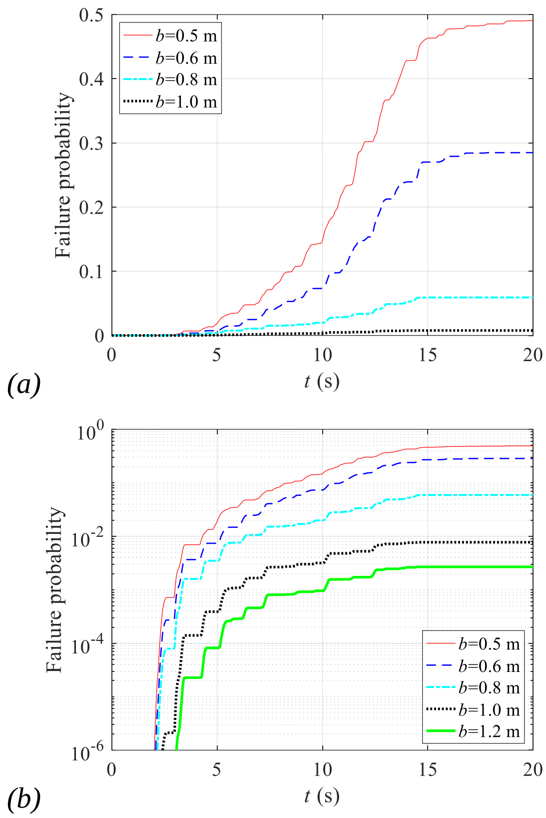

Figure 2: Time-variant probabilities of failure. (a) Linear coordinate; (b) Logarithmic coordinate.

Table 1: Probabilities of failure.

Threshold (m)	*Frequency*	*GE-GDEE*
0.5	*0.50*	*0.49065*
0.6	*0.27*	*0.28488*
0.8	*0.04*	*0.05923*
1.0	*0.00*	*0.00774*
1.2	*0.00*	*0.00268*

4. CONCLUSIONS

In the present paper, a methodology for seismic reliability analysis of real-world RC structures subjected to stochastic ground motion are developed via synthesis of the refined nonlinear FE dynamic analysis and the physically-driven GE-GDEE. In the developed method, the uncertainty characterization of concrete parameters is modelled by vine-copula-based probabilistic dependence quantification, and the uncertainty propagation in a real world physical stochastic system is governed by the physically-driven GE-GDEE. The intrinsic drift coefficients in the GE-GDEE are the physical-driving force for the uncertainty propagation, and can be identified via the data obtained by representative dynamic analyses of the refined FE model of the structure. Solving the GE-GDEE yields the time-variant PDF and dynamic reliability under various thresholds. A practical engineering case of a high-rise RC shear wall structure with nearly 280,000 DOFs is conducted to illustrate the engineering application of the proposed method.

In the future work, more engineering applications can be conducted based on the developed methodology in building, bridge, and geotechnical engineering.

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