Active learning-based structural design optimization under constraints on first-passage probability

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ABSTRACT: In efforts to manage the risk of unexpected failures caused by stochastic loads, e.g., earthquakes and wind loads, the first-passage probability often needs to be evaluated. With the growing complexity of modern engineering systems, estimating the first-passage probability with low computational costs in the structural design process is essential. This paper presents a new active learning-based framework to incorporate constraints on the first-passage probability into reliability-based design optimization (RBDO) of stochastic dynamical systems. An alternative mixture-distribution-based formulation of the first-passage probability is utilized to handle the high-dimensional sequences of stochastic excitations during the optimization. The design parameter sensitivity of the first-passage probability is introduced to facilitate the use of a gradient-based optimizer in the RBDO iterations. These procedures employ heteroscedastic Gaussian process-based surrogates and active learning scheme to reduce the high computational costs in the first-passage probability estimation. The numerical example dealing with the optimal design of an eight-story building system subjected to stochastic wind excitations demonstrates the accuracy and efficiency of the proposed method.

1. INTRODUCTION

Finding the optimal design of a structural system concerning safety, cost or performance is one of the most essential tasks in engineering practice. The optimal design should achieve the primary design objectives of managing reliability operation even under stochastic loads caused by natural and human-made hazards, e.g., wind loads and earthquakes. Therefore, appropriate strategies are required for uncertainty quantification and optimization in the structural design process.

To this end, reliability-based design optimization (RBDO) has received significant attention in recent years (Dubourg et al. 2011; Kim 2022). RBDO aims to achieve optimal structural designs satisfying the probabilistic constraints on the structural performance indicators. In particular, when the structural systems are subjected to stochastic excitations, the first-passage probability, i.e., the probability that the maximum structural response exceeds a prescribed threshold over a given time interval, has been widely adopted as a critical reliability measure (Yi and Song 2021). Thus, structural design optimization under constraints on first-passage probability needs to be investigated to assure the target reliability of stochastic dynamical systems in the design process.

However, obtaining a reliable optimal design is challenging since RBDO needs to evaluate the system's performance repeatedly. Furthermore, the evaluation of the first-passage probability given design parameters usually should handle a large number of random variables. Several methods such as simulated-based approaches (Dubourg et al. 2011; Suksuwan and Spence 2018), surrogate-based methods (Zhang et al. 2017; Kim and Song 2021), and system-reliability-based scheme (Chun et al. 2019), have been proposed to alleviate the computational burden of RBDO. Their applications to practical problems, however, still require a substantial number of model evaluations. Therefore, minimizing the number of system performance evaluations is an essential task with the growing complexity of today's structural systems.

To overcome the technical challenges, this study proposes a new active learning-based method for RBDO under constraints on the first-passage probability. An alternative mixture-distribution-based formulation of the first-passage probability is utilized to handle the high-dimensional sequences of stochastic excitations during the optimization. A sampling-based design parameter sensitivity of the first-passage probability is introduced to facilitate the use of a gradient-based optimizer in the RBDO iterations. In addition, the heteroscedastic Gaussian process (HGP) surrogate model and its active learning process are employed to reduce the high computational costs in the first-passage probability estimation. A numerical design example of an eight-story building structure subjected to stochastic wind excitations demonstrates the performance and merits of the proposed method.

The paper first provides a brief overview of the general RBDO problem subjected to stochastic loads. Section 3 provides the detailed procedures of the proposed active learning-based RBDO approach. Section 4 presents the numerical investigations of the proposed method through an eight-story building system. Lastly, the paper concludes with a summary and discussion.

2. PROBLEM DEFINITION

The conventional RBDO can be formulated as follows:

$$\boldsymbol{\theta}_* = \underset{\boldsymbol{\theta} \in \boldsymbol{\Omega}_{\boldsymbol{\theta}}}{\operatorname{argmin}} \; f(\boldsymbol{\theta})$$
$$subject\ to\text{:}\ P_{f_i}(\boldsymbol{\theta}) \leq P_{f_i}^t, \quad i = 1, \dots, n_c \quad (1)$$

where $\boldsymbol{\theta}$ denotes the vector of design parameters used to define the structural system, which often includes the mean vector of the random variables; $f(\boldsymbol{\theta})$ denotes the cost function representing the objective of optimizing the structural system; $P_{f_i}(\boldsymbol{\theta})$ and $P_{f_i}^t$ respectively denote the probability of i-th failure mode with given design parameters and the corresponding threshold value of the failure probability associated with the target reliability of the structural system, $i = 1, \dots, n_c$; and $\boldsymbol{\Omega}_{\boldsymbol{\theta}}$ is an admissible set of $\boldsymbol{\theta}$.

In general, the assessment of the performance constraints in Eq. (1) requires the evaluation of the following multidimensional integral:

$$P_f(\boldsymbol{\theta}) = \int \int_{g(\boldsymbol{X},\boldsymbol{Z}) \leq 0} f_{\boldsymbol{X}|\boldsymbol{\theta}}(\boldsymbol{x}; \boldsymbol{\theta}) f_{\boldsymbol{Z}}(\boldsymbol{z})\, d\boldsymbol{z}\, d\boldsymbol{x} \quad (2)$$

where $f_{\boldsymbol{X}|\boldsymbol{\theta}}(\boldsymbol{x}; \boldsymbol{\theta})$ and $f_{\boldsymbol{Z}}(\boldsymbol{z})$ are the probability density functions (PDFs) of random vectors $\boldsymbol{X}$ and $\boldsymbol{Z}$, respectively; $\boldsymbol{X} \in \mathbb{R}^{n_x}$ denotes the vector of *basic* random variables including time-invariant random parameters associated with the structural systems and hazard model, e.g., damping coefficients, story stiffness, and basic wind speed; $\boldsymbol{Z} \in \mathbb{R}^{n_z}$ denotes the vector of *environmental* random variables often used to represent the external stochastic loads in time or frequency domain, e.g., the sequence of random phase angles in a random process model. Note that the environmental random vector $\boldsymbol{Z}$ is usually not considered as design parameters. Meanwhile, the design parameters may include the distribution parameters of the basic random variables in $\boldsymbol{X}$.

The limit-state function for the structural performance of interest, $g(\cdot)$ can be written as:

$$g(\boldsymbol{X}, \boldsymbol{Z}) = u_0 - \max_{0 < t \leq \tau} |u(\boldsymbol{X}, \boldsymbol{Z}, t)| \quad (3)$$

where $u(\cdot)$ is the structural response time history, which is affected by the two vectors of random variables $\boldsymbol{X}$ and $\boldsymbol{Z}$; u_0 is the prescribed threshold on the structural response; $|\cdot|$ denotes absolute value expression; τ is a time duration considered for the evaluation. Note that the random vector $\boldsymbol{Z}$ in Eq. (2) and (3) usually incorporates the high-dimensional sequence of stochastic excitations and eventually makes the performance assessment a high-dimensional stochastic problem.

3. PROPOSED ACTIVE LEARNING-BASED RBDO METHOD

3.1. Mixture-distribution-based formulation of first-passage probability

The estimation of the first-passage probability in Eq. (2) requires a high-dimensional integral at each design optimization iteration, which may entail prohibitive costs of computational simulations. Thus, this study adopts an alternative formulation of the first-passage probability recently proposed by Kim et al. (2023). The probability of failure in Eq. (2), without loss of generality, can be expressed as

$$P_f(\boldsymbol{\theta}) = \int_{\boldsymbol{x}\in\mathbb{R}^{n_x}} P_{f|\boldsymbol{X}}(\boldsymbol{x}) f_{\boldsymbol{X}|\boldsymbol{\theta}}(\boldsymbol{x};\boldsymbol{\theta})\, d\boldsymbol{x} \quad (4)$$

where $P_{f|\boldsymbol{X}}(\boldsymbol{x})$ is the conditional first-passage probability given basic random variables, where the peak response is typically approximated as a lognormal random variable in the performance-based engineering framework (Ellingwood 2004; Kim et al. 2021). Thus, the first-passage probability in Eq. (2) can be estimated using Monte Carlo (MC) integration as

$$\hat{P}_f(\boldsymbol{\theta}) \cong 1 - \frac{1}{n_k}\sum_{k=1}^{n_k} \Phi\left(\frac{\ln u_0 - \lambda(\boldsymbol{x}_k)}{\zeta(\boldsymbol{x}_k)}\right) \quad (5)$$

where $\boldsymbol{x}_k$ is k-th random sample generated from $f_{\boldsymbol{X}|\boldsymbol{\theta}}(\boldsymbol{x};\boldsymbol{\theta})$, $k = 1, \dots, n_k$; $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard Gaussian distribution; $\lambda(\cdot)$ and $\zeta(\cdot)$ are respectively the mean and standard deviation of the natural logarithm of the peak response $\mathcal{M}(\boldsymbol{x},\boldsymbol{z}) = \max_{0<t\le\tau}|u(\boldsymbol{X},\boldsymbol{Z},t)|$. Figure 1 illustrates the alternative formulation of first-passage probability.

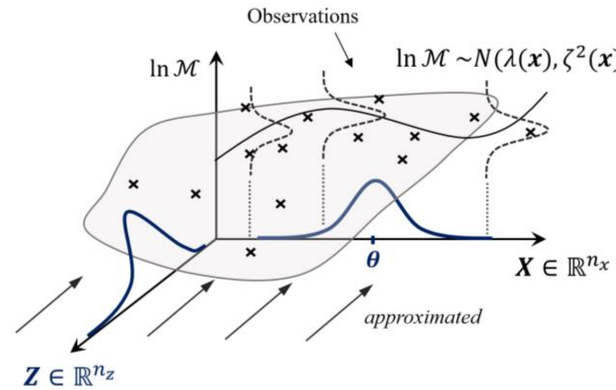

Figure 1: Illustration of alternative formulation of first-passage probability given design parameters

3.2. Design parameter sensitivity of the first-passage probability

In RBDO, the design parameter sensitivities of probabilistic constraints are required to use an efficient gradient-based optimization scheme. To derive the sensitivity of the first-passage probability with respect to the design parameters, which is often defined as distribution parameters, e.g., mean and standard deviation, the score function approach is adopted in this study (Lee et al. 2011).

Since the alternative formulation of the first-passage probability in Eq. (5) is expressed as a function of only time-invariant random vector $\boldsymbol{X}$, and not the high-dimensional random vector $\boldsymbol{Z}$, the design parameter sensitivity of the first-passage probability for i-th design parameter can be approximated by MC samples as

$$\frac{\partial \hat{P}_f(\boldsymbol{\theta})}{\partial \theta_i} = \frac{1}{n_k}\sum_{k=1}^{n_k}\left[1 - \Phi\left(\frac{\ln u_0 - \lambda(\boldsymbol{x}_k)}{\zeta(\boldsymbol{x}_k)}\right)\right] s_{\theta_i}(\boldsymbol{x}_k;\boldsymbol{\theta}) \quad (6)$$

where $s_{\theta_i}(\boldsymbol{x}_k;\boldsymbol{\theta})$ is the score function for $\boldsymbol{\theta}$, whose analytical calculation can be obtained by the joint PDF of the random vector $\boldsymbol{X}$, defined as

$$s_{\theta_i}(\boldsymbol{x};\boldsymbol{\theta}) = \frac{\partial \ln f_{\boldsymbol{X}|\boldsymbol{\theta}}(\boldsymbol{x};\boldsymbol{\theta})}{\partial \theta_i} \quad (7)$$

It should be noted that the score function in Eq. (7) can be computed as a post-processing of the samples generated for the first-passage probability estimation, and thus design parameter sensitivity in Eq. (6) can be evaluated without additional performance evaluations.

3.3. Active learning of heteroscedastic Gaussian process-based surrogates in RBDO

The proposed formulation of the first-passage probability in Eq. (5) and its corresponding design parameter sensitivity in Eq. (6) are determined by the distribution parameter functions $\lambda(\cdot)$ and $\zeta(\cdot)$. Thus, accurate estimations of these parameter functions are essential for valid predictions of the first-passage probability and optimization. The recent study by the authors (Kim et al. 2023) demonstrated that the Gaussian process-based surrogates with heteroscedastic noises can efficiently predict these distribution parameters capturing the variability of stochastic excitation sequence whose influence varies over the input space $\boldsymbol{x}$. The predictions of distribution parameters by HGP models, i.e., $\hat{\lambda}_{HGP}(\cdot)$ and $\hat{\zeta}_{HGP}(\cdot)$ are derived as (Lázaro-Gredilla and Titsias 2011)

$$\hat{\lambda}_{HGP}(\boldsymbol{x}) = \boldsymbol{k}_y^{\mathrm{T}}(\boldsymbol{K_y} + \boldsymbol{R})^{-1} \ln \boldsymbol{\mathcal{M}_{\mathcal{D}}} \tag{8}$$

$$\hat{\zeta}_{HGP}(\boldsymbol{x}) = \sqrt{\exp(\eta + \zeta^2/2) + \alpha^2} \tag{9}$$

where $\boldsymbol{K_y}$ and $\boldsymbol{k_y}$ are covariance matrix and vector of the covariance function, respectively; $\boldsymbol{\mathcal{M}_{\mathcal{D}}}$ is the maximum response observations; $\boldsymbol{R}$ is a diagonal matrix with elements $R_{i,i} = \exp\left(m_i - V_{i,i}/2\right)$, $i = 1, \dots, n$; m_i and $V_{i,i}$ are respectively ith elements of the parametric models; η, ζ and α are a set of parameters described as kernel matrices of latent functions for signal and noises (See Kim et al. 2023 for details). Therefore, the HGP-based predictions of distribution parameters, i.e., $\hat{\lambda}_{HGP}(\boldsymbol{x})$ and , $\hat{\zeta}_{HGP}(\boldsymbol{x})$, in Eq. (8) and Eq. (9), facilitate efficient calculations of the first-passage probability and sensitivity in RBDO.

To increase the computational efficiency for the RBDO of complex engineering systems, the active learning scheme is employed (Kim and Song 2020; Wang and Broccardo 2020). The following learning function is introduced to adaptively train the surrogates:

$$L_\alpha(\boldsymbol{x}) = \Phi\left(\frac{\ln u_0 - \hat{\lambda}_{HGP}(\boldsymbol{x})}{\hat{\zeta}_{HGP}(\boldsymbol{x})}\right) \tag{10}$$

The HGP surrogates are refined by adaptively selecting simulation points that are identified by minimizing the learning function in Eq. (10). This refinement process is repeated n_l times at each optimization iteration.

Thus, the proposed method identifies the reliable optimal solutions by combining the adaptive training process of HGP surrogates with the design optimization procedure guided by the design parameter sensitivities. Figure 2 shows the flowchart of the proposed active learning-based RBDO method.

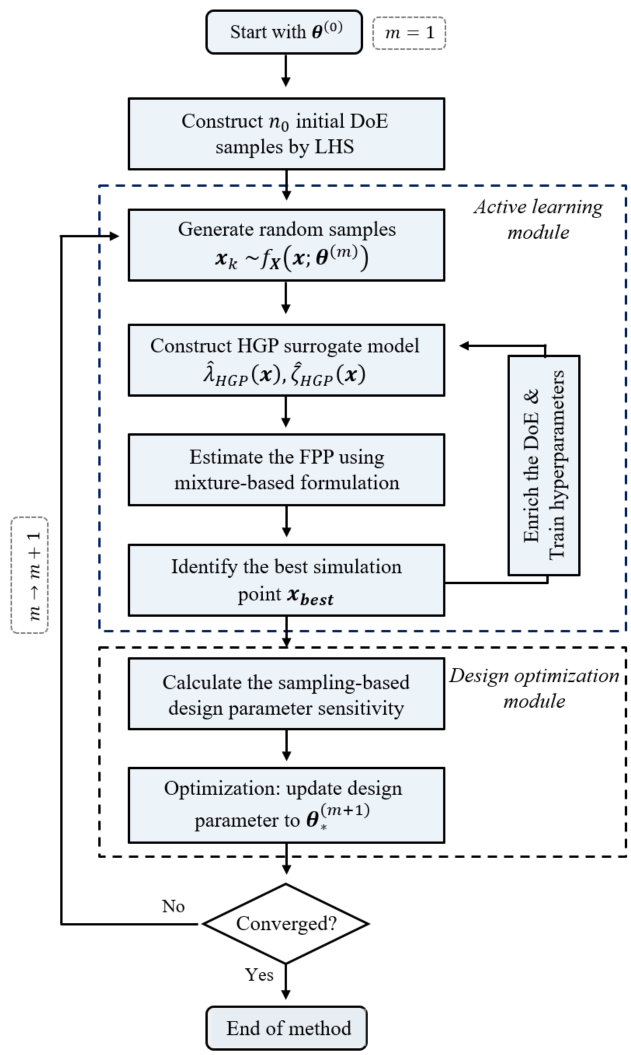

Figure 2: Flowchart of the proposed active learning-based RBDO method

4. NUMERICAL EXAMPLE: VISCOUS DAMPER DESIGN OF AN EIGHT-STORY BUILDING STRUCTURE AGAINST STOCHASTIC WIND EXCITATIONS

4.1. Structure and wind hazard model

To demonstrate the efficiency and accuracy of the proposed method, we investigate a design of viscous dampers of an eight-story building structure showing bilinear behavior under stochastic wind loads (Figure 3). The building structure is subjected to the dynamic forces caused by fluctuating winds. The properties of structural systems are provided in Table 1 (FEMA 2012).

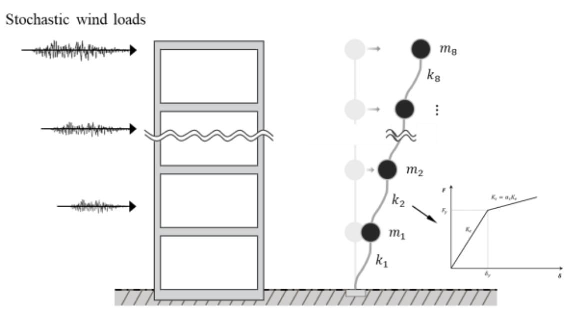

Figure 3: Structural archetype

Table 1: Structural parameters of an eight-story building system.

Parameter	Value
Height (ft)	106
Width (ft)	120
Weight (kip)	12,276
Story elastic stiffness (kip/in)	6,000
Hardening stiffness ratios	0.05
Modal damping ratios	0.02

The stochastic wind hazard model (Suksuwan and Spence 2018) is used to characterize the stochasticity of winds based on the quasi-steady model. The following power spectral density (PSD) model (Kaimal et al. 1972) is employed to generate uncertain components of the wind speed at height h_i:

$$S(\omega) = v_f^2 \frac{a_s h_i}{\bar{v}(h_i)} \frac{1}{\left(1 + b_s \omega h_i / \bar{v}(h_i)\right)^{5/3}} \tag{11}$$

where ω is turbulence frequency; a_s and b_s are the spectrum constants; $\bar{v}$ is the mean wind speed; and v_f is the friction velocity of the wind flow (ASCE 2017). The Davenport coherence model (Davenport 1967) is used to introduce the correlation between the uncertain components of the wind speeds at different heights.

Based on the wind hazard model, the random histories of fluctuating winds are simulated by the spectral representation method (Deodatis 1996). Figure 4 presents the comparison between the target and simulated normalized PSD of wind velocity.

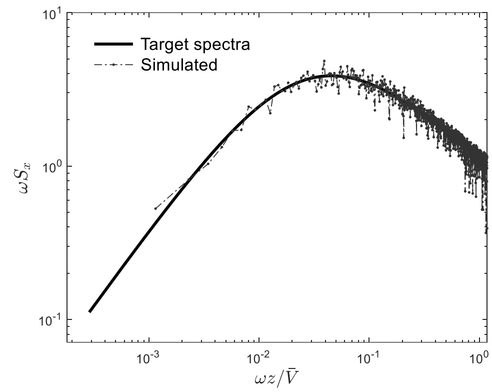

Figure 4: Target and simulated normalized PSD of the wind velocity

4.2. Random variables and performance objective

Suppose linear viscous dampers are considered for the purpose of protection (or retrofitting) of the structure against stochastic wind loads. The ten parameters associated with the structural systems (capacity of dampers) and wind hazard model (basic wind speed and air density) are considered as basic random variables $\boldsymbol{X}$. The mean values of the damper's capacity are considered as design parameters $(\theta_1, \ldots, \theta_8)$ in the problem. The environmental random vector $\boldsymbol{Z}$ includes 16,384 random sequences of standard Gaussian random variables. The target first-passage probability is set as $P_f^t = 6.5 \times 10^{-3}$.

The objective of the design is to minimize the damper costs, which are assumed to be proportional to their mean capacity, while satisfying the constraint on the first-passage probability (Zhang et al. 2017). The response of interest is the eighth-floor (top) relative drift, whose threshold value is $u_0 = 0.11\ (in)$, and the time duration τ is set to 10 minutes.

4.3. Results

After the proposed method is initiated with 150 initial observations by dynamic simulations, five points are adaptively incorporated into the surrogate refinement process at each optimization step. Figure 5 and Figure 6 confirm the cost and design parameters are successfully converged to the final values through a small number of design iterations.

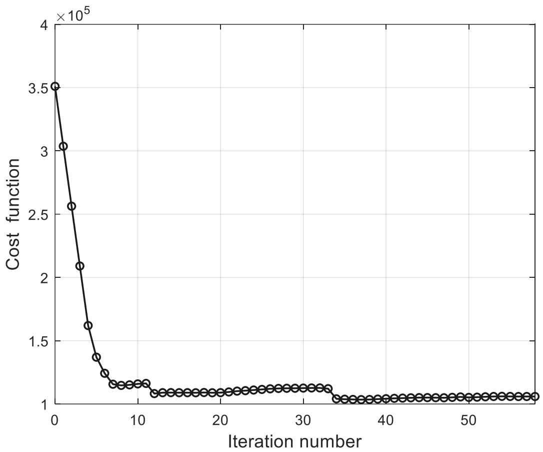

Figure 5: Convergence history of the cost function for the eight-story building example

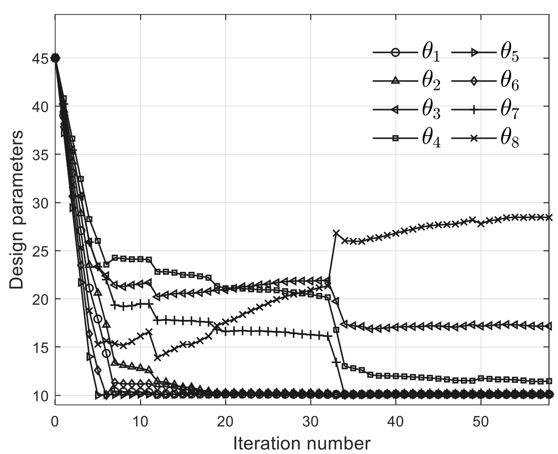

Figure 6: Convergence histories of design parameters for the eight-story building example

Figure 7 shows the convergence history of the first-passage probability estimated by the HGP surrogate model during optimization. The results confirm that the proposed method enables convergence toward the optimal designs satisfying first-passage probability constraints through only 440(=150+58*5) structural performance evaluations.

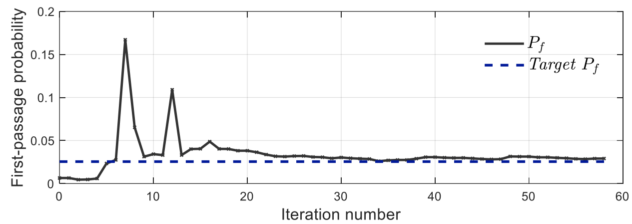

Figure 7: History of the first-passage probability estimated by surrogates for the eight-story building example

5. CONCLUSIONS

This paper proposed a new active learning-based RBDO method to identify the reliable optimal designs of structural systems subjected to stochastic loads. An alternative first-passage probability formulation and its corresponding design parameter sensitivity were introduced to facilitate an efficient gradient-based optimizer in RBDO. The HGP surrogates and its adaptive training process were proposed to enhance computational efficiency. The application to an eight-story building system demonstrated the superb performance and merits of the proposed approach.

6. ACKNOWLEDGMENTS

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