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Determination of load combination factors for the assessment of existing bridges

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ABSTRACT: There is a difference between the calibration of partial factors of safety for a new code of practice intended for design, and the development of factors for assessment. In particular, the treatment of load combination factors for code calibration is often a source of conservatism, as is reasonable. However, for the assessment of an existing structure, this conservatism is not desirable. Consequently a means of deriving load combination factors that simultaneously satisfy several limit states is sought. This paper examines the various methods used for load combination factors in the literature and assesses them for use in the assessment of bridges. A simple formula is proposed and demonstrated using several case studies, and compared with the results from the usual heuristic practice. It is shown that the proposed method offers a valuable means of deriving load combination factors for bridge assessment.

1. INTRODUCTION

Modern structural design codes are mainly based on reliability calibration. Many of the loading variables in the design problem are time-varying. Using the First-Order Reliability Method (FORM), the appropriate partial factors for each load can be determined. However, the reduced probability of the simultaneous occurrence of extreme realizations of multiple time-varying loads must be considered through a load combination factor. While there is much guidance in the literature on the estimation of partial factors, far less attention has been given to the estimation of load combination factors.

Melchers and Beck (2017) provides an excellent development of the load combination problem. In particular, the simplifications of the more general (and complex) problem that were introduced by Ferry Borges and Castanheta (1972) and Turkstra and Madsen (1980) are commonly used in code calibration work. However, while these describe means to combine loading processes, they do not specify the determination of partial and load combination factors. Indeed, many structural reliability textbooks describe the determination of partial factors, but not load combination factors; e.g. Nowak and Collins (2013); Melchers and Beck (2017); Haldar and Mahadevan (1999). Thoft-Cristensen and Baker (1982) provides one of the earliest descriptions of both partial and load combination factor calibration, in particular describing the heuristic design value method we examine later. Sørensen (2004) provides an comprehensive description of both partial and load combination factor calibrations using the results of reliability analysis.

We review the calibration problem and the determination of partial and load combination factors. We highlight the methods used to-date for the calibration of load combination factors, and suggest a new method. We apply the approaches to some prototypical examples and illustrate the differences. A particular concern of the new approach is for the calibration of suitable load combination factors for the assessment of existing structures, and this aspect is highlighted.

2. BACKGROUND

2.1. *FORM Basis*

This section briefly introduces the aspects of FORM relevant to the calibration of partial and load combination factors. Refer to structural reliability textbooks for more details (e.g. Nowak and Collins (2013) and Melchers and Beck (2017)).

At the most probable failure point, and in physical coordinates, the limit state equation is

$$g(x_1^*, x_2^*, \ldots) = 0 \tag{1}$$

where x_i^* is the design point of the ith random variable, X_i. In general the distributions of the random variables are non-normal, and so the isoprobabilistic transformation used is

$$x_i^* = F_{X_i}^{-1}[\Phi(u_i^*)] \tag{2}$$

in which $u_i^* = \alpha_i \beta$ is the corresponding design value in standard normal (or u-) space, and α_i is its direction cosine.

For design purposes, a nominal or characteristic value of the variable X is used, x_k. Since the design values satisfy the limit state function (Equation (1)), they will yield a design with reliability, β. Thus, the partial factor to apply to the characteristic value of the variable, to yield the appropriate design value, is

$$\gamma_i = \frac{x_i^*}{x_{i,k}}. \tag{3}$$

2.2. *Code Format*

The prototypical equation to be satisfied in structural design is

$$\phi R_k \geq \sum_i \gamma_{S_i} S_{k_i}, \tag{4}$$

in which R_k is the characteristic value of the resistance variable, and S_{k_i} that of the ith loading variable. Each variable has its respective partial factor, γ, often denoted ϕ in the case of the resistance variable.

The loading variables, S_i, can be classified as permanent and temporal, or varying. That is, permanent loads, such as dead loads—although uncertain—have just one realization from its distribution, for the lifetime of the structure. Conversely, varying loads, such as traffic or wind loads, have realizations occurring from the parent distribution continuously. Of course, this can include periods where the value of loading is zero, such as when there is no traffic on the bridge (i.e. zero-inflated distributions).

For a designer to satisfy Equation (4), the combination of permanent and varying loads should allow for each varying load achieving a maximum value while the others remain at their point-in-time values, within the time frame being considered. As a result, Equation (4) will have to be checked n times, as each varying loading variable achieves a maximum.

As a special (but important) case, consider there to be a permanent dead load, G, a varying live load Q, and a varying wind load W. Equation (4) then becomes:

$$\phi R \geq \gamma_G G_k + \gamma_Q Q_k + \gamma_W W_k. \tag{5}$$

The difficulty here is that the live and wind loads are varying, and (most likely) uncorrelated. So to satisfy Equation (5) when either varying load reaches a maximum within the period being considered, the designer will then seek to satisfy two load cases:

$$\text{LC1:} \quad \phi R \geq \gamma_G G_k + \gamma_{Q_{\max}} Q_k^{\max} + \gamma_{W_{\text{pit}}} W_k^{\text{pit}} \tag{6}$$

$$\text{LC2:} \quad \phi R \geq \gamma_G G_k + \gamma_{Q_{\text{pit}}} Q_k^{\text{pit}} + \gamma_{W_{\max}} W_k^{\max} \tag{7}$$

where the live and wind loads achieve their maximum (max) distribution respectively, while the other remains at its point-in-time (pit) value.

In Equation (6), each varying load is now characterized by four values rather than two: its characteristic point-in-time and maximum values, and their corresponding partial factors. This significant increase in the number of parameters (i.e. $4n$) required to represent each stochastic variable is clearly undesirable for practical structural design codes. To rationalize the number of parameters required, most code formats have adopted a model which can be expressed as

$$\phi R \geq \gamma_G G_k + \gamma_{S_d} S_{d,k} + \psi \gamma_{S_a} S_{a,k}, \tag{8}$$

in which d is the dominating varying load, a is the accompanying varying load, and ψ is a load combination factor. This representation means that each load is parameterized by just a single partial factor γ and a characteristic value S_k. It is the load combination factor ψ that accounts for the reduced probability of both loads achieving their maximum value simultaneously. With this rationalization, Equation (6) can be expressed as:

$$\text{LC1:} \quad \phi R \geq \gamma_G G_k + \gamma_Q Q_k + \psi_W \gamma_W W_k, \tag{9}$$

$$\text{LC2:} \quad \phi R \geq \gamma_G G_k + \psi_Q \gamma_Q Q_k + \gamma_W W_k. \tag{10}$$

Some code formats will merge the load combination and partial factor for the accompanying load and specify a reduced (composite) partial factor (i.e. $\psi\gamma$) when used in combination. Other code formats will retain the partial factor but provide for a reduced characteristic value (i.e. ψS_k) when used in combination. Whatever means is used to represent the problem in codes, the implication for code calibration is that the load combination factor must be estimated.

3. Load Combination Factors

3.1. Design Value Method

Thoft-Cristensen and Baker (1982) gives an empirical estimate of load combination factors. When the variables are ranked in order of their directional cosines α_i, the ith variable can be considered to have a relative direction cosine of:

$$\tilde{\alpha} = \sqrt{i} - \sqrt{i-1}. \tag{11}$$

This recommendation can be coupled with the $\alpha_R = -0.8$ and $\alpha_S = 0.7$ estimate (from ISO 2394:2015 (2015)) to yield estimates of partial and combination factors (Equations (3) and (12)). Although approximate, experience has shown that this approach gives reasonable results.

3.2. Coefficient Method

Sørensen (2004) explains this method as the obvious extension of the FORM result for a single limit state function (or loadcase), to that for two loadcases. Consider the two limit state functions of Equation (9). For calibration, the design values for both loadcases are found for a single target reliability index, β_T. This yields a set of partial factors (following Equation (3)) for each variable, for each limit state. That is, for example, the wind load will have $\gamma_{W,1}$ and $\gamma_{W,2}$, corresponding to the calibration for each loadcase. Rationalization of these partial factors is then done such that a single value of the partial factor for each variable results, e.g. γ_W. With the single set of partial factors and the design point known, the load combination factor for the secondary varying loading becomes:

$$\psi_X = \frac{x^*}{\gamma_X x_k} \tag{12}$$

from which it is clear that the required design point is retrieved from the stipulated partial factor and characteristic value.

While this method gives the precise results for a problem with two time-varying loads, extension to problems with more becomes problematic. It results in unique estimates for partial load factors γ but estimates of ψ and ϕ that vary per load case. In such cases, there are too many degrees of freedom in the problem and a trial and error estimation results. The multiplicity of estimates compounds the difficulties of further rationalization of partial and combination factors using optimization or calibration.

4. General Method

4.1. Matrix Solution

Here we propose an extension to the coefficient method in which a unique set of load combination factors is found; a single one for each varying load. It is supposed that a single load combination factor is desired for each variable, and that a set of partial factors has been determined.

To introduce the method, we consider an example with three time-varying loads, and then show how it can extend to the general case. We consider that each time-varying load must be considered acting at its maximum (i.e. dominating), and so for n such loads, there will be n limit state functions to be satisfied:

$$\phi R \geq \gamma_G G + \gamma_1 S_1 + \psi_2 \gamma_2 S_2 + \psi_3 \gamma_3 S_3 \tag{13}$$

$$\phi R \geq \gamma_G G + \psi_1 \gamma_1 S_1 + \gamma_2 S_2 + \psi_3 \gamma_3 S_3 \tag{14}$$

$$\phi R \geq \gamma_G G + \psi_1 \gamma_1 S_1 + \psi_2 \gamma_2 S_2 + \gamma_3 S_3 \tag{15}$$

in which the subscript k is dropped for clarity, it being understood that these variables are at their characteristic values. Aligning the varying load terms containing the unknown ψ_i to the left hand side of the equation, we get

$$0\psi_1 + \gamma_2 S_2 \psi_2 + \gamma_3 S_3 \psi_3 \leq \phi R - \gamma_G G - \gamma_1 S_1 \quad (16)$$

$$\gamma_1 S_1 \psi_1 + 0\psi_2 + \gamma_3 S_3 \psi_3 \leq \phi R - \gamma_G G - \gamma_2 S_2 \quad (17)$$

$$\gamma_1 S_1 \psi_1 + \gamma_2 S_2 \psi_2 + 0\psi_3 \leq \phi R - \gamma_G G - \gamma_3 S_3 \quad (18)$$

Now rewrite this in matrix form, separating the unknown ψ_i from the remaining known values as follows:

$$\begin{bmatrix} 0 & \gamma_2 S_2 & \gamma_3 S_3 \\ \gamma_1 S_1 & 0 & \gamma_3 S_3 \\ \gamma_1 S_1 & \gamma_2 S_2 & 0 \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{Bmatrix} \leq \begin{Bmatrix} \phi R - Q_1 \\ \phi R - Q_2 \\ \phi R - Q_3 \end{Bmatrix} \quad (19)$$

in which the total permanent and dominating design loads are denoted

$$Q_i = \gamma_G G + \gamma_i S_i. \quad (20)$$

For the limiting equality case, clearly Equation (19) can then be easily solved for the ψ vector by matrix inversion to yield a single set of load combination factors, ψ_i, that satisfy Equation (13).

In a reliability calibration (see later examples), each loadcase in Equation (19) will be separately calibrated. Consequently, there will (in general) be a different design point for the resistance variable, $r^* = \phi R$ in each row. Of course, for structural design purposes, the maximum calibrated resistance must be used in order to satisfy all loadcases. Calculation of the load combination factors can be done with the individual resistance design points, or with the maximum resistance design point.

4.2. *Closed-Form Expression*

While Equation (19) can be extended to consider $n \geq 2$ varying loads, it is desirable to determine an algebraic expression for the general case. On the assumption of a single value for the resistance design point, Equation (19) can be solved for the general case in closed form using Gaussian elimination. The result is:

$$\psi_i = 1 - \frac{\sum_j^n \gamma_j S_j + \gamma_G G - \phi R}{(n-1)\gamma_i S_i}. \quad (21)$$

This provides a closed-form expression for n load combination factors given calibrated partial factors and characteristic design values.

5. EXAMPLES

5.1. *Two Varying Loads*

5.1.1. *Description*

Here we consider an example from Sørensen (2004, p. 190) with two time-varying load sources. The limit state function is

$$g = zR - (0.4G + 0.6Q + 0.3W), \quad (22)$$

in which z is a scalar design parameter (e.g. a deterministic nominal strength design to achieve the target reliability) and the other variables have their usual interpretations. The basic variables are given in Table 1. The distributions specified for the varying loads are the annual maximum distributions. From the number of occurrences in the 1-year reference period, it can be found that the point-in-time distributions for the imposed and wind loads have expected values of 0.89 and 0.77 respectively (the CoV remains unchanged).

Equation (22) is used to calibrate the design parameter z for two loadcases: when the imposed load has its annual maximum distribution, and the wind load is at its point-in-time distribution, and vice-versa. A target reliability index of $\beta_T = 4.3$ is used for the calibration. The resulting design points and design variable values for the two calibration loadcases are shown in Table 1. Further, it is considered that the characteristic values of the imposed and wind loads are given by the 98-percentiles of the annual maximum distributions, and so are $Q_k = 1.518$ and $W_k = 2.037$. With these values, the partial factors are:

$$\gamma_Q = \frac{Q_1^*}{Q_k} = \frac{1.624}{1.518} = 1.069, \quad (23)$$

$$\gamma_W = \frac{W_2^*}{W_k} = \frac{2.246}{2.037} = 1.103. \quad (24)$$

5.1.2. *Coefficient Method*

The load combination factors can be worked out using the coefficient method as:

$$\psi_Q = \frac{Q_2^*}{\gamma_Q Q_k} = \frac{1.513}{1.624} = 0.9262, \quad (25)$$

$$\psi_W = \frac{W_1^*}{\gamma_W W_k} = \frac{2.017}{2.246} = 0.9037. \quad (26)$$

*Table 1: Random variables of Example 1; τ is the occurrence rate of the varying load, and r is the number of such of such occurrences within the reference 1-year period. (*Note: although 360 recurrences per year is reasonable and sated in Sørensen (2004, p. 190), the problem was actually worked out using 2, and this 'error' is retained here for comparison with the source.)*

Symbol	Load	Distribution	E[X]	CoV	τ	r	x_1^*	x_2^*
z	Design parameter	-	-	-	-	-	3.043	3.048
R	Capacity	Lognormal	1.0	0.15	-	-	0.655	0.655
G	Dead	Normal	1	0.10	-	-	1.037	1.037
Q	Imposed	Gumbel	1.0	0.20	0.5 years	2	1.624	1.513
W	Wind	Gumbel	1.0	0.40	1 day	2 (360*)	2.017	2.246

5.1.3. *Design Value Method*

For comparison, the design value method described in Thoft-Cristensen and Baker (1982) gives (see Equation (11)):

$$\alpha = 0.7 \times (\sqrt{2} - \sqrt{1}) = 0.29 \tag{27}$$

which compares to the actual values of $\alpha = 0.54$ (which are the same for both the imposed and wind loads). Nevertheless, when dominating, the design points for the imposed and wind loads are 1.945 and 2.661; and when secondary 1.173 and 1.117, respectively. From which, the partial factors are $1.945/1.518 = 1.28$ for imposed and $2.661/2.037 = 1.31$ for wind. Finally, the load combination factors become $1.173/1.945 = 0.60$ for imposed and $1.117/2.661 = 0.42$ for wind. These values are clearly quite different from the coefficient method.

5.1.4. *Matrix Method*

Using the individual calibrations for each load case, renders Equation (19) in the following form:

$$\begin{bmatrix} 0 & 0.6737 \\ 0.9741 & 0 \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix} = \begin{Bmatrix} 0.6081 \\ 0.9077 \end{Bmatrix} \tag{28}$$

from which the solution for the load combination factors is easily found to be:

$$\psi = [0.9318 \quad 0.9026] \tag{29}$$

which are not very different from those of the coefficient method.

5.1.5. *Closed-Form Equation*

To calculate the result for the closed-form equation (Equation (21)) we stipulate that the characteristic values of the resistance and dead load are at the 5- and 95-percentiles of the distributions, giving $R_k = 0.774$ and $G_k = 1.164$. The design points for these variables are at 0.655 and 1.037 respectively, giving partial factors of $\phi = 0.655/0.774 = 0.847$ and $\gamma_G = 1.037/1.164 = 0.891$ (impractically less than 1.0). The partial factors for the remaining variables are otherwise as for the coefficient method. Using Equation (21), and taking the design parameter as $z = \max(z_1, z_2)$, the load combination factors are:

$$\begin{aligned} K &= 0.6 \times 1.624 + 0.3 \times 2.246 \\ &\quad + 0.4 \times 1.037 - 3.048 \times 0.655 \\ &= 0.066 \end{aligned}$$

$$\psi_Q = 1 - \frac{K}{0.6 \times 1.624} = 0.9326 \tag{30}$$

$$\psi_W = 1 - \frac{K}{0.3 \times 2.246} = 0.9026. \tag{31}$$

These are practically the same as those from the coefficient and matrix methods, and match the results given in Sørensen (2004, p. 191) to two decimal places.

5.2. *Three Varying Loads*

5.2.1. *Description*

Here we extend the previous example and demonstrate that the coefficient approach does not yield a single set of load combination factors; a problem resolved using the proposed matrix approach. The limit state function for the problem becomes:

$$g = zR - (0.2G + 0.6Q_1 + 0.35Q_2 + 0.25Q_3) \quad (32)$$

and the variables are described in Table 2. For the calibration of the design parameter z, a target reliability of $\beta_T = 4.8$ is used. The design points for the three load cases in which each time-varying load is considered as combination in turn are shown in Table 2.

5.2.2. *Coefficient Method*

Similar to Equation (19), we can write the coefficients of the time-varying loads as a matrix. Dividing the design point values by their characteristic values yields:

$$\begin{bmatrix} \gamma_1 & \psi_{2,1}\gamma_2 & \psi_{3,1}\gamma_3 \\ \psi_{1,2}\gamma_1 & \gamma_2 & \psi_{3,2}\gamma_3 \\ \psi_{1,3}\gamma_1 & \psi_{2,3}\gamma_2 & \gamma_{3,3} \end{bmatrix} \quad (33)$$

from which the partial factors are found as the diagonal elements to be:

$$\gamma_Q = [1.3634 \quad 1.1072 \quad 1.2269] \quad (34)$$

Consequently, dividing each column by its partial factor yields the coefficient method estimate of the load combination factors:

$$\begin{bmatrix} 1.0000 & 0.7291 & 0.8627 \\ 0.7743 & 1.0000 & 0.9463 \\ 0.8273 & 0.7925 & 1.0000 \end{bmatrix} \quad (35)$$

As is evident, there is not a single solution for the load combination factor for each time-varying loads. Although not evident from the literature, presumably past practice has then yielded the load combination factors from these results as $\psi_i = \max_j \psi_{i,j}$ giving:

$$\psi = [0.8273 \quad 0.7925 \quad 0.9463] \quad (36)$$

Proceeding with these values, and using the design parameter $z = \max_i z_i$ yields achieved reliabilities of

$$\beta = [5.0028 \quad 5.0708 \quad 5.1493] \quad (37)$$

which are all quite in excess of $\beta_T = 4.8$.

5.2.3. *Matrix Method*

Applying the matrix approach to the problem, and using the individual resistance design points in each row, yields:

$$\begin{bmatrix} 0. & 0.6044 & 0.4668 \\ 1.1233 & 0. & 0.4668 \\ 1.1233 & 0.6044 & 0. \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{Bmatrix} = \begin{Bmatrix} 0.8434 \\ 1.3421 \\ 1.4754 \end{Bmatrix},$$

from which the unique solution for the load combination factors is:

$$\psi = [0.8787 \quad 0.8079 \quad 0.7607]. \quad (38)$$

Using the maximum calibrated design parameter z as before, with these combination factors gives achieved reliabilities for each load case as:

$$\beta = [4.8499 \quad 4.9150 \quad 4.9931]. \quad (39)$$

Although these still exceed the target reliability (as desired), they do so by a lower amount than the traditional coefficient method. The proposed matrix approach offers a least-squares optimum solution for the load combination factors.

5.2.4. *Closed-Form Equation*

Selecting the largest design parameter value $z = \max_i z_i = 3.5045$ and its associated resistance, variable $R = 0.6194$, gives the resistance design point as $\phi R = 2.1707$. From which, we then find:

$$\begin{aligned} K &= 1.1233 + 0.6044 + 0.4668 \\ &\quad - (2.1707 - 0.2 \times 1.0207) \\ &= 1.9665 \end{aligned}$$

$$\psi_1 = 1 - \frac{K}{2 \times 1.1233} = 0.8920 \quad (40)$$

$$\psi_2 = 1 - \frac{K}{2 \times 0.6044} = 0.7992 \quad (41)$$

$$\psi_3 = 1 - \frac{K}{2 \times 0.4668} = 0.7400. \quad (42)$$

Using the maximum calibrated design parameter z as before, with these combination factors gives achieved reliabilities for each load case as:

$$\beta = [4.8197 \quad 4.8843 \quad 4.9623]. \quad (43)$$

These again exceed the target reliability as desired, but to a smaller degree than the traditional coefficient method.

Table 2: Random variables of Example 2.

	Distribution	Annual Max.		Point-In-Time		Characteristic		Design Points		
		E[X]	CoV	E[X]	CoV	Percentile	x_k	x_1^*	x_2^*	x_3^*
z	-	-	-	-	-	-	-	3.5045	3.4546	3.3951
R	Lognormal	1.0	0.15	-	-	0.05	0.7738	0.6194	0.6137	0.6124
G	Normal	1.0	0.10	-	-	0.50	1.0000	1.0194	1.0202	1.0207
Q_1	Gumbel	1.0	0.20	0.887	0.183	0.95	1.3732	1.8722	1.4497	1.5489
Q_2	Gumbel	1.0	0.30	0.828	0.278	0.95	1.5597	1.2591	1.1270	1.3686
Q_3	Gumbel	1.0	0.40	0.802	0.416	0.90	1.5218	1.6108	1.7667	1.8671

6. Discussion & Conclusions

6.1. Discussion

All of the literature on the calibration of partial and load combination factors has focussed on just one or two time-varying variables. This is certainly appropriate for code-calibration for generic limit state functions, typical of those in building or other simple structures. Bridges on the other hand—especially railway bridges—must cope with a wide variety of time-varying loading variables and loadcases. For example, rail bridges must be designed or assessed for wind loads, live loads, braking forces, and track nosing forces; all of which vary in time. Little attention seems to have been given to the calibration of load combination factors for such cases of many $n > 2$ time-varying loads. For design codes this has not necessarily caused a problem, since it is relatively inexpensive to build in some conservatism at the construction stage. However, as the population of bridges around the world ages dramatically, probability-based bridge assessment offers a rational and quantitative means of ensuring safety. But in doing so, it is no longer acceptable to rely on ad-hoc methods of load combination factor calibration, such as the traditional coefficient method for $n > 2$.

6.2. Conclusions

In this work on the calibration of load combination factors, we have demonstrated a matrix-based approach for the simultaneous satisfaction of n loadcases when there are n time-varying load components in a linear limit state system. The general matrix method is shown when the individual loadcase resistance design points are used. A closed-form expression is also derived for the usual case when a single resistance is known. The implications of these methods are demonstrated against current practice, and it is shown that they yield coefficients that render the calibrated limit state functions closer to the target reliability index, than the current ad-hoc coefficient approach. Consequently, these expressions, or the ideas more generally, can be used towards a more rational calibration of load combination factors for linear limit state functions with multiple time-varying load components. This problem is particularly appropriate for the calibration of partial and load combination factors for existing rail bridges.

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