Reliability of statically indeterminate timber structures: impact of connection non-linearity and overstrength

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ABSTRACT: In statically indeterminate timber structures, the stiffness of the members and the semi-rigid behavior of the joints influence the force distribution in the structure, affecting its performance and reliability. In this paper the impact of the non-linear load-deformation behavior of connection on a simple statically indeterminate timber structure is evaluated. The basis of the definition of the non-linear load-deformation behavior of connections in terms of stiffness, load-carrying capacity, and ductility is reviewed. The reliability of a beam supported by rotational spring is determined in a Monte-Carlo analysis. The impact of different typical variation of the connection design as well as overstrength and excessive stiffness are evaluated. It is shown that by considering sufficient ductility of the joints a gain in reliability of structural indeterminate structures compared to the reference condition of a simply supported beam can be achieved. More detailed specifications on how to consider the connection non-linearity in design should be introduced in design codes.

1. INTRODUCTION

Wood is one of the oldest building materials and timber has been used for remarkable and long-lasting structures already in the past. However, traditionally the design of these timber structures was mainly based on experience rather than engineering science and understanding. With the marked development of new engineered wood products, such as cross-laminated timber, and the creation of more and high-performance timber structures, engineers are facing new challenges in their design works with these new products and construction types.

It was the collapse of the formwork of the Sandö-bridge in northern Sweden at the eve of second world war that initiated major research on the non-linear load-deformation behavior of nailed connections between small dimensional timber members carried out at Chalmers University of Technology (Granholm 1949). The outcomes of this research resulted in the so-called European Yield Model, which was a major step forward in technology and understanding of connections in timber. Today, 80 years later, it is still the basis for the design of timber structures and is implemented in Eurocode 5. However, the recent developments of new wood-based products, new fastener types and technologies and not least the digitalization and automation of planning, design and production have created new generations of highly complex and structurally indeterminate timber structures. Though, modern timber structures have little in common with the formwork of the Sandö-bridge, we still use the same design approaches and rely on the same old knowledge that was developed from that triggering event. In order to benefit from the full potential of modern, high performance timber structures, it is now necessary to get an up to date understanding of the load deformation behavior of modern connections and their impact on the structural behavior of the entire timber structure.

One of the crucial aspects that needs to be better understood is the impact of the non-linear connection behavior on the force distribution and reliability of these structures. Design guidance has to be derived on how to better consider this aspect in design.

2. BACKGROUND ON CONNECTIONS IN TIMBER STRUCTURES

2.1. Connection resistance

Connections with laterally loaded dowel type fasteners are among the most common connection types for large timber structures used in practice. The design basis in Eurocode 5 EN 1995-1-1:2004 (CEN 2004) of these connections is the so-called European Yield Model (EYM), which considers the ductile behavior (yielding) of both the steel and timber. The EYM however, provides only the load-carrying capacity under the assumption of perfect plasticity. The stiffness or deformation capacity cannot be derived by Eurocode 5.

Besides laterally loaded fasteners, modern connections often rely on axially loaded fasteners such as self-tapping screws or screwed-in or bonded-in rods. The load-carrying capacity of these axially loaded fasteners depends on the effective anchorage or withdrawal length. Similarly, to laterally loaded fasteners, only the load-carrying capacity can be derived with the equations in Eurocode 5.

2.2. Connection stiffness

2.2.1. Stiffness in the serviceability limit state

When the deformation of a structure needs to be determined or in statically indeterminate structures, the stiffness of the members needs to be considered. The design recommendations regarding stiffness of joints in the 2004 version of Eurocode 5 is very basic and focusses on the serviceability limit state. The connection stiffness is defined on the basis of a so-called slip modulus K_{ser} that is specified for the serviceability limit state. Eurocode 5 gives formulas for different types of fasteners. For predrilled fasteners, such as dowels, bolts, screws or nails, Eurocode 5 proposes

$$K_{ser} = \frac{\rho^{1.5} d}{23} \tag{1}$$

No distinction is made between fasteners loaded parallel or perpendicular to the grain. Furthermore, the failure mode of the fastener is not included.

In SIA 265 (SIA 2012) a distinction is made for dowels, bolts and predrilled nails for the loading directions parallel or perpendicular to the grain: In the latter case the stiffness is considered to be 50% of the former case.

More detailed background of the derivation of Equation (1), benchmarking by experiments and determination of distribution characteristics can be found in (Jockwer & Jorissen 2018, Jockwer et al. 2021). Different parameters of influence have been analyzed in this literature, such as the effect of multiple fasteners in a connection on the slip modulus, or the effect of spacing between fasteners on the ductility.

2.2.2. Stiffness in the ultimate limit state

The load-deformation behavior of laterally loaded fasteners is highly non-linear and, hence, it is not possible to define one single stiffness value valid for different load levels. The load-deformation curve shows a convex shape. While the stiffness in the serviceability limit state follows approximately a tangent to the linear-elastic branch of the load-deformation behavior, the stiffness a higher load levels, which relevant in the ultimate limit state, is typically simplified by a secant from the origin to the respective load level on the load-deformation curve.

The test standard EN 26891 (CEN 1991) does not provide any guidance on the determination of the secant stiffness in the ultimate limit state. Eurocode 5 specifies a simple reduction of the stiffness in the serviceability limit state by 1/3 for the ultimate limit state.

$$K_u = \frac{2}{3} K_{ser} \tag{2}$$

However, no further background or explanation are given in Eurocode 5 on how to treat different connection types and ductility levels. One of the main questions relates for example to the impact of the variability of the connection stiffness: the stiffness value in Equation (1) is a mean value and it is not clear if the simple reduction in Equation (2) considers only reduction of mean stiffness or also reflects to a percentile value commonly used in design.

The German national annex to Eurocode 5 (DIN 2013) specifies a further reduction of the stiffness in the ultimate limit state by reduction by the partial safety factor for the material:

$$K_u = \frac{2}{3} \cdot \frac{K_{ser}}{\gamma_M} \tag{3}$$

It can be questioned if the application of the general partial safety factor, which is calibrated for a beam in bending, is adequate also for the stiffness of connections. The failure behavior and deformation capacity of the connection and the consequences in the failure of the structure are currently not considered in the design and the related definition of the partial safety factors for the stiffness for statically indeterminate structures.

2.3. Ductility of connections

Due to the brittle failure behavior of timber in tension and shear, the ductile behavior of connections is often the only way to introduce the potential for load redistribution into a structure in order to achieve robustness (Dietsch 2011). In order to achieve the sufficient deformation capacity and ductility of the connection, however, sufficient spacing, end-and edge distances have to be satisfied in order to prevent premature failure in the timber surrounding the connection. In practice such large spacing and distance are often not satisfied, which might be considered as acceptable when looking at the load-carrying capacity alone. However, insufficient ductility, and hence, reduced ability for load-redistribution can result in failures with greater or at least different consequences.

In Eurocode 5 no specifications of ductility levels are given. It is only requested that connections should have an "adequate" ductility for the internal load-redistribution. Other standards or literature provide some guidance on ductility levels, such as the Swiss Standard SIA 265 (SIA 2012), where basically two levels of lower and higher ductility are distinguished. The ductility definition is based on the ratio between the deformations at failure v_f and at yielding v_y.

$$D_f = \frac{v_f}{v_y} \tag{4}$$

Other literature such as (Smith et al 2006) proposes ductility definition based on deformations at ultimate load v_u:

$$D_u = \frac{v_u}{v_y} \tag{5}$$

According to these two definitions, values of $D \approx 1$ correspond to no (brittle failure) or very limited ductility. Smith et al (2006) specify value of $D_u \leq 2$ as brittle, $2 < D_u \leq 4$ as low ductility, $4 < D_u \leq 6$ as moderate ductility, and $6 < D_u$ as high ductility. In Standard SIA 265 the values $1 \leq D_f \leq 2$ are specified for laterally loaded fasteners with less than two plastic hinges and $3 < D_f$ for those with two plastic hinges. Within this study a linear elastic- perfectly plastic behavior was assumed and hence a simplified ductility definition similar to Equation (4) was chosen.

The deformation capacity and consequently the ductility of connections is often reduced by overstrength of the fasteners.

2.4. Sources of overstrength in timber structures

Different sources for overstrength in timber structures can be identified, which can be related to the material properties of the components as well as the design regulations applied in the design.

Steel strength of the dowels: Common dowels typically exhibit a considerably higher strength than declared, which results in increased fasteners resistance. This in turn can provoke brittle timber failure in the surrounding material around the fastener and connection and it is particularly problematic in the case that ductility is needed, such as for robustness or for the seismic situation.

The design models of the EYM are simplified regarding the material behavior (rigid-plastic behavior is assumed) as well as regarding the second order effects of the fastener (rope effect). This results in a typical underestimation of the experimental by the analytical models.

In addition, also friction in the connection can result in an increase in stiffness or load-carrying capacity.

3. MATERIALS AND METHODS

3.1. Structural Model

A simple statically indeterminate structure is evaluated in this study: a beam supported by rotational springs at both ends Figure 1. This model can be considered as a representation of a roof beam of a portal frame with semi-rigid joints. The same model was used by Caprio et al. (2022) and it is referred there for further information about the implementation.

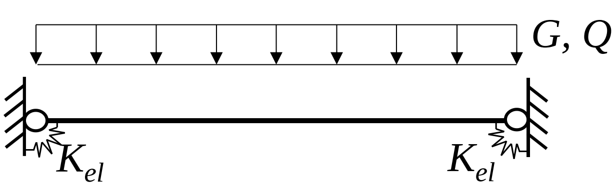

Figure 1: Illustration of the structural model of a beam supported by rotational springs.

The relevant action effects considered in this study are the support (joint, $M_{E,Joint}$) and field ($M_{E,Beam}$) moments, that can be expressed as a function of the coefficients α and β as follows:

$$M_{E,Joint} = \alpha \cdot q_E \cdot L^3 \quad (6)$$

$$\alpha = \frac{K_{el}}{24 \cdot EI + 12 \cdot K_{el} \cdot L} \quad (7)$$

$$M_{E,Beam} = \beta \cdot q_E \cdot L^3 \quad (8)$$

$$\beta = \frac{1}{8} - \alpha \quad (9)$$

The distributed load q_E results from permanent (G) and variable (Q) loads. A load ration $\alpha_E = Q/(Q+G) = 0.8$ was chosen in this study. The length $L = 10\ m$ and cross-section of the beam $b/h = 160\ mm/600\ mm$ are constant.

The joint at the support is represented by its rotational stiffness K_{el}. For sake of simplicity a dimensionless joint stiffness k_b can be defined in relation to the beam stiffness EI:

$$k_b = \frac{K_{el} L}{EI} \quad (10)$$

The wide range of connections in timber structures shows typically a semi-rigid to rigid behavior when applied for rotational supports with values ranging from $k_b = 1 - 30$ as discussed in (Caprio et al., 2022). Roughly the connection types can be assigned with the stiffness as following: dowelled connection $k_b \approx 3$, axially loaded fasteners $k_b \approx 6$, glued joints $k_b > 9$.

The moment resistance of the beam is based on its bending strength:

$$M_{R,Beam} = f_m \cdot \frac{b \cdot h^2}{6} \quad (11)$$

The resistance of the connection is based on the fraction $M_{R,joint} = \Phi_{R,joint} \cdot M_{R,Beam,mean}$ of the mean beam capacity and the value of $\Phi_{R,joint} = \alpha/\beta$ results in the full utilization of the required connection capacity. In the model a capacity reduction of 70% for assumed for the joint, i.e. the connection had a 30% lower strength than assumed in the design. That way the ductility can be utilized.

3.2. Reliability analysis

The design equation according to the load and resistance factored design can be expressed as follows:

$$z \frac{k_{mod}}{\gamma_M} R_k - \gamma_G G_k - \gamma_Q Q_k \geq 0 \quad (12)$$

In this study the partial factors proposed in Eurocode are used, i.e. $\gamma_M = 1.3$, $\gamma_G = 1.35$, and $\gamma_Q = 1.5$. Duration of load and moisture effects were disregarded; hence, the modification factor was chosen to be $k_{mod} = 1$. The design variable z is to be chosen by the designer to satisfy the condition. For the case of the beam with different failure modes and locations the design equations has to be solved for all these different cases and the minimal respective design variable is chosen.

For the reliability analysis the following limit state equation is defined, using the random variables of $\boldsymbol{R}$, $\boldsymbol{G}$, and $\boldsymbol{Q}$ and an additional model uncertainty $\boldsymbol{M}$.

$$g = z \cdot \boldsymbol{M} \cdot \boldsymbol{R} - (1-\alpha) \cdot \boldsymbol{G} - \alpha \cdot \boldsymbol{Q} \qquad (13)$$

The probability of failure can be described as follows for the random variable $\boldsymbol{X}$:

$$P_f = P(g(\boldsymbol{X}) \leq 0) = \int_{g(X)\leq 0} f_x(x)dx \qquad (14)$$

For the case of the beam supported by rotational springs, not only one limit state equation can be defined but instead a variety of different failure combinations including brittle and ductile failure as well as the possibility for load-redistribution. The failure combinations are described in detail in (Caprio et al., 2022) and can be summarized as follows:

- Brittle failure of the beam either at midspan or at the supports and linear elastic state of the connection
- Failure of the beam after achieving ductility of the connections and consequently load-redistribution
- Excess of the deformation capacity of the connection.

In all cases final failure of the beam was defined by exceedance of the moment resistance of the beam at joint location or midspan or of the deformation capacity of the joint.

Crude Monte Carlo Method was used with $2 \cdot 10^7$ realizations per configuration. The probability of failure was derived as the number of failures divided by the number of total realizations.

3.3. Material and modelling parameters

The distribution characteristics of all modelling parameters are summarized in Table 1. The parameters where chosen based on (JCSS 2001, Kohler 2005, Köhler et al. 2007). Reference to stiffness and ductility values can be found in (Jockwer and Jorissen 2018, Jockwer et al. 2021).

Table 1: Distribution characteristics and values of modelling parameters and properties.

Prop.	*Distr. funct.*	*Unit*	*Mean value*	*CoV*	*Char. value*
G	Normal	kN/m	3.0		50%
Q	Gumbel	kN/m			98%
α_E	-	-	0.8		
f_m	Logn.	N/mm	37	25%	5%
h	-	mm	600		
b	-	mm	160		
L	-	m	10		
E_0	Logn.	N/mm^2	11’500	10%	
k_b	Logn.	Nmm/rad	3-9	30%	
D	Logn.	-	1-5	30%	
$\Phi_{R,joint}$	Logn.	-		20%	
X	Logn.	-	1	10%	

4. RESULTS AND DISCUSSION

4.1. Calibration and reference case

A calibration of the model can be performed for the case of zero connection stiffness, resulting in the case of a simply supported beam. The failure of this beam is expected to occur in midspan with a probability of failure of $P_f \approx 1{,}2 \cdot 10^{-5}$ for the given conditions. Based on this reference probability of failure, the impacts of different variations in the model are studied.

4.2. Initial design considering full stiffness

The probability of failure can be calculated for the statically indeterminate system of the beam with the two rotational spring supports at its end. The input parameters in Table 1 are used and design according to Equation (12) is performed using the actual elastic stiffness of the joint. The actual connection resistance is only 70% of the one assumed in the design.

The resulting probability of failure are shown in Figure 2. The probability of failure for ductile connection failure is on a similar level as for the reference case, i.e. loads can be redistributed. However, for low ductility connections, the probability of failure is considerably higher than in the reference case. The specific value of the probability of failure depends on the stiffness of the connections, the lowest failure probability is achieved for very stiff connections. In all cases a ductility value of $D \geq 2$ should be used.

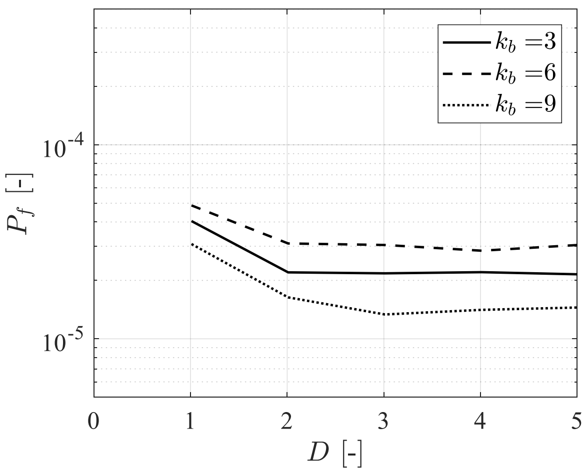

Figure 2: Probability of failure P_f. in dependency of connection ductility D and connection stiffness k_b for the initial design situation with consideration of the full elastic connection stiffness in the design.

4.3. Consideration of design stiffness

According to Eurocode 5 the connection stiffness in the ultimate limit state should be considered as 2/3 of the stiffness in the serviceability limit state. Hence, in the ULS design a lower stiffness than the actual elastic stiffness should be used. In the German National Annex to Eurocode 5, the stiffness is further reduced by the partial material factor γ_M.

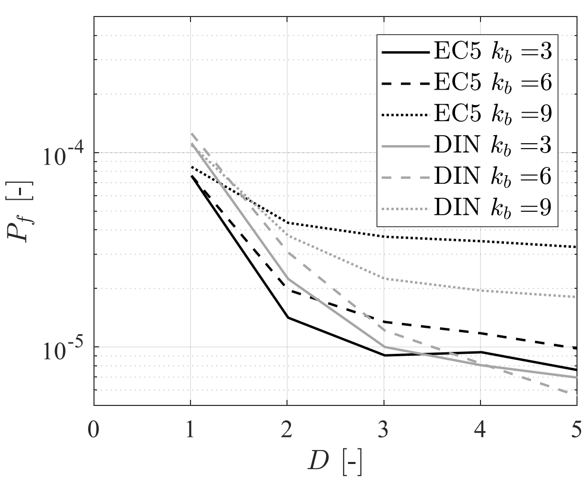

Figure 3: Impact of connection ductility D and connection stiffness k_b on the probability of failure P_f when considering a reduced design strength on the probability of failure in dependency of connection ductility.

The consideration of a different stiffness in design leads to the fact that the design failure mode can be misinterpreted, especially for cases of low connection ductility ($D \approx 1$) as shown in Figure 3. If sufficient ductility is available, a lower probability of failure than in the initial design case can be achieved. However, it should be considered that according to the DIN approach the design load-carrying capacity of the structure reduced by 6-7% compared to the Eurocode 5 design and up to 16% compared to the initial design considering full connection stiffness. For Eurocode 5 and DIN ductility values of $D \geq 3$ can be recommended.

4.4. Increased stiffness

The background of the regulations for connection stiffness in Eurocode 5 are vague and the same equation is given for a broad range of fasteners. Different studies found considerably different stiffness values. Especially for modern, axially loaded self-tapping screws, the specified values are not adequate. Another issue can be observed for dowelled connections with slotted in steel plates. For these connections Eurocode 5 gives the same stiffness as for timber-timber connections whereas the Swiss Standards SIA 265 proposes twice its stiffness.

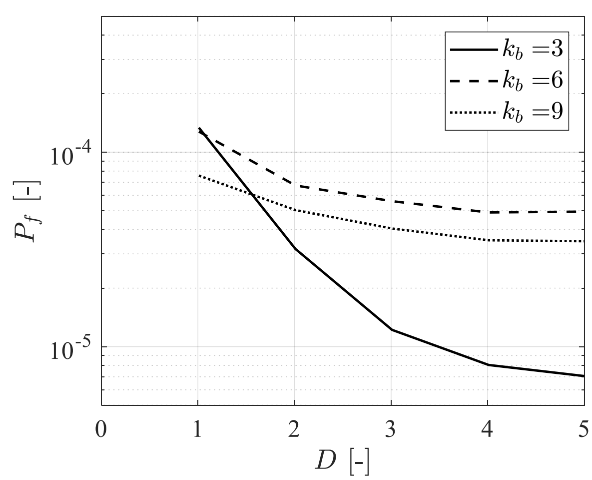

Figure 4: Impact of connection ductility D and connection stiffness k_b on the probability of failure P_f when considering an actual connection stiffness twice the value used in design.

In this study the connection design was performed with the basic (single) stiffness and in the reliability analysis the stiffness was doubled as it can be expected for slotted-in steel plate connections. The results are shown in Figure 4. As a result the loads acting on the connection are underestimated in the design. Ductility of the connection is required in order to compensate this error and redistribute the forces in the beam. Especially for connections with low stiffness ($k_b \approx 3$) it can be benefited from the high ductility through a reduction of the probability of failure. In these cases, loads are effectively redistributed from the connection to the beam.

4.5. Overstrength of connection

Specified steel qualities of fasteners are often minimum properties, that is why often considerable overstrength of the actual steel fasteners can be observed. This fact can be problematic if certain failure modes are envisioned, which for timber structures is often the case in seismic design where ductile connection failure is used to dissipate energy. Hence, other more brittle failure modes might occur and load-redistribution within the structure may be prevented.

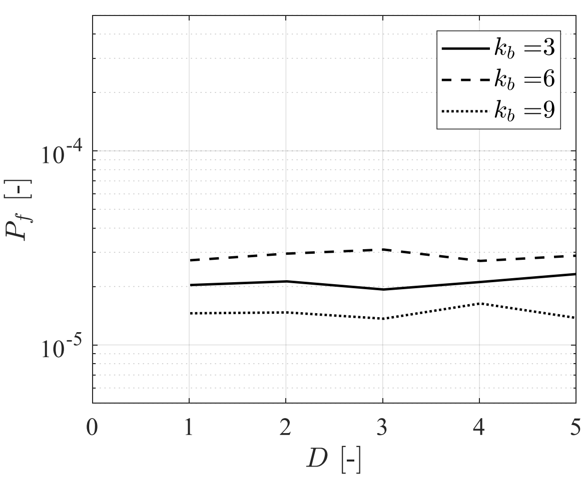

Figure 5: Impact of connection ductility D and connection stiffness k_b on the probability of failure P_f when considering a connection overstrength of 40%.

In this study it was assumed that the actual fastener steel strength is twice the declared value, leading to an increase of connection resistance of approximately 41%. It should be considered that in such a case a reduced connection ductility can be expected, which is not considered in this example. As shown in Figure 5, the overstrength of the connection induces the failure modes in the timber elements and, hence, the related failure probabilities are achieved. Since the connections remain largely in the elastic state it cannot be benefited from the ductility and load-redistribution.

4.6. Comparison

A comparison of the different configurations is shown in Figure 6. It can be observed that the probability of failure is relatively constant for different ductility values for the reference configuration and in case of connection overstrength. However, there are major changes of the failure probability in case of a higher actual stiffness than assumed in design (overstiffness) and according to the procedures given in Eurocode 5 or the German National Annex to Eurocode 5.

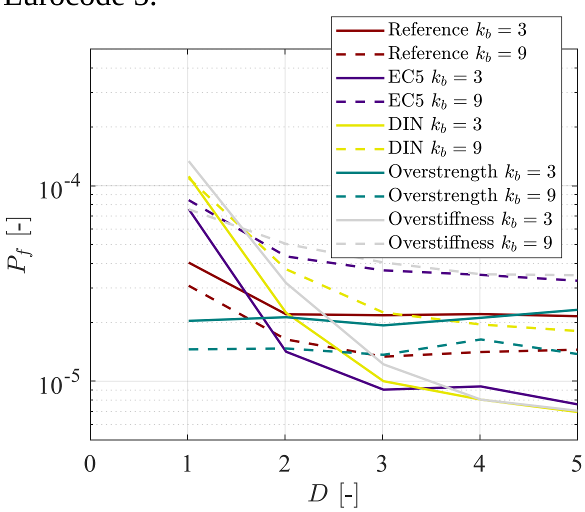

Figure 6: Impact of different configurations on the probability of failure P_f in dependency of connection ductility D.

5. SUMMARY AND CONCLUSIONS

The following main conclusion can be drawn:

- Connections in timber structures show a highly non-linear behavior that is currently not appropriately considered in the semiprobabilistic design concept using load and resistance factored design in Eurocode 5. Different definitions of stiffness values in the serviceability (elastic) and ultimate limit state are unclear.

- Ductility is needed in order to compensate for undesirable effects from excessive connection stiffness or overstrength. Especially for low stiffness connections it can be benefited in these cases from a considerably reduced failure probability. Overstrength of connections can stimulate other failure modes in the timber with the associated failure probabilities. Special emphasis should be paid to brittle failure modes in the surrounding of the joints such as net area, block-, plug-, or row-shear failure, that are often disregarded in design, in order to avoid detrimental effects.
- The consideration of the reduced connection stiffness in the ultimate limit state as defined in Eurocode 5 or the German National Annex to Eurocode 5 is conservative only for high ductility connections with approximately $D \geq 3$. Risks related to this definition are the misinterpretation of the governing failure mode in the design.

It can be concluded that the definitions of stiffness that is used in the ultimate limit state design in Eurocode 5 should be reviewed and that the non-linear connection behavior and particularly the connection ductility should be considered and specified more in detail.

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