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Fast perturbation-dependent reliability curves in traffic networks

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ABSTRACT: Uncertainty characterisation and reliability analysis for high-fidelity models is often prohibitive due to the large analysis efforts it demands. This is particularly prevalent in highly complex systems that require costly simulations, such as that case of traffic networks. If reliability of traffic networks is to be evaluated for different perturbations, regardless of how it is defined, then prohibitive analysis times and efforts should be expected. Traffic networks are network systems composed of multiple sub-systems and components. When changes in the system intrinsic variables occur, these result in operational changes in the network that can only be understood in an holistic form.
In the present work, a perturbation-dependent fast reliability assessment is proposed. It considers reliability as a variation in travel time to the reference time, which is often used to characterize reliability in traffic. In the present work it is discussed in a full probabilistic context, with reliability curves being characterised using a lower-fidelity model that uses a kriging-based sequential learning approach approach. This metamodeling approach enables the characterisation of different levels of reliability for a perturbation, through a N threshold modelling approach, that uses probability density functions and that sets reliability curves in a form of a fragility curve. With such implementation it is possible to enable a fast characterisation of reliability, and its probabilistic behaviour, in traffic. The implementation is researched in two reference traffic networks with uncertain demands, and results show that this technique can inform multiple purposes of decision-making, ranging from reduced order modelling tools to operational management of the system.

1. Introduction

The probabilistic analysis of traffic involves characterising the influence of its different uncertainties with posterior quantification in a probabilistic form of their effects on traffic performance. This is done under a number of user demand and defined origin destination trips. There are different modelling approaches to traffic, nonetheless, and regardless of the approach used, it can be said that the more realistic traffic models become the more time and effort-consuming is their analysis. This results from increasing complexity in modelling, as well as, larger data availability for the creation of progressively more involved models. In practice, this progressive increase in complexity makes probabilistic approaches quite challenging in practice. As a result there is a demand to enable, not only accurate but also pragmatical modelling techniques that allow to perform highly relevant probabilistic evaluations in systems. Teixeira et al. (2022a) highlighted this before when discussing the relevance of working at different fidelities to explore the full potential of progressively more realistic computational models, *e.g.*, digital-twins of engineering systems. The same rationale is used in (Zhang et al., 2023), where a high-fidelity model is used to extrapolate domains of operational conditions.

In the present work, modelling fidelities are explored to perform a probabilistic evaluation of traffic, enabling damage-dependent reliability curves that would be unpractical due to the sheer number of traffic simulations required to define these. For such, simulated traffic is used to create an uncertainty quantifier that uses a lower-fidelity metamodel, a kriging, that allows, after efficiently defined, to characterize levels of operational reliability in a spectrum of perturbation scenarios at virtually no cost. For this approach to be efficient (in effort and accuracy), and due to the complexity of the simulation model, the approach to construct the lower-fidelity model is key. Hence, an approach to define a lower-fidelity surrogate that uses k-means for exploration and uncertainty in the prediction for exploitation of the uncertain variable space is proposed to support a probabilistic evaluation of traffic. It is then successfully implemented in two representative traffic networks. In both case a user-equilibrium is applied, that despite its medium fidelity, is a proxy for validation of the proposed approach, which is well-suited to more complex traffic models.

1.1. Traffic reliability

Nogal et al. (2019) discusses alternatives to define reliability in the context of transportation. According to Mattsson and Jenelius (2015), reliability in transportation is related to the certainty and predictability of travel conditions, which can vary due to a number of factors, such as, daily demand fluctuations, weather conditions or perturbation with non-natural causes. Transportation network reliability can be measured in different forms and, such as in most reliability problems, the separation between what is failure or not depends on the rationale that is applied. In transport networks, capacity reliability (e.g., (Chen et al., 2002)), travel time reliability (e.g., (Wakabayashi, 2012)) and connectivity reliability (e.g., (Bell and Iida, 1997)) can be identified.

In the present work, reliability is assessed using the travel cost, or travel time reliability (Nogal et al., 2019). Its operational definition is as follows; travel time reliability measures the feasibility that road users reach a destination within a certain travel time under the operating conditions encountered. To measure the reliability of a given Origin-Destination (OD) pair ij or at the network level, the actual travel cost experienced by users travelling from origin(s) i to the destination(s) j, t_{ij}, is compared with the associated travel time in free flow conditions, t_{ij_0}.

$t_{ij}(x)$ is the function that defines the travel cost t_{ij} for an uncertain operational condition x, where x is a vector of size n that defines the number of traffic network random or uncertain variables, such as, capacities, number of users in OD pairs, uncertainties related to choice of routes, traffic cost function, among other. Then R_{ij} is the reliability associated with the OD pair(s) ij dependent on the x operational conditions,

$$R = R_{ij}(x) = \frac{t_{ij_0}}{t_{ij}(x)} \tag{1}$$

that is upper bounded with the value of 1. Because, as defined, R depends on x, then it is possible to define the OD ij travel time reliability probability density function $p_R(x)$ and cumulative density function $P_R(x)$, with reliability as in Equation (1). This $P_R(x)$ can be applied to solve an operational probabilistic characterization problem considering stochastic behaviour for x.

Having this representation of reliability has high relevance for decision-making, because it can inform about deviations in the levels of reliability in the network, or network members, if any imposed or accidental changes occur.

Despite of high relevance, defining reliability levels using a probabilistic approach that depends on simulations of a high-fidelity model can become challenging due to the large number of network performance evaluations that may be required to address the network's multi-component probabilistic functionality and dependence. This assessment, to be feasible, demands the need to enable techniques that facilitate its implementation; being one of the most promising in this context the application of multiple levels of fidelity in the simulation of traffic performance.

2. On the usage of multiple levels of fidelity in traffic

One important remark in the context of using multiple-fidelities in the modelling of traffic is related to the existence of different alternatives that can be potentially applied, each with its own assumptions. In the present implementation metamodels are applied, and a Kriging is implemented due to its capability to act as a robust surrogate of complex problems. Other alternatives exist which are capable of performing as global surrogates. However, their applicability may depend on the complexity of the function to be approximated. In traffic, the performance function dependence on systems variables is expected to be highly non-linear. In such cases, of high non-linearity, kernel-based metamodels, such as the Kriging, have proven to perform.

The idea of using multiple fidelities with a metamodel is that of surrogating a performance function $f(x)$ dependence on variables x (in the case of probabilistic analysis, uncertain variables x), that usually demands large efforts to be evaluated, with a lower fidelity model $G(x)$ that has adequate accuracy. The latter, can be used to approximate queries of $f(x)$ at virtually no cost. Such approach is highly relevant in the context of probabilistic analysis, as it allows to perform analyses that are frequently hindered by large analysis efforts, such as probabilistic analysis. This rationale is used to create an approach that allows to define traffic reliability curves in a cost-effective manner for any intrinsic operational condition in the network (*e.g.*, loss of functionality in network links).

2.1. *Generating reliability curves using surrogates of traffic*

In the context of using multiple fidelities with metamodels, iterative experimental designs (EDs) have proven to be highly effective for the development of accurate surrogates. The significant interest captivated by iterative EDs has generated a spectrum of methods for the creation of efficient surrogates in different contexts. Iterative or sequential designs rely fundamentally in enriching a preestablished ED, frequently in accordance with some notion of improvement related to the metamodel expected performance in approximating a specified function (Teixeira et al., 2022b). Recently the idea of using learning functions to iteratively add new points to the ED became popular. This approach, which pursues to use the strictly necessary points to set an accurate metamodel is frequently called to as adaptive metamodeling. It is noted that in the context of probabilistic assessment, several methods have been actively researching the unique character of this type of analysis, with global and local metamodelling, or multi-fidelity approaches being used for this effect. As highlighted in (Teixeira et al., 2022a) a choice of a global or local approach may depend on the necessary responsiveness of the decision-making scheme.

To define a lower-fidelity model, as a metamodel, that is representative of the uncertain performance of the traffic network with reference to different uncertain variables, a k-clustered approach that builds on the uncertainty of the predictions applied in (Teixeira et al., 2022b) is used. In it $S = [s_1, s_2, \ldots, s_N] \in \mathbb{R}^d$ is a Sobol Sequence of size N in a d-dimensional space (representing the number of support variables), with N being the ED budget. Then an ED enrichment sequence based on k-clusters $\hat{S}_{o=\{1,\ldots,k\}} = \{s_j^o\} \subset S$, each $\hat{S}_o$ with $j = \{1, \ldots j_1; \ldots; 1 \ldots j_k\}$ elements, is used to construct a $\hat{X}_h \in \mathbb{R}^d$ that is a set of operational points that considers traffic performance information and that includes the most uncertain candidates in $\hat{S}_o$.

The $\hat{X}_h$ is used to iteratively enrich a subset of the S sample, $\hat{X}_i \subset S$ in iteration i, that defines a low-fidelity model $\mathbb{M}_i(x)$, that uses a kriging, and that after a number of i is able to describe the probabilistic performance of the network or any of its ED considered components with low error. In consecutive i, the progressively enriched sample $\hat{X}$ that defines the kriging is built as,

$$\hat{X}_{i+1} = \{\hat{X}_i\} \cup \{\hat{X}_{h_i}\} \tag{2}$$

with

$$\hat{X}_{h_i} = \{x_o\} \in \hat{S}_o^i = \{\hat{S}_1, \ldots, \hat{S}_k\} \tag{3}$$

and in each o cluster,

$$x_o = \arg\max\{\eta(\mathbb{M}_i(s_j^o))|\eta\left(\mathbb{M}_i(\hat{S}_o^i)\right) > \eta_0\} \tag{4}$$

with $\hat{S}_o^i = [\hat{S}_1, \ldots, \hat{S}_k]$ being obtained by a k-cluster classification of $\hat{S}^i = \{S | \eta\left(\mathbb{M}_i(S)\right) > \eta_0\}$ in iteration i with

$$\eta\left(\mathbb{M}_i(s_j^o)\right) = \frac{Z\, \mathbb{M}_i^{\sigma}(s_j^o)}{\mathbb{M}_i^{\mu}(s_j^o)} \tag{5}$$

and η_0 being a threshold of uncertainty in the kriging prediction. $\mathbb{M}^{\mu}$ and $\mathbb{M}^{\sigma}$ refer respectively to the kriging mean prediction and standard deviation. $\hat{X}_i$ is updated with the additional points at i from X_{h_i} that are predicted by the model with $\pm Z$ margin of uncertainty larger than η_0, that is in relative deviation to the mean prediction. Z relates to the uncertainty of the prediction and at 1.645 provides a a relative prediction deviation from the mean with likelihood lower than 5%. η_0, in the other hand, will evaluate the probabilistic interest of new points in the ED, avoiding simulating points that provide limited information to the definition of the lower-fidelity model in the current iteration. It encloses the relative (maximum) deviation from the mean that is accepted in accordance to the probabilistic representation of Z.

the update of $\hat{X}$ continues until no further interest in exploring and exploiting points in S is identified as given by the kriging predictions, with,

$$\max\{\eta\left(\mathbb{M}_{i-1}(S)\right)\} < \eta_0 \tag{6}$$

being true, where η_0 is an evaluator of relative deviation from the mean. If this condition is true, the $\mathbb{M}_i$ used to predict S is assumed to be an accurate lower-fidelity model of the traffic network performance indicator of interest.

The rationale for this approach is straightforward. Because performing the large number of evaluations that characterize the probabilistic response of a traffic network is costly, then there is interest in replacing the traffic network performance model with another model that is less costly to evaluate. Therefore, defining a probability density function of any output of interest in this new function can be done at virtually no cost. However, there is a need to ensure that this new function is an accurate representation of the higher fidelity traffic performance. For such, a series of performance points are generated, randomly or quasi-randomly, then an iterative search is conducted for

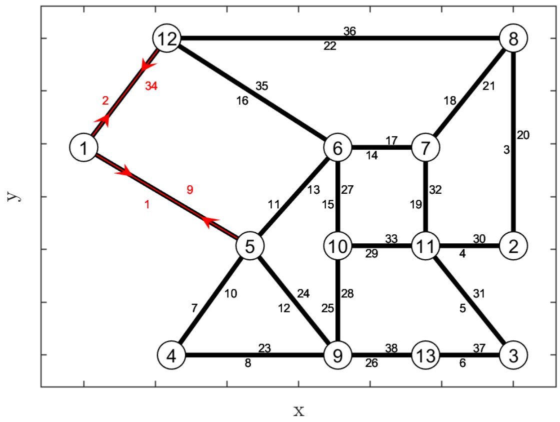

Figure 1: Nguyen-Dupuis network as defined in the present work.

the further performance points that enrich and improve the accuracy of this secondary, lower-fidelity, model. This avoids the simulation of points that have limited contribution to improve the accuracy of the metamodel, making its definition highly efficient. The k-clustered algorithm ensures efficient exploration of the space (*i.e. that new points are further apart and cover the space of possible operational points, ensuring global coverage of the space*) and the kriging uncertainty guarantees that the new iterated points are of interest to improve the accuracy of the currently built model. When no further interest is identified, it is assumed that the lower-fidelity model is an adequate approximation of the traffic network performance, and it can be used to perform a probabilistic analysis. Such performance is confirmed by the results obtained.

Figure 1 presents the traffic network used to build a representative example of application of the approach presented, with results in Figure 2, where the usage of an iterative design is illustrated. In order to study the problem defined the traffic user equilibrium model is applied.

In Figure 2, it is possible to infer the first two iterations of the search (I-II) for an accurate kriging, with the (black) filled circles representing the sample $\hat{X}_i$ and the (red) filled squares representing the $\hat{X}_{h_i}$ for the current i (the coloured cross marks represent each a subset $\hat{S}$ where $\hat{X}_{h_i}$ are selected), that will define the $i+1$ experimental design in II. It is possible to infer how the space of potential candidates is constrained from I to II when the sample $\hat{X}$ is enlarged.

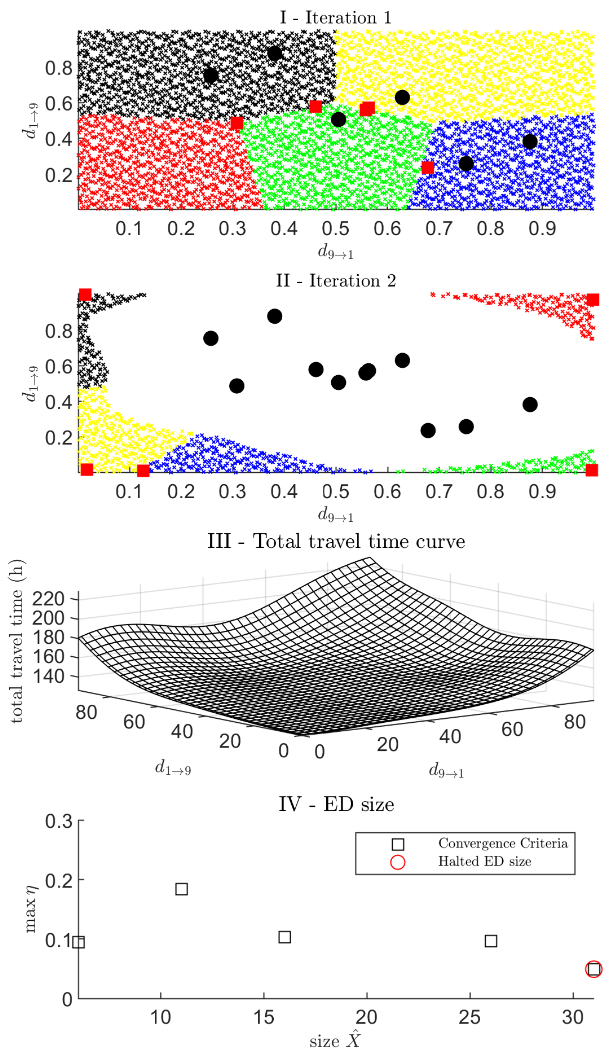

Figure 2: Example of metamodel fit to a two-dimensional example enclosing links 1 and 9 damage in the ED, respectively $d_{1\to 9}$ and $d_{9\to 1}$, subscript node direction travel. I-II - Experimental design of iterations 1 and 2. III - Two-dimensional representation of the variations in travel time, depending on $d_{1\to 9}$ and $d_{9\to 1}$. IV - ED size and relationship to the Sobol sequence in the present example metamodel definition.

In III it is possible to analyse the final converged surrogate for the total travel time with reference to changes in the capacity of the links 1 and 6, according to Figure 2 nomenclature. In IV, the convergence criteria results are evaluated, where the Experimental Design is halted after $\hat{X}$ has been extended to 31 points. It is noted that the initial space is started with 6 points.

In the context of defining the surrogate, as the dimensions of the space are key in its capability to approximate the traffic performance accurately and convergence of the iterative approach used to define it, it sufficiently large enough initial sample $\hat{X}$ is required, and hence an adopted size of at least $3d$ is recommended for reliable performance, with d being the size of the dimensional space. Manache and Melching (2008) use the same sample size for the purpose of reliable validation of a sensitivity analysis. Other al;alternative may consider an initial sample of Latin Hypercube Points, which was identified before to provide an adequate global description of traffic networks (Martinez-Pastor et al., 2020). After definition of an accurate lower fidelity model of the network, it can be used to define probability distribution functions (PDFs) of the performance dependence as a function of any uncertain variables; using for example, Monte Carlo Simulation (MCS). In Figure 3 the PDF of total travel time with dependence on links 1, and link 9 is presented. These PDFs include the probabilistic response of the traffic network performance (in travel time) with respect to a uncertain number of users in 15 of its Origin-Destination (OD) pairs. Users in OD pairs are assumed to follow a normal distribution with Coefficient of Variation of 10%, and all the variables are fitted in uniform space $x \in]0,1[$. All newly defined metamodel use a sample $\hat{X}$ of 51 function evaluations, defined at each level of link damage (or reduction of capacity).

Results show good concordance of the PDFs for the two cases. While the metamodel based PDF only uses 51 evaluations (for each of I-III) of the traffic model simulation, each of the Real PDF uses 1250 function calls generated using MCS, making it substantially more effort consuming. In practice this has the potential to constitute a reduction of approximately $\frac{\text{size}\{\hat{X}\}}{N_{MCS}\times N_{PDF}}$ if the metamodel is defined for different levels of damage in a single simulation, with N_{MCS} being the size of the Monte Carlo Simulation samples, and N_{PDF} the number of PDF extracted in the analysis at different operational conditions. While it is noted that the traffic assignment here used is already a simplified approximation of more complex traffic models, this same rationale can be used to model traffic in other more time consuming evaluation models, such as traffic modelling that uses agents, and that are more representative of what can be a digital-twin of a traffic network performance in the future.

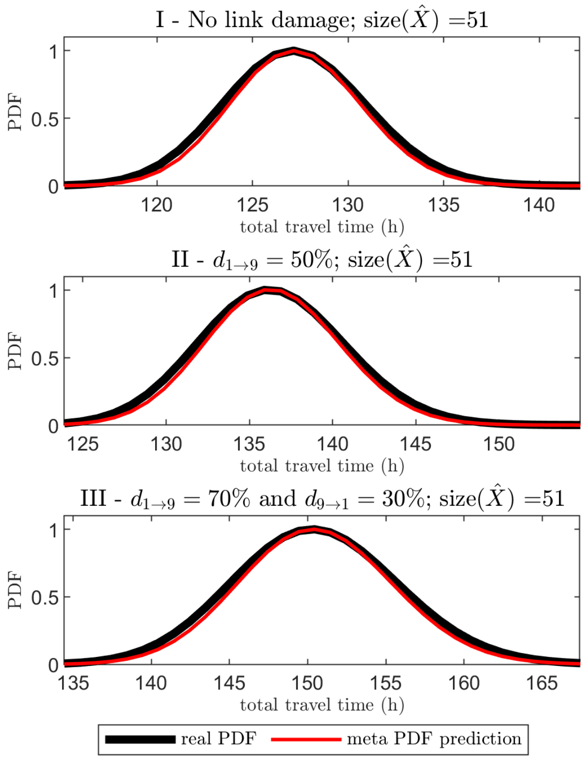

Figure 3: Prediction of the probability density function for the Nguyen-Dupuis network total travel time considering three operational scenarios, and considering random number of OD users in 15 OD pairs. I - no damage in the network. II - 50% loss of capacity in the link 1 connecting nodes 1 to 9 ($d_{1\to9}$). III - 70% loss of capacity in link 9 connecting nodes 9 to 1 ($d_{9\to1}$), combined with a 40% loss of capacity in link 1 that operates the reverse direction ($d_{1\to9}$).

In the present work, the Cuenca traffic network is applied to illustrate how the proposed methodology can be used to build perturbation dependent reliability curves for traffic networks, that are a form of fragility curve at different reliability levels. Gauchy et al. (2022) use a similar rationale with a global estimator that defines a fragility curve, using its uncertainty also to characterize confidence intervals.

3. Reference Example: Multiple-link reliability analysis for the Cuenca traffic network

In this example the Cuenca network is considered. Cuenca is a city in the south of Spain and its network is composed of 232 nodes and 672 links. This same network was used earlier to study link criticality in (Martinez-Pastor et al., 2022), and considers 199 OD pairs in 207 routes. For the analysis performed a pair OD that has at least one alternative route is selected. The Bureau of Public Roads (BPR) function is applied to model the relationship between link service capacity, the demand and travel time in each link, with $\alpha = 0.286$ and $\beta = 5.091$ determining the shape of this function. For effects of uncertainty quantification, a coefficient of variation of 5% is considered in both parameters.

Perturbation dependent reliability curves are studied in three links, 425, 430 and 431, as part of two routes through the mentioned OD, Figure 4 .

Reliability damage-dependence is studied considering uncertainty in the number of users that pass through this OD pair (10% in coefficient of variation), and in the BPR shape parameters α and β, as referred.

The idea here is to study how uncertainty affects the reliability of traffic for it, considering a full probabilistic response. With the approach mentioned it is possible to define an accurate surrogate for the three links considered in this analysis with $\eta_0 = 0.1$ using 845 evaluations of the traffic network.

In Figure 5 it is possible to infer on the reliability curves for link 430 (the second link in the 4 links route of Figure 4). In this curve, each evaluation of a damaged scenario implies sampling the PDF of R to characterize the level of probability at which a certain reliability is surpassed. Therefore, each reliability curve at a certain value of R represents a probabilistic transition of R with the reduction of capacity. That is, below that least value of capacity for which the probability $P[R(x) > R]$ is larger than 0, the probability of the reliability being lower that R is 100%. These curves are highly informative in decision-making, if their definition is enabled. It is interesting to note the increasingly faster transi-

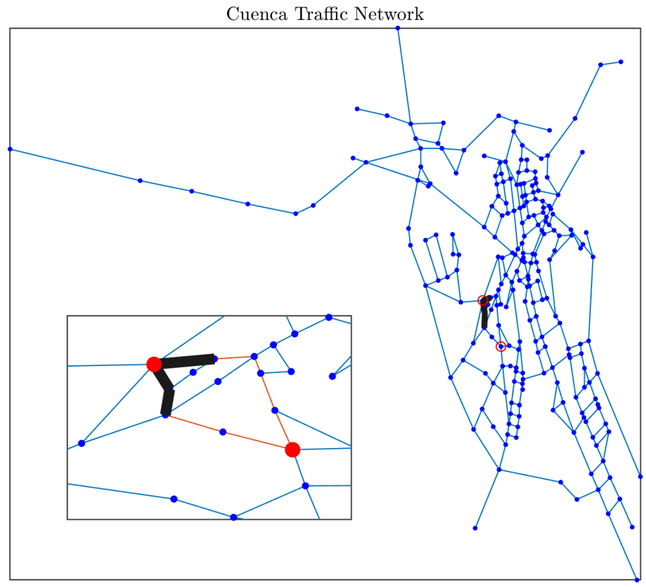

Figure 4: Cuenca network and reference OD pair researched. Highlighted OD pair, routes and Links 425, 430, and 431.

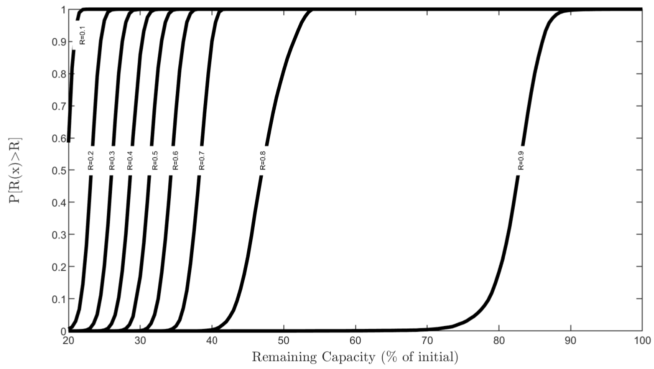

Figure 5: Reliability curves for damage dependence, as a form of capacity of link 430, in the network link.

tion (i.e. lower reduction of capacity is required) that occurs for a certain value of R with decreasing capacity, which is likely to be related to shape of the travel cost function. To understand the degree of complexity in defining these curves that relate the probability of R with decreases in capacity, $\mathscr{C}_a$, it is important to emphasize that for each value of $\mathscr{C}_a$ a MCS is required to describe the PDF of R and hence quantify the probabilistic behavior of it at different R thresholds. In practice, this requires $t \times N_{MCS}$ samples, with t being the number of reduction capacities to be inferred. In the present case, because evaluations of the traffic performance have zero cost in the lower-fidelity model of the Cuenca, a fine resolution for the reduction of capacity was used, with 160 values of $\mathscr{C}_a$ being used to draw these curves. In practice, and without the use of a lower fidelity surrogate, more than 1 million evaluations of the Cuenca traffic network model would be required to achieve the same level of probabilistic significance, which is highly challenging in practice to do. If different links are considered in the analysis this becomes unfeasible.

It is noted that a surrogate can be defined for multiple links such that individual or coupled behavior can be studied. In the present example, with 845 evaluations of the network it is possible to define the individual and coupled performance of the links considered. In Figure 6 a mapping for coupled behavior of links with different reductions of capacity is presented, where as expected the accuracy loss at larger reliability is larger. The coupled effects are not significant at lower reduction of capacity, however, when $R \leq 0.7$ the coupled effects of reduction in capacity may have an impact of 100% or more in the value of R (*i.e.* R becomes less than half due to coupled effects than a same level of damage individually considered in the link)

The present implementation is indicative of how working with multiple fidelities may help decision-makers to have insight into information that would be unfeasible to obtain in a practical basis otherwise.

4. Conclusions

The present work investigated how different fidelities of modelling can be paired to perform highly effort consuming analysis in traffic networks, such as a probabilistic evaluation of traffic reliability when defined as a fraction between the free-flow travel time and the system state travel time. An approach is proposed to define global surrogates that define traffic reliability for a spectrum of damage scenarios, that then form the basis of the creation of damage-dependent reliability curves for the network as whole, or individualised at component level. Results show that only a fraction of evaluations are necessary, when a lower-fidelity model, in the present a kriging, is used to surrogate

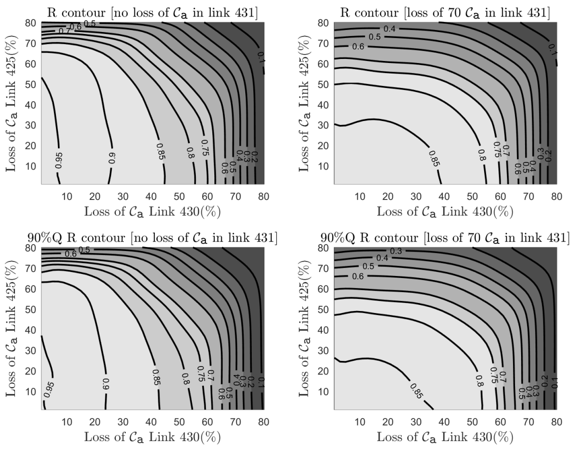

Figure 6: Reliability surface for coupled effects in Link 425, Links 430, and Link 431.

the higher-fidelity traffic model; enabling, an otherwise, highly effort consuming computations of reliability curves. Validation is made in two networks, and the approach presented is shown to be an effective way to compute damage-dependent reliability curves for traffic at virtually no cost and with high accuracy, which are a powerful tool for decision-making of damaged systems. Future works may consider adaptive thresholds at different reliability levels, such as considered in (Teixeira et al., 2021) to enable an even further faster computation of these curves, instead of using global sampling.

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