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Life Quality Index for Optimizing a Safety Improvement Program

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ABSTRACT: The Life Quality Index (LQI), a composite social indicator, was proposed as a basis for evaluating the effectiveness of investments in safety measures to enhance life quality. In the structural reliability field, the life cycle cost (LCC) minimization method is widely used to optimize the structural safety and LQI has often been considered to provide a lower bound to the target safety. This study presents a systematic formulation of an LQI-based approach to optimize structural safety and demonstrates that the LCC minimization that excludes life safety considerations is a subset of the LQI optimization problem. This unified approach shows that a consideration of the life safety objective is the dominant criterion for optimization in contrast to the minimization of the LCC function alone. In other words, the LQI approach demonstrably leads to a higher level of target reliability than that obtained from the LCC approach.

1. INTRODUCTION

Strategic principles for managing risk and improving human safety include accountability, maximizing of net benefit to society and life measure that reflects the health and safety benefit by increased life expectancy in good health. The Life Quality Index (LQI) is a compound social indicator intended to capture the life safety benefit and can be interpreted as the total utility of consumption derived over the lifetime, T_h, by a statistically representative individual in the society. Using the gross domestic product (GDP), g \$/year/person, as a measure of available personal consumption rate, and g^q, $q < 1$, as an associated utility function, the total lifetime utility can be quantified as, $g^q T_h$. Since the lifetime utility is a function of a single random variable, T_h, the LQI is defined as its expected value,

$$L = g^q \, \mathbb{E}\left[T_h\right] = g^q \, e \tag{1}$$

where $e = \mathbb{E}\left[T_h\right]$ is the mean lifetime or life expectancy (LE) at birth. Although different ways to derive LQI have been presented in the literature (Nathwani et al., 1997; Pandey et al., 2006), this is the most concise derivation of the index. LQI is also considered as a composite social indicator, as it includes two commonly used measures of social development, g and e.

The LQI-based optimization of a safety program includes both the economic and life safety impacts through changes in g and e. For example, if the implementation of a safety improvement program is expected to change the GDP by an amount dg \$/person/year, and the life expectancy by de years, then the change in LQI, dL can be estimated as

$$\frac{dL}{L} = q\frac{dg}{g} + \frac{de}{e} \tag{2}$$

It should be noted that dg and de are algebraic quantities, i.e., a positive or negative sign needs to be assigned to them to distinguish an increase or decrease in their magnitudes.

In the literature, Eq.(2) is widely utilized as a basis to optimize a safety program and structural design codes (Van Coile et al., 2019; Fischer et al., 2019). However, what is lacking in the literature is a clear discussion about definitions of the base case and an alternate case to allow a rigorous quantification of a change in LQI via changes in g and e. However, what is lacking in the literature is a clear discussion about definitions of the base case and an alternate case to allow a rigorous quantification of a change in LQI changes in dg and de. Without a clear elucidation of the base case and the alternatives, use of the LQI method for optimization can lead to counter-intuitive results. An example of this can be seen in the LQI-based optimization of structural safety. Since LQI integrates a life measurfe of safety in addition to economic costs, it is expected to yield a more stringent level of reliability than that obtained by a pure cost-based optimization. However, some studies in the literature report a contradictory result, i.e., a pure cost-based optimization dominates the LQI-based optimization of structural design (Rackwitz, 2002). A detailed discussion on compatibility between the economic and societal optimal solutions was presented by Van Coile and Pandey (2017).

Thus, the central objective of this paper is to resolve this anomaly that exists in the literature through reformulating the LQI optimization criterion in a clear manner that is also consistent with the stochastic nature of a hazard process and the definition of life expectancy.

2. LQI Optimization: Conceptual Approach

2.1. *Background*

When a regulatory authority or a decision maker identifies an existing hazard that poses an increased risk of mortality, a safety improvement program is often undertaken to reduce the the risk. For example, the impacts of seismic hazard can be reduced by design improvements, chemical hazards can be reduced by banning the usage of toxic substances, and the risk of fire hazard can be reduced by several interventions, namely, installing of fire detection and extinguishing systems or stand-by capability for limiting consequences.. A specific safety related intervention is expected to cost money but at the same time it will enhance life safety by reducing the mortality risk. For simplicity, assume that a safety measure can be optimized with respect to a single design parameter, y.

2.2. *Problem Definition*

This section describes three cases that are relevant to the formulation of the LQI-based optimization of a safety improvement program, with respect to a design variable, y.

1. *A hypothetical state in absence of the hazard.* When the hazard in question is completely absent from the society, the GDP, the mortality rate, and the LE at birth are denoted as g_0, m_0, and e_0, respectively. This is a hypothetical state, since the hazard in question, such as a seismic hazard, is already present in the society for a long period of time. Therefore, actual values of these (hidden) variables are unknown to the decision maker. This state is introduced in the discussion for the sake of clarity in defining the optimization problem. Actual values of these variables are not required for LQI optimization.

   Although the mortality rate depends on a person's age (u), it is not written here explicitly for the sake of brevity. In short, $m_0(u)$ is denoted as m_0.

2. *Base Case: Present state with the hazard.* Suppose the existing hazard has increased the mortality risk by a magnitude, Δm_1, such that the prevailing mortality risk in the society can be thought of as a sum, $m_1 = m_0 + \Delta m_1$, which leads to current LE, e_1. The hazard may cause some economic losses, Δg_1 \$/person/year, such that the current GDP can be

written as, $g_1 = g_0 - \Delta g_1$. Information about g_1 is available from current economic data. Similarly, current mortality risk, m_1 and LE, e_1 are known from current life table data.

The LQI optimization is performed with reference to this case. It is important to note that only the information about g_1, m_1, and e_1 are needed and actual magnitudes of Δm_1 and Δg_1 are not required to formulate the LQI optimization problem.

3. *Alternate case: Safety improvement program* The safety program is expected to reduce the hazard induced mortality from Δm_1 to $\Delta m_2(y)$. In other words, $\Delta m_2(y)$, is the residual mortality risk caused by the hazard, which depends on the design parameter. Also note that $0 \leq \Delta m_2(y) \leq \Delta m_1$.

   The overall mortality rate in the alternate case is expected to be $m_2 = m_0 + \Delta m_2(y)$. Since m_0 is unknown, m_2 can be re-expressed as

$$\begin{aligned} m_2(y) &= (m_0 + \Delta m_1) - \Delta m_1 + \Delta m_2(y) \\ &= m_1 + (\Delta m_2(y) - \Delta m_1) \end{aligned}$$

   The reason for using this expression is that m_1 is known from the current life table in the base case and $\Delta m_2(y)$ can be estimated from an analysis of the safety measure. For example, expected mortality risk after a seismic retrofitting of a structure can be estimated using models for structural collapse and building usage (FEMA, 2018).

   With a similar reasoning, the expected GDP in this case can be estimated as, $g_2(y) = (g_0 - c(y) = (g_0 - \Delta g_1) - c(y) + \Delta g_1$, which leads to

$$g_2(y) = g_1 - c(y) + \Delta g_1, \quad (\$/\text{person/year}) \tag{3}$$

   where $c(y)$ denotes the annualized losses (i.e., $\Delta g_2(y)$) per person associated with the safety program over its life cycle, $(0,t]$. These losses can be estimated by the life cycle cost analysis, as shown in Section 4.2.

   It will be shown in Section 4.3 and Eq. (21) that if increments in the mortality rate, Δm_1 and $\Delta m_2(y)$ are constant and independent of the age, the modified LE can be estimated as

$$e_2(y) \approx e_1 \; [1 - \mu_A (\Delta m_2(y) - \Delta m_1)] \tag{4}$$

   where μ_A is referred to as the average age of the life table (or stationary) population in the base case.

2.3. *Formulation of LQI Optimization*

2.3.1. *Derivation*

Here the objective is to find an optimum value of the design parameter, $y = y*$, which will maximize the LQI in the alternate case as compared to the base case. The analysis starts with the LQI expression in the alternate case,

$$\begin{aligned} L_2(y) &= [g_2(y)]^q \; e_2(y) \\ &= [g_1 - (c(y) - \Delta g_1)]^q \quad e_1 \; [1 - \mu_A (\Delta m_2(y) - \Delta m_1)] \end{aligned} \tag{5}$$

Since it is expected that $(c(y) - \Delta g_1) << g_1$, the binomial approximation, $(1+\varepsilon)^q \approx (1+q\varepsilon)$ for $\varepsilon << 1$, can be used to simplify the first term and noting that $L_1 = g_1^q \; e_1$, the above equation can be rewritten as

$$\begin{aligned} \frac{L_2}{L_1} \approx \left(1 - q\frac{c(y) - \Delta g_1}{g_1}\right) \\ [1 - \mu_A (\Delta m_2(y) - \Delta m_1)] \end{aligned} \tag{6}$$

Now the product of the above two terms in parentheses is approximated by keeping only the first order terms in the multiplication, which leads to an expression for the first-order change in LQI in alternate case with reference to the base case as

$$\begin{aligned} \frac{L_2(y) - L_1}{L_1} = \frac{\Delta L_{12}(y)}{L_1} \approx \\ \underbrace{-\left(q\frac{c(y)}{g_1} + \mu_A \Delta m_2(y)\right)}_{\text{Decrease}} + \underbrace{\left(q\frac{\Delta g_1}{g_1} + \mu_A \Delta m_1\right)}_{\text{Increase}} \end{aligned} \tag{7}$$

In summary, the change in LQI has the following two components:

1. A potential decrease in LQI due to the cost of safety program, $c(y)$, and the residual mortality risk due to hazard, $\Delta m_2(y)$. Since this decrease alone is a function of the design parameter, y, it can solely serve as the optimization function.

2. Since the potential increase in the LQI, $(q\Delta g_1/g_1) + (\mu_A \Delta m_1)$, is independent of the design parameter, this term does not play any role in the optimization.

Based on the expression (7) and above observations, the total reduction in LQI, ΔL_R, is proposed as an objective function for the design optimization:

$$\frac{\Delta L_R}{L_1} \approx q\frac{c(y)}{g_1} + \mu_A \Delta m_2(y) \tag{8}$$

To conclude, Eq.(8) defines a normalized loss function and its minimization with respect to the design parameter would ensure that LQI achieves its optimum value in the alternate case as compared to the base case. Further, this loss minimization problem primarily requires information about the expected annual losses associated with the safety program, $c(y)$, and the residual mortality rate, $\Delta m_2(y)$ due to the hazard after the implementation of the safety program, which is bounded as $0 \leq \Delta m_2(y) \leq \Delta m_1$. In addition, the life table in the base case is required for computing the mean age, μ_A (see Section 4.3).

2.3.2. Remarks

- If the residual risk can be completely eliminated for all values of y, i.e., $\Delta m_2(y) = 0$, then the LQI optimization reduces to the cost minimization problem.
- If life safety effects are achieved by a safety investment but ignored within the optimization, i.e., $\Delta m_2(y) - \Delta m_1 = 0$, then the resulting optimum design would be less stringent as compared to that obtained by including the life safety considerations.

3. Stochastic Reliability Analysis

3.1. Basic Terminology

A structure is exposed to recurring hazards throughout its service life, such as those resulting from earthquakes, wind gusts, and flooding.

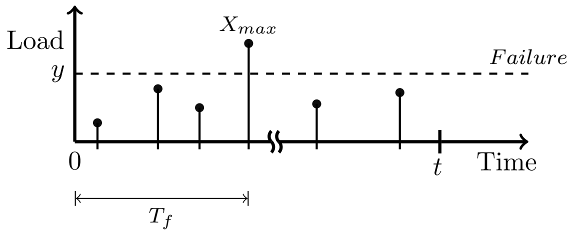

Figure 1: A stochastic model of a hazard

Such randomly occurring events are modelled as a stochastic shock process, as shown in Figure 1. A shock process consists of independent and identically distributed (*iid*) sequences of random variables (RVs), X_i and $T_i, i = 1, 2, \ldots$, representing the magnitude and inter-arrival time of load events, respectively (Pandey and van der Weide, 2017).

For any random variable X, $F_X(x)$ and $\overline{F}_X(x) = 1 - F_X(x)$, denote the cumulative distribution (CDF) and the survival function, respectively.

In this study, the Homogeneous Poisson Process (HPP) is used to model a stochastic hazard. The distribution of the number of events, $N(t)$, in a time interval, $(0,t]$, is given by the Poisson probability mass function as

$$f_N(k;t) = \frac{[\lambda t]^k}{k!} e^{-\lambda t}, \quad (0 \leq k < \infty) \tag{9}$$

where λ is the rate of occurrence of events. The expected number of events in this interval is given as λt. The inter-arrival time follows an exponential distribution with a survival function, $\overline{F}_{T_i}(t) = e^{-\lambda t}$.

3.2. Structural reliability analysis

Consider a structural component of strength, y, which is assumed to be a deterministic variable to simplify the analysis. The probability of failure of this component under a random shock load, X_i, is given as, $p_{fx}(y) = \mathbb{P}[X_i > y] = \overline{F}_X(y)$. A failure event, $X > y$, does not necessarily mean a complete destruction of the structure. Rather, it is expected to cause some damage that would require repair to restore its condition to the original state after each shock. In short, the structural damage/failure events are appropriate to model as a stochastic renewal process.

A structural failure can result in fatalities. The conditional probability, $p_{d|f}$, of occurrence of a fatality given a failure event can be estimated using models of the structure and its usage.

Based on the decomposition property of the Poisson process presented by Serfozo (2009, Section 3.9, Theorem 36), the original stochastic hazard process can be decomposed into into various independent sub-processes, such as the following two that are relevant to the LQI optimization problem:

1. A sub-process causing structural failures and fatalities with the rate, $p_{d|f}\ p_{fx}(y)\lambda$, which is relevant to the life safety analysis.

2. A sub-process causing structural failures (with and without fatalities) with a rate, $p_{fx}(y)\lambda$, which is relevant to the life cycle cost analysis.

There is another sub-process of events with a rate, $(1-p_{fx}(y))\lambda$,that does not not cause any structural failure, i.e., $X_i \leq y$, and it is not relevant to the reliability analysis.

The expected number of failures, $N_F(t)$, is required to estimate cost of repair over the life cycle of the structure. It is given based on a decomposed HPP as

$$\mathbb{E}\left[N_F(t)\right] = p_{fx}(y)\lambda t \tag{10}$$

The lifetime reliability of the structure is given as $e^{-p_{fx}(y)\lambda t}$.

3.3. *Life Safety Considerations*

The life safety impact of the hazard is a result of the sub-process that causes a failure and fatality with the rate, $p_{d|f}\ p_{fx}(y)\lambda$. The lifetime distribution of a person facing this hazard is equivalent to the distribution of the first time of occurrence of the fatality event, denoted as T_d. Note that the LQI does not consider "renewals" or "rebirths" after a fatality, as it is based on a single lifetime of a statistically representative individual. In other words, only a "single" generation model is implicit in the definition of LQI.

The survival function of T_d follows an exponential distribution:

$$\overline{F}_{T_d}(t) = e^{-p_{d|f}\ p_{fx}(y)\lambda t} = e^{-\delta(y)t} \tag{11}$$

where $\delta(y) = p_{d|f}\ p_{fx}(y)\lambda$ is a constant residual mortality rate to a person associated with a design of strength y over an interval, $(0,t]$.

4. LQI Optimization of Structural Safety

4.1. *Approach*

This Section will apply the LQI-based objective function, Eq.(8), to optimize the structural strength, y. This requires the estimation of the annualized cost of structural safety and the change in life expectancy over a reference time interval of the analysis, $(0,t]$. For simplicity, no discounting is considered as this adds compexity to the formulations without adding to the essence of the discussion presented in this paper.

4.2. *Cost of Structural Safety*

The total cost of a structure, C_{tot} over its life, t, includes the initial cost, $C_I(y)$, and the cost of repairing damages caused by the stochastic load process, as discussed in the previous Section.

The initial cost is commonly modelled as a linear function of the design parameter (Fischer et al., 2019):

$$C_I(y) = C_0 + C_1\, y \tag{12}$$

The fixed initial cost, C_0, includes the cost of land, cost of design and other services that are independent of y. The second components, $C_1 y$, represents the construction cost and depends on the design parameter.

In case of each failure event, the repair cost includes two components. The first one is equal to construction cost, $C_1 y$, and the second is the consequential economic losses, C_F, which includes the loss of contents and services provided by the structure. It can also includes other losses, such as loss of reputation, litigation cost, and compensations. Economic benefits generated by the structure are typically assumed to be independent of the design parameter, y. Therefore, they are not included in the life cycle cost analysis. Thus, the total life cycle cost (LCC) considering $N_F(y,t)$ potential failures can be computed as

$$C_{tot}(y,t) = (C_0 + C_1 y) + (C_1 y + C_F) N_F(y,t) \tag{13}$$

As the only element of randomness in this function is $N_F(y,t)$, the expected LCC is evaluated as

$$\mathbb{E}\left[C_{tot}(y,t)\right] = (C_0 + C_1 y) + (C_1 y + C_F)\mathbb{E}\left[N_F(y,t)\right] \tag{14}$$

Substituting from Eq.(10) and omitting the expectation sign for the sake of brevity leads to the following expression:

$$C_{tot}(y,t) = (C_0 + C_1\,y) + (C_1\,y + C_F)\,p_{fx}(y)\lambda t \tag{15}$$

The design optimization can be carried out purely on basis of minimization of LCC using the first-order condition as,

$$\frac{\mathrm{d}}{\mathrm{d}y}(C_{tot}(y,t)) = 0$$

which leads to

$$C_1 + C_1\lambda t p_{fx}(y) + (C_1\,y + C_F)\,\lambda t \frac{\mathrm{d}p_{fx}(y)}{\mathrm{d}y} = 0 \tag{16}$$

Finally, the optimum design parameter, $y = y^*$, is obtained by either minimizing the LCC function, Eq.(15), or by directly solving Eq.(16) for y.

Since discounting of cost is not considered in this paper, the annualized cost can be simply obtained as

$$c_{tot}(y) = \frac{C_{tot}(y,t)}{Nt} \quad (\$/\text{person/year}) \tag{17}$$

where N is the number of people who are exposed to hazards due to structural failures and expected to share the cost of improving the structural safety.

4.2.1. *Remarks on Time Horizon of Analysis*

The time horizon of optimization is also an important element of the problem (Fischer et al., 2013). Most studies have adopted an infinite time horizon for the optimization, since costs and benefits of many infrastructures are expected to continue over a very long period of time (Rackwitz, 2000). In the infinite time horizon, the asymptotic limits of cost rate derived for a renewal process model are used. However, the life expectancy is defined as the mean of a single lifetime. Therefore, the computation of a change in life expectancy caused by a renewal hazard process over an infinite time horizon must be carefully formulated as a "first-failure" problem, as illustrated in Section 3.3.

In a general setting, the consideration of a finite time horizon is necessary for a safety program for which the cost is amortized over t_1 years and its life safety benefits are expected to last for t_2 years, $t_1 \neq t_2$. In this case, an evaluation of the life safety impact would consider a change in the mortality rate over an age interval of length t_2. As an example, consider a safety instrumentation system, which enhances safety by providing an advanced warning of an accident. Such a system is likely to have a finite life, since a more advanced technology is expected to replace this system after t_1 years.

In this paper, LQI optimization is formulated over a finite time horizon with an equal period of costing and mortality reduction,i.e., $t_1 = t_2 = t$. Further, the time horizon of the analysis, t, is assumed to be close to the mean human lifetime, of the order of 80- 90 years. This simplifies the analytical evaluation of change in life expectancy, as discussed in the next Section.

4.3. *Life Safety Analysis*

In the mathematical demography, the human lifetime is modelled as a random variable, T_h, and the corresponding survival curve is given in the form of a life table of the population. Since the human survival function is related to the mortality rate, it can be defined for the base case in terms of $m_1(u)$ as (Maes et al., 2003),

$$\overline{F}_{T_h}(u) = e^{-\int_0^t m_1(u)\mathrm{d}u} \tag{18}$$

The life expectancy (LE), or the mean lifetime, $\mathbb{E}[T_h] = e_1$, is defined as

$$e_1 = \int_0^\infty \overline{F}_{T_h}(t)\mathrm{d}t = \int_0^\infty e^{-\int_0^t m_1(u)\mathrm{d}u}\,\mathrm{d}t \tag{19}$$

Suppose an HPP hazard process increases the mortality rate by an amount, δ, over the entire lifetime of the individual. The distribution of lifetime, T_d, under this hazard would follow an exponential distribution. The distribution of lifetime, T_c, under the prevailing mortality, $m_1(u)$, and an additional mortality, δ, can be obtained as a "competing risk" problem:

$$\begin{aligned} \mathbb{P}[T_c > s] &= \mathbb{P}[T_d > s, T_h > s] \\ \Rightarrow \overline{F}_{T_c}(s) &= \overline{F}_{T_d}(s)\overline{F}_{T_h}(s) \end{aligned}$$

The second equation is based on an assumption that T_d is independent of T_h. The corresponding life expectancy, e_2, can be evaluated by substituting the survival function of T_d as

$$e_2 = \int_0^\infty e^{-\delta s}\overline{F}_{T_h}(s)\mathrm{d}s \tag{20}$$

Note that in reality the upper limit of integration should be the upper limit of human lifetime, of the order of 100 years. Since this analysis is using a continuous exponential distribution, ∞ is used as the upper limit to simplify the integration.

Using a first order approximation, $e^{-\varepsilon} \approx 1-\varepsilon$ for $\varepsilon < 1$, an approximate expression for e_2 can be derived as

$$\begin{aligned} e_2 &\approx \int_0^\infty (1-\delta s)\overline{F}_{T_h}(s)\mathrm{d}s \\ &\approx \int_0^\infty \overline{F}_{T_h}(s)\mathrm{d}s - \delta\int_0^\infty s\overline{F}_{T_h}(s)\mathrm{d}s \\ &\approx e_1 - \delta\, e_1 \int_0^\infty s f_A(s)\mathrm{d}s \end{aligned}$$

where $f_A(s) = \overline{F}_{T_h}(s)/e_1$ denotes the stationary age distribution. This leads to the final expression for the modified LE as

$$e_2 \approx e_1(1-\delta\,\mu_A) \tag{21}$$

with μ_A the mean age of the stationary life table population in the base case.

4.4. LQI Optimization Criterion

In the objective function, Eq. (8), the following substitutions are made, $c(y) = c_{tot}(y)$ from Eq. (17) and $\Delta m_2(y) = \delta(y) = p_{d|f}\ p_{fx}(y)\lambda$ from Eq.(11), which lead to the following function:

$$\frac{\Delta L_R}{L_1} \approx q\frac{C_{tot}(y,t)}{N\,t\,g_1} + \mu_A p_{d|f}\ p_{fx}(y)\lambda \tag{22}$$

This expression can be rearranged as:

$$Ng_1 t\frac{\Delta L_R(y)}{qL_1} \approx C_{tot}(y,t) + K_L(n_{d|f}\,p_{fx}(y)\lambda t) \tag{23}$$

Here, $n_{d|f} = p_{d|f}N$, denotes the expected number of fatalities conditioned on a structural failure under a shock load. $K_L = \frac{g_1\mu_A}{q}$ is a socio-economic constant. The minimization of the above function with respect to y would lead to an LQI optimal design strength of the structure.

4.4.1. Remarks

As seen from Eq.(23), the LQI optimization criterion includes the life cycle cost (LCC) minimization criterion and a function that corresponds to the minimization of the residual risk to life safety. Thus, LQI optimization criterion is expected to require a higher level of reliability than that based on the minimization of LCC alone. This aspect will be further illustrated by an example.

5. EXAMPLE

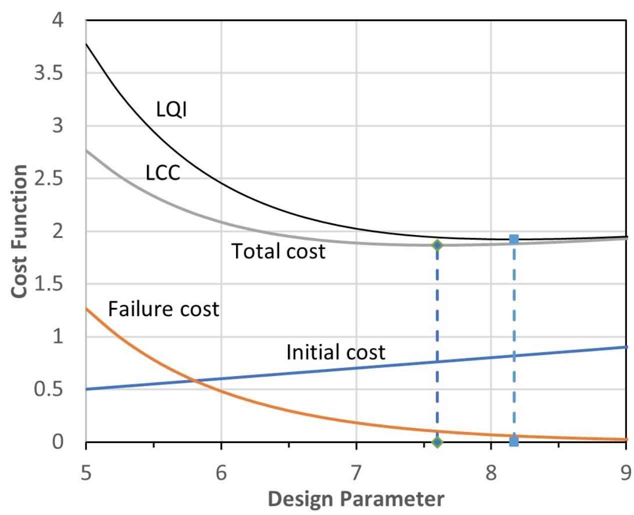

Figure 2: Variation of cost functions with the design parameter

An illustrative example is presented using the following normalized cost parameters: $C_1/C_0 = 0.1$, $C_F/C_0 = 2$. The loading frequency is assumed as $\lambda = 1$/year and the time horizon as $t = 75$ years. The shock load is assumed to follow an exponential distribution with a unit mean value, such that the probability of failure per shock is given as, $p_{fx} = e^{-y}$. The LQI parameters are based on 2019 Canadian data as, g_1 = 46,291 \$/person, $e_1 = 82$ years, and $q = 0.2$. The socio-economic parameter is also normalized as $K_L/C_0 = \theta_L = 2$. The life cycle cost (LCC) optimization leads to an optimal value of the design parameter as $y_C^* = 7.59$ and the corresponding lifetime reliability index of 1.78.

The LQI optimization problem with expected number of fatalities, $n_d = 1$, leads to an optimal design parameter, $y_L^* = 8.17$, and the corresponding lifetime reliability index of 2.03. Figure 2 shows the variation of LCC and LQI cost functions with the design parameter. If the expected number of fatalities is increased to $n_d = 10$, it leads to $y_L^* = 9.75$ and the lifetime reliability index of 2.62.

This example shows that for any value of $n_d > 0$, the design value and the reliability level obtained by the LQI optimization would exceed than those obtained from the LCC optimization.

6. CONCLUSIONS

The Life Quality Index (LQI) was proposed as a basis for optimizing investments in a safety program by balancing the objectives of enhancing life safety and minimizing the program cost. For improving the integrity and reliability of structures, LQI has been applied to optimize structural safety targets, in addition to a more commonly used method of minimizing the life cycle cost (LCC). In this context, some studies in the literature have reported a rather counter-intuitive result that the LQI approach, despite an explicit consideration of life safety, leads to a lower safety target as opposed to a pure cost based optimization embedded in the LCC method. To investigate this matter, this study presents a logical formulation of the LQI-based safety optimization problem.

The paper presents clear definitions of the base case and the alternate case to document a rigorous basis for the LQI optimization problem. An analytical expression is derived for a first-order change in LQI in the alternate case with reference to the base case. Using this expression, a minimization function is developed, which implies the maximization of LQI in the alternate case.

A remarkable results of this analysis is that the LCC minimization turns out to be a subset of the LQI optimization problem, as seen from Eq.(23), with LQI additionally including a function that represents the life safety impact of the program. Thus, a consideration of the life safety objective will increase the optimum safety level as compared to the minimization of LCC alone. In conclusion, the LQI approach leads to a higher -or more stringent- level of target reliability than that obtained using a pure cost-based minimization.

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