Stochastic Incremental Dynamics Methodology for Nonlinear Structural Systems Endowed with Fractional Derivative Terms Subjected to Code-compliant Seismic Excitation

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ABSTRACT: A novel stochastic incremental dynamics analysis methodology is developed for nonlinear structural systems with fractional derivative elements exposed to seismic excitation consistently aligned with contemporary aseismic codes provisions (e.g. Eurocode 8). Rendering to the concept of non-stationary stochastic processes, the vector of the imposed seismic excitations is characterized by evolutionary power spectra compatible in a stochastic sense with elastic response acceleration spectra of specified modal damping ratio and scaled ground acceleration. The proposed stochastic dynamics technique relies on a combination of the stochastic averaging and statistical linearization methods, which permits the determination of the response displacement probability density function in an efficient and comprehensive manner. The commonly encountered in the literature incremental dynamics analysis curves have been replaced by a stochastic incremental dynamics analysis surface providing with reliable higher order statistics of the system response. A significant attribute of the method pertains to the derivation of an associated response evolutionary power spectrum as a function of spectral acceleration. The method retains the coveted attribute of a particularly low associated computational cost. A structural system comprising the nonlinear model endowed with fractional derivative terms subject to a Eurocode 8 elastic design spectrum serves as a numerical example for demonstrating the reliability of the proposed methodology, whose accuracy is demonstrated by comparisons with pertinent Monte Carlo simulation data.

1. INTRODUCTION

In the engineering discipline of earthquake resistant structures nonlinearities arise naturally in various forms. In this setting, there is a well-detected need for rigorous and consistent representation of the system model by considering thoroughly the underlying mechanisms which determine the overall system behavior. Constant needs for enhanced modeling purposes dictate a reasonable transition to more advanced mathematical tools such as fractional calculus. Notably, structural engineering has significantly benefited from the utilization of fractional calculus concepts. The emerged number of research efforts pertaining to seismic isolation, vibration control and energy harvesting applications reveals the capabilities of fractional calculus to offer upgraded system modeling services in numerous cases of structural engineering interest (Makris and Constantinou, 1991; Rüdinger, 2006; Koh and Kelly, 1990; Kougioumtzoglou et al., 2022; Di Paola et al., 2013; Rossikhin and Shitikova, 2010). Further, an appropriate stochastic representation of seismic excitation in conjunction with nonlinear system modeling and in alignment with aseismic codes provisions secures a solid basis for formulating a realistic structural analysis procedure (Mitseas and Beer, 2021).

Reliable numerical estimations related to the performance of structural systems necessitate a proper quantitative treatment of uncertainties. The emerging concept of performance-based earthquake engineering (PBEE) advocates the assessment of the structural system performance in a comprehensive and rigorous manner by properly accounting for the presence of uncertainties (Mitseas et al., 2016; Mitseas and Beer, 2020). Specifically, basic notions pertaining to PBEE comprise the definition of excitation related variables, known as intensity measures (IMs) (e.g., spectral acceleration, peak ground acceleration, etc.), and of system response related variables known as engineering demand parameters (EDPs) (e.g., peak story drift, inter-story drift ratio, etc.). Moreover, the information provided via the functional relationship between the IMs and the EDPs in conjunction with judicially defined damage-state rules (DSs), is utilized for quantifying a decision variable (DV) (e.g. financial loss).

In the earthquake engineering field, one of the customarily employed methodologies for estimating the functional relationship between the IMs and the EDPs is the incremental dynamic analysis (IDA) (Vamvatsikos and Cornell, 2002). IDA aims at assessing the structural performance of systems subject to a suite of ground motion records, each scaled to several levels of seismic intensity; thus, conducting a nonlinear response time-history analysis (RHA) for each and every scaled record. It is noteworthy that each IDA curve is related to a specific ground motion record whereas each point of the curve corresponds to a specific ground motion intensity level and respective structural system response magnitude. Clearly, the determination of the above-mentioned functional relationship is associated with a significant computational cost. Further, IDA provides with simple statistics of the selected EDP such as the standard deviation and the mean whereas potential higher order statistics requirements under a fully probabilistic framework could render the whole process computationally prohibitive. Notably, some recent research efforts have been made in the area harnessing the potential of advanced random vibration concepts (dos Santos et al., 2016; Mitseas and Beer, 2021).

The developed stochastic incremental dynamics analysis methodology pertains to nonlinear structural systems with fractional derivative elements exposed to seismic excitation consistently determined with contemporary aseismic codes provisions (e.g. Eurocode 8). Specifically, the imposed scaled seismic excitation is characterized by a series of evolutionary power spectra (EPS) compatible in a stochastic sense with an elastic response acceleration spectrum of specified modal damping ratio and scaled ground acceleration (Cacciola, 2010). At the core of the proposed technique lies a combination of the stochastic averaging and statistical linearization methodologies (Roberts and Spanos, 2003; Fragkoulis et al., 2019), which permits the determination of the response displacement probability density function (PDF) in an efficient manner. The generated stochastic incremental dynamics analysis surface provides with reliable higher order statistics of the system response. In addition, a significant attribute of the proposed method is the derivation of the associated response EPS as a function of spectral acceleration. Notably, the method keeps the associated computational cost at a minimum level. An illustrative numerical example pertaining to a bilinear hysteretic structural system with fractional derivative elements subject to a Eurocode 8 elastic design spectrum serves as a numerical example for demonstrating the reliability of the proposed methodology, while comparisons with relevant Monte Carlo simulation (MCS) data are included as well for assessing its accuracy.

2. MATHEMATICAL FORMULATION

2.1. Equivalent linear system determination

The governing equation of motion of a nonlinear single-degree-of-freedom (SDOF) system endowed with fractional derivative elements subject to a non-stationary excitation is given by

$$\ddot{x}(t) + \beta D_{0,t}^{\alpha} x(t) + g(t,x,\dot{x}) = a_g(t), \qquad (1)$$

where x is the response displacement and a dot over a process denotes differentiation with respect to time. $g(t,x,\dot{x})$ is an arbitrary nonlinear/hysteretic function and $D_{0,t}^{\alpha}(\cdot)$ represents the Caputo fractional derivative of fractional order α ($0 < \alpha < 1$)

$$D_{0,t}^{\alpha} x(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot{x}(\tau)}{(t-\tau)^{\alpha}} d\tau, \qquad (2)$$

where $\Gamma(\cdot)$ is the Gamma function and β is a damping coefficient. Lastly, $a_g(t)$ is a stochastic seismic excitation process whose evolutionary power spectrum (EPS) $S_{a_g}(\omega,t)$ is compatible with a prescribed design spectrum $S(\omega,\zeta_0)$ (Cacciola, 2010).

Next, a recently developed approximate analytical technique (Fragkoulis et al., 2019), which relies on a combination of statistical linearization and stochastic averaging methods, is applied to determine the non-stationary response amplitude PDF of the oscillator in Eq. (1). Considering that the oscillator is lightly damped, its response follows a pseudo-harmonic behavior, given by

$$x(t) = A(t)\cos(\omega(A)t + \psi(t)), \qquad (3)$$

$$\dot{x}(t) = -\omega(A)A(t)\sin(\omega(A)t + \psi(t)), \qquad (4)$$

where $\psi(t)$ and $A(t) = A$ denote the response phase and amplitude, respectively. The latter vary slowly with respect to time, and thus, can be regarded as constant over one cycle of oscillation (Roberts and Spanos, 1986). Taking into account Eqs. (3)-(4), a decoupling of the corresponding differential equations is attained in the form

$$A^2(t) = x^2(t) + \left(\frac{\dot{x}(t)}{\omega(A)}\right)^2, \qquad (5)$$

$$\psi(t) = -\omega(A)t - \arctan\left(\frac{\dot{x}(t)}{x(t)\omega(A)}\right). \qquad (6)$$

Then, applying a statistical linearization scheme, Eq. (1) is recast into

$$\ddot{x}(t) + (\beta_0 + \beta(A))\dot{x}(t) + \omega^2(A)x(t) = a_g(t), \qquad (7)$$

where $\beta_0 = 2\zeta_0\omega_0$ with ω_0 and ζ_0 denoting the natural frequency and damping ratio of the corresponding linear oscillator. Further, defining an error function as the difference between Eqs. (1) and (7) and minimizing it in a mean square sense, leads to the equivalent linear amplitude-dependent elements

$$\beta(A) = -\beta_0 + \frac{S(A)}{A\omega(A)} + \frac{\beta\sin\left(\frac{\alpha\pi}{2}\right)}{\omega^{1-\alpha}(A)}, \qquad (8)$$

$$\omega^2(A) = \frac{F(A)}{A} + \beta\omega^{\alpha}(A)\cos\left(\frac{\alpha\pi}{2}\right), \qquad (9)$$

where

$$S(A) = -\frac{1}{\pi}\int_0^{2\pi} g_0(A,\phi)\sin\phi d\phi, \qquad (10)$$

$$F(A) = \frac{1}{\pi}\int_0^{2\pi} g_0(A,\phi)\cos\phi d\phi, \qquad (11)$$

with $g_0(A,\phi) = g(A\cos\phi, -A\omega(A)\sin\phi)$ and $\phi(t) = \omega(A)t + \psi(t)$. Since Eqs. (8) and (9) are amplitude-dependent, $\beta(A)$ and $\omega(A)$ are also non-stationary processes. Hence, taking expectations on Eqs. (8) and (9) yields the time-varying mean values (Kougioumtzoglou and Spanos, 2009)

$$\beta_{eq}(t) = \int_0^{\infty} \beta(A)p(A,t)\mathrm{d}A, \qquad (12)$$

$$\omega_{eq}^2(t) = \int_0^{\infty} \omega^2(A)p(A,t)\mathrm{d}A, \qquad (13)$$

where $p(A,t)$ denotes the non-stationary response amplitude PDF. In passing, note that Eqs. (12) and (13) correspond to the equivalent linear system

$$\ddot{x}(t) + \left(\beta_0 + \beta_{eq}(t)\right)\dot{x}(t) + \omega^2(t)x(t) = a_g(t). \qquad (14)$$

Clearly, $p(A,t)$ is required for evaluating the time-varying equivalent elements in Eqs. (12)-(13), which is given by (Fragkoulis et al., 2019)

$$p(A,t) = \frac{\sin\left(\frac{\alpha\pi}{2}\right) A}{\omega_0^{1-\alpha} c(t)} \exp\left(-\frac{\sin\left(\frac{\alpha\pi}{2}\right)}{\omega_0^{1-\alpha}} \frac{A^2}{2c(t)} \right). \tag{15}$$

In Eq. (15), $c(t)$ denotes an unknown time-dependent coefficient, which is determined by resorting to a stochastic averaging treatment of Eq. (14). In this regard, first, a first-order stochastic differential equation for $A(t)$ is derived. Then, substituting Eq. (15) into the corresponding Fokker-Planck partial differential equation governing the evolution in time of $p(A,t)$, i.e.,

$$\begin{aligned} \frac{\partial p(A,t)}{\partial t} = &-\frac{\partial}{\partial A}\left\{ \left(-\frac{1}{2}(\beta_0 + \beta_{eq}(t))A \right.\right. \\ &\left.\left. + \frac{\pi S_{a_g}(\omega_{eq}(t),t)}{2\omega_{eq}^2(t)A} \right) p(A,t) \right\} \\ &+ \frac{1}{4}\frac{\partial}{\partial A}\left\{ \frac{\pi S_{a_g}(\omega_{eq}(t),t)}{\omega_{eq}^2(t)} \frac{\partial p(A,t)}{\partial A} \right. \\ &\left. + \frac{\partial}{\partial A}\left(\frac{\pi S_{a_g}(\omega_{eq}(t),t)}{\omega_{eq}^2(t)} p(A,t) \right) \right\}, \end{aligned} \tag{16}$$

and manipulating yields

$$\begin{aligned} \dot{c}(t) = &-\left(\beta_0 + \beta_{eq}(c(t))\right) c(t) \\ &+ \left(\frac{\sin\left(\frac{\alpha\pi}{2}\right)}{\omega_0^{1-\alpha}} \right) \frac{\pi S_{a_g}(\omega_{eq}(c(t)),t)}{\omega_{eq}^2(c(t))}. \end{aligned} \tag{17}$$

Eq. (17) constitutes a deterministic first-order nonlinear ordinary differential equation, which can be readily solved by the Runge-Kutta numerical integration scheme; the interested reader is directed to Fragkoulis et al. (2019); Kougioumtzoglou et al. (2022); Fragkoulis and Kougioumtzoglou (2023) for more details on the derivation of Eqs. (7)-(17).

Lastly, considering Eqs. (8)-(9) and following Kougioumtzoglou (2013), the amplitude-dependent response EPS is determined by

$$S_{xx}(\omega,t) = \int_0^\infty \frac{S_{a_g}(\omega,t)p(A,t)dA}{(\omega^2(A)-\omega^2)^2 + (\omega\beta(A))^2}. \tag{18}$$

2.2. *Code-compliant stochastic incremental dynamics analysis methodology*

Numerous systems of real engineering interest can be modeled adequately as SDOF systems (Roberts and Spanos, 2003). Consider a quiescent nonlinear SDOF system base-excited by a response spectrum compatible acceleration stochastic process $a_g(t)$ whose dynamic behavior is governed by Eq. (1). Following Cacciola (2010), the non-stationary acceleration process $a_g(t)$ is characterized in the frequency domain by an associated EPS $S_{a_g}(\omega,t)$, compatibly defined with Eurocode 8 provisions. An incremental mechanization analogous to that used in normal IDA is adopted herein, where a_g^0 stands for the scaled image of the excitation magnitude leading to the introduction of the definition of $S_{a_g}(\omega,t;a_g^0)$. In the present study, the selected EDP is that of the response displacement amplitude A at the most critical time instant t_{in}, which stands for the time instant when the parameter $c(t)$ found in Eq. (17) reaches its maximum value. In this regard, the response amplitude PDF at t_{in} with respect to specific level of the scaled excitation a_g^0 is given by

$$p(A,t_{in};a_g^0) = \frac{\sin\left(\frac{\alpha\pi}{2}\right) A}{\omega_0^{1-\alpha} c(t_{in})} \exp\left(-\frac{\sin\left(\frac{\alpha\pi}{2}\right)}{\omega_0^{1-\alpha}} \frac{A^2}{2c(t_{in})} \right). \tag{19}$$

The generated $p(A,t_{in};a_g^0)$ for each and every scaled level of the excitation a_g^0 leads to the efficient determination of the stochastic IDA response amplitude PDF surface, comprising valuable higher order statistics under a fully probabilistic consideration. Manipulating Eqs. (18)-(19) yields the response power spectrum with respect to a specified level of excitation a_g^0 and time instant,

$$\begin{aligned} S_{xx}(\omega,a_g^0) = \int_0^\infty &\frac{S_{a_g}(\omega,t_{in};a_g^0)}{(\omega^2(A)-\omega^2)^2 + (\omega\beta(A))^2} \\ &\times p(A,t_{in};a_g^0)\, dA, \end{aligned} \tag{20}$$

where $S_{xx}(\omega,a_g^0)$ is the system response EPS at the time instant when the response variance reaches its maximum value for a given ground acceleration a_g^0.

The above quoted relation leads to the efficient determination of the coveted response power spectrum which pertains evolutionary characteristics as a function of spectral acceleration. The mechanization of the proposed methodology is provided in the following steps:

1. Derive the excitation EPS $S_{a_g}(\omega,t;a_g^0)$ in a stochastically compatible manner with an assigned elastic response acceleration spectrum of specified modal damping ratio and scaled ground acceleration a_g^0; see Cacciola (2010) for more details.
2. Following the proposed stochastic averaging and linearization method shown in section 2.1, determine the maximum value $c_{max}(t_{in})$ and the corresponding time instant t_{in}. Practically, this is achieved by employing Eqs. (12)-(13) and Eq. (17).
3. For a specific level of excitation a_g^0, estimate the response EDP PDF and the response EPS at t_{in} by Eqs. (19) and (20), respectively.
4. Repeat steps 1-3 for all scaled images of the excitation a_g^0 to determine the stochastic IDA response amplitude PDF surface and the response EPS as function of the spectral acceleration.

3. NUMERICAL APPLICATION

Employing the bilinear hysteretic force-deformation law is a common practice to capture the behavior of structural members and structures under seismic excitation (Mitseas and Beer, 2019; Giaralis and Spanos, 2010). Therefore, in this section, a bilinear hysteretic oscillator with fractional derivative elements subject to a Eurocode 8 elastic pseudo-acceleration response spectrum is utilized to demonstrate the reliability of the proposed stochastic IDA framework. The obtained results are compared and found in good agreement with corresponding results derived from nonlinear RHA in a MCS-based context.

3.1. Bilinear hysteretic SDOF system with fractional derivative elements

The equation of motion of a nonlinear bilinear hysteretic SDOF system with fractional derivative elements is considered. The restoring force of the system is given by

$$g(t,x(t),\dot{x}(t)) = \gamma\omega_0^2 x(t) + (1-\gamma)\omega_0^2 x_y z(t), \quad (21)$$

$$x_y\dot{z}(t) = \dot{x}\{1-\Phi(\dot{x}(t))\Phi(z(t)-1) - \Phi(-\dot{x}(t))\Phi(-z(t)-1)\}, \quad (22)$$

where $\Phi(\cdot)$ denotes the Heaviside step function. Further, $z(t)$ is an auxiliary state representing the hysteretic component, γ denotes the post- to pre-yield stiffness ratio and x_y is the yielding displacement.

Next, taking into account Eq. (21), Eqs. (10) and (11) become

$$S(A) = \begin{cases} \frac{4x_y\omega_0^2}{\pi}\left(1-\frac{x_y}{A}\right), & A > x_y \\ 0, & A \leq x_y \end{cases} \quad (23)$$

$$F(A) = \begin{cases} \frac{A\omega_0^2}{\pi}\left(\Lambda - \frac{1}{2}\sin(2\Lambda)\right), & A > x_y \\ A\omega_0^2, & A \leq x_y \end{cases} \quad (24)$$

with $\cos(\Lambda) = 1 - \frac{2x_y}{A}$. Thus, Eq. (12) yield

$$\begin{aligned} \beta_{eq}(c(t)) &= -\beta_0 + \frac{\beta\sin^2(\frac{\alpha\pi}{2})}{\omega_0^{1-\alpha}c(t)} \\ &\times \int_0^\infty \frac{A}{\omega^{1-\alpha}(A)}\exp\left(-\frac{\sin(\frac{\alpha\pi}{2})}{\omega_0^{1-\alpha}}\frac{A^2}{2c(t)}\right)dA \\ &+ \frac{4x_y\omega_0^2(1-\gamma)\sin(\frac{\alpha\pi}{2})}{\pi\omega_0^{1-\alpha}c(t)} \\ &\times \int_{x_y}^\infty \frac{1-\frac{x_y}{A}}{\omega(A)}\exp\left(-\frac{\sin(\frac{\alpha\pi}{2})}{\omega_0^{1-\alpha}}\frac{A^2}{2c(t)}\right)dA, \end{aligned} \quad (25)$$

whereas Eq. (13) leads to

$$\begin{aligned} \omega_{eq}^2(c(t)) &= \omega_0^2 - (1-\gamma)\omega_0^2 \\ &\times \left\{\exp\left(-\frac{{x_y}^2\sin(\frac{\alpha\pi}{2})}{2c(t)\omega_0^{1-\alpha}}\right) - \frac{\sin(\frac{\alpha\pi}{2})}{\pi\omega_0^{1-\alpha}c(t)} \right. \\ &\left. \times \int_{x_y}^\infty \frac{2\Lambda-\sin(2\Lambda)}{2A^{-1}}\exp\left(-\frac{\sin(\frac{\alpha\pi}{2})}{\omega_0^{1-\alpha}}\frac{A^2}{2c(t)}\right)dA\right\} \\ &+ \frac{\beta\sin(\frac{\alpha\pi}{2})\cos(\frac{\alpha\pi}{2})}{\omega_0^{1-\alpha}c(t)} \\ &\times \int_0^\infty \omega^\alpha(A)A\exp\left(-\frac{\sin(\frac{\alpha\pi}{2})}{\omega_0^{1-\alpha}}\frac{A^2}{2c(t)}\right)dA. \end{aligned} \quad (26)$$

3.2. Response statistics stochastic IDA surfaces determination

The elastic pseudo-acceleration design spectrum $S(\omega, \zeta_0 = 0.05)$ for soil type B according to Eurocode 8 is selected as the reference input spectrum. In addition, the recorded time history at the El Centro site corresponding to the SOOE (N-S) component of the Imperial Valley earthquake of May 18, 1940, is used to model the excitation's non-stationary attributes. The scaled images of the excitation are determined as $a_g^0 = 0.35g \times [0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6]$ where g stands for the acceleration of gravity.

The following parameters pertaining to the bilinear SDOF system under consideration have been employed: $m = 1$, $\omega_0 = 8$, $\zeta_0 = 0.05$, $a = 0.5$, $\gamma = 0.2$ and $x_y = 0.02$ m. It can be readily seen that following the mechanization presented in section 2.2, the stochastic IDA response EDP PDF surface and the response EPS stochastic IDA surface can be efficiently determined at a particularly low computational cost. Note, in passing, that the corresponding time instants t_{in} differ with respect to the scaled image of the ground acceleration a_g^0 following the criterion of the maximum value $c_{max}(t_{in})$.

Fig. 1 shows the response EDP PDF surface of the bilinear SDOF system with fractional derivative order $\alpha = 0.5$. Note that the red solid line depicts the modes of the EDP. To assess the accuracy of the developed approach, the response EDP PDF surface from MCS data is plotted as well in Fig. 2. In this regard, utilizing the spectral representation method of Liang et al. (2007), an ensemble of $10,000$ acceleration time histories is generated, compatible with the reference design spectrum corresponding every time to a specified scaled image of the excitation a_g^0. Subsequently, the governing equation of motion Eq. (1) subject to the above ensemble of accelerograms is numerically solved by resorting to an L1-algorithm (Koh and Kelly, 1990). Considering the approximations involved in the proposed approach, it can be clearly stated that the results obtained by the proposed methodology are in good agreement with the MCS-based estimates.

The response EPS stochastic IDA surface is shown in Fig. 3. It is noted that exceeding an intensity threshold signals a gradual transition from elastic into the plastic region. The noted break, which is expressed with a transition to lower values of frequency, is indicative of the system stiffness degradation. It is noteworthy that the proposed method provides with an insight into the underlying dynamic character of the system; this significant operation cannot be determined following typical nonlinear RHA.

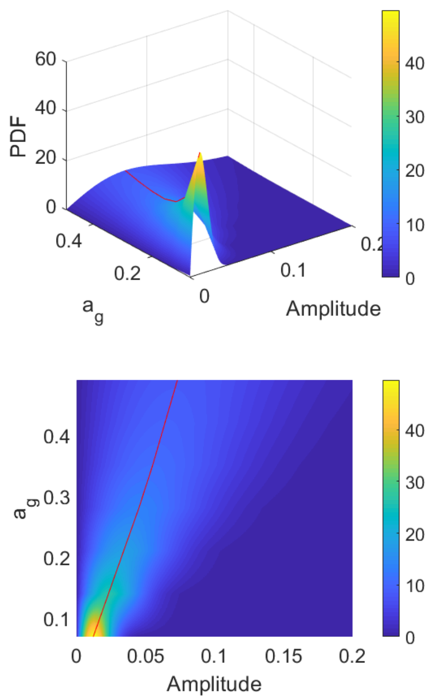

Figure 1: Response EDP PDF surface of a bilinear hysteretic oscillator with fractional derivative elements by the proposed method: (a) 3D view; (b) planar view.

4. CONCLUSIONS

In this paper, a novel stochastic incremental dynamics analysis methodology has been developed for nonlinear systems with fractional derivative elements subject to a seismic excitation vector consistently aligned with contemporary aseismic codes provisions. In this regard, an incremental mechanization analogous to the one used in normal incremental dynamic analysis is adopted to ensure the necessary compatibility for pertinent applications in structural engineering field. Specifically, rendering to the concept of non-stationary stochastic

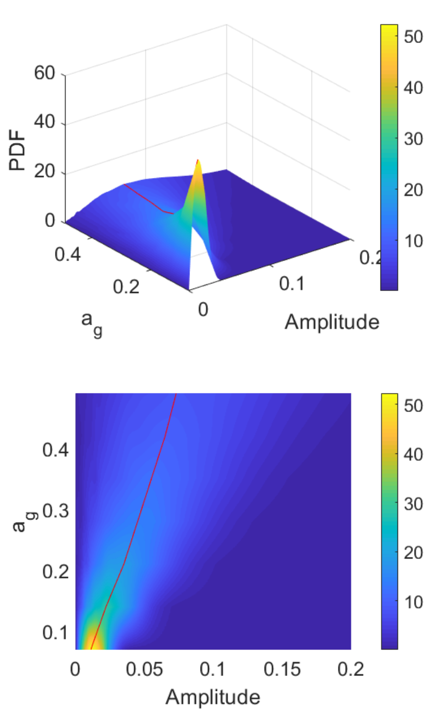

*Figure 2: Response EDP PDF surface of a bilinear hysteretic oscillator with fractional derivative elements by MCS method (*10,000 *realizations): (a) 3D view; (b) planar view.*

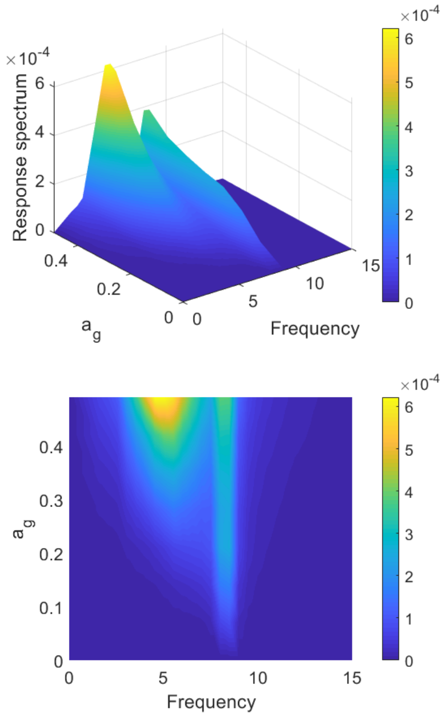

Figure 3: Response EPS stochastic IDA surface of a bilinear hysteretic oscillator with fractional elements: (a) 3D view; (b) planar view.

processes, the vector of the imposed seismic excitations is characterized by EPS stochastically compatible with elastic response acceleration spectra of specified modal damping ratio and scaled ground acceleration. Harnessing the potential of a combination of the stochastic averaging and statistical linearization methods, the response displacement PDFs are determined in an efficient and comprehensive manner. The proposed methodology provides with reliable higher order statistics of the selected EDP rather than simple estimates only of the mean and standard deviation, which is currently the norm in the IDA relevant literature. Further, a particularly interesting attribute of the proposed methodology is the derivation of the associated response EPS as a function of spectral acceleration. This coveted element has a twofold meaning; it performs structural behavior monitoring considering intensity, whereas it provides with an insight into the underlying dynamic character of the system. The efficient identification of the latter cannot be determined following nonlinear RHA. Lastly, the associated low computational cost renders the proposed methodology particularly useful for related performance-based engineering applications. A structural system comprising the bilinear model endowed with fractional derivative elements serves as a numerical example for demonstrating the reliability of the proposed methodology, whereas comparisons with relevant MCS data demonstrate the accuracy of the proposed code-compliant stochastic IDA technique.

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