Probabilistic Design Optimization of TMDI System for Seismic Mitigation Subjected to Stochastic Ground Motions

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ABSTRACT: The tuned mass damper Inerter (TMDI), consisting of mass blocks, springs, dampers and two-terminal inertial elements, is a passive control device on the basis of the tuned mass damper (TMD). A large number of studies on the design optimization and performance evaluation of the TMDI system have been conducted in recent year. But they are almost analyzed from the perspective of deterministic analysis. This is far from non-stationary and non-Gaussian processes that feature the characteristics of engineering excitations such as earthquake ground motions and typhoons. Aiming at such a challenge, the variance-based design optimization of TMDI system for seismic mitigation and probability density evolution analysis subjected to stochastic ground motions are performed in this study. First, based on the theoretical formula and the numerical optimization via genetic algorithm (GA), the design method of TMDI system with respect to the primary structure frequency ratio and the TMDI damping ratio subjected to Gaussian white noise is proposed. On this basis, the probability density evolution method and physically-motivated stochastic ground motion model are used. Combining with numerical examples, the optimal parameter design of structure-TMDI system and probability density evolution analysis under the action of stochastic ground motion are carried out. The seismic mitigation performance and structural reliability of structure-TMDI system with different cases of parameters are then explored. Numerical results reveal the technical advantages of TMDI system, i.e., high performance, mass reduction and stroke limitation in a probabilistic sense.

Tuned mass damper inerter (TMDI) is a passive control device based on tuned mass damper (TMD), which consists of mass blocks, springs, dampers and two-terminal inertial elements (Marian and Giaralis 2013). Many scholars have conducted studies for the optimal design and seismic mitigation performance of TMDI. For example, Marian and Giaralis applied the proposed TMDI to undamped single-degree-of-freedom (SDOF) structures under white noise support excitation and damped multi-degree-of-freedom structures excited by stationary colored noise, respectively. They optimized the parameters of TMDI with the objective of minimizing the variance of relative displacement of primary structure. By comparison with TMD, the TMDI has a better seismic mitigation performance (Marian and Giaralis 2014). Giaralis proposed an optimal design framework for TMDI based on the effect of higher order modes on the dynamic performance of the structure, where the TMD and Tuned Inerter Damper (TID) were used as special cases of TMDI, and the analytical results proved the superiority of TMDI in seismic mitigation (Giaralis and Taflanidis 2018). Wang et al. conducted a study on the single and multiple TMDI control of wind vibration in conjoined supertall buildings under strong winds by theoretical analysis and wind tunnel tests where the structural first-order modal response is taken as the control objective (Wang et al. 2021). Djerouni et al. used the H_2 optimization criterion for the optimal design and seismic performance analysis of shear-frame structure-TMDI systems under impulsive ground shaking (Djerouni et al. 2022).

Studies have shown that structural performance analysis is the basis of structural design and control, and a reasonable way is to perform structural performance evaluation based on reliability, so as to guarantee the overall safety and applicability of the structure. However, the existing structural-TMDI system performance evaluation often uses the structural response index under deterministic input, or mean or variance of the structural response under deal white noise. Very few performance analyses of structural-TMDI system based on reliability measure also consider Gaussian white noise as the input, which is far from the non-stationary and non-Gaussian stochastic excitation such as earthquake ground motions and typhoons. While the probabilistic analysis and reliability assessment by conventional stochastic simulation methods require a huge computational effort, which faces serious challenges in practical applications. In response to the above challenges, the probability density evolution method has been proposed in recent years (Li and Chen 2009), which widely used in stochastic response analysis of linear and nonlinear structural systems, dynamic reliability and reliability-based structural control.

In this paper, the probabilistic design optimization of TMDI system for seismic mitigation and probability density evolution analysis subjected to stochastic ground motions are addressed. Firstly, based on the theoretical formula and numerical optimization algorithm, the design method of TMDI system with respect to the primary structure frequency ratio and the TMDI damping ratio subjected to Gaussian white noise input is proposed. On this basis, the probability density evolution method and physically-motivated stochastic ground motion model are used. Combining with numerical examples, the optimal parameter design of structure-TMDI system and probability density evolution analysis under the action of stochastic ground motion are carried out. Further, the seismic mitigation performance and structural reliability of TMDI with different cases of parameters are explored.

1. EQUATIONS OF MOTION OF STRUCTURE-TMDI SYSTEM

Consider a linear damped structural-TMDI configuration where the primary structure is a SDOF system with mass m_1 connected to the ground via a linear spring k_1 and a viscous damper c_1, and the additional structure mass m_2 is connected to the primary structure mass m_1 through a linear spring k_2 and a viscous damper c_2, and also connected to the ground through an inerter with the mass-equivalent constant of proportionality b.

Assuming that the linear dynamical system is subjected to external excitation $\ddot{y}_0$, the governing equations of motion of the structure-TMDI system is:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = -m_1\ddot{y}_0 + c_2\left(\dot{x}_2 - \dot{x}_1\right) + k_2\left(x_2 - x_1\right) \quad (1)$$

$$m_2\ddot{x}_2 + c_2\left(\dot{x}_2 - \dot{x}_1\right) + k_2\left(x_2 - x_1\right) + b\ddot{x}_2 = -m_2\ddot{y}_0 \quad (2)$$

Then divide both Eq. (1) and (2) by m_1, and simplify to write the following form:

$$\begin{cases} \ddot{x}_1 + 2\xi_1\omega_1\dot{x}_1 + \omega_1^2 x_1 = -\ddot{y}_0 + \left(f_{\text{tmd}}/m_1\right) \\ \mu\ddot{x}_2 = -\mu\ddot{y}_0 - \left(f_{\text{tmd}}/m_1\right) - \left(f_{\text{I}}/m_1\right) \\ f_{\text{tmd}}/m_1 = 2\xi_2\omega_1\nu\mu\left(\dot{x}_2 - \dot{x}_1\right) + \omega_1^2\nu^2\mu\left(x_2 - x_1\right) \\ f_{\text{I}}/m_1 = \left(b/m_1\right)\ddot{x}_2 = \beta\ddot{x}_2 \end{cases} \quad (3)$$

where $\omega_1 = \sqrt{k_1/m_1}$ and $\xi_1 = c_1/\left(2m_1\omega_1\right)$ are the natural frequency and the critical damping ratio of the primary structure, respectively; $\omega_2 = \sqrt{k_2/m_2}$ and $\xi_2 = c_2/2m_2\omega_2$ are the natural frequency and the critical damping ratio of the TMDI, respectively; $\mu = m_2/m_1$ and $\nu = \omega_2/\omega_1$ are the mass ratio and the dimensionless frequency ratio, respectively; $\beta = b/m_1$ is the inertance-to-mass ratio, i.e. the ratio of the inerter constant b by the mass of the primary structure.

2. METHOD FOR OPTIMIZING PARAMETERS OF STRUCTURE-TMDI SYSTEM

The optimal design of a TMDI system means that after the mass ratio μ and inertance-to-mass ratio

β of the structure-TMDI system are determined, the frequency ratio υ and damping ratio ξ_2 are optimized, so that the design values are taken to optimize the vibration control effect of the TMDI on the primary structure.

2.1. Theoretical formula

Assuming that the input is a stationary stochastic excitation with a power spectrum of $S(\omega)$, the variance of the relative displacement of the primary structure can be expressed as:

$$\sigma_1^2 = \int_{-\infty}^{+\infty} \left|H(\omega)\right|^2 S(\omega) d\omega \tag{4}$$

where $H(\omega)$ is the transfer function; ω is frequency of the excitation.

Then the optimization objective function can be written as follows:

$$J = \min\left(\sigma_1^2\right) \tag{5}$$

Meanwhile the optimal parameters need to satisfy the following conditions:

$$\frac{\partial \sigma_1^2}{\partial \xi_2} = 0 \; ; \; \frac{\partial \sigma_1^2}{\partial \nu} = 0 \tag{6}$$

Assuming that the primary structure is undamped, i.e., $\xi_1 = 0$, and the input is ideal white noise, i.e., $S(\omega) = S_0$, then we can obtain the optimal parameters as follows (Marian and Giaralis 2014):

$$\begin{cases} \nu^* = \dfrac{1}{1+\beta+\mu} \dfrac{\sqrt{(\mu+\beta)\left[\beta(\mu-\beta)+(2-\mu)(1+\mu)\right]}}{\sqrt{2\mu(1+\mu)}} \\ \xi_2^* = \dfrac{(\beta+\mu)\sqrt{\beta(3-\mu)+(4-\mu)(1+\mu)}}{2\sqrt{2\mu(1+\beta+\mu)\left[\beta(1-\mu)+(2-\mu)(1+\mu)\right]}} \end{cases} \tag{7}$$

2.2. Numerical optimization method

In the derivation process of the above theoretical formulation, some assumptions are used, such as the primary structure damping is not considered. In fact, for the primary structure damping problem examined in this paper, the design parameters obtained from the theoretical formulation will produce large errors. For this reason, Genetic Algorithm (GA) (Holland 1975) is further introduced for the numerical optimization of TMDI system. The optimization criterion is as follows:

$$\begin{aligned} \left\{\nu^*, \xi_2^*\right\} &= \arg\min\left(\sigma_{\max}(\nu, \xi_2)\right) \\ &= \arg\min\left(\max\left|\sigma_1(\nu, \xi_2, t)\right|\right) \end{aligned} \tag{8}$$

3. PROBABILITY DENSITY EVOLUTION METHOD

3.1. Probability density evolution method

The essence of the stochastic dynamic response analysis is to solve the probabilistic information of multi-degree-of-freedom systems. The probability density evolution method obtains the full probability information of the system response through solving the equation of the generalized probability density evolution which is derived based on the basic idea of physical stochastic system and the principle of probability conservation (Li and Chen 2009).

For an n-degree-of-freedom nonlinear structural system, the equations of motion are as follows:

$$\begin{gathered} \mathbf{M}(\mathbf{\Theta})\ddot{\mathbf{X}} + \mathbf{C}(\mathbf{\Theta})\dot{\mathbf{X}} + \mathbf{f}(\mathbf{\Theta}, \mathbf{X}) = \mathbf{F}(\mathbf{\Theta}, t) \\ \dot{\mathbf{X}}(t_0) = \dot{\mathbf{x}}_0 \;,\; \mathbf{X}(t_0) = \mathbf{x}_0 \end{gathered} \tag{9}$$

where $\ddot{\mathbf{X}}, \dot{\mathbf{X}}, \mathbf{X}$ are the n-dimensional acceleration, velocity and displacement vectors; $\mathbf{M}$, $\mathbf{C}$ are mass and damping matrices of order $n \times n$, $\mathbf{f}(\mathbf{\Theta}, \mathbf{X})$ is the n-dimensional restoring force vectors; $\mathbf{F}(\mathbf{\Theta}, t)$ is the stochastic excitation vector; $\dot{\mathbf{x}}_0$ and $\mathbf{x}_0$ are the initial velocity and initial displacement vector; and $\mathbf{\Theta} = (\Theta_1, \Theta_2, \cdots, \Theta_s)$ is the basic random vector composed of the random factors in the structural system and the excitation. In this study, only the random sources in the random excitation are included.

Based on the principle of probability conservation, the following generalized probability density evolution equation can be obtained through a series of derivations (Li and Chen 2009):

$$\frac{\partial p_{Z\mathbf{\Theta}}(z, \mathbf{\theta}, t)}{\partial t} + \dot{Z}(\mathbf{\theta}, t) \frac{\partial p_{Z\mathbf{\Theta}}(z, \mathbf{\theta}, t)}{\partial z} = 0 \tag{10}$$

where $p_{Z\Theta}(z,\boldsymbol{\theta},t)$ is the joint probability density function of the extended physical quantities $(Z(t),\boldsymbol{\Theta})$.

When the initial and boundary conditions are given, the probability density functions of the physical quantities of interest are obtained as follows:

$$p_Z(z,t)=\int_{\Omega_\Theta} p_{Z\Theta}(z,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta} \qquad (11)$$

To solve the Eq. (10) the random vector of probability space is first dissected to obtain the set of random event sample points $\{\boldsymbol{\theta}_i\}_{i=1}^{N_{\mathrm{RP}}}$ and their assigned probabilities $\{P_i\}_{i=1}^{N_{\mathrm{RP}}}$, then Eq. (10) can be solve via the finite difference formats.

3.2. Structural Reliability Analysis

In scientific research and practical engineering, structural reliability is an important index for evaluating structural performance. This problem can be conveniently studied in the framework of probability density evolution method. For example, based on the first-passage failure criterion, dynamic reliability of structures under stochastic ground motion can be defined as (Li and Chen 2009):

$$R(t)=\Pr\{Z(\tau)\in\Omega_s,\ 0\le\tau\le t\} \qquad (12)$$

where Ω_s is the security domain. If the physical quantities exceed the safety c boundary, the structure fails, then the probabilities carried by the corresponding random events will no longer return to the security domain, i.e., the following absorbing boundary condition is imposed:

$$p_{Z\Theta}(z,\boldsymbol{\theta},t)=0,\ z\in\Omega_f \qquad (13)$$

Where Ω_f is the security domain.

Under this boundary condition, the residual joint probability density function $\breve{p}_{Z\Theta}(z,\boldsymbol{\theta},t)$ can be obtained, then the reliability of the structure is:

$$\begin{aligned}R(t)&=\int_{\Omega_s} p_Z(z,t)\mathrm{d}z\\&=\int_{\Omega_s}\int_{\Omega_\Theta} p_{Z\Theta}(z,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta}\mathrm{d}z\\&=\int_{-\infty}^{\infty}\int_{\Omega_\Theta} \breve{p}_{Z\Theta}(z,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta}\mathrm{d}z\end{aligned} \qquad (14)$$

4. NUMERICAL EXAMPLE

In this section, the optimized design parameters are obtained following the methods discussed in Section 2, and the probability density evolution analysis of the controlled structural response is then performed. A SDOF primary structure is considered whose natural frequency is ω_1 =6.28 rad/s, and damping ratio is ξ_1 =0.05. The TMDI mass ratio μ =0.01 and the inertance-to-mass ratio β =0.09 (low inertance-to-mass ratio), β =0.19 (medium inertance-to-mass ratio), β =0.49 (high inertance-to-mass ratio) are taken, respectively, in this study.

The input action is stochastic earthquake ground motions, and their response spectrum is compatible in the “mean sense” with the spectrum of the Chinese code for seismic design of buildings (GB 50011-2016) for site condition class type “III”, the design seismic group type “II” and the fortification intensity degree 8 (GB 50011-2010 Code 2010). It can be derived by the methodology of the physically-motivated stochastic ground motion model (Peng and Li 2019). The mean and standard deviation response spectra of stochastic earthquake ground motions and the normative spectrum response are plotted in Figure 1. It can be seen that the mean spectrum of the stochastic ground motions fits well with the standard response spectrum of the code.

4.1. Optimization of TMDI parameters

In order to perform the probability density evolution analysis of the TMDI system, the optimization of TMDI parameters is required first. In this section, stationary white noise input is used to reflect the influence of primary structure damping on the optimal parameter design, and the theoretical formulas discussed in Section 2.1 and the numerical optimization methods based on genetic algorithms discussed in Section 2.2 are used for the optimal design of TMDI parameters, respectively. The power spectrum S_0 of white noise input is determined according to the Chinese code for seismic design of building, i.e. GB 50011-2016 (Zhang et al. 2007).

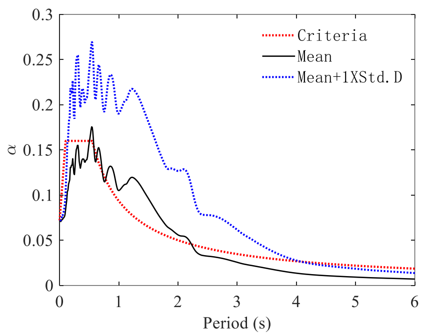


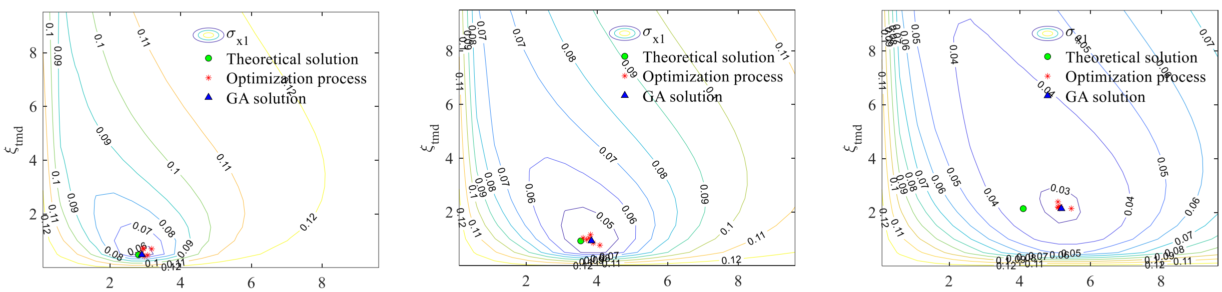
Figure 1 : Mean and standard deviation of response spectra of stochastic earthquake ground motion and standard response spectrum of the code

From the previous analysis, it can be seen that the optimal solution of the parameters given by the theoretical formulas is independent of the primary structural damping because of the assumption in the derivation process. While the numerical optimization method based on the genetic algorithm (GA) considers the influence of the primary structure damping on the optimal parameters (in this study, the optimal parameters are obtained by minimizing the variance of relative displacement of the primary structure subjected to the ideal white noise input). And the parameters of the genetic algorithm are set as follows: the population size is 100, the crossover probability is 0.8, the mutation probability is 0.08, and iteration number is 100, and the parameters optimization intervals are: $\upsilon \in (0.01,10)$ and $\xi_2 \in (0.1,10)$.

The TMDI parameters obtained by the theoretical formulas and the GA optimization for the three inertance-to-mass ratios are shown in Figure 2. The solid green dots are the solutions of the theoretical formulas (for low inertance-to-mass ratio, υ =2.80, ξ_2 =0.4824, for medium inertance-to-mass ratio, υ =3.53, ξ_2 =0.9336, for high inertance-to-mass ratio, υ =4.09, ξ_2 =2.1420), the red stars represent the process of the GA, and the blue triangular blocks are the optimal solutions of the GA (for low inertance-to-mass ratio, υ =2.90, ξ_2 =0.4825, for medium inertance-to-mass ratio, υ =3.84, ξ_2 =0.9337; for high inertance-to-mass ratio, υ =5.17, ξ_2 =2.1421). The results reveal that with the increase of inertance-to-mass ratio, the solutions of the theoretical formula gradually deviate from the optimization results of GA, and the error generated by the solution of the theoretical formula is mainly the value of frequency ratio, and the error is even as high as 20% when β =0.49. To further verify the control efficiency of the TMDI designed based on the optimal solution of GA, the variation of the maximum value of the relative displacement of the primary structure with respect to the frequency ratio υ and the damping ratio ξ_2 under the stationary white noise input is analyzed. The contour plot of the standard deviation of extreme value of the relative displacement response of primary structure is also shown in Figure 2, from which it can be seen that the optimal solution (υ^*, ξ_2^*) of the GA is always in the minimum contour region (i.e., the TMDI control significantly reduces the structural displacement response and meets the design expectation) under different cases of TMDI parameters. The solution of theoretical formula falls within the optimal solution range at low inertance-to-mass ratio; while at high inertance-to-mass ratios, the solution of theoretical formula even completely is in the larger contour region, which further indicates that the use of the theoretical formula cannot reasonably design the TMDI system when the inertance-to-mass ratio is large.

4.2. Probability density evolution analysis

The optimal parameter obtained by the GA are used for the TMDI design, then the probability density evolution method is used to carry out the probability density evolution analysis of the displacement response of the structure-TMDI system under different inertance-to-mass ratios. In Figure 3, the probability density curves of the displacement of the primary structure at typical 2s times are plotted for uncontrolled, low inertance-to-mass ratio (β = 0.09), medium inertance-to-mass ratio (β = 0.19), and high inertance-to-mass ratio (β = 0.49) conditions. It can be seen that (1) At low inertia ratios (β = 0.09), the displacement response of the controlled structure decreases, and comparing the probability density curve at 2s typical moments with the uncontrolled condition, the probability amplitude increases from 0.028 to 0.038, indicating that the variability (distribution width) of the structural response decreases by 35% (the area governed by the probability density curve is always 1). (2) Under the medium and high inertance-to-mass ratios (β = 0.19, 0.49), the displacement response of the controlled structure is significantly reduced; comparing the probability density curves at typical instant 2s with the uncontrolled condition, the probability density amplitude increases from 0.028 to 0.046 (medium inertance-to-mass ratio) and 0.061 (high inertance-to-mass ratio), and the variability of the structural response decreased by 64% (medium inertance-to-mass ratio) and 118% (high inertance-to-mass ratio), indicating a significant improvement of the structural performance.



(a) μ =0.01, β =0.09 **(b) μ =0.01, β =0.19** **(c) μ =0.01, β =0.49**

Figure 2 : Contours of standard deviation of extreme responses of structure-TMDI system with respect to ν and ξ_2 under different inertance-to-mass ratios and ground motions as code

4.3. Analysis of control device parameters

In order to compare and analyze the seismic mitigation performance of TMDI and TMD, parameter optimization and probability density evolution analysis of small mass TMD (μ = 0.01), and large mass TMD (μ = 0.1, 0.2, 0.5) with the same equivalent mass ratio (mass ratio plus inertance-to-mass ratio) of TMDI are further carried out, and all the results are shown in the Figure 3 and Table 1.

It is seen that: (1) the frequency ratio of small mass TMD (μ = 0.01) is about 1.0, which is consistent with the conventional TMD frequency tuning rule. However, for large mass TMDs, the damping ratio increases with the increase of the mass ratio, while the frequency ratio decreases with the increase of the mass ratio. This indicates that for small mass TMDs, when the mass ratio is small, smaller damping elements are required; while the frequency ratio is tuned to about 1.0 to achieve the damping effect by using resonance. When the mass ratio is large, additional larger damping elements are required, and the energy consumption of the damping elements accounts for a more prominent effect. (2) The optimal parameters of TMDI with a same mass ratio of equivalent mass ratio as the mass ratio of the large mass TMD show a different pattern from that of large mass TMD. The damping ratio and frequency ratio of TMDI increase with the increase of inertance-to-mass ratio, which indicates that when the inertance coefficient of inerter is larger, it is necessary to provide larger damping ratio and frequency ratio, and use the energy dissipation of damping and resonance effect for vibration mitigation at the same time. In addition, the conditions with significant resonance effects appear in the medium inertance-to-mass ratio.

The probability density evolution method is used to carry out the probability density evolution analysis of structural displacement response under the control of small-mass and large-mass TMDs, as also shown in Figure 3. It is easy to see that the TMDI with a same equivalent mass ratio (especially the high mass ratio) as the large-mass TMD but with smaller physical mass, has a larger probability density amplitude, i.e., the TMDI with the same mass ratio has better seismic mitigation

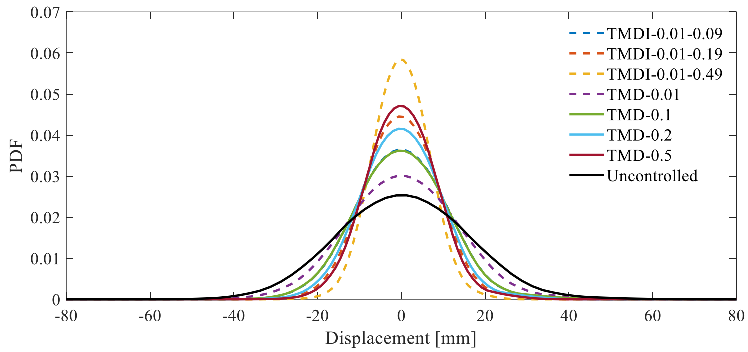


Figure 3 : Comparison of probability density curve at typical instant 2s under various conditions

performance than the TMD. For example, when the equivalent mass ratio is 0.50, the probability density amplitude of the TMDI is 0.061; while the probability density amplitude of the TMD is 0.048. The variability of the structural response is thus reduced by 22%. Comparing the uncontrolled and the TMDs with small mass ratio, the enhancement of the structural performance by the small-mass TMD is not obvious, so for the objective examined in this study, a large-mass TMD or a TMDI design with the same equivalent mass ratio is necessary.

To further illustrate the enhancement effect of TMDI and TMD on structural safety, the analysis of reliability of structural displacement is carried out using probability density evolution method and imposing absorption boundary conditions. The values of reliability of different conditions is shown in Table 1. The results show that: (1) The small-mass TMD has no significant effect on increasing the reliability of the structure. (2) The effect of large-mass TMD on increasing the reliability of the structure is significant, and the reliability changes from 0.8890 to 0.9996, increasing by 12.4%. (3) TMDI with a same equivalent mass ratio of large-mass TMD can increase the reliability of structural displacement to 0.9999, which meets the requirements of safety design. However, the physical mass ratio of TMDI is only 0.01, which is much smaller than the mass ratio of TMD of 0.50. And this indicates that by deploying the inerter, a better control effect is obtained using 1/50 of the physical mass of the original TMD design, and the TMD mass reduction is efficiently achieved.

The technical advantages of TMDI in limiting stroke are also investigated, and the results are also shown in Table 1. Comparing the TMDs with same equivalent mass ratios, the means of stroke of TMDI with small, medium and high inertance-to-mass ratios are reduced by 18.10%, 27.27% and 41.54%, respectively. While under same physical mass ratios, the means of stroke of TMDI with small, medium and high inertance-to-mass ratios are reduced by 45.06%, 60.69% and 75.15%, respectively. The technical advantages of TMDI on mass reduction and stroke limitation are revealed in a probabilistic sense.

5. CONCLUSIONS

The parameter optimization and seismic mitigation performance analysis of the different inertance-to-mass ratios of linear structure-TMDI system subjected to the stochastic earthquake motions have been carried out in this study. Firstly, the results of the theoretical formulas and the genetic optimization algorithm are compared and analyzed, the influence of the primary structure damping on the optimization results is studied, and the applicability conditions of both methods are determined. Then, the damping performance and structural reliability of the structure-TMDI system without inertance-to-mass ratio are comparatively studied according to the optimization results. Finally, the damping performance of small-mass TMD, large-mass TMD and TMDI are compared and analyzed from the view of reliability. The following conclusions are obtained as follows:

1. Under the condition of low inertance-to-mass ratio, the primary structure damping has less influence on the results of the optimal parameters of the structure, and the theoretical formula can be used to solve the optimal parameters at this time. Under the condition of high inertance-to mass ratio, the primary structure damping has a greater influence on the results of parameter optimization, and the GA method should be adopted for optimization at this time.

Table 1 : Optimal parameters and strokes of TMD and TMDI and reliabilities of controlled structures

Cases	*Parameters of control devices*		*Optimal parameters*		*Stroke*		*Reliability*
	μ	β	ν	ξ_2	*Mean [mm]*	*C.O.V*	*p*
Uncontrolled	-	-	-	-	-	-	*0.8890*
Small-Mass TMD	*0.01*	-	*0.96*	*0.10*	*89.36*	*1.28*	*0.9205*
Large-Mass TMD	*0.10*	-	*0.85*	*0.15*	*59.94*	*1.73*	*0.9229*
	0.20	-	*0.74*	*0.21*	*48.30*	*1.82*	*0.9987*
	0.50	-	*0.52*	*0.32*	*37.99*	*2.13*	*0.9996*
TMDI	*0.01*	*0.09*	*2.90*	*0.48*	*49.09*	*1.62*	*0.9937*
	0.01	*0.19*	*3.84*	*0.93*	*35.13*	*1.87*	*0.9998*
	0.01	*0.49*	*5.17*	*2.14*	*22.21*	*2.25*	*0.9999*

2. Comparing the uncontrolled structure, the TMDI system has obvious seismic mitigation effect, and also has obvious improvement for the reliability of the structure, In addition, the larger the inertia ratio, the more significant the control effect.
3. Comparing the TMD with same equivalent mass ratios, the TMDI system requires only 1/50 of the physical mass of the TMD to achieve or even surpass the seismic mitigation performance and structural reliability of the TMD. At the same time, TMDI has a significant stroke limiting effect.

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