Basis of serviceability limit state target reliability: Fact or fiction?

Andrew C. Way
Postdoctoral Research Fellow, Dept. of Civil Engineering, Stellenbosch University, Stellenbosch, South Africa

Christina H. McLeod
Senior Lecturer, Dept. of Civil Engineering, University of KwaZulu-Natal, Durban, South Africa

Celeste Viljoen
Professor, Dept. of Civil Engineering, Stellenbosch University, Stellenbosch, South Africa

ABSTRACT: Target reliability forms the basis of modern structural design and finds its root in cost optimisation. Although ultimate limit state target reliability is well-founded, serviceability limit state (SLS) target reliability has received comparatively little attention and its basis is unclear. This is concerning, especially where SLS criteria governs the design of structural members structures or entire structures. The widely accepted, annual (lifetime) reversible and irreversible SLS target reliabilities of 2.2 (0) and 2.9 (1.5), respectively, are said to be a result of generic cost optimisation. The SLS consequences of failure and costs of increasing safety in existing literature are at best qualitatively defined, which begs the question: What really forms the basis of SLS target reliability? This research investigates the basis of SLS target reliability in concrete structures using cost optimisation, across a range of failure consequences and cost of safety values. Quantitative SLS failure costs implied by current target reliability values are back-calculated from the cost optimisation, which could aid in the process of future code-making and calibration. Selected examples of SLS failure costs in practice are compared to the back-calculated failure costs and indicate that current SLS target reliability values may be too general. A greater choice of SLS target reliability and information thereabout could therefore be beneficial. A proposal in the choice of SLS target reliability as a function of safety and failure costs is presented.

1. INTRODUCTION

Target reliability is the point of departure in modern structural design codes and represents the balance point between the cost and the safety of a structure. Inappropriate target reliability values thus either result in a compromise in the safety of structures, or in structures that are more expensive than they should be. As such, target reliability values should be derived based on cost optimisation, considering the failure costs and the cost of (increasing) safety, given that societal risk constraints do not typically govern target reliability in new-build structures in developed or developing countries (Sykora *et al* 2017, Way *et al* 2022). Target reliability values are defined both for ultimate- (ULS) and serviceability limit states (SLS). The failure costs are distinctly different between ULS and SLS, as ULS failures are more catastrophic in nature and typically include costs associated with loss of life and limb and reconstruction, whereas SLS failures typically only incur costs related to repair and loss of service. The cost of safety is generally the same or are similar in both ULS and SLS cases, given that the same material is used to increase safety in both (reinforcing steel, concrete etc.).

The derivation of target reliability for ULS in many current design codes and standards considers a quantitative measure of the failure costs, such as EN1990 (CEN, 2004), or both the failure costs and cost of safety as in ISO 2394:2015 (ISO, 2015), the JCSS Probabilistic Model Code (JCSS, 2001) and the *fib* Model Code 2010 (*fib*, 2010). However, a quantitative basis for the failure costs in the derivation of SLS target reliability does not seem to exist or is only qualitatively defined in existing literature. As a result, whether current SLS target reliability values are near optimal or not in terms of cost and risk is uncertain. This is particularly pertinent in structures where the design is governed by limiting SLS criteria. The matter is further complicated by the subjectivity of the limiting SLS criteria themselves (deflection limits, crack width limits etc.), as well as the distinction between target reliability for irreversible and reversible SLS.

This research aims to highlight the need for, and to initiate an investigation into the basis for SLS target reliability in modern structural design codes.

2. TARGET RELIABILITY

In new-build structures, target reliability is determined through cost optimisation, which is typically given in a form similar to that in Rackwitz (2000). The total cost of a structure, normalised by the initial construction cost, C_0, is a function of the decision parameter (d), and is given by Eq. (1). Note that Eq. (1) considers a generic limit state, whether ULS or SLS.

$$z = 1 + \frac{C_1}{C_0} \cdot d + \frac{\omega}{\gamma} \cdot \left(1 + \frac{C_1}{C_0} \cdot d + \frac{A}{C_0}\right) + \frac{C_F}{C_0} \cdot \frac{p_f}{\gamma} \quad (1)$$

$$p_f = P(R(d) < S) \quad (2)$$

The total cost is most notably dependent on the normalised cost of increasing safety, C_1/C_0, the normalised failure costs of the limit state, C_F/C_0, and the probability of failure, p_F. As shown in Eq. (2), the p_f is the probability that the action effect, S, exceeds the resistance, $R(d)$, of the limit state under consideration. The total cost is only marginally affected by typical ranges of the obsolescence rate, ω , the normalised obsolescence costs, A/C_0, and the age-averaged societal discount rate, γ (Rackwitz, 2000; Rackwitz *et al*, 2005). Future costs are discounted back to current value through the approximation of $1/\gamma$, which assumes a stationary Poisson process for obsolescence event occurrences. Target reliability is the point at which the total normalised cost of the structure is minimized, i.e. the value of d where $\partial z/\partial d = 0$.

Failure costs are quantitatively defined for ULS cases and are classified as (JCSS PMC, 2001):

Minor	$C_F/C_0 < 2$
Moderate	$2 \leq C_F/C_0 < 5$
Great	$5 \leq C_F/C_0 < 10$

The cost of increasing safety can also be quantified as (Fischer *et al*, 2019):

Low	$10^{-4} < C_1/C_0 \leq 10^{-3}$
Medium	$10^{-3} < C_1/C_0 \leq 10^{-2}$
High	$10^{-2} < C_1/C_0 \leq 10^{-1}$

Target reliability is defined in the form of the reliability index, β_t, for ULS based on these quantitative values as shown in Table 1.

Table 1: Annual ULS Target reliability (β_t) for failure costs (C_F/C_0) and costs of safety (C_1/C_0) from the JCSS PMC and ISO 2394:2015.

	C_F/C_0		
C_1/C_0	*Min.*	*Mod.*	*Great*
Low	*3.1*	*3.3*	*3.7*
Medium	*3.7*	*4.2*	*4.4*
High	*4.2*	*4.4*	*4.7*

The basis for SLS target reliability, however, is less clear. The SLS costs of safety are likely to be the same or similar to those associated with ULS, given that the same material is used to increase safety (concrete beam depth, area of reinforcement etc.), however, little guidance is given with regard to the failure costs associated with SLS. Some qualitative descriptions are given in various research, codes and standards, as shown in Table 2 for annual, irreversible SLS. No quantitative values of these qualitative descriptions are given however, with the exception of Rackwitz (2000). In the derivation of ULS β_t values in Rackwitz (2000), SLS failure costs of $C_F = 0.2C_0$ are implied, with a comment that these may be typical for bridges. Rackwitz (2000) also notes that the SLS values obtained (shown in Table 2) appear to be conservative and that current practice (as of the year 2000) appears to accept larger risks, i.e. β_t lower than those in the *Rackwitz/JCSS PMC/MC2020* column of Table 2. From this, it is interesting to note that EN1990 and MC2010 recommend SLS β_t values that are $\Delta\beta = 1.2$ *higher* than those from Rackwitz (2000). The latest revision of ISO 2394 (2015) does not propose SLS target reliability values.

Table 2: Annual irreversible SLS target reliability (β_t) values.

Source	*Rackwitz, JCSS PMC, MC2020 draft*	*MC2010, EN1990*	*ISO 2394:1998*
	C_F/C_0 qualitative descriptions		
C_1/C_0	*"Insignificant"*	*"Small"*	*"Small"*[1]
Low	*2.3*	-	*3.5*
Medium	*1.7*	*2.9*	*2.9*
High	*1.3*	-	*2.2*

1 – Converted from lifetime to annual

Further obscurity is introduced through the subjectivity of limiting criteria in the various SLS limit states, through which β_t are calculated. In ULS limit states, a clear line exists between failure and non-failure, whereas the limits for SLS cases are not as clear. A consensus on appropriate crack width limits to limit leakage in water retaining structures has not yet been reached, for example, and similarly for deflection limits in concrete beams to ensure acceptable levels of serviceability.

Additionally, there is often confusion as to what reversible SLS refers to, particularly in concrete structures. Due to the nature of reinforced concrete, the effects of most SLS failures are not truly reversible. Although caused by temporary actions, most SLS failures have irreversible effects, whether they be permanent reductions in concrete stiffness or increased permeability due to cracking, for example. These are often considered as being reversible, even though they are irreversible. Irrespective, no qualitative or quantitative distinction currently appears to be made between the assumed failure costs to warrant a differentiation between the reversible and irreversible SLS β_t values.

Given the discussion above, what is the basis for the adoption of annual/lifetime β_t values of 2.2/0 and 2.9/1.5 for reversible and irreversible SLS, respectively? Without an answer to this, it is uncertain as to whether current code calibration to these SLS target reliability values actually result in cost-optimality or not. This research therefore aims to investigate whether current SLS target reliability provisions are in fact optimal.

3. COST OPTIMISATION METHODOLOGY

In order to investigate the appropriateness of current SLS target reliabilities, a SLS cost optimisation is performed in conjunction with FORM reliability analyses using Eq. (1) and (2). Typical parameter values from Rackwitz (2000) are used:

$$\omega = 0.02 \qquad \gamma = 0.035 \qquad A/C_0 = 0.2$$

The resistance and action effects are both considered as being lognormally distributed, as a lognormal distribution effectively represents resistances (Holický, 2009), as well as the combination of permanent (normal distribution) and variable (extreme value distribution) actions where the coefficient of variation does not exceed 0.3 (Fischer *et al*, 2019; Steenbergen *et al*, 2018). Two cases of parameter variation are considered; One with typically low coefficients of variation for resistance and load ($V_R = 0.1$, $V_S = 0.15$) and one with higher parameter variation ($V_R = 0.3$, $V_S = 0.3$). The former would be appropriate for cases where loads and resistances contain little to moderate uncertainty, such as SLS stress limitation in a prestressed concrete bridge, whereas the latter is appropriate to cases where notable uncertainty in load and/or resistance is present, such as the limitation of crack widths to minimize leakage in water retaining structures (McLeod & Viljoen, 2019; Way & Viljoen, 2021).

The costs of increasing safety are varied in the cost optimisation, using the middle of each range given in the previous section from Fischer *et al* (2019):

Low $\quad C_1/C_0 = 5 \times 10^{-4}$
Medium $\quad C_1/C_0 = 5 \times 10^{-3}$

High $C_1/C_0 = 5 \times 10^{-2}$

Failure costs for SLS are notably different from those for ULS. The non-catastrophic nature of SLS failures translate into failure costs lower than for ULS cases, as is indicated by the qualitative descriptions in Table 2. This is because SLS failures seldom result in a loss of human life or limb. Typical SLS failures only incur costs related to a loss of service and for repair or strengthening. As a result, SLS failure costs are typically well below the initial cost of the structure ($C_F/C_0 < 1$), except perhaps for major bridges or other structures where repair or a loss of service results in notable costs; these are not considered in this research. In the same vein, cases of minor SLS failures such as non-structural crack repair can incur costs as insignificant as $C_F/C_0 < 0.01$. The SLS failure costs are thus varied for a range of $0.01 \leq C_F/C_0 \leq 1$ in the cost optimisation.

4. RESULTS AND DISCUSSION

The results of the SLS cost optimisation are shown in Figure 1. The annual SLS target reliability values are shown as a function of the failure costs for low, medium and high costs of safety, for low (top) and high (bottom) parameter variation. The typical β_t values of 2.2 and 2.9 for reversible and irreversible SLS are indicated by the blue and red lines, respectively, for reference.

The β_t values are shown to vary by as much as $\Delta\beta = 1.8$ between insignificant ($C_F/C_0 \approx 0.01$) and notable SLS failure costs ($C_F/C_0 \approx 1$). The cost of safety also notably affects the β_t value, increasing it by as much as ≈ 1 for an order of magnitude decrease in the cost of safety. As expected, lower parameter variation results in an increase in β_t, in the order of 0.5. The SLS failure cost implications from Figure 1 can be evaluated in one of two ways. Either, it is assumed that existing practice SLS β_t values are near-to-optimal and failure costs are back-calculated or, ranges of failure costs are adopted based on practical experience and/or engineering judgement and β_t values are proposed using those failure costs. The latter option is preferable, as it uses the true cost of SLS failures and gives β_t values that most transparently reflect cost-optimality. There are drawbacks to this, however, in that:

- Failure costs vary between SLS limit states and,
- Engineering practice is often skeptical of changes to current design parameters and,
- Engineering practice is often resistant toward changes to the current design *status quo*.

Due to this, the approach of back-calculating from existing SLS β_t values is adopted here.

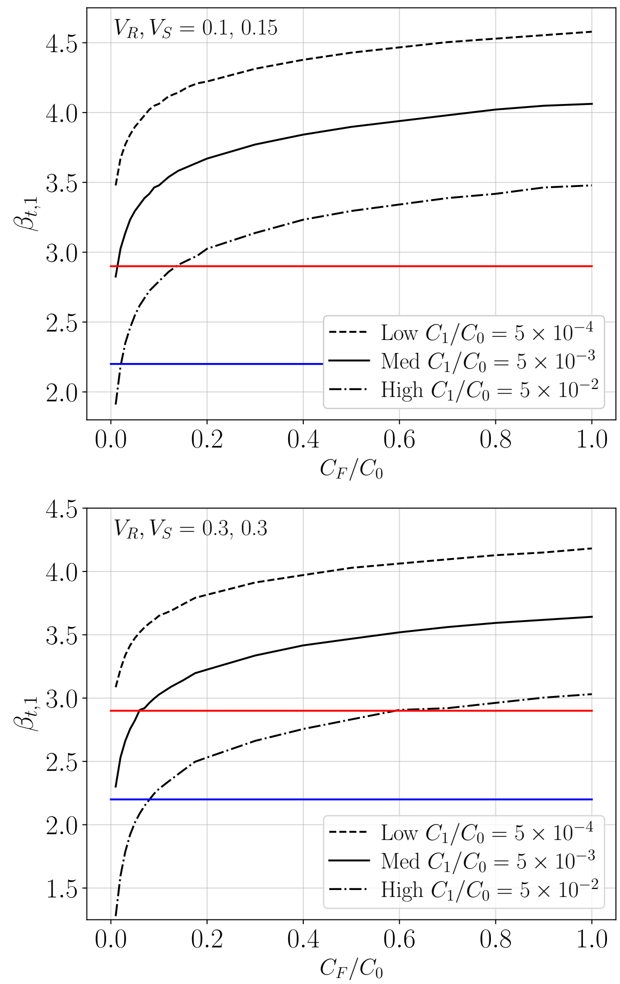

Figure 1: Annual SLS target reliability from cost optimisation for low (top) and high (bottom) parameter variation.

4.1. Back-calibrating C_F/C_0 from current β_t

Existing codes/standards typically adopt an annual irreversible SLS β_t value of $\beta_t = 2.9$ (red lines in Figure 1). The JCSS PMC suggests that the *medium* safety cost case is considered as being typical. For this case, the cost optimisation results in Figure 1 imply an irreversible SLS C_F/C_0 range from ≈ 0.02 to 0.06 for low to high parameter variation. The corresponding β_t values for the same C_F/C_0 range for low and high safety costs of $\approx$ 3.5 and 2.1 (averaged between low and high parameter variation), respectively, agree well with those proposed by ISO 2394:1998, EN1990 and MC2010 in Table 2. The annual target reliability for reversible SLS (blue lines in Figure 1) of $\beta_t =$ 2.2 for *medium* safety costs seems to appropriately be associated with insignificant failure costs $(C_F/C_0 < 0.01)$.

Although the back-calculation seems to yield results similar to those from ISO 2394:1998 for typical SLS failure costs, a more complete proposal of β_t values for failure cost ranges would be helpful to designers, especially for cases where failure costs may not be "typical". Based on the back-calculation of the cost optimisation results in Figure 1, proposals for annual SLS β_t are made and shown in Table 3. These are proposed as a means of giving designers and code-makers more information with which to make an informed decision of an appropriate value of SLS β_t.

Table 3: Proposed annual SLS target reliability (β_t) from back-calculation to existing practice.

Failure cost	*Cost of safety* C_1/C_0		
	Low	*Med.*	*High*
Insignificant or Reversible SLS $C_F/C_0 < 0.01$	*2.2*		
Minor SLS (typical) $0.01 < C_F/C_0 \le 0.05$	*3.5*	*2.9*	*2.2*
Moderate SLS $0.05 < C_F/C_0 \le 0.20$	*3.9*	*3.3*	*2.6*
Great SLS $0.20 < C_F/C_0 \le 1.0$	*4.2*	*3.6*	*3.0*

In Table 3, the failure cost ranges are proposed using the same terminology as the ULS cases for familiarity, but adding the *SLS* suffix to avoid confusion between ULS and SLS failure costs. The *insignificant* range $(C_F/C_0 < 0.01)$ is added, which denotes the case where SLS failure costs are practically neglectable. Given that cases of reversible SLS have no practical cost implications, these also fall into this category which has a constant β_t value of 2.2, irrespective of safety costs. The *minor* case is the typical case of SLS as back-calculated from existing practice, which accordingly has β_t values unchanged from those in ISO 2394:1998, EN1990 and MC2010. Two higher classes of *moderate* and *great* SLS failure costs are proposed, with failure cost ranges of $0.05 < C_F/C_0 \le 0.20$ and $0.2 < C_F/C_0 \le 1.0$, respectively. The *great* class has a wide range of cost, given that the target reliability changes little for $0.5 < C_F/C_0$. Cases where the SLS failure costs exceed $1C_0$ are highly unlikely, given that this range verges on ULS failure cost ranges.

4.2. Discussion

As previously mentioned, the SLS β_t values in Table 3 are proposed from back-calculation to existing $\beta_t Comments$ values. This has obvious benefits in that current SLS β_t values remain unchanged and are merely supplemented with greater choice and a quantitative basis. There is a notable disadvantage however, that while the proposed values are in line with current practice, they are only accurate insofar as current practice actually represents cost-optimality. In a case where it does not, the back-calculation merely perpetuates non-optimality; the back-calibrated C_F/C_0 ratios may not be representative of those in reality. As such, it is important to compare the theoretical, back-calculated C_F/C_0 ratios with those from industry practice. This is carried out only indicatively here below. Future research should focus on developing a more comprehensive consideration of SLS failure costs and compare them with the back-calculated C_F/C_0 ranges to definitively evaluate whether existing SLS β_t values are in fact cost-optimal.

Consider the case of SLS design to limit cracking in concrete structural bridge components. Components are designed so that crack widths are limited to within acceptable bounds (typically 0.3mm or less). In this case, SLS failure would occur when cracks that are greater than this specified limit develop and would necessitate crack repair. Instinctively, one would assume crack repair to be a relatively inexpensive task and that the failure costs would therefore be low in comparison to the construction cost of a bridge (assuming no traffic accommodation is required). To get an indication of SLS failure costs in practice, a sample of 1200 bridges with crack repair items for structural components was considered from an analysis of bridge management system (BMS) data in South Africa. The ratio of the crack repair cost to bridge construction cost estimates from the BMS data yielded mean C_F/C_0 ratios of between 0.003 and 0.007, depending on locality. Based on the proposed values from Table 3, the *insignificant* failure cost category applies ($C_F/C_0 < 0.01$), with $\beta_t = 2.2$, which corroborates the instinctive assumption of an inexpensive repair (failure cost). Current codes, however, would allocate a value of $\beta_t = 2.9$.

Consider a second SLS case of cracking in a concrete water retaining structure (WRS). In comparison to cracking in a bridge, excessive cracking in a WRS is likely to result in leakage of the stored water and potentially a loss of service. Cracking would thus incur failure costs in terms of both crack repair (or the installation of a waterproof lining) and potentially a loss of service of the WRS. Cracks may need to simply be repaired or the reservoir may need to be lined, and the reservoir may need to be emptied, depending on the severity of the cracking. Instinctively, the cost of failure in this case is likely to be higher than the previous case. In a case where a liner needs to be procured and installed (not considering the subjective cost of loss of service), failure cost estimates ranged from $C_F/C_0 = 0.01$ to 0.3, whereas lesser crack repairs ranged from $C_F/C_0 = 0.002$ to 0.06. Loss of service costs will obviously lead to increased failure cost ratios. These cost estimates were derived using limited data on the costs of repair of existing South African WRS. With reference to Table 3, lesser repairs are likely to be in the *minor* category ($\beta_t = 2.9$), while failure costs for middle-of-the-range crack repairs would be in the *moderate* category ($\beta_t = 3.3$) and those that require linings are likely to be in the *great* category ($\beta_t = 3.6$). Compared to the current code value of $\beta_t = 2.9$, lesser crack repairs would have an appropriate target reliability, while typical and greater repairs would require a higher target reliability than currently allocated.

While only indicative, the two examples indicate that current SLS β_t values may be too general and that greater cost optimality could be achieved through a wider range of choice in SLS β_t values, depending on failure costs. Future research should focus on developing a more comprehensive consideration of SLS failure costs and compare them with the back-calculated C_F/C_0 ranges to definitively evaluate whether existing SLS β_t values are in fact cost-optimal or not. Additionally, these estimates are for a developing country economy, however, because the failure costs are in a ratio form, they should not be dissimilar to those from developed countries but this also requires further research to confirm.

5. CONCLUSIONS

Appropriate target reliability values help to ensure that structures are both cost optimal and display acceptable levels of reliability. Given the unclear basis of SLS target reliability in concrete structures, this research presents an analysis of current SLS target reliability through cost optimisation. The failure costs implied by SLS design to current practice are back-calculated from the cost optimisation, providing a quantitative basis with which to use in future code-making, as well as to compare to values from engineering practice. A high-level comparison between the SLS failure costs implied from back-calculation and those from engineering practice is performed. Results indicate that current SLS target reliability specifications may be too general and could benefit from a greater range of choice in SLS target reliability, as a function of safety and failure cost. A proposal is made which provides a greater choice in the selection of SLS target reliability, as well as guidance in this regard. A more complete comparison between theoretical and practical SLS failure costs should be the subject of future research.

6. REFERENCES

CEN (2002). *EN 1990:2002*. *Eurocode – Basis of Structural Design.* Brussels: European Committee for Standardization (CEN).

fib MC2010 (2010): *fib* Model Code for Concrete Structures 2010. *Ernst & Sohn*, Berlin, Germany.

fib MC2020 draft (2022): fib Model Code for Concrete Structures 2020 (2nd draft).

Fischer, K., Viljoen, C., Köhler, J., and Faber, M.H. (2019). "Optimal and acceptable reliabilities for structural design". *Structural Safety*, 76, 149-161.

Holický, M. (2009). "Reliability analysis for structural design". Stellenbosch: *SunMedia*.

ISO (2015). ISO 2394:2015. General Principles on Reliability for Structures. Geneva, Switzerland: *ISO*.

ISO (1998). ISO 2394:1998. General Principles on Reliability for Structures. Geneva, Switzerland: *ISO*.

JCSS (2001). JCSS Probabilistic Model Code. *Joint Committee on Structural Safety,* www.jcss.byg.dtu.dk.

McLeod, C., and Viljoen, C. (2019). "Quantification of crack prediction models in reinforced concrete under flexural loading". *Structural Concrete*, 20, 2096– 2108.

Rackwitz, R. (2000). "Optimization - the basis of code making and reliability verification." *Structural Safety*, 22, 27–60

Rackwitz, R., Lentz, A. and Faber, M., 2005. "Socio-economically sustainable civil engineering infrastructures by optimization". *Structural safety*, *27*(3), 187-229.

Steenbergen, R., Rózsás, Á. and Vrouwenvelder, A. (2018). "Target reliability of new and existing structures: A general framework for code making". *Heron*, 63(3): 219.

Sykora, M., Diamantidis, D., Holicky, M. and Jung, K. (2017). "Target reliability for existing structures considering economic and societal aspects". *Structure and Infrastructure Engineering*, *13*(1), 181-194.

Way, A., and Viljoen, C. (2021). "Reliability in tension governed RC reservoirs considering autogenous self-sealing". *Engineering Structures*, 248, 113235.

Way, A.C., de Koker, N., and Viljoen, C. (2022). "Target reliability for new road bridges in South Africa". *Journal of the South African institution of civil engineering*, *64*(3), 10-19.