14th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP14
Dublin, Ireland, July 9-13, 2023

Application of structural reliability methods for the safety assessment of autonomous vehicles

Thomas Most & Roland Niemeier
ANSYS Germany GmbH, Weimar, Germany

Maximilian Rasch & Paul Tobe Ubben
Mercedes Benz AG, Sindelfingen, Germany

ABSTRACT: In this paper, the presented process is based on event-based simulations where specific traffic scenarios are parametrized, simulated and analyzed by a set of criteria. By using predefined distribution functions for each input parameter a safety statement can be given by approximating the probability of failure for each traffic scenario by determining the unsafe region in the parameter space. Therefore, multiple steps of different algorithms are combined to ensure trustworthy results by being very efficient in reducing the number of required simulation runs.

1. Scenario-based safety assessment

As discussed in Kalra and Paddock (2016), the safety assessment of autonomous vehicles would require an evaluation of several billions of miles to assure that the failure rate is similar or less then this of a human driver. Since this is impossible to reach in field operational tests, in this study a scenario based approach is applied as recommended in Wood et al. (2019). Within this approach the required mileage needed to proof an assistance system is subdivided in critical scenarios and no-event situations, where no critical event occurs.

The validation of Advanced Driver Assistance Systems is performed with a simulation for each logical scenario. Simulation in this context means that the control device, on which the ADAS are running, is present as a simulation tool, running the real ECU code and thus software-in-the-loop simulations are performed. All inputs for the simulated controller are generated by a simulation environment. These include sensors, vehicle data as well as data from other ECU's installed in the vehicle. In order to generate plausible input data, a virtual environment is simulated in which the system vehicle moves and other road users (objects) are detected by sensor models. Thus, the virtual world is processed and captured, and control quantities calculated therefrom are delivered back to the vehicle model. For the scenario-based approach, a number of logical scenarios describable by parameters are defined Menzel et al. (2018). The scenarios are derived from the system requirements, from the research project PEGASUS (Joint project to develop new methods for validating and testing ADAS) as well as observations from the field. A logical scenario is typically a specific traffic situation. For instance, a cut in maneuver of other objects or a jam end situation on a highway as shown in Figure 1. To describe such a logical scenario the 6-Layer model can be used Bock et al. (2018). For demonstration purposes, only the road layer and the moving objects layer are considered in this study. With the help of the corresponding parameters, these logical scenarios can be varied in their characteristics. Hence it is possible to vary speeds of the vehicles, distances from objects or the dynamics of lane change maneuvers. These so-called specific scenarios resulting from different parameter combinations are simulated and the system reaction of the ADS is evaluated. This is done through evaluation criteria that reflect the criticality of a specific scenario. For example, the Time-To-Collision (TTC) or the

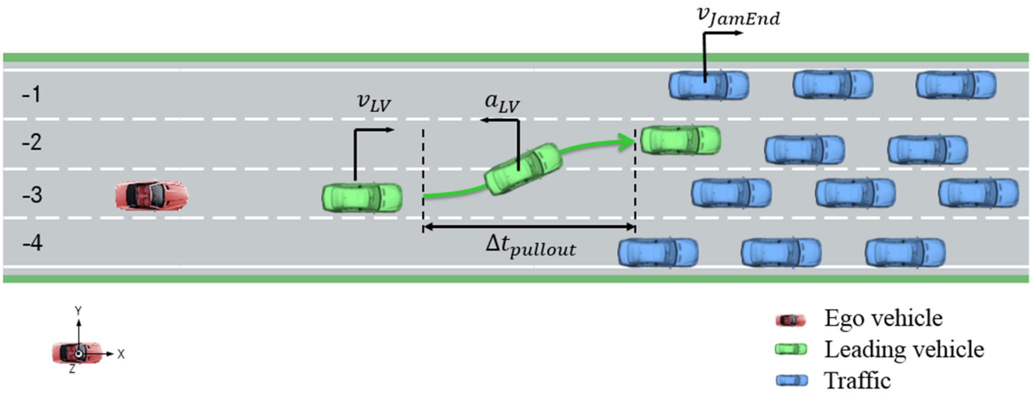

Figure 1: Jam end traffic scenario on the highway. By altering the input parameters this logical scenario can be varied in its characteristic.

distance between two vehicles can be used as evaluation criteria. The intention of the methodology described in the following is to determine the probability of failure for each logical traffic scenario by using well-known reliability methods (Rasch et al., 2019). Therefore, the parameter space is investigated with search and sampling algorithms to determine the probability that a critical situation or even an accident can occur. The probability distributions of the input parameters as well as the probability of occurrence of the respective scenario are determined on the basis of real measured data and by using the PEGASUS database (Pütz et al., 2017).

2. Definition of uncertainties

In our study, the individual parameters of a logical scenario are defined as scalar random parameters. Additionally to the parameter distributions, the dependence of the scattering parameters needs to be considered in the uncertainty model. The individual parameters are assembled in a random vector

$$\mathbf{X} = [X_1, X_2, \ldots, X_k], \tag{1}$$

which contains the continuous and discrete random numbers X_i. The marginal distributions of the individual random numbers are derived directly from categorized fleet data by using standard distribution types, such as normal, log-normal, truncated normal and uniform types as well as more flexible types, such as the beta distribution, the generalized lamba distribution (Karian and Dudewicz, 2000) and a piecewise-uniform distribution function, which is called here multi-uniform distribution. The parameters of this multi-uniform distribution are directly derived from the histogram of the investigated data. Discrete parameters are respresented using either Bernoulli or general discrete distribution types. In Dynardo GmbH (2022) a detailed overview of different continuous and discrete distribution types is given.

For normally-distributed variables, the dependence between the scalar random input parameters can be represented with the Gaussian copula in closed form

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^k |\mathbf{C_{XX}}|}} \exp\left[-\frac{1}{2}(\mathbf{x}-\bar{\mathbf{X}})^T \mathbf{C}_{\mathbf{XX}}^{-1}(\mathbf{x}-\bar{\mathbf{X}})\right], \tag{2}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of random vector $\mathbf{X}$ and $\bar{\mathbf{X}}$ is the corresponding mean vector and $\mathbf{C_{XX}}$ the covariance matrix. In this study, the Nataf model (Nataf, 1962) is used to extend the Gaussian correlation model to non-Gaussian distribution types. In the Nataf approach the marginal distributions of the random variables are transformed to the standard Gaussian distribution. In this transformed space, a Gaussian copula as given in equation 2 is assumed. The correlation coefficients of the standardized Gaussian spaced are obtained from the correlation coefficients of the original distributions by an iterative procedure (Bucher, 2009) or analytical regression function (Liu and Der Kiureghian, 1986). The Nataf approach requires a unique mapping of the mariginal distributions from the original to the Gaussian space and back. Therefore, it is not applicable for the discrete distribution types and therefor all discrete input parameters are assumed to be independent in this study.

In Figure 2 original measurements of a two-dimensional random vector are compared with the randomly generated samples using the Nataf model. As marginal distributions, a piecewise-linear distribution function and a truncated normal distribution have been chosen. The figure indicates, that although linear dependencies are assumed in the standard-normal space, the non-linear dependencies in the original space can be sufficiently represented by the Nataf model.

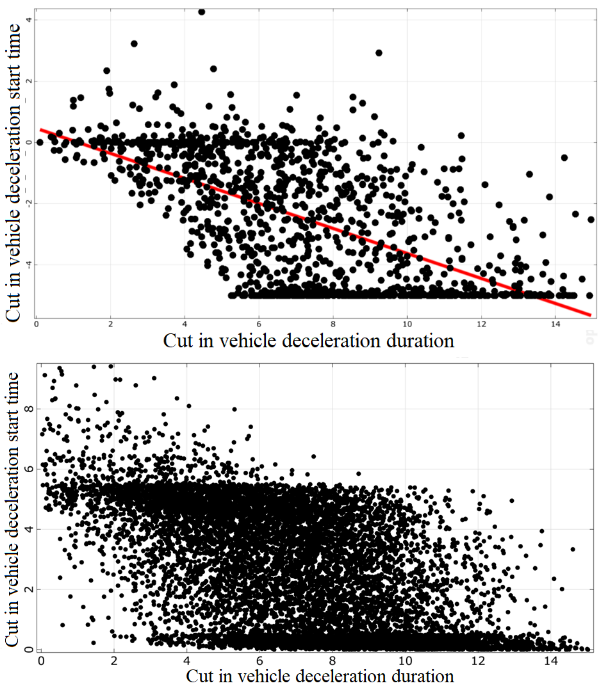

Figure 2: Example of a 2D joint probability distribution using the Nataf model: measurements (top) and generated samples (bottom) of two random parameters of a cut-in scenario

3. Reliability analysis

For a given set of jointly distributed random variables X_i and a limit state function $g(\mathbf{X})$ the probability of failure P_F can be determined via integration

$$\begin{aligned} P_F &= P\left[\mathbf{X} : g(\mathbf{X}) \leq 0\right] \\ &= \int \cdots \int_{g(\mathbf{X})\leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}. \end{aligned} \tag{3}$$

The limit state function divides this random variable space into a safe domain $S = \{\mathbf{x} : g(\mathbf{x}) > 0\}$ and a failure domain $F = \{\mathbf{x} : g(\mathbf{x}) \leq 0\}$. The computational challenge in determining the integral of Eq. (3) lies in evaluating the limit state function $g(\mathbf{x})$ at a specific position $\mathbf{x}$, which for non-linear systems usually requires an incremental/iterative numerical approach.

The most simple and robust method for the evaluation of Eq. (3) is the Monte Carlo Simulation (Rubinstein, 1981) where the estimated failure probability is obtained from a set of N samples $\mathbf{x}_i$ as

$$\hat{P}_F = \frac{1}{N} \sum_{i=1}^{N} I\left(g(\mathbf{x}_i)\right) \tag{4}$$

where the indicator function $I\left(g(\mathbf{x}_i)\right)$ is one if $g(\mathbf{x}_i)$ is negative or zero and zero else. The variance of the MCS estimator depends directly on the failure probability

$$\sigma^2_{\hat{P}_F} = \frac{P_F}{N}, \tag{5}$$

which results in an increasing number of samples for decreasing failure probability. However, the Monte Carlo Simulation can be applied independently of the model non-linearity and the number of input parameters. This method is very robust and can detect several failure regions with highly non-linear dependencies. In the scenario based safety assessment the MCS will require an extremely large number of model evaluations since we have to proof quite rare events, which limits this methods mainly for verification purposes.

In our study more advanced sampling strategies like Importance Sampling have been applied, where the sampling density is adapted in order to cover the failure domain sufficiently and to obtain more accurate probability estimates with much less solver calls. Other methods like the First or Second Order Reliability Method (FORM and SORM) are still more efficient than the sampling methods by approximating the boundary between the safe and the failure domain, the so-called limit state. In contrast to a global low order approximation of the whole response, the approximation of the limit state around the most probable failure point (MPP) is much more accurate. Classically, only one dominant failure point could be found and evaluated. This limitation holds even for the Importance Sampling Procedure Using Design points (ISPUD), where the non-linearity of the limit state can be considered by a sampling around the MPP. A good overview of these "classical" methods is given in Bucher (2009). Recently, some of these methods have been extended for multiple failure regions (Der Kiureghian and Dakessian, 1998), Geyer et al. (2019).

In reliability analysis where small event probabilities have to be estimated, we have to pay special attention that the algorithms obtain an acceptable level of confidence in order to detect the important regions of failure. Otherwise, they may estimate a much smaller failure probability and the safety assessment will be much too optimistic. The available methods for an efficient reliability analysis try to detect where the dominant failure regions are located and concentrate the simulation effort in those regions in order to drastically reduce the necessary CAE simulations. Of course, there is always a risk that experimenting with such approaches will lead to inappropriate short cuts, perhaps missing the failure domain and providing too optimistic an estimation of failure probability. Therefore, we strongly recommend that at least two different types of reliability methods are used to verify variance-based estimates of the failure probability in order to make reasonable design decisions based on CAE-models.

3.1. Adaptive Importance Sampling

In our study we have investigated several methods. Since the simulation results may contain numerical distortions and bifurcations, response surface-based approaches could be used only for a first estimate. One reliable and robust method for our application is the adaptive importance sampling strategy (Bucher, 1988). The failure probability is estimated from a modified importance sampling density as follows

$$\hat{P}_F = \frac{1}{N}\sum_{i=1}^{N} w(\mathbf{x}_i) I\left(g(\mathbf{x}_i)\right), \quad w(\mathbf{x}_i) = \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{h_{\mathbf{X}}(\mathbf{x}_i)}. \tag{6}$$

The corresponding variance of the estimator can be derived as

$$\sigma^2_{\hat{P}_F} = \frac{1}{N^2}\sum_{i=1}^{N} w^2(\mathbf{x}_i) I\left(g(\mathbf{x}_i)\right) - \frac{\hat{P}_F^2}{N}. \tag{7}$$

In the adaptive importance sampling approach the modified importance sampling density $h_{\mathbf{X}}(\mathbf{x}_i)$ is obtained by iterative adjustment of a modified sampling density. Based on an initial step, where the original sampling density is blown-up in the standard normal space, the position of the most dominant failure region is estimated. In the following steps, the sampling density is adapted according the estimated center and covariance of the previous failure samples.

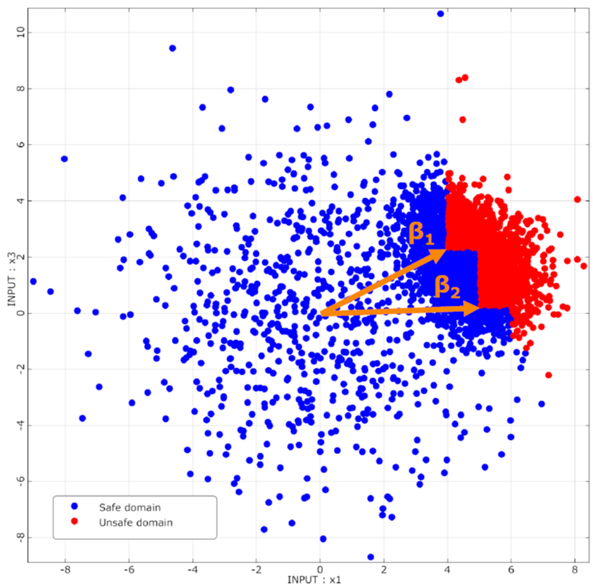

Figure 3: Adaptive Importance Sampling for a linear limit state function considering discrete random variables (samples in the standard Gaussian space)

This method becomes inefficient with increasing number of random variables due to the less accurate estimates of the density statistics. Therefore, it is recommended to apply this method for problems with up to twenty random variables. Furthermore, it can analyze only one dominant failure region.

Extensions for multiple failure regions have been published recently e.g. in Geyer et al. (2019). In our studies, where discrete distribution types have been used together with continuous random variables, we observed an additional numerical effort to obtain a similar accuracy of the failure probability estimates as in pure continuous problems. This is caused in artificial discontinuities of the limit state function in the standard normal space as shown in Figure 3. Even for continuous limit state functions such discontinuities occur due to the discrete distributions. This phenomenon causes multiple most probable failure points, which makes the normal sampling density less efficient.

3.2. *Importance Sampling using Design Points*

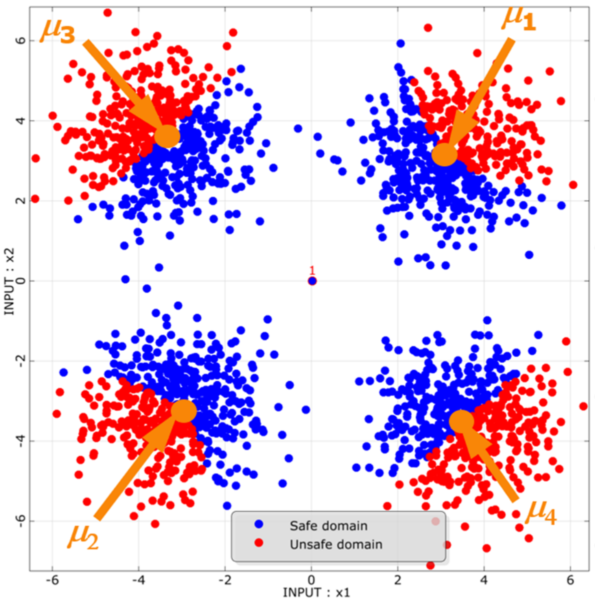

Figure 4: Importance Sampling using designs points by using a multi-modal sampling density which consists of several standard normal densities

In order to overcome the limitation of a single adapted failure region, we extended the original Importance Sampling using Design Point (ISPUD)(Bourgund and Bucher, 1986) by a multi-modal density according to Geyer et al. (2019). The modified sampling density is generated in the standard Gaussian space by a given number of individual standard normal sampling densities m with corresponding center points μ_j. In Figure 4 the sampling is shown for the Katsuki test function Katsuki and Frangopol (1994) with four individual failure regions. The corresponding original and modified sampling density in the standard Gaussian space reads

$$f_{\mathbf{X}}(\mathbf{x}_i) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\|\mathbf{x}_i\|}{2}\right),$$
$$h_{\mathbf{X}}(\mathbf{x}_i) = \frac{1}{m\sqrt{2\pi}} \sum_j^m \exp\left(-\frac{\|\mathbf{x}_i - \mu_j\|}{2}\right). \tag{8}$$

In order to detect the individual failure regions with sufficient confidence, we extended the multiple FORM algorithm (Der Kiureghian and Dakessian, 1998): Based on given start points or an initial presampling similar to the first iteration of the Adaptive Importance Sampling approach, we perform a local optimization several times. With help of a local gradient-based optimizer the closest point, where the limit state turns from safe to unsafe and which has the smallest distance to the median point on the standard normal space, is detected. Since the start points are selected using a density criterion by considering the previous optimization runs, we can assure that with a given number of local optimization runs, the important failure regions could be found. In situations where some of the input parameters are modeled with discrete distribution types, the local optimization is performed only in the reduced continuous subspace, but different combinations of the discrete values are investigated. In Figure 5 such a search is illustrated: from 100 presamples 10 candidates are selected for local optimization runs until all four failure regions are found. After the most important failure regions have been detected, the corresponding most probable failure points are used as centers for the sampling densities in the multi-modal ISPUD approach. Since the failure probability is not estimated by the beta-distance analogous FORM but by the more accurate Importance Sampling, even non-linear limit state functions can be accurately evaluated. Furthermore, the local optimizer needs not to be very accurate in the estimate of the local most probable failure point.

4. Application example: jam end scenario

In this example a jam end scenario as shown in Figure 1 is investigated. In this scenario an ego ve-

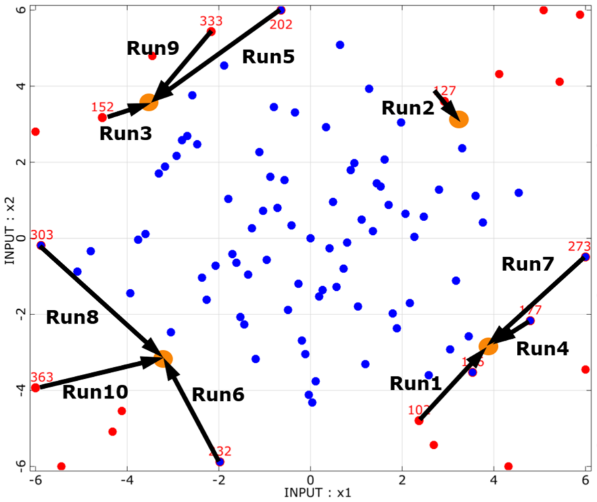

Figure 5: Multiple design point search by using the First Order Reliability Method with multiple start points

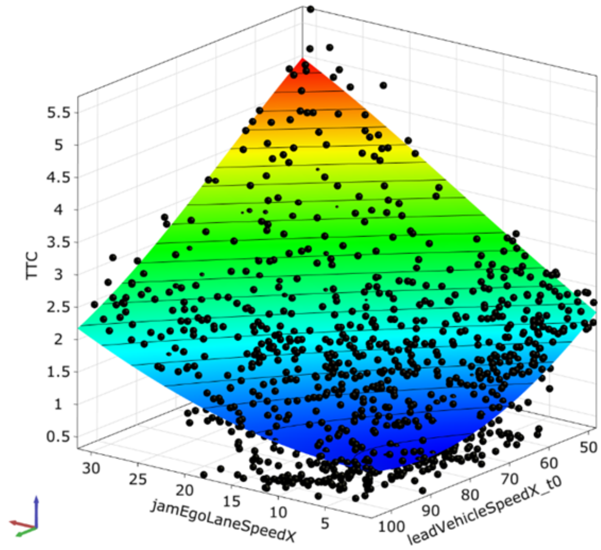

Figure 6: Jam end scenario: adaptive meta-model used for the verification of the reliability algorithms

hicle including a lead vehicle drive to the end of a traffic jam on a highway. At a certain time, the lead vehicle will change the lane and the ego vehicle has to detect the last vehicle of the jam in order to perform an accident-free braking. In the simulation software the Time-To-Collision (TTC) is estimated w.r.t. the given input parameters. We consider this TTC as limit state and investigate several limits with the reliability algorithms. As input scatter we assume nine continuous scattering parameters as lead vehicle and jam end speed, pull out time, lead vehicle braking deceleration as well as a lane offsets of the traffic jam and the lead vehicle. The number of road lanes, the lead vehicle class and the pull out direction have been modeled with discrete random distributions.

In order to perform the analysis and verification more efficiently, in a first step a global meta-model was created based on 1000 samples. In order to obtain more samples and thus higher accuracy in the relevant regions a local adaptation strategy was used (Adaptive Metamodel of Optimal Prognosis, Most and Will (2011); Most (2011)). Based on this fast meta-model we investigated the multimodal and the adaptive importance sampling in comparison to the brute-force Monte Carlo Simulation. In figure 6 one subspace of the 12-dimensional meta-model is shown. As result of the sensitivity analysis, the lead vehicle speed and the jam end speed are indicated to be most important in this scenario. Furthermore, the relation of the Time-To-Collision and the input parameters is almost monotonic. Thus, we would expect to obtain different failure regions mainly due to different combinations of the discrete parameters.

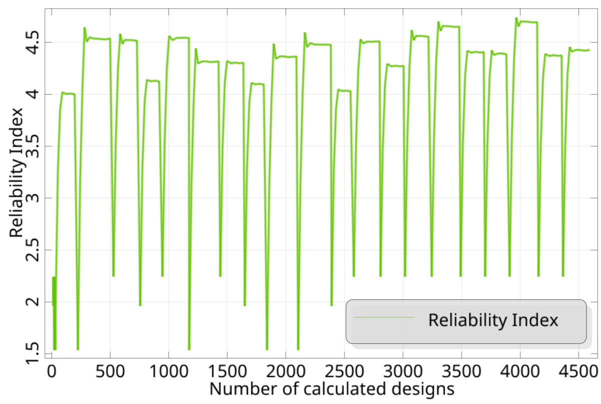

Figure 7: Jam end example: convergence of the multi FORM-search assuming a limit of 0.5s for the Time-To-Collision

In Figure 7 the convergence of the multiple FORM is shown for one specific failure limit. It can be seen, that the optimizer converged to different reliability index values, which correspond to different most probable failure points. Altogether, 20 failure points have been detected which are used as sampling centers for the importance sampling. In Figure 8 the convergence of the corresponding

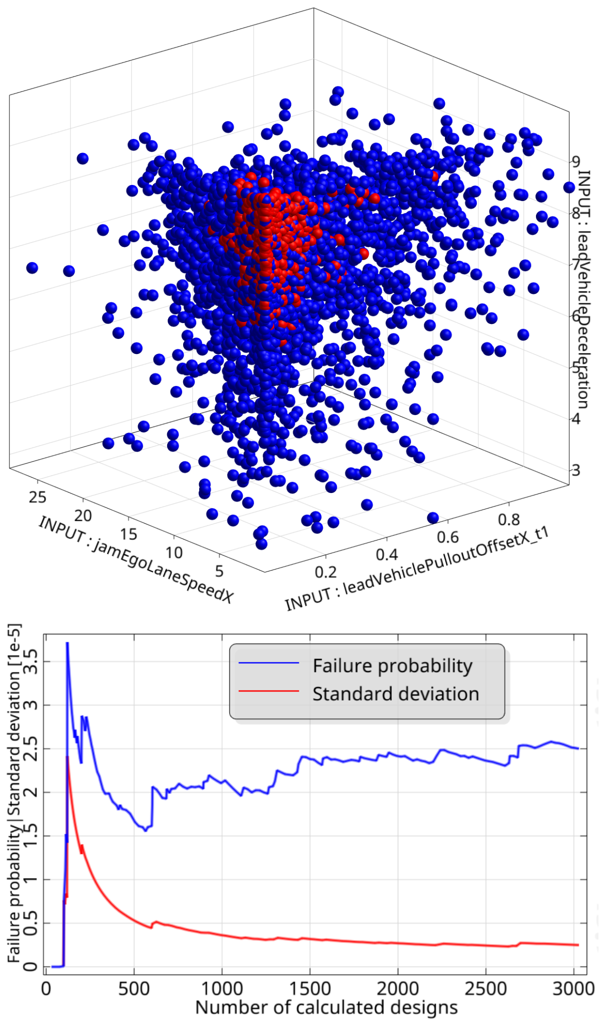

Figure 8: Jam end example: identified failure region and convergence of the multi-modal importance sampling assuming a limit of 0.5s for the Time-To-Collision

multi-modal importance sampling is shown. The figure clearly indicates, that in the subspace of the three most important parameters, all failure samples are located in a concentrated domain.

In Table 1 the obtained estimates of the failure probability are given for the different limit values. The multi-modal and adaptive Importance Sampling strategies are compared to the results of the Monte Carlo Simulation. As indicated in the table, we could obtain a very excellent agreement of the results. Furthermore, the multi-modal ISPUD is the most efficient algorithm, especially for small failure probabilities, which is the expected application field. Next, the multi-modal and adaptive Importance Sampling are applied using the traffic simulation software directly. The Monte Carlo Simulation could not be applied due to the large numerical effort. In Table 1 the results are compared. Again, the results of both methods agree very well, while the ISPUD approach needs less samples. Since the FORM method is applied on the meta-model only, all together 1000 samples for the meta-model plus 5000 importance samples are needed. However, the estimates with the real solver indicate a much larger failure probability as estimated using the meta-model. Therefore, in our applications we always apply the ISPUD approach using the direct solver. If the most probable failure points are not estimated very accurately, we obtain still valid results since the ISPUD algorithms is running the sampling procedure until a certain accuracy of the estimated failure probability is obtained.

	Samples	$\hat{P}_F$	CoV $\hat{P}_F$	β
Meta-model				
Limit state $TTC \leq 1.0$				
MCS	30.000	$1.58 \cdot 10^{-2}$	4.6%	2.15
AS	8.000	$1.54 \cdot 10^{-2}$	4.8%	2.16
FORM + ISPUD	7.300 + 3.000	$1.51 \cdot 10^{-2}$	4.4%	2.17
Limit state $TTC \leq 0.5$				
MCS	4.250.000	$2.35 \cdot 10^{-5}$	10.0%	4.07
AS	12.000	$2.71 \cdot 10^{-5}$	8.1%	4.04
FORM + ISPUD	4.500 + 3.000	$2.50 \cdot 10^{-5}$	10.0%	4.06
Limit state $TTC \leq 0.4$				
MCS	31.170.000	$3.12 \cdot 10^{-6}$	10.0%	4.51
AS	18.000	$3.19 \cdot 10^{-6}$	9.2%	4.51
FORM + ISPUD	5.500 + 9.000	$2.90 \cdot 10^{-6}$	9.7%	4.53
Traffic simulation				
Limit state $TTC \leq 0.5$				
MCS	Not possible	-	-	-
AS	26.000	$5.30 \cdot 10^{-3}$	9.2%	2.55
FORM + ISPUD	(4.500) + 5.000	$4.40 \cdot 10^{-3}$	20.1%	2.62

Table 1: Jam end example: estimated failure probabilities for different limit state limits using the global meta-model and the the traffic simulation software

5. Conclusions

In this paper we have presented an automatic approach for the reliability evaluation of specific traffic scenarios for the validation of Advanced Driver Assistance Systems. In this analysis the control device is represented as a simulation model using software-in-the-loop technology. Specific inputs of this simulated controller are modeled as random inputs in a stochastic analysis. Based on a definition of a failure criterion well known reliability algorithms could be applied. In our study we have used classical Monte Carlo Simulation only for verification due to its enormous numerical effort to proof small event probabilities. In order to reduce the number of necessary simulation runs, variance reduced importance sampling was applied. For this purpose, we used a multiple design point search approach to detect the important failure regions. Based on this result a multi-modal importance sampling density was automatically generated in order to quantify the contribution of each failure region to the overall failure probability. Based on a confident error estimate we could ensure, that the sampling loop was continued until a required accuracy of the probability estimate was obtained. The presented approach enables the automatic reliability proof of an Advanced Driver Assistance System for a specific scenario with minimum manual input. However, one very important point is the quantification of the input uncertainties of the investigated scenario. These assumptions strongly influence the finally estimated failure rate, therefore, attention should be paid in order to derive suitable assumptions about distribution type, scatter and event correlations from real world observations.

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