

Considerations for the Development of Evidence Theory Applications

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ABSTRACT: Evidence theory provides a natural framework for analyzing imprecise probabilities. Evidence theory enables the explicit recognition of ignorance and is therefore suited for the analysis of missing or incomplete data. This paper examines the characteristics of uncertainty beyond traditional probability theory, and is motivated by three primary objectives: (i) introducing appropriate roles for uncertainty theories and their associated relevance, (ii) developing guidance and methods for data combination applications using evidence theory, and (iii) developing evidence theory applications within structural design. The paper reviews the development and applications of evidence theory in order to motivate further development for specific practical applications. Potential structural design applications for evidence theory include dynamic loading and performance-based design. The continued development of data combination methods using evidence theory can reduce the additional efforts required to apply evidence theory and make future evidence theory applications more efficient and effective.

1 INTRODUCTION

The need for an improved assessment of uncertainty within our society has never been greater. The design of infrastructure has been constrained recently by limited resources and tested frequently by extreme weather events. More frequent and destructive hazards, driven by anthropogenic practices that increase both possible damages and our vulnerability to hazard, pose a threat to our communities and infrastructure not adequately addressed by our prior development and design practices. The development of a method for an improved assessment of uncertainty would be greatly beneficial to infrastructure design for which many uncertainties exist.

Infrastructure owners face uncertainties with respect to past, present, and future states of

their assets. They are uncertain of the past because of incomplete or insufficient records. They are uncertain of the present because of variable material properties, construction idiosyncrasies, and unpredictable deterioration. They are uncertain about the future because of variable environmental conditions, loads, and maintenance funding. Infrastructure owners are required, however, to make important decisions regarding critical assets within this state of uncertainty. The recognition and proper management of uncertainty is therefore necessary in order to best inform decision making, reduce exposure to risk, and guarantee continual functioning of infrastructure. The acknowledgement of this fact has led to the adoption of uncertainty analysis using mathematical formulations to represent

uncertainties, resulting in a quantitative analysis of uncertainty based on probability theory.

Probability theory-based methods used in the analysis of uncertainty are difficult to apply in situations characterized by ignorance, where lack of information makes estimates of initial probabilities or probability distributions to use in the quantitative analysis difficult to justify (Faes et al. 2019; Shafer 2016). The influence of ignorance on uncertainty is recognized by the definition of two types of uncertainty: aleatory and epistemic uncertainty. Aleatory uncertainty is the uncertainty pertaining to randomness and chance, also referred to as irreducible uncertainty, stochastic uncertainty, and variability uncertainty. Aleatory uncertainty is defined as the inherent variation associated with a system (Oberkampf et al. 2002). Epistemic uncertainty is the state of imperfect knowledge that arises from ignorance and has been de-fined as a potential inaccuracy in any activity that is due to lack of knowledge (Oberkampf et al. 2002). Aleatory uncertainty is well addressed by existing probability theory, because of the applicability of probability distributions to model the different states of the system (Oberkampf and Helton 2002). Epistemic uncertainty, the state of imperfect knowledge, however, remains difficult to accurately analyze using probability theory. This is because judgments based on probability theory suggest there is precise information not only about the event itself, but also about its contrary, which is often not appropriate in cases of limited quantitative knowledge (Corotis 2015).

Novel methods of uncertainty assessment are needed to address this shortcoming of probability theory. One field that inspires the development of novel methods of uncertainty assessment is the field of Artificial Intelligence (AI), notably in the development of knowledge representation in expert systems. In fact, the application of algorithms developed for use in expert systems could provide significant improvements in engineering system reliability and design (Bonissone 1989). One framework of assessing uncertainty is evidence theory, also

known as Dempster-Shafer theory or the theory of belief functions. Evidence theory was originally conceived in the 1970s (Dempster 1968; Shafer 1976), and saw initial applications and concept development within the AI community. Evidence theory has recently seen expanded applications to practical problems typically addressed by traditional engineering methods (Behrouz and Alimohammadi 2018; Seites-Rundlett et al. 2022a). Despite some practical applications (Behrouz and Alimohammadi 2018; Xu et al. 2018; Zhou et al. 2018), Evidence Theory still lacks clear methods of application (Seites-Rundlett et al. 2022b). Evidence Theory requires further guidance regarding data combination methods and additional practical applications, such as structural design, in order to expand its applications and use within the infrastructure design community.

The motivation of this paper is (i) introducing appropriate roles for uncertainty theories and their associated relevance, (ii) developing guidance and methods for data combination applications using evidence theory, and (iii) developing evidence theory applications within structural design. This paper echoes the call during recent times for an improved understanding of uncertainty, particularly as it relates to infrastructure design (Gardoni and LaFave 2016). An improved understanding of uncertainty needs to address the inability of probability theory to adequately capture incomplete knowledge, i.e., epistemic uncertainty. Evidence theory is one such method that offers the ability to systematically incorporate incomplete knowledge into an analysis. This paper will explore evidence theory through a critical review of practical applications of evidence theory in published research, in order to guide future research into the use of evidence theory for infrastructure systems. In so doing, this paper will identify a direction for future research into practical applications of evidence theory.

2 BACKGROUND

2.1 Aleatoric and Epistemic Uncertainty

The assessment of uncertainty continues to be dominated by the application of probability theory. The assessment of uncertainty, however, addresses both aleatoric and epistemic uncertainty. Aleatoric uncertainty utilizes probability theory to address inherent variation or the randomness of input data. The possible states of this variation can be modeled with a specified distribution once sufficient information or prior knowledge of the system is available to estimate the distribution (Agarwal et al. 2004). Aleatoric uncertainty is described as irreducible because once the distribution is well known, obtaining more information will not change the range or shape of the distribution. The applicability of probability theory to assess aleatoric uncertainty has resulted in probability theory as the preferred method of addressing all uncertainty, including epistemic uncertainty (Agarwal et al. 2004; Behrouz and Alimohammadi 2018; Khalaj et al. 2018; Oberkampf et al. 2002).

Epistemic uncertainty, however, remains difficult to incorporate into probability theory because lack of information makes it difficult to estimate a distribution and its parameters. These limitations have been widely recognized and are summarized well by Martz and Waller (1988) “the quality and quantity of more-or-less relevant available data for use in a probabilistic safe-ty assessment is almost never of the precise form and format required for using classical statis-tical methods.” The recognition of these limitations has fostered a debate over the proper method of calculating probability and performing statistical inference. The debate originally focused on the difference between the classical objective frequentist interpretation of probability and the Bayesian subjective degree of belief interpretation of probability (Martz and Waller 1988). The classical frequentist interpretation considers that probability is objective and represents the long-run relative frequencies of events. The Bayesian subjectivist interpretation considers probability as

a subjective degree-of-belief about events, based on available information and reasoning.

Many have advocated for the application of the Bayesian interpretation to treat epistemic uncertainty. Arguments supporting the applicability of the Bayesian interpretation include the inability of the frequentist interpretation to treat low probability, high consequence events, the uncertainty present in estimating parameters for distributions, and the ability to incorporate additional sources of information such as expert opinions or qualitative assessments (Martz and Waller 1988). These arguments have resulted in epistemic uncertainties traditionally modeled as a random variable with subjective probability distributions (Oberkampf and Helton 2002).

The Bayesian interpretation is a recognized improvement on the frequentist interpretation in many applications of uncertainty analysis (Apostolakis 1990). This approach, however, has still been challenged as inadequate for the treatment of epistemic uncertainty (Faes et al. 2019; Shafer 2016). Challenges to the Bayesian interpretation include the existence of mental biases present in calculating subjective probabilities (Capen 1976; Tversky and Kahneman 1974), the inability to quantify ignorance (Khalaj et al. 2018), and the lack of complete prior knowledge makes the choice of an initial distribution and parameters difficult to justify, often contributing to misleading results (Agarwal et al. 2004; Oberkampf and Helton 2002). Recognition of these limitations motivated research into a broader conception of uncertainty and the development of new statistical methods to apply to epistemic uncertainty (Corotis 2015). One such method is evidence theory.

2.2 Evidence Theory

One method of assessing epistemic uncertainty is evidence theory, also known as Dempster-Shafer theory or the theory of belief functions. The theory was conceptualized initially by Dempster (1968) who interpreted statistical inference based

on the concepts of upper and lower probabilities, as opposed to the confidence intervals developed by Neyman (Lehmann 2011). The theory was then further developed by Shafer (1976) with his introduction of a theory of evidence based on belief functions. Dempster had interpreted upper and lower probabilities as bounds on degrees of knowledge, however Shafer interpreted these upper and lower probabilities as bounds on degrees of belief and renamed these limits belief functions. The historical development of the theory, including a collection of published research critical to its development, is provided by Yager and Liu (2008).

Evidence theory is often described as a generalization of the Bayesian subjective degree of belief interpretation. A distinguishing feature of evidence theory is that belief functions allow the calculation of three probabilities for any given event: the probability for, the probability against, and the probability of don't know (i.e., ignorance) (Dempster 2008). This explicit recognition of ignorance as a probability to quantify is a special feature of evidence theory, as the probabilities for and against (i.e., its complement) for a given event must sum to unity in probability theory. Another beneficial feature of evidence theory is the calculation of probabilities on the power set (all possible sets) of potential outcomes. The calculation of probabilities on sets allows information to be applied to a set of events, without a complete distribution to individual events themselves, as is done in probability theory. The mechanics of the theory are best summarized by Shafer (2016): "The theory of belief functions suggest that we try break our evidence down into such relatively simple components that we make probability judgements separately on the basis of each of these components, and that we then think about how to combine these judgements."

The computational complexity and demand required to compute belief functions on power sets of many variables was not practical when the theory was first proposed in the 1970s (Reineking 2014). These computational

limitations kept initial practical applications of evidence theory in the field of artificial intelligence, i.e., the development of expert systems (Bonissone 1989). Fortunately, recent advances in computational power and the development of algorithms have made evidence theory more practically viable. These advancements include the application of Monte Carlo simulation to approximate belief functions and the capability to share and expand evidence theory calculation packages using open source software, such as Python (Behrouz and Alimohammadi 2018; Reineking 2014). This new-found computational viability has seen an expansion of the application and exploration of evidence theory to practical engineering problems in recent research. Although, evidence theory has yet to be employed in actual engineering applications, as its application has been mostly confined to conceptual re-search.

2.3 *Practical Applications of Evidence Theory*

Evidence theory has seen many uses for uncertainty analysis in engineering applications in recent years. The following section provides an example of many of these practical applications. The list is not comprehensive nor complete, but provides an overview of practical applications of evidence theory. These applications cover many topics important to an infrastructure asset owner, including system reliability, structural assessment, natural hazard impact as-assessment, and multicriteria optimization.

Initially, evidence theory was primarily applied to engineering system safety and reliability. Bogler (1987) looked at evidence theory for the fusion of data from multiple sensors on an aircraft. Inagaki (1993) looked at the use of evidence theory in decision making using the Challenger space shuttle explosion as an example. Hester (2012) analyzes aircraft maintenance times by combining expert opinions of failure sources using evidence theory.

Recently, evidence theory has been incorporated into aspects of structural capacity

assessments. Ballent et al. (2019) apply evidence theory in the estimation of structural damages following an earthquake by combining estimates of the extent of damage by different groups. Bao et al. (2012) experimentally analyze a rigid truss structure and demonstrate the applicability of evidence theory in the process of damage detection, by using evidence theory to combine the damage at multiple elements in the truss to quantify overall damage. Xu et al. (2018) predicted vortex-induced vibration of bridges by combining wind sensor data across the span of a bridge using evidence theory. Fetz et al. (2000) explored to application of evidence theory to uncertain parameters included in finite element models of foundations.

Evidence theory has even seen particularly beneficial applications to natural hazard assessment. Alim (1988) explored the use of evidence theory in seismic analysis, motivated by the inherent imprecision of seismic parameters and the frequent use of linguistic labels to confer quantitative data. Behrouz and Alimohammadi (2018) analyze flood design problems by first calculating design discharge using probability theory and then comparing the results to a range for design discharge calculated using evidence theory, citing the design advantage of applying the ranges created using evidence theory to communicate information about possible flood levels.

Evidence theory has also been applied to multi-criteria optimization problems. Agarwal et al. (2004) apply evidence theory to optimization, using belief functions as constraints in an example sizing an aircraft subject to performance requirements. Chen and Rao (1998) apply evidence theory to multi-criteria optimization as well, analyzing a four-bar mechanical linkage for an optimum path of travel. Fetz et al. (2000) analyze queuing times for transport vehicles given constraints on excavator capacity.

The discussion above demonstrates the numerous broad applications of evidence theory to engineering systems. Evidence theory, however, has yet to be applied to many

applications relevant to infrastructure design, as most applications simply combine incomplete information using evidence theory. Recent research has produced many other practical and applied methods of uncertainty analysis, but evidence theory has not yet been used in conjunction with these methods. These other methods of uncertainty analysis include Markov models of deterioration (e.g. Corotis et al. (2005)), multicriteria optimization (e.g. Bocchini and Frangopol (2012)), and risk-based asset management (e.g., Yang and Frangopol (2019)), which have been applied in performing analyses of infrastructure network and structural reliability.

Network and structural reliability analyses, however, must incorporate significant epistemic uncertainty, which limits the utility of current methods of uncertainty analysis. The presence of epistemic uncertainty in these analyses has changed the analytic paradigm away from an assumption of a stationary and prescribed future and motivated the development of methods of uncertainty analysis that can account for the existence of multiple plausible futures (Yang and Frangopol 2019). Research in structural reliability has led to the development of many innovative and practical methods of accounting for uncertainty, but still this research has yet to explicitly explore the broad benefits of evidence theory to improve these methods. The application of evidence to practical problems has been mostly limited to the fusion of data using evidence theory's rule of combination, i.e., Dempster's rule of combination (Altieri et al. 2017; Ballent et al. 2019; Hou 2021; Souza et al. 2016; Xu et al. 2018).

3 DEVELOPMENT OF DATA COMBINATION METHODS FOR FUTURE APPLICATIONS

3.1 *Data Combination in Evidence Theory*

Practical applications of evidence theory in research have demonstrated the beneficial ability of evidence theory to combine information from different sources (Ballent et al. 2019; Xu et al.

2018). The ability to incorporate data not restricted by the constraints of probability theory offers an opportunity for expanded analysis in cases of high epistemic uncertainty. Evidence theory has the capability to incorporate expert opinions into quantitative analysis (Hester 2012). Evidence theory provides a framework to combine expert estimates to address in-complete data using set-based mathematics. The benefit of evidence theory in this case is the explicit acknowledgment of the use of experts and a systematic justification for how their beliefs are incorporated into assessments.

One of the primary differences between probability theory and evidence theory is in the combination of evidence. This difference will be illustrated using the example provided by Xu et al. (2018). In the example, the probabilities of vortex-induced vibration from three sensors were 67.7%, 59.4%, and 64.3%. The combination of these probabilities using probability theory would result in the application of an averaging method, such as the arithmetic mean or the geometric mean. The averaging method would produce an estimate of the overall probability of vortex-induced vibration that is not greater than the maximum probability calculated by any single sensor, in this case 67.7%. The combination rules of evidence theory allow each piece of evidence to be represented by a separate belief function. Each belief function is then combined to capture the total belief provided by the sensors. The result of the combination using evidence theory is an estimated probability of 84.6% for vortex-induced vibration. This result is greater than the probability calculated by any single sensor, demonstrating how aggregation of belief in the vortex-induced vibration resulted in a probability greater than that calculated by any single sensor.

The combination rule of evidence theory, however, has been criticized for its inability to intuitively combine conflicting evidence (Reineking 2014). This criticism was first raised by Zadeh (1979), by providing an example where two doctors are attempting to diagnose a patient given three diseases, A, B and C. Doctor one

diagnoses a 99% probability of disease A and a 1% probability of disease B. Doctor 2 diagnoses a 1% probability of disease B and a 99% probability of disease C. The combination rules of evidence theory result in the conflicting evidence nullifying any belief in disease A and C, resulting in a 100% probability of disease B.

The non-intuitive nature of Zadeh's (1979) combination result has motivated research into new combination schemes for evidence theory. Some popular combination schemes include Yager's rule of applying all conflicting evidence to the universal set or the conjunctive rule of not applying a normalization factor to conflicting evidence (Yager and Liu 2008). Several additional rules have been proposed, however, no combination rule has been universally accepted or found to be devoid of any non-intuitive results. This is not because already proposed rules are all deficient, but rather each combination method proposed offers a tradeoff between analytical precision and explicit recognition of ignorance. The default combination rule in evidence theory reassigns belief associated with conflict, producing a convergence of belief on certain outcomes (Seites-Rundlett et al. 2022b). Alternative rules, such as Yager's rule, retain belief associated with conflict in ignorance, thereby explicitly recognizing the potential for epistemic uncertainty in the analysis. The application of several combination rules, therefore, provides a means of sensitivity analysis and evaluation of output from certain methods (Seites-Rundlett et al. 2022b). Future research providing guidance on combination rules selection addressing specific engineering applications of evidence theory are needed to both demonstrate the viability of the theory and motivate further practical applications of evidence theory.

3.2 Evidence Theory to Address Weaknesses in Current Uncertainty Analysis Methods

Evidence theory offers a promising framework that can be used in conjunction with recently developed practical methods of uncertainty

analysis used in the assessment of infrastructure network and structural reliability. These methods have dramatically improved and expanded our ability to assess uncertainty, yet they still have recognized weaknesses in their assessment of epistemic uncertainties.

Markov models, for example, have such weaknesses, which could be tackled using evidence theory to quantify partial observability. Corotis et al. (2005) discusses the tradeoff between accuracy and cost of data collected to inform Markov models. Markov models, however, cannot be used to provide an assessment of the net value of information gathered from different sources and future costs to obtain information in order to best determine a management policy of an infrastructure asset. Evidence theory provides a framework that can enhance the analysis of potential information sources and update estimates on future cost and information needs as partial evidence becomes available. Research in other fields has explored the use of evidence theory in Markov models, particularly hidden Markov models (Pieczyński 2007). Despite these advances, limited research has yet been conducted into practical applications of evidence theory in Markov models of deterioration commonly used in infrastructure asset management.

Multicriteria optimization is another example of a method that could be enhanced with the use of evidence theory. A recognized deficiency of optimization is that the outcome can be highly dependent on a few key parameters, such as discount rate in the case of cost-based optimization (Yang and Frangopol 2019). Evidence theory offers the possibility of modeling these key parameters with belief functions and updating their values, which could prove more useful than traditional sensitivity analysis. Furthermore, evidence theory has been applied in research to multicriteria optimization (Agarwal et al. 2004; Chen and Rao 1998), yet not in these infrastructure applications.

Finally, evidence theory offers the ability to combine improved assessments of

infrastructure network reliability in regard to natural hazards, the analysis of which introduces many un-certainties. The presence of significant epistemic uncertainty in hazards means that linguistic or qualitative data are often the best data available to describe certain processes or parameters (Alim 1988). The use of evidence theory can address these uncertainties and data restrictions through the incorporation of new sources of information. For example, Souza et al. (2016) fuse the evidence provided by tweets with historic traffic data in order to update traffic flow predictions. This method is recognized to be particularly useful during times of disaster, when historic data are less reliable for predictions (Souza et al. 2016). This application could also be expanded to address more complex problems, such as evacuations. Indicators of social vulnerability that have been shown to affect evacuation behavior are known to be excluded from evacuation models (Seites-Rundlett et al. 2020). The incorporation of these indicators into models of evacuation behavior can improve evacuation planning and community vulnerability analyses. Additionally, output from transportation network analyses (e.g., Bocchini and Frangopol 2012 a, b) could be expanded from quantitative measures related only to delay times and population affected by the incorporation of affected population vulnerability.

3.3 Future Research in Evidence Theory

Evidence theory offers many benefits to the analysis of uncertainty. However, there remain some areas of the theory which could be improved upon, in order to facilitate the real application of evidence theory to structural design. One of the primary differences between commonly applied probability theory-based methods and evidence theory-based methods is in the collection and processing of data (Seites-Rundlett et al. 2022b). Probability theory-based methods require one to pre-process data in order to address imprecise or incomplete data included in the analysis. Evidence theory-based methods, however, utilize

the imprecision and incompleteness of data to assign belief to the appropriate sets. One important branch of future re-search, therefore, is continued application to real practical problems in order to establish procedures and guidance to facilitate future applications. Well defined methods of applying evidence theory need to be developed for specific fields and tasks.

One of the primary areas of research is in the use of evidence theory to make decisions (Yager and Liu 2008). Significant efforts have been made to identify decision making procedures using evidence theory when making decisions under ignorance, i.e., epistemic uncertainty (Yager and Liu 2008). These efforts have focused on how to use belief functions to make decisions. A common method developed with evidence theory is the concept of pignistic transformations, which use belief functions to maximize expected utility (Smets 2005). The output of a pignistic transformation is defined as a probability that can be used in decision making as if one were placing a wager (Smets 2005). The original concept of evidence theory did not consider the problem of decision making essential, and no particular method of decision making has been proposed. Nonetheless, the original theory has been applied to multiple-criteria optimization and the creation of confidence intervals, which can be used to inform decisions (Dempster 2008).

Moreover, given the variety of calculations that can be performed with evidence theory and the variety of decisions that are supported by existing probabilistic methods, the creation of general methods for using evidence theory for decision making can be difficult to explain and justify to practitioners. Therefore, in order for evidence theory to be used in infrastructure systems, specific outcome-oriented decision-making procedures using evidence theory are needed.

4 FUTURE APPLICATIONS OF EVIDENCE THEORY IN STRUCTURAL DESIGN

4.1 Application of Evidence Theory Suitable to Risk Management and Communication

The day-to-day side of structural design requires the acceptance of a great deal of epistemic uncertainty. For example, epistemic uncertainty could stem from uncertain material quality, weld strength, or loads. One must generally accept these uncertainties, however, to complete daily tasks of structural analyses and calculations. But at an organizational or societal level, one must take the time to develop methods that analyze these epistemic uncertainties in order to mitigate disasters and structural losses over the long term. Therefore, there is a need to develop alternative methods of uncertainty analysis, such as evidence theory, in order to guaran-tee long-term robustness and resilience of structural designs.

The development of alternative methods of uncertainty analysis is hindered by the both the extra effort required in analysis and the perceived potential for overdesign. The identification and explicit recognition of epistemic uncertainty will always require extra effort during engineering analysis and design. Furthermore, the novelty and difference in application compared to probabilistic methods (Seites-Rundlett et al. 2022b) will require extra time and quality control efforts when applying alternative methods of uncertainty analysis. Furthermore, since evidence theory and alternative methods of uncertainty analysis are often used to analyze low frequency events, there is a perceived potential for overdesign. Modern structures are increasingly designed up to their performance limits, and therefore there is an emphasis on recognizing conservative design philosophies and optimizing structural designs (Faes and Valdebenito 2020). This highlights, however, the role of these alternative methods to characterize risk, as they can evaluate potentially overlooked risks and potentially underestimated design capacity.

The practical application of evidence theory, therefore, may not improve upon current ana-lytic or computational aspects employed within structural design. However, the explicit

recognition and quantification of epistemic uncertainty, enabled by applying evidence theory, could improve risk management and the ability of engineers to communicate risk to the public. This is because of the ability of evidence theory to incorporate epistemic uncertainties pertaining to future events, for which one only has incomplete knowledge (Riley et al. 2016). Evidence theory goes beyond simple integrated updating approaches, which compare statistical modeling and observational data (Riley et al. 2016), and improves upon probabilistic outputs created from incomplete data sources. The application of Evidence Theory is recognized to facilitate modeling and processing uncertainty (Beer et al. 2013). Evidence Theory can also be applied to better capture mental habits and characterize uncertainty, reliability, and conflicting evidence (Huang et al. 2017). Communication of risks arising from identifying epistemic uncertainty can facilitate communication between engineers, various stakeholders, and the public.

4.2 *Dynamic Loads*

Significant epistemic uncertainty can arise when evaluating loads, and particularly dynamic loads. Epistemic uncertainty stems from both the characterization of loadings, including intensity, duration, point of application, and the unknown possibility of certain loading conditions, such as a change in building use or unanticipated natural hazards. Since many loading conditions can be characterized by epistemic uncertainty, evidence theory is a suitable method of comprehensively evaluating potential loading conditions over the design life and beyond of a structure. The application of evidence theory can provide richer context to an analysis of loading conditions. This context provides additional information concerning risk by quantifying epistemic uncertainty, which can inform future redevelopment or rehabilitation plans.

The combination of data using evidence theory is motivated by the logical process of gathering evidence to determine an uncertain

outcome (Shafer 2016). Evidence theory evaluates uncertainty by gathering and collecting data to update initial beliefs. Therefore, evidence theory is a suitable method to evaluate and update initial beliefs and evaluate organizational performance over the long-term. Potential uncertainties could be analyzed to determine the possibility of a given loading condition inducing peak stresses and deflection. Although such a possibility may not be determined by probabilistic approaches, the identification of related epistemic uncertainty, can identify the risk and allow for discussion for the need of mitigating measures.

4.3 *Performance Based Structural Design*

Evidence theory has seen many applications in system reliability. Evidence theory applications therefore offer an opportunity to evaluate the reliability of each element of a structural system, for example reliability of the loads or reliability of the materials. Evidence theory, therefore, has potential for application to performance based structural design. Epistemic uncertainties that could affect the achievement of performance objectives can be identified and analyzed within an evidence theory application. Furthermore, these initial beliefs concerning performance objectives can be evaluated and updated over time. Such an analysis can influence the management of infrastructure over its design life and provide guidance on potential maintenance and design modifications.

Evidence Theory also has potential applications in reliability-based optimization. One important aspect of reliability-based optimization is the nested nature of both deterministic optimization and reliability/risk analysis. The decoupling of these nested problems can reduce computational complexity and the number of analyses required (Faes and Valdebenito 2020). The decoupling of the reliability and optimization analysis is also an area of application for Evidence Theory. Evidence Theory is well suited for reliability-based optimization based on its generalizability (Huang et al. 2017). If

information is complete and does not conflict, the results of an evidence theory analysis will generalize to the same results as a probabilistic approach. The versatility of evidence theory facilitates its use in reliability-based optimization, both in the evaluation of reliability characterized by epistemic uncertainty and the optimization analysis characterized by a need for a more deterministic approach.

5 CONCLUSION

As entities responsible for continuous functioning of infrastructure, engineers have a mandate to account for uncertainty in order to ensure the ultimate sustainability of communities and the built-environment included therein. However, in order to ensure the adequacy of community infrastructure, engineering decision-makers need novel methods of assessing uncertainty. Evidence theory offers one promising method, although there is still a need for further research into the use of evidence theory to make decisions and combine sources of conflicting evidence. The improvement of these concepts can lead to the creation of an applied and practical form of evidence theory that can transform the safety and security of our built environment and our understanding of uncertainty.

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