Functional Connectivity Analysis (FCA): An efficient method to model infrastructure functionality

Juanya Yu
PhD Candidate, Dept. of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA

Neetesh Sharma
Postdoctoral Scholar, Dept. of Civil and Environmental Engineering, Stanford University, Stanford, CA, USA

Paolo Gardoni
Professor, Dept. of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA

ABSTRACT: The performance loss of infrastructure caused by hazards can disrupt regular economic activities, obstruct emergency responses, and be detrimental to society's recovery. Network analysis methods, including topology-based and flow-based methods, are valuable tools for infrastructure functionality assessment. Topology-based methods capture connectivity patterns of network components with relatively low computational costs. However, topology-based methods fail to model the flow of resources from source facilities to consumers. On the other hand, flow-based methods are computationally intensive but can provide on the flow performance of infrastructure by modeling the operational dynamics. This paper introduces a novel hybrid approach to provide comparable functionality assessment to flow-based methods with computational efficiency by introducing flow-related characteristics into topological connectivity metrics. This hybrid approach supports efficient optimization and probabilistic analysis of infrastructure life cycles. This approach is then illustrated with a real-world example modeling the functionality of potable water infrastructure in Shelby County, Tennessee, in a post-earthquake scenario.

1. INTRODUCTION

Infrastructure reliability ensures community well-being and prosperity. However, efforts to improve infrastructure functionality subject to disruptions involve making complex decisions and large investments requiring risk assessment (Gardoni et al. 2016). An essential element in risk assessment is the functionality assessment of damaged infrastructure to capture the immediate impact of hazards and the ability of infrastructure to recover (Boakye et al. 2019).

Various methods are available for functionality assessment, such as network-based, statistical, and hybrid methods (Vaiman et al. 2012). Network-based methods can describe detailed topological properties and flow patterns of infrastructure. Most of the current literature classifies network-based methods into topology-based and flow-based methods (Ouyang 2014).

Topology-based methods capture properties such as network connectivity and its changes. These methods demand limited data because they do not require physical attributes of components (Agathokleous et al. 2017). Furthermore, topology-based methods have low computational costs (Pagano et al. 2019) because they do not model the flow of resources. Therefore, topological methods only provide partial and approximate results (Yazdani et al. 2011). Weighted topology-based network models can use the information on component attributes to quantify the component criticality in vulnerability analysis (Hajibabaei et al. 2022) and resilience assessment (Herrera et al. 2016). However, few studies have used weighted topology-based network models to assess the time-varying performance of disrupted networks.

Flow-based methods adopt equations that govern the flow of services or commodities on networks (Lee et al. 2007). These methods model the performance and resilience of infrastructure to disruptions (Liu et al. 2020; Sharma et al. 2020). Flow-based methods can offer high-fidelity results of the ability of infrastructure to deliver services to communities and their abilities to withstand and recover from disruptions. However, considering all the physical attributes of components and detailed operating mechanisms of infrastructure networks make flow-based methods data and computationally intensive, especially for large infrastructure networks (Yazdani and Jeffrey 2012). Thus, the granularity and resolution should be carefully selected when adopting flow-based methods (Nocera and Gardoni 2022; Sharma and Gardoni 2022).

This paper proposes Functional Connectivity Analysis (FCA) to pursue a trade-off between providing functionality information comparable to flow analysis and saving computational costs in modeling the functionality of infrastructure. We first briefly describe the current methods and then present the general procedure to perform FCA of infrastructure. Next, building on the general procedure of FCA, we develop a specific FCA for potable water infrastructure, which we illustrate as an example of the potable water infrastructure in Shelby County, Tennessee, subjected to earthquake hazards. Finally, we compare the results from connectivity analysis, flow analysis, and FCA regarding the information on functionality and computational cost for the immediate impact analysis and the recovery process.

2. CURRENT APPROACHES IN INFRASTRUCTURE PERFORMANCE ANALYSIS

This section briefly describes the current formulations in topology-based and flow-based methods.

2.1. Network connectivity analysis

Topological methods represent infrastructure using graph theory. These methods capture specific features of the graph and infer their performance and reliability (Hernandez-Fajardo and Dueñas-Osorio 2011). One way to analyze these topological features is to study the connectivity of the nodes (i.e., connectivity analysis).

Two of the most popular metrics of network connectivity in the literature are *diameter* (also known as characteristic path length) and *efficiency* (Albert et al. 1999). *Nodal efficiency* can describe the connectivity of a specific node to the rest of the nodes in the network and is more useful to assess the extent of the loss of connectivity than *nodal diameter*. Nodal efficiency η_i is defined as the average of the inverse of the shortest path between the given node i and the rest of the nodes in the network, shown as (Latora and Marchiori 2001)

$$\eta_i = \frac{1}{(n-1)} \sum_{\substack{j=1 \\ j \neq i}}^{n} h_{ij} \tag{1}$$

where $h_{ij} = 1/d_{ij}$ for $i \neq j$ and $h_{ij} = 0$ otherwise. η_i ranges from 0 (no connections between node i to any other node) to 1 (node i is connected with all the other nodes of the network). To capture the nodal importance in calculating η_i, Guidotti et al. (2017) proposed weighted nodal efficiency for node i, $\eta_{w,i}$, written as

$$\eta_{w,i} = \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} h_{ij} \times W_i \tag{2}$$

2.2. Flow analysis

Flow analysis, also known as flow-based methods, can accurately measure component functionality as their ability to deliver goods and services to satisfy demand. For example, hydraulic flow analysis can provide information on whether potable water infrastructure can meet desired pressure head and demand.

Flow analyses also use graph theory to represent infrastructure systems; however, to model the dynamics, the nodes, and links have additional attributes describing their operation/function. These additional attributes are in the form of vectors representing the *state variables* $\mathrm{x}(t)$, *capacities* $\mathrm{C}(t)$, *demands* $\mathrm{D}(t)$, and *supply* $S(t)$ (Sharma and Gardoni 2022). The performance assessment of infrastructure carried out by flow analysis is developed on these measures. Sharma and Gardoni (2022) define the derived performance measure $\mathrm{Q}(t) = [\mathrm{S}(t) \oslash \mathrm{D}(t)] \odot 1_{\{\mathrm{D}(t)>0\}}$ as the fraction of demand served at the demand nodes, where $\oslash$ the element-wise division operator, and $1_{\{\mathrm{D}(t)>0\}}$ ensures $\mathrm{Q}(t)$ is defined for non-zero demands. For example, in potable water infrastructure, $\mathrm{Q}(t)$ refers to the demand satisfaction of an end-user in terms of flow or pressure head of water.

3. FUNCTIONAL CONNECTIVITY ANALYSIS (FCA)

This section introduces Functional Connectivity Analysis (FCA) which uses performance-weighted connectivity metrics for comprehensive and computationally efficient assessment of infrastructure performance.

Step 1: Define the states of functionality. Performance assessment should serve the need of the consumers. In general, such a performance can be measured on a continuous scale, say, $Q \in [0,1]$, where $Q = 0$ defines a completely not functional state, $Q = 1$ defines a fully functional state and $Q = (0,1)$ denotes some partial levels of functionality. In fact, during a post-disaster scenario, most consumers are concerned about whether they have access to goods and services (e.g., food, potable water) and whether these resources satisfy certain minimum requirements rather than the exact amounts. Furthermore, in most infrastructure systems, the functionality states of $Q = 0$ and $Q = 1$ are much more likely than the intermediate intervals. We can reduce the continuous performance assessment scale to a small set of ordinal states. We define the performance measure of demand node i, $Q_{o,i} \in \{q_1, q_2, \dots, q_m\}$, $m = 1, 2, 3, \dots$, where q_m are ordinal states denoting the levels of functionality. Compared with a continuous scale, an ordinal scale can help create simpler models for functionality. Therefore, we reduce the computational cost while sufficiently encoding consumers' utility from the functionality assessment by defining a small set of functional states $Q_{o,i}$.

Step 2: Select the relevant metrics for network connectivity and define their performance-weighted forms. For functionality assessment of networks, other than topological connectivity, we incorporate performance-related information of network components by introducing nodal weight based on performance to connectivity metric. For commodity flow networks, they account for the connectivity between sinks and sources rather than connectivity between each pair of nodes (Ouyang et al. 2009). Hence, for a commodity flow network, we can write weighted nodal efficiency for demand node i, $\eta_{\kappa,i}$, as

$$\eta_{\kappa,i} = \frac{1}{N_s} \sum_{j \in V_s} h_{ij} \times W_i \tag{3}$$

where N_s is the number of source nodes; V_s is the set of source nodes. for node i. We use W_i to represent different performance related information for different types of networks. For example, if we are interested in the pressure at each demand node for potable water infrastructure, then the nodal weight is pressure related.

Step 3: Define the mapping from performance-weighted connectivity metric to categorical states of functionality. Performance-weighted connectivity metric alone cannot directly predict the functional state of demand nodes. We define a surjective mapping from the metric to the ordinal states of functionality. If we have the performance-weighted connectivity metric, $C_{w,i}$, for each demand node i in a network, then we define the following mapping function M from $C_{w,i}$ to $Q_{o,i}$:

$$M: C_{w,i} \mapsto \begin{cases} Q_{o,i} = q_1, & \text{Not functional} \\ Q_{o,i} = q_2, & \text{Partially functional} \\ \quad \cdots & \cdots \\ Q_{o,i} = q_m, & \text{Fully functional} \end{cases} \quad (4)$$

Step 4: Limited simulations to calculate the nodal weights. In FCA, we use limited number of simulations to estimate the supply at node i for a damaged network without performing a detailed flow analysis. Then, W_i at demand node i can be calculated as the proportion of the supply under damaged conditions to the supply under normal operation conditions, written as $W_i = S_i(t = 0^+)/S_i(t = 0^-)$, where $S_i(t = 0^+)$ is the supply measure at demand node i when the infrastructure network is damaged; $S_i(t = 0^-)$ is the supply measure at demand node i when the infrastructure network is under normal operating conditions.

Step 5: Use weighted connectivity analysis to predict functionality. Once we have determined the nodal weight W_i for each demand node i, we can calculate $C_{w,i}$. Then, for each demand node i, $Q_{o,i}$, can be determined through M.

4. FUNCTIONAL CONNECTIVITY ANALYSIS (FCA) FOR POTABLE WATER INFRASTRUCTURE

In this section, we present an application of the general theory of FCA in the case of potable water infrastructure. One of the essential functional performances in potable water infrastructure is whether the demand nodes are satisfied with the required pressure (Yang et al. 1996). Hence, in this paper, we mainly discuss the functional state of a demand node regarding its pressure. We treat reservoirs as source nodes and customers as demand nodes. Furthermore, we represent pumping stations and pipes as links connecting sources and demand nodes.

Step 1: Define the states of functionality in potable water infrastructure. When the pressure head at a demand node i, P_i, is above the required pressure P_{req}, consumers can receive the desired water supply (Wagner et al. 1988). When the pressure at a demand node is below the minimum pressure P_{min}, consumers cannot receive any water (Wagner et al. 1988). When the pressure is between P_{min} and P_{req}, consumers can only receive part of the desired amount. Hence, here we use 3 ordinal functional states to represent the consumer perspective, shown in Eq. (5), that is, not functional, corresponding to the performance state when P_i is smaller than P_{min}; partially functional, corresponding to the performance state when P_i is greater than P_{min} while smaller than P_{re}; fully functional, corresponding to the performance state when P_i is greater than P_{req}.

$$\begin{cases} P_i < P_{min}, & \text{Not functional} \\ P_{re} > P_i \geq P_{min}, & \text{Partially functional} \\ P_i \geq P_{req}, & \text{Fully functional} \end{cases} \quad (5)$$

In this paper, P_{req} is set to 15 psi, and P_{min} is set to 10 psi.

Step 2: Select the relevant metrics for water network connectivity and define their performance-weighted forms. We select weighted nodal efficiency for demand node i, $\eta'_{\kappa,i}$, as the performance-weighted connectivity metric for water networks, written as

$$\eta'_{\kappa,i} = \frac{1}{N'_s} \sum_{j \in V_s} h_{ij} \times W_i \quad (6)$$

where N'_s is the number of the chosen nearest source nodes, V'_s, to demand node i. V'_s is only a subset of all the source because for a damaged water network, even though the demand node may be connected to a distant source, this source may not supply water to the demand node i because of pipe damage. Based on our computational experiments, we recommend using 2 or 3 nearest sources for an urban potable water network.

Step 3: Define the mapping from performance-weighted connectivity metric to categorical states of functionality. We propose to use the weighted nodal efficiency loss L_i to reflect the loss of the weighted nodal efficiency of demand node i when there is damage and no damage to the water network. We define L_i as L_i=1-$\eta_{i,dg}/\eta_{i,undg}$, where $\eta_{i,dg}$ is the weighted nodal efficiency of demand node i when there is damage to the water network, and $\eta_{i,undg}$ is the weighted nodal efficiency of demand node i when there is no damage in the water network. Here we propose to use the $P_{min}/P_{op,i}$ and $P_{req}/P_{op,i}$ as two thresholds of the ordinal functional states of demand node i, shown as Eq. (7).

$$M: \begin{cases} L_i > \frac{P_{req}}{P_{op,i}}, & \text{Not functional} \\ \frac{P_{min}}{P_{op,i}} \leq L_i < \frac{P_{req}}{P_{op,i}}, & \text{Partially functional} \\ L_i \leq \frac{P_{min}}{P_{op,i}}, & \text{Fully functional} \end{cases} \quad (7)$$

Step 4: Limited simulations to calculate the nodal weight for functionality. We define the nodal weight W_i for demand node i as $W_i = P_{dg,i}/P_{op,i}$, where $P_{op,i}$ is the pressure of demand node i under normal operating conditions, and $P_{dg,i}$ is the pressure of demand node i when there are only damages to pipes. First, we obtain $P_{op,i}$ by performing the hydraulic flow analysis of the water network under normal operating conditions (without damage to any components). Next, we obtain $P_{dg,i}$ by performing a simplified hydraulic flow analysis, referred to as pipe damage analysis, of the water network to reduce computational costs. That is, we only consider damage to pipes, including leaks and breaks (assuming other elements, like pumping stations and storage tanks, are undamaged).

Furthermore, hydraulic flow analysis requires a high temporal resolution (hourly or higher) for the convergence of the dynamic flow equations (Sharma and Gardoni 2022). In contrast, FCA supports using low temporal resolution (daily or lower) because it is good enough for mitigation decision-making compared to the high temporal resolution required for flow analysis. In this paper, we use daily resolution in FCA. For a damaged water network, the worst performance within a day occurs when the water demand is high and the damage to the network is severe. When assessing the performance immediately after the damaging event, we only perform pipe damage analysis for peak demand periods. During the recovery process, we perform the flow analysis for peak demand periods and the first two hours of that day. Because for the first few hours, we have the worst status of damaged pipes within that day, and that is when the minimum W_i might occur.

Step 5: Use the nodal weight in the weighted connectivity analysis to predict the functional state. Once we have W_i, we calculate L_i. Then, we determine $Q_{o,i}$ for each demand node i in the water network through the mapping function M.

5. PREDICTION OF THE PHYSICAL STATE OF POTABLE WATER INFRASTRUCTURE DURING IMPACT AND RECOVERY

FCA predicts the functionality and performance of infrastructure in conjunction with predicting the physical states of infrastructure components. There are two main processes, deterioration (gradual and shock processes resulting from natural and anthropogenic hazards) and recovery, affecting the physical states of components (Jia and Gardoni 2018).

5.1. Modeling of impact analysis

The analysis to assess the performance immediately after a damaging event is referred to impact analysis. First, we estimate the direct physical damage to the network components. We use seismic fragility curves for nodal elements, including tanks (FEMA 2020), pumping stations and booster pumps (Hwang et al. 1998) and repair rate curves (ALA 2001) for pipelines. Once we have determined the physical damage to network components, we can assess the immediate performance of the potable water network using connectivity analysis, flow analysis, and FCA.

For connectivity analysis, the failure of the pipe occurs when the pipe needs at least one repair (Poljanšek et al. 2011), thus the probability of failure of the pipe, P_f, can be calculated as

$$P_f = 1 - P(N = 0) = 1 - e^{-\lambda L} \quad (8)$$

where N is the number of ruptures for a pipeline with length L; λ is the repair rate, which is the number of ruptures per unit length. Failed elements, for example, storage tanks, pumping stations, and pipes, are directly removed from the water network.

For flow analysis, a pressure-dependent hydraulic flow analysis (Klise et al. 2017) is performed, and the hydraulic flow network model is from Sharma et al. (2020).

5.2. Modeling of the recovery process

In this paper, the recovery durations of damaged pumping stations, booster pumps, and storage tanks are from HAZUS (FEMA 2020). The detailed recovery schedules of damaged pipelines are from Sharma et al. (2020).

6. COMPARING CONNECTIVITY, FLOW, AND FCA FOR POTABLE WATER INFRASTRUCTURE IN SHELBY COUNTY

We illustrate the proposed FCA using an example of modeling the potable water infrastructure in Shelby County, Tennessee, USA. In the example, we consider a 7.7 magnitude scenario earthquake with epicenter at $35.93°N$ and $89.92°W$, which is approximately the north-west of Shelby County (details in Sharma et al. 2020).

In the case of connectivity analysis, if a demand node i is connected with at least one source node, its functional state is defined as connected. Otherwise, its functional state is defined as disconnected. For flow analysis, the interested performance measure is the pressure head at demand nodes. To see whether FCA can predict the functional state as accurately as flow analysis instead of showing the exact number of P_i, we categorize P_i into three states, which are, P_i is greater than P_{req}; P_i is greater than P_{min} while smaller than P_{req}; and P_i is smaller than P_{min}. For FCA, as described in Section 4, we define three functional states, which are fully functional, partially functional, and not functional, corresponding to the above three functional states in flow analysis.

Tables 1-3 present the number of demand nodes in the most likely functional state of flow analysis and FCA for $t = 0^+$ day (immediate impact), $t = 3$ days, and $t = 21$ days respectively. The diagonal terms in the three tables are the number of demand nodes analyzed by FCA that match the corresponding state analyzed by flow analysis. For example, on $t = 0^+$ day, there are 8 demand nodes that are predicted by FCA to be fully functional, meanwhile, flow analysis also gives out that the pressure of these 8 demand nodes is most likely to be greater than P_{req}. The large number of diagonal terms in the three tables shows that FCA predicts each functional state of demand nodes nearly as accurately as flow analysis.

Table 1: The match of each functional state for FCA and flow analysis (t = 0^+ day: immediate impact).

		FCA		
		Fully functional	Partially functional	Not functional
Flow analysis	$P_{req} \leq P_i$	8	0	15
	$P_{min} \leq P_i < P_{req}$	0	0	1
	$P_i < P_{min}$	1	0	930

Table 2: The match of each functional state for FCA and flow analysis (t = 3 days).

		FCA		
		Fully functional	Partially functional	Not functional
Flow analysis	$P_{req} \leq P_i$	35	0	37
	$P_{min} \leq P_i < P_{req}$	0	0	2
	$P_i < P_{min}$	14	0	867

Table 3: The match of each functional state for FCA and flow analysis (t = 21 days).

		FCA		
		Fully functional	Partially functional	Not functional
Flow analysis	$P_{req} \leq P_i$	795	0	0
	$P_{min} \leq P_i < P_{req}$	0	5	0
	$P_i < P_{min}$	0	0	155

Figure 1 shows the ratio of CPU time needed per day for FCA and connectivity analysis compared to flow analysis. Compared with flow analysis, FCA can save substantial computational costs because the main part of computational cost comes from the calculation for W_i . When calculating W_i, we only perform the pipe damage analysis for required range of hours within a day, while flow analysis needs to be performed for the entire 24 hours to obtain the worst performance, P_{min}, for the demand nodes. Moreover, for the overall trend, the ratio of computational time needed per day for FCA compared to flow analysis decreases with the increasing number of days, illustrating that performing FCA can save more cost with the increasing days of the recovery process. This is because if we want to assess the functional performance of the water network for $t = n$ days, FCA can be performed for $t = n$ days only, while we need to perform flow analysis from $t = 0^{+}$ day to $t = n$ day continuously to obtain the same results.

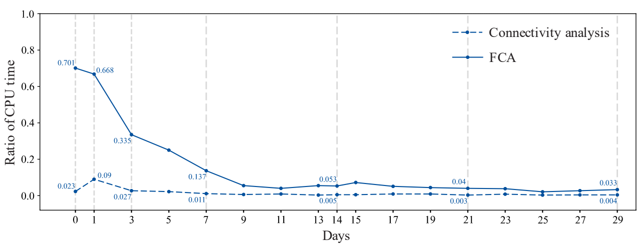

Figure 1: The ratio of CPU time needed with respect to traditional flow analysis.

7. CONCLUSIONS

This paper proposed a novel Functional Connectivity Analysis (FCA) to achieve a trade-off between providing information on functionality comparable to flow analysis and saving computational costs in modeling the functionality loss and recovery process of infrastructure subjected to hazards. Then, following the general procedure of FCA, the paper developed a specific FCA for potable water infrastructure. An example then illustrated the implementation of FCA on the potable water infrastructure in Shelby County subjected to earthquake hazard. In the example, we made comparisons of connectivity analysis, flow analysis, and FCA in terms of computational costs and the functional state of demand nodes in immediate impact analysis and the recovery process. The results indicated that FCA can provide comparable results of the functional states of demand nodes to flow analysis while saving huge computational costs. The proposed FCA allows for the assessment of the infrastructure functionality in probabilistic analysis and optimization applications for mitigation.

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