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Identifying dominant failure sequences and key elements in progressive collapse analysis

Trisha Chakravorty
Graduate Student, Dept. of Civil Engineering, IIT Kharagpur, Kharagpur, India

Minangshu Baidya
Assistant Professor, Dept. of Civil Engineering, Techno India University, Kolkata, India

Aritra Chatterjee
Assistant Professor, Dept. of Civil Engineering, IIT Kharagpur, Kharagpur, India

Baidurya Bhattacharya
Professor, Dept. of Civil Engineering, IIT Kharagpur, Kharagpur, India

ABSTRACT: Identification and strengthening of weak elements, and mitigation of element failure sequences that may lead to progressive collapse, should account for uncertainties and incorporate system level performance criteria and initial damaged states. A reliability-based methodology of assessing threat-independent progressive collapse is developed in this work. Intact and surrogate structures are analyzed incorporating non-linearities and dynamic transients. Mutually exclusive minimal cut sets among ordered 'histories' are identified and only statistically significant ones (dominant sequences) are followed up to system failure. The framework is applied to a benchmark truss. Certain 'key' elements that are unlikely to fail in the intact structure, if somehow removed, lead to progressive collapse up to system failure, which highlights the significance of surrogate structures. Asymmetry is observed in ordered failure sequences. Ignoring dynamic effects changes failure probability estimates but not dominant failure sequences. Neglecting nonlinearities, however, may change or eliminate key elements. The outlined framework eliminates the need to consider bounds on the system failure probability since the ordering makes the minimal cut sets mutually exclusive.

1. INTRODUCTION

Structural reliability provides a framework to treat uncertainties associated with a structure (loads, materials, topologies) rationally to ensure adequate safety through design. Though it is easier to model and quantify the reliability of structural elements, the true measure of a structure's safety is its system reliability. Progressive or "disproportionate" collapse, i.e., collapse of a structure triggered by failure of a relatively small part of it, constitutes an important failure mechanism of a structural system. An early study by Leyendecker and Burnett (1976) concluded that nearly 15-20% of all building collapses belong to the category of disproportionate collapse, and that these failures typically occur without any warning. It is initiated by a relatively minor failure that does not stay confined locally, but creates a chain of failures leading to instability or collapse of the whole or a significant part of the system. The triggering event may be caused by natural hazards, accidents, intentional harm, etc.

Ellingwood and Leyendecker (1978) proposed an LRFD type design procedure for unusual structures to limit their progressive collapse probability (conditioned on damage A) to 10^{-2} over the design life. The Probabilistic Model Code JCSS (2001) incorporated reliability-based provision against progressive collapse with the damage A specified as "one element removed." During that time, however, structural analysis tools and computational power available were insufficient for the repeated non-linear dynamic analysis of real structures required for implementation.

Provisions to incorporate progressive collapse explicitly in design or assessment in design codes require probabilistic quantification of redundancy, continuity, strengthening etc. In this paper, a reliability-based framework of assessing progressive collapse accounting for system level performance criteria is developed. Given a structure, apart from its intact state, a surrogate structure corresponding to each of its initial damaged states is defined. A 'history' of the structure is defined as an ordered set of failed elements up to a given time. Not all histories will lead to system failure; mutually exclusive minimal cut sets consisting of ordered element failure histories (including simultaneous failure of two or more elements) are identified and only the statistically significant ones are followed up to system failure. Geometric and material non-linearities and transient dynamic effects are incorporated.

2. METHODOLOGY

2.1. Elements and system representation

Structures are idealized as active load-sharing redundant systems comprising of interconnected elements with dependent failures. An 'element' may not be a single physical unit like a structural member, rather it is a logical unit representing a single failure mode at a particular location. Due to load sharing and load path dependence, elements in a structural system are usually dependent. In this work, elements are defined binary in nature– surviving or failed, which is determined by its limit state equation obtained mechanics.

Defining system failure is critical in reliability analyses. In progressive collapse modeling, system failure may be defined as global instability, or related to performance limits such that maximum inter-storey drift or minimum tangent stiffness. Owing to the binary nature of elements, Boolean logic can be used to relate system failure to element failure events.

Two common methods to determine statistically dominant sequences are the incremental load method and the branch and bound method. The former, since it works quasi-statically, fails to capture dynamic effects that are characteristic to progressive collapse. The latter is suited to progressive collapse analysis, but failure probability may only be expressed with upper and lower bounds since failure sequences are dependent.

In this work, element failure 'histories' are ordered by time (including simultaneous failures of two or more elements), mutually exclusive minimal cut sets of the system are defined and the statistically significant ones followed up to system failure. The outlined framework eliminates the need to consider bounds on the system failure probability since the ordering makes the minimal cut sets mutually exclusive.

2.2. History and stages

Given a structural system consisting n_e elements numbered 1, 2,...,n_e, let event e_j denote failure of element j and e_j^l denote the event that element j is the l^{th} to fail in the structure. A "history" of the structure, H, is defined as an ordered set of failed elements up to a given time. With the state 0 denoting the intact structure, a history involving k failed elements is:

$$H = \{0, e_{j_1}^1(\tau_1), e_{j_2}^2(\tau_2), ..., e_{j_k}^k(\tau_k)\} \qquad (1)$$

which means element $e_{j_1}^1$ fails in the intact structure at time τ_1 and element $e_{j_k}^k$ fails k^{th} at time τ_k. ($0 \leq \tau_1 \leq \tau_2 \leq ... \leq \tau_k$). A "stage" is defined as the interval between two successive failures, i.e., time 0 to τ_1 denotes intact stage and τ_k to τ_{k+1} denotes k^{th} stage.

If simultaneous failures are ignored, *k!* histories are possible for the same failed elements. Moreover, these *k!* histories involving the same elements may not lead to the same end system state due to nonlinearities. The structure is subjected to dynamic effects in every stage following an element failure and the damaged structure needs to have residual strength to survive the peak demand following the current failure and reach equilibrium; otherwise, the next failure occurs, and so on.

2.3. Minimal cut sets and their probabilities

A history does not necessarily end in structural failure. A cut set is a collection of elements, all of which must fail for the system to fail. A minimal cut set is one from which removal of any one constituents implies it no longer remains a cut set. Since progressive collapse is a sequence driven event, system failure depends not only on the elements of a minimal cut set but also on the order in which they occur. Thus, to define a minimal cut set, we include the order in which its constituent elements fail:

$$C_i^0 = \{e_{j_1}^1, e_{j_2}^2, ..., e_{j_{n_i}}^{n_i}\} \tag{2}$$

where superscript in C_i^0 is the history at the start of the sequence (0 signifies intact state), n_i is number of elements in the minimal cut set and $e_{j_k}^k$ denotes that j_k is the k^{th} element to fail. This formulation does not include simultaneous failure of two or more elements, which can be included by defining the events slightly differently ($e_{j_k\&j_k}^l$ for j_k and j_{k+1} failing simultaneously at the start of stage j). Now, for a given element j, the two events e_j^l and e_j^m are mutually exclusive for $l \neq m$ making the minimal cut sets defined in Eq. (2) mutually exclusive. Their union, i.e., system failure probability, is simply the sum of the individual probabilities without the need to consider probabilities of joint events. The trade-off is that for large n, the combinatorial aspect of the formulation could become prohibitive. This problem may be simplified by implementing an efficient search strategy to find the probabilistically dominant sequences, or by reducing effective number of elements by appropriate scaling and discretization of structural members. Probability of occurrence of a single minimal cut set may be estimated as followed, involving load in each element at every stage of the history.

$D_j^{H_k}$ and $S_j^{H_k}$ represents the demand and capacity of element j at the end of history H_k. The first line of Eq. (3) implies that element j_1 fails first from the intact stage, the second line that element j_2 survives the intact state but fails next, and so on, and the last line that all elements x not belonging to C_i^0 survive until the end.

$$\begin{aligned} P[C_i^0] = P[e_{j_1}^1, e_{j_2}^2, ..., e_{j_{n_i}}^{n_i}] \\ = P[S_{j_1}^0 < D_{j_1}^0 \\ S_{j_2}^0 \geq D_{j_2}^0, S_{j_2}^{j_1} < D_{j_2}^{j_1} \\ S_{j_3}^0 \geq D_{j_3}^0, S_{j_3}^{j_1} \geq D_{j_3}^{j_1}, S_{j_3}^{j_1,j_2} < D_{j_3}^{j_1,j_2} \\ \dots \\ S_{j_x}^0 \geq D_{j_x}^0, S_{j_x}^{j_1} \geq D_{j_x}^{j_1}, ..., S_{j_x}^{j_1,j_2,...,j_{n_i-1}} \\ \geq D_{j_x}^{j_1,j_2,...,j_{n_i-1}} \\ x \notin C_i^0] \end{aligned} \tag{3}$$

If element strengths are independent of the history, as in several cases, the expression can be simplified as follows.

$$\begin{aligned} P[C_i^0] = P[e_{j_1}^1, e_{j_2}^2, ..., e_{j_{n_i}}^{n_i}] \\ = P[S_{j_1} < D_{j_1}^0 \\ D_{j_2}^0 \leq S_{j_2} < D_{j_2}^{j_1} \\ max(D_{j_3}^0, D_{j_3}^{j_1}) \leq S_{j_3} < D_{j_3}^{j_1,j_2} \\ \dots \\ max(D_{j_x}^0, D_{j_x}^{j_1}, ..., D_{j_x}^{j_1,...,j_{n_i-1}}) \leq S_{j_x} \\ x \notin C_i^0] \end{aligned} \tag{4}$$

Randomness in the demands arises from uncertainties in loads, constitutive properties, boundary conditions and in analysis models while that in element strengths is owed to uncertainties in material properties, aging effects and idealizations made in mechanistic models. Statistical dependence between demands, element strengths, and sometimes mutual dependence between $D_j^{H_k}$ and S_j for the same element j are considered while evaluating the probability of a cut set.

2.4. System reliability representation

System failure in progressive collapse may be viewed in different ways. The first approach involves estimating the system failure probability of the intact structure under a given loading, $\underline{L}_d$, within the design envelope.

$$P[F_{sys}|\underline{L}_d] = P\cup_i^{n_c}[C_i^0|\underline{L}_d] = \sum_i^{n_c} P[C_i^0|\underline{L}_d] \quad (5)$$

Often used in traditional system reliability analysis, statistically dominant sequences in the intact structure can be determined this way. The second approach looks at a surrogate structure created by removing elements $\underline{I}$ from the original system. The loading $\underline{L}_d$ is typically milder than that in design basis (due, among others, to a reduced time horizon), but dynamic loads and falling debris, if any, caused by the removal of $\underline{I}$ should be added to it.

$$P[F_{sys'}|\underline{L}_{d'}] = P\cup_i^{n_c'}[C_i^I|\underline{L}_{d'}] = \sum_i^{n_c'} P[C_i^I|\underline{L}_{d'}] \quad (6)$$

The minimal cut sets are redefined based on the new intact condition. In general, $P[F_{sys'}|\underline{L}_{d'}]$ in Eq. (6) is different from $P[F_{sys}|\underline{L}_d, \underline{I}]$ obtained by conditioning failure probability in Eq. (5) on $\underline{I}$ even if $\underline{L}_d$ were made equal to $\underline{L}_{d'}$. This approach is a threat-independent assessment of progressive collapse. The third approach is threat-dependent analysis, which is outside the scope of this work.

2.5. *Opening of stages*

In order to make the computation efficient, only those histories which would lead to probabilistically significant minimal cut sets are followed up to failure and other are eliminated at the earliest. The probability of an arbitrary cut set $P[C_i^0] = P[e_{j_2}^2, e_{j_3}^3, ..., e_{j_{n_i}}^{n_i}|e_{j_1}^1]P[e_{j_1}^1]$ cannot exceed probability of element j_1 failing in intact stage $P[e_{j_1}^1] = P[S_{j_1} < D_{j_1}^0, S_x \geq D_x^0, x \notin C_i^0]$, which in turn cannot exceed the joint probability of any subset of $\{S_{j_1} < D_{j_1}^0, S_x \geq D_x^0, x \notin C_i^0\}$.

$$P[C_i^0] \leq P[e_{j_1}^1] \leq min\{P[S_{j_1} < D_{j_1}^0], P[S_x \geq D_x^0, x \notin C_i^0]\} \quad (7)$$

In the intact stage, probabilities of element failure occurring singly, then if necessary jointly, and so on with increasing order are computed till histories worth pursuing into the first stage are identified. The decision criterion can be a simple predetermined lower cut-off value for $P[C_i^0]$, or based on ranking of the individual $P[e_{j_1}^1]$'s, or sophisticated network search strategies. However, the decision should not be determined only on single stage probabilities, which may miss sequences that would be conditionally likely later on. The algorithm can be extended into the k^{th} stage (for some k) to decide upon histories worth pursuing into the $(k+1)^{th}$. A smaller set of elements $(e_{j_1}^{1*}, e_{j_2}^{1*}, ..., e_{l_1}^{k*}, e_{l_2}^{k*}, ...)$ would now constitute first k failures and we need only be concerned with minimal cut sets starting with $e_j^{1*}, ..., e_l^{k*}$. Thus, in the first stage, the structure would be reanalyzed with e_j^{1*} removed and subject to operating loads $\underline{L}_d$ and loads caused by the removal, and so on up to collapse and each probabilistically significant minimal cut set is identified to completion.

The approach should be adopted for each surrogate structure, too. Not only the elements identified for high failure probability in the intact stage should be chosen as $\underline{I}$ for creating surrogate structures, since it is possible that an element unlikely to fail first under operating loads in the intact structure, if somehow removed, leads to unsuspected weak failure sequences.

3. ILLUSTRATIVE EXAMPLE—SINGLE STORY RECTANGULAR TRUSS

3.1. *System definition*

A six-membered single story rectangular steel truss (Figure 1) is taken up to illustrate the framework. This truss configuration, first considered by Murotsu et al. (1980) for system reliability bounds, has been analyzed thereafter, among others, by Melchers and Tang (1984) for finding dominant failure modes, Felipe et al. (2018) for progressive collapse analysis, etc. Dimensions of the truss is 1.2 m (horizontally) x 0.9 m (vertically); cross sectional area of horizontal and vertical members is 133 sq mm while that of diagonal members is 149 sq mm.

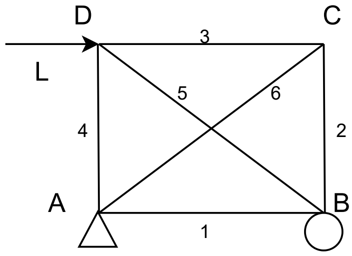

Figure 1: Truss configuration used

3.2. *Minimal cut sets in intact and surrogate structures*

Truss members take only axial load, so each of the 6 members is represented by one reliability element. Degree of indeterminacy is 1, hence any two members failing would lead to global instability, which we define as failure. There are 30 cases of sequential failure. If simultaneous failure is considered, we would have 15 cases of two members failing together in the intact stage. There can be other combinations, such as more than two failing in the intact stage, or in the first stage, but these cut sets are expected to be unlikely in progressive collapse situations.

For illustration, we take the first minimal cut set in the intact state, the sequence that element 1 fails first and element 2 fails second, $C_1^0 = \{e_1^1, e_2^2\}$. Assuming strength of members to be independent of the history, this event expanded in terms of the 6 elements gives (x=3,4,5,6):

$$C_1^0 = [S_1 < D_1^0, D_2^0 \leq S_2 < D_2^1, max(D_x^0, D_x^1) \leq S_x] \tag{8}$$

Surrogate structures are constructed by sudden removal of a single element (say element I) in the normally loaded truss. The surrogate structures here are statically determinate. 31 minimal cut sets are possible for each surrogate structure, of which 5 are single element failure events $C_i^I = \{e_j^1\, j \neq I, i = 1, \ldots 5\} = [S_j < D_j^I, S_k \geq D_k^I, k \neq I, j]$ and 10 are two-element simultaneous failures $C_i^I = \{e_{j\&l}^1\, j, l \neq I, i = 6, \ldots 15\} = [S_j < D_j^I, S_l < D_l^I, S_k \geq D_k^I, k \neq I, j, l]$. Only a few of these cut sets are expected to be statically significant.

3.3. *Analysis including dynamic effects*

Yield stress and Young's modulus are assumed deterministic at 276 MPa and 206 GPa. A tri-linear constitutive model as recommended by ECCS (European Convention for Constructional Steelwork) and discussed by Yun and Gardner (2017) is used, where stress is horizontal after yield up to the strain hardening threshold $\varepsilon_{sh} = 10\varepsilon_Y$ (ε_Y being yield strain), followed by strain hardening at constant modulus of $E_{sh} = E/50$. Complete fracture is assumed to take place when axial member stress exceeds failure stress in tension or compression, with no residual resistance. Along with material non-linearity, geometric nonlinearity and transient dynamic effects after sudden failure of a member (with 5% structural damping) are included. An implicit dynamic time history analysis is performed with in ABAQUS DassaultSystems (2009) using the Hilbert-Hughes-Taylor time integration scheme.

A horizontal force L is applied at node D, ramped up quasi-statically from zero to its final value and sustained thereafter, and response history of every element is recorded. No element fails during this stage. In the next stage, a pre-selected member, I, is suddenly removed and remaining elements' histories are recorded under the sustained load L. The structure being one-degree indeterminate, analysis is terminated after first stage. The process is repeated for all I's.

Figures 2, 3 and 4 shows strain and stress time histories for each member. The sustained load is the mean load considered by Murotsu et al. (1980), L=45.5 kN. The intact stage corresponds to the duration 0–2s (ramping up to 1 s, held constant afterwards). Stage 1 commences at 2 s when the SE-NW diagonal element 5 is suddenly removed. Dynamic transients caused by sudden failure are visible at 2 s which die down at around 2.2 s. From Figure 2 without material non-linearity to Figure 3 with it, transient dynamics become more prominent, but stress response does not change much. The strain response, however, changes with the overloaded surviving diagonal and top chord experiencing plastic deformation. With introduction of geometric nonlinearity in Figure 4, strain and stress in the surviving diagonal increases noticeably while those in the top chord decreases. Large non-linear deformations affect the bottom chord too, which no longer remains a zero force member (Figure 2) but carries a significant load (Figure 4) in stage 1. Nonlinearities are expected to have a significant role in progressive collapse reliability.

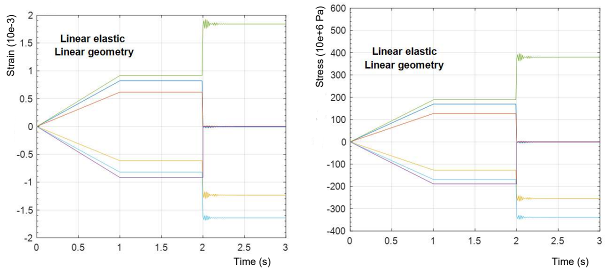

Figure 2: Strain and stress history with linear material and geometry, member 5 removed at 2 s

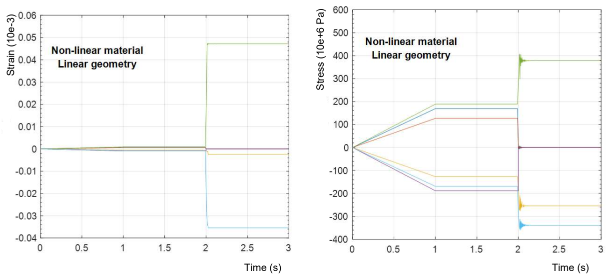

Figure 3: Strain and stress history with non-linear material and linear geometry, member 5 removed at 2 s

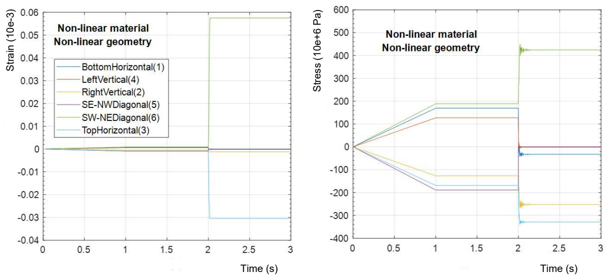

Figure 4: Strain and stress history with non-linear material and geometry, member 5 removed at 2 s

3.4. Probability computations

When strengths are mutually independent and independent of demands, probability of occurrence of a cut set is estimated from Eq. (3). This expression is evaluated using Monte Carlo simulations.

A stress-based approach defines member failure. Load L is assumed random following a Gumbel distribution with mean 31.5 kN and COV 20% and 6 member capacities (equal in tension and compression) each lognormally distributed with mean 430 MPa and COV 20%). All seven random variables are mutually independent. Load is simulated using Latin Hypercube sampling; for each realization of L, the structure is analyzed, element demands recorded, and probabilities of minimal cut sets estimated.

3.5. Reliability of intact structure

Reliability of intact structure is estimated using Eq. (5). From intact state, six single element failures and 57 two or more simultaneous element failures are possible. The minimal cut sets involving simultaneous failures are found to be unlikely. The single element failure probabilities in intact state are respectively $2.07x10^{-7}$, $6.68x10^{-11}$, $2.02x10^{-7}$, $7.56x10^{-11}$, $2.57x10^{-6}$ and $2.59x10^{-6}$. Probability of failure of elements 2 and 4 are negligible compared to others; histories starting with them failing first may be neglected in estimating the intact structure's reliability. For the sake of completeness, however, they are all analyzed in this example.

Top 10 dominant minimal cut sets for the system (out of an exhaustive list of 243) account for 96% of the system failure probability of $2.46x10^{-6}$. Eight of these, including the top four, involve two element sequential failures; only the fifth and sixth involve the failure of one element followed by the simultaneous failure of two elements (accounting for only 7% of total system failure probability).

The diagonal members, in spite of having 12% more area than the four peripheral members contribute most to the system's failure; sequential failure of the two diagonals contributes 61%. Asymmetry is observed in the two sequences (5,6) and (6,5); although element 5 and 6 are equally likely to fail in intact condition, (5,6) is thrice more likely to occur than (6,5). Considering all 30 sequences (Table 1), asymmetry is observed everywhere, which emphasizes the importance of ordering of sequences. Further studies show the asymmetry arises from nonlinearity.

Table 1: Probability of two element ordered failure sequences starting from intact state (row: first element to fail, column: second element to fail

	1	2	3	4	5	6
1	-	2.83 e-10	2.23 e-08	6.70 e-18	8.38 e-14	6.69 e-08
2	6.15 e-12	-	4.32 e-18	2.96 e-13	1.10 e-11	3.41 e-17
3	3.00 e-08	4.30 e-31	-	1.81 e-09	4.41 e-08	7.87 e-14
4	2.67 e-18	8.05 e-15	6.19 e-15	-	6.19 e-21	9.25 e-14
5	7.82 e-18	1.09 e-08	1.23 e-07	0.00 e-00	-	1.09 e-06
6	3.72 e-07	6.21 e-17	4.98 e-18	2.60 e-08	4.14 e-07	-

3.6. The surrogate structures

Analysis of the intact structure shows that the two diagonals are the elements likeliest to fail first. (5,6) and (6,5) are the top two sequences leading to system failure. If we are interested only in the reliability of the intact structure, strengthening the diagonals will make the structure safer. If we are concerned about the damaged structure, too, surrogate structures must be taken into account.

The surrogate structure is created by removing one element (I) from the intact structure with dynamic effects added to operating load L. There are 6 statically determinate surrogate structures with 31 minimal cut sets, five involving single element failure and the rest two or more simultaneous failures. The system failure probability of the surrogate structure is obtained from Eq. (6) with $n_c = 31$. Setting I=1, the individual minimal cut sets can be expanded as $C_1^1 = \{S_1 < D_1^1, S_x \geq D_x^1, x = 2, ...6\}$, $C_6^1 = \{S_x \geq D_x^1, S_6 < D_6^1, x = 1, ...5\}$ up to $C_{31}^1 = \{S_x < D_x^1, x = 1, ...6\}$.

System failure probabilities of surrogate structures are shown in Figure 5. Two highest contributing cut sets have been marked in each column; unsurprisingly, in most cases the likeliest to fail is 5 or 6. System failure probabilities are roughly four orders higher than failure probability of the intact system. Clearly, reliability is affected substantially by one degree of redundancy. The fourth column, 2.5 to 5 times larger than the rest is interesting–even though element 4 is amongst the unlikeliest to fail in the intact stage, if it is removed, the structure is almost certain to collapse, making element 4 the strongest contender for the key element.

Hence, if the intact structure has sufficient reliability, it would probably be better to strengthen element 4 and leave the diagonals as they are.

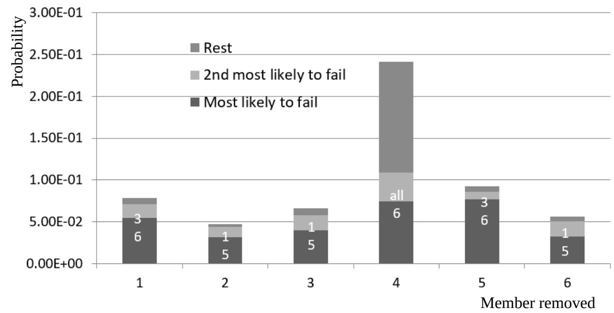

Figure 5: System failure probabilities of the six surrogate structures (two likeliest to fail element(s) indicated in white font; nonlinearities and dynamics included)

3.7. Effects of ignoring nonlinear and dynamic effects

Computation of progressive collapse analysis can be simplified by (a) assuming the loading to be quasi-static or (b) ignoring geometric and material non-linearities. General trend of implementing these simplifications is studied.

Ignoring dynamic transients caused by sudden member failure is studyied by reducing the load carried by the member to be removed quasi-statically to zero and keeping structural load constant at L_d. Material and geometric nonlinearities are retained. Since peak loads are decreasing, expectedly system failure probability reduces by about 30%. However, top sequences, become more dominant. The sequence (5,6) now contributes 51% of system failure probability (vs 44% before); the top 5 sequences remain unchanged but contribute 91% to $P_{f_{sys}}$ (vs 85% before). Asymmetry between different ordered sequences with same elements (i.e., (6,5) vs (5,6) and (6,1) vs (1,6)) remains. For the six surrogate structures, system failure probabilities reduce (slightly for I = 4, by about 20%

for the others): $6.44x10^{-2}$, $3.59x10^{-2}$, $3.28x10^{-2}$, $2.36x10^{-1}$, $7.22x10^{-2}$ and $3.47x10^{-2}$ respectively. Element 4 becomes a stronger contender for the key element status. Thus, neglecting dynamic effects does not change the main conclusions, at least qualitatively.

Neglecting non-linearities, however, alters the nature of the problem, whether dynamics is included or not. System failure probability of intact structure goes up by about 20% in either case. The sequences (5,6) and (6,5) become equally likely, eliminating asymmetry, and contribute about 27%-30% each to system failure probability. The effect on the surrogate structure becomes more telling. Failure probability for each I is no longer distinct and there is no identifiable key element.

4. CONCLUSIONS

The reliability-based framework of assessing threat-independent progressive collapse developed in this work accounts for credible initial damages, sequential and simultaneous failures of elements, and relevant uncertainties and incorporates geometric and material non-linearities and transient dynamic effects. Statistically significant minimal cut sets (dominant failure modes) are identified. Since the different minimal cut sets are mutually exclusive due to ordering of element failures, the requirement of bounds on the system failure probability is eliminated. The methodology is demonstrated on a benchmark indeterminate 6-member planar steel truss. Six surrogate structures, each corresponding to one member loss, are analyzed in addition to the intact structure.

Only a few minimal cut sets contributed to most of the system failure probability of the intact structure and each surrogate structure, with the most dominant one contributing as much as 50% in some cases. Simultaneous failures were rare. Several ordered sequences were asymmetric, emphasizing the importance of ordering of element failures in defining cut sets. The key element in the truss (element 4), obtained by comparing the six surrogate structures, was among the least likely to fail in normal conditions, all failure sequences starting with it being negligible in the intact structure. This suggests that picking dominant sequences in the intact structure for strengthening, without looking at the surrogate structures, could be counterproductive.

Ignoring non-linearities changed the nature of problem, the dominant sequence losing its prominence and there being no indentifiable key element anymore. Neglecting transient dynamics reduced failure probabilities, but did not change the qualitative nature of the conclusions. This suggests that it may be possible to substitute dynamic analysis by a carefully calibrated static equivalent.

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