*14th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP14*
*Dublin, Ireland, July 9-13, 2023*

# Influence of Spatial Variability on the Failure Probability to Initiation Time of Reinforced Concrete Structures under Chloride-induced Corrosion

Romain Clerc
*Associate Professor, Nantes Université, École Centrale Nantes, CNRS, GeM, UMR 6183, F-44000 Nantes, France*

Charbel-Pierre El-Soueidy
*Associate Professor, Nantes Université, École Centrale Nantes, CNRS, GeM, UMR 6183, F-44000 Nantes, France*

Franck Schoefs
*Professor, Nantes Université, École Centrale Nantes, CNRS, GeM, UMR 6183, F-44000 Nantes, France*

ABSTRACT: Reinforced concrete structures are subject to the penetration of chemical agents, such as chlorides, which lead to their degradation through the corrosion of their rebars. The reduction of uncertainties about the properties of concrete allows to better predict these phenomena and to optimize maintenance. In case of large structures such as harbor and marine ones, the characterization of the degradation mechanism's spatial variability is thus a major issue, supported by the development of structural health monitoring methods. This paper intends to contribute to the decision-making process for the implementation of such systems, through a sensitivity analysis of the corrosion initiation failure probability of reinforced concrete structures with respect to the spatial variability of their degradation parameters. The study-case is the reinforced concrete pier beam of the French APOS project (IFSTTAR & CEREMA 2016). Results show that the characterization of this spatial variability is superfluous to gain precision on the failure probability according to the Durability Limit State and that it is more efficient to focus on the assessment of the correlation between the average surface concentration and the average diffusivity of chlorides.

## INTRODUCTION

Today, 60% of the French coastal infrastructures have reached their lifespans (Bastidas-Arteaga and Schoefs 2015). In this context and because of their high exposure to the risk of corrosion-induced degradation, the definition of maintenance strategies for reinforced concrete (RC) structures has become a major concern.

Due to our imperfect knowledge of structures' environment and degradation processes, there are global, spatial, and temporal uncertainties and variabilities in the definition of their degradation parameters, whose neglect may imply a risk of premature and unanticipated failure. In the case of Limit States calculations of marine structures subjected to chloride-induced corrosion, many researchers have thus highlighted the increase in failure probability with the consideration of the spatial variability of the input parameters (Karimi 2002; Kenshel 2009; Li 2004; Stewart 2009). From these works, the recommendation is to take into account the spatial variability, i.e. to model the input parameters as Random Fields (RF) according to a geostatistical modeling (Chilès and Delfiner 2012).

In the generally used case of weak-stationary RF, such a field $Z$ is defined on a domain $D$ of $\mathbb{R}^{\mathrm{n}}$ and on a probability space $(\Omega, A, P)$ such that:

- $\forall \boldsymbol{x} \in D^n$ and $\forall (p,q) \in [1;n]^2$, $Z(\boldsymbol{x},\cdot)$ is a random variable of $\omega \in \Omega$ with joint probability density $f_Z\,(m,\boldsymbol{\Sigma})$ of mean $m$ and covariance matrix $\Sigma_{pq} = v \times \rho\,(\parallel x_p\ \ -$

$x_q \|, \theta)$, with $v$ its variance, $\rho$ its *correlation function* and $\theta$ its *fluctuation scale* ;

- $\forall x_j \in D, j \in [1;n]$, $Z(x_j,\cdot) = Z_j$ is a random variable with marginal probability density $f_{Z_j}\,(m, v)$ ;
- $\forall \omega \in \Omega\,,\ Z(\cdot,\omega) = Z^{\omega}$ is a realization of $Z$ called *trajectory*.

This formalism is illustrated on Figure 1. Like random variables, RFs can be discretized by different methods, the description of which is out of the scope of this article.

Under such a model, the failure probability is evaluated from the simulation of a large number of similar structures or structural components such as beams ($NoS$,index $i$), spatially discretized into sufficiently numerous elements to restore the spatial variability ($N_{el}$, index $j$, in 1D; Figure 2). Let $S_j^i(t)$ and $R_j^i(t)$ the applied solicitation and the resistance of element $j$ of a sample $i$ at instant $t$. Eq. (1) expresses the failure probability at the limit state $k$ and at time $t$, noted $p_f^{EL_k}(t)$.

$I(\cdot)$ is the indicator function and $Q_f^i$ is the ratio of failed elements at which the Limit State is reached. At the Durability Limit State (DLS) of chloride-induced corrosion initiation, $R_j^i(t) = C_{cr,j}^i(t)$ is a critical concentration of total chlorides, $S_j^i(t) = C_j^i(z = d, t)$ is the total chloride concentration at the rebar depth, and $Q_f^i \in [1/N_{el}\,; 1]$ (parallel modeling).

The problem of defining uncertainties then arises in order to evaluate $p_f$, since a major stake of the maintenance strategies is to maximize the added value of uncertainties' reduction.

Thus, even if a deterministic definition of the less sensitive parameters is sufficient, it is necessary to define, for all the sensitive parameters, the type and the parameters of the marginal law as well as the spatial correlation function $\rho(\theta)$ in case of spatial variability. These parameters can come either from *a priori* conservative definitions, at the risk of increasing the computational costs for a limited uncertainty reduction, or from measurements, at the risk of generating important inspection or SHM costs.

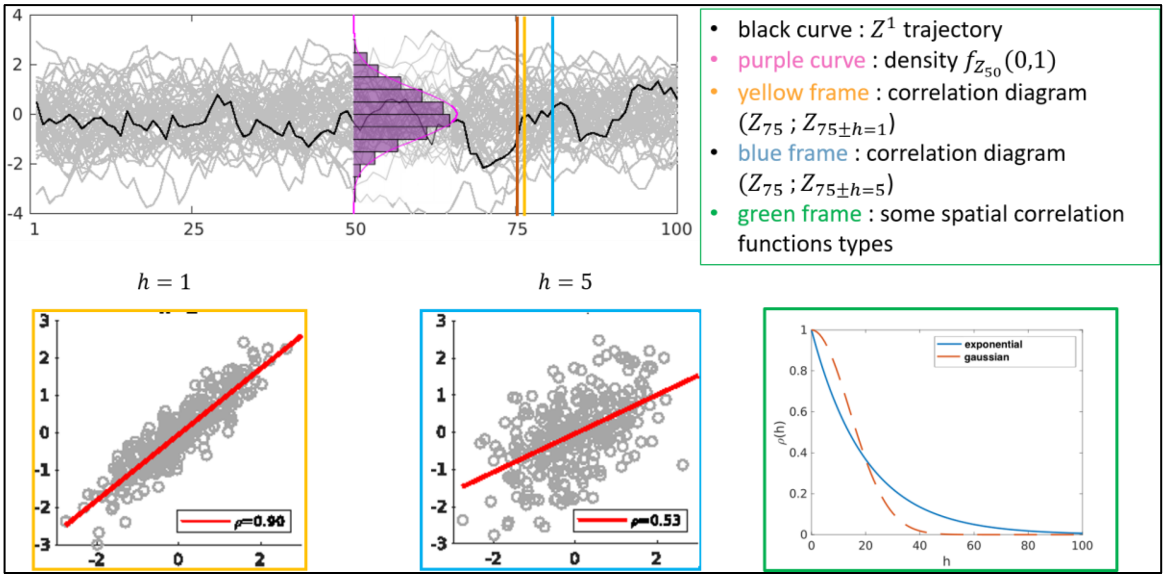

*Figure 1 - Illustration of the geostatistical formalism*

$$p_f^{EL_k}(t) = \frac{1}{NoS} \sum_{i=1}^{NoS} I\left( Q_f^i - \frac{1}{N_{el}} \sum_{j=1}^{N_{el}} I(R_j^i(t) - S_j^i(t) \leq 0) \leq 0 \right) \tag{1}$$

It is then necessary to have an idea of the sensitivity ratio $\frac{\text{marginal law parameters}}{\text{fluctuation scale}}$ of $p_f$ to make the optimal choice. Such information can come from expert opinions or from upstream Sensitivity Analyses (SA). Indeed, SA allows to rank input hyper-parameters of a model (noted HP) according to the influence of their respective variations on the global variations of the outputs. This classification is made from the Sensitivity Indices (SI).

In following work, we consider the case of a RC beam and the DSL of chloride-induced corrosion initiation. We seek to address the problem of identifying model parameters for which $\theta$ is particularly sensitive compared to all hyper-parameters. For this purpose, a quantitative and robust All At Time (AAT) SA is implemented (Pianosi et al. 2016). In order to combine structural and environmental representativeness with speed of execution, we place the study in the framework of a geostatistical analytical model of $p_f^{ELS}$ and we consider the study-case of the wharf beam of the APOS project (see 1.1).

The paper is organized as follows: (1) definition of the case study, (2) configuration of the Sensitivity Analysis, (3) results, (4) conclusions.

## 1. METHOD: STUDY-CASE DEFINITION

### 1.1. Structure

The study-case support is a RC CEMI 45N beam of W/C~0.4 with dimensions 9.34x0.86x0.4 m located at the end of the Montoir-de-Bretagne coal terminal, built from 1981 to 1983 and 7 km-distant from the Atlantic Ocean. This beam is in a tidal zone. Following the APOS project conducted in 2011 (Desbois et al. 2012), a rich database is available for it.

That allows the precise (geo)statistical description of many corrosion parameters, among which the concrete cover $d$ (366 measurement points distributed in 6 trajectories on both sides) as well as the average surface concentration $C_{sa}$ and the average diffusivity $D_a$[1] at 28 years on the EXTerior, exposed, and INTerior, sheltered faces (37 values from 37 profiles (Othmen et al. 2018)).

### 1.2. Chlorides penetration model

The semi-empirical model of chloride concentration $C(z,t)$ called false-erfc and presented by Eq. (2) is chosen to limit the computational time. It is adapted from the exact solution of the second Fick's law (Frederiksen et al. 2008) under the hypotheses of (i) unidirectional diffusion in a saturated semi-infinite medium, (ii) negligible interactions of chlorides with other ions in solution, and (iii)

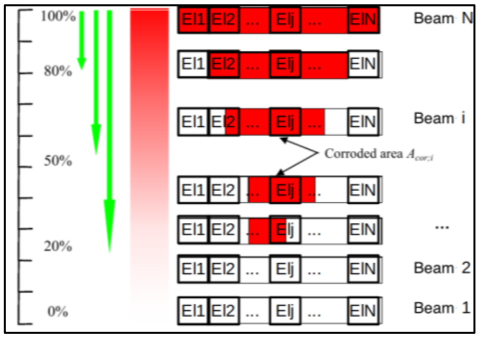

*Figure 2 - Structural sampling for the calculation of $p_f$ - adapted from (Li 2004 fig. 5.9)*

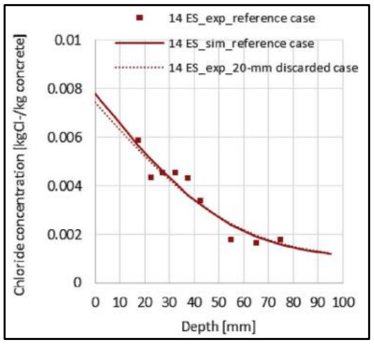

*Figure 3 - Estimation of $C_s$ and $D_{ar}$ on a trimmed total chloride profile (Othmen et al. 2018 fig. 7.b)*

---

[1] average values between a reference time $t_r$ reference time and the $t$ of calculation

temporal evolution of the chlorides diffusion parameters (decrease of $D$ and growth of $C_s$ due to hydration and progressive chlorides fixation (Tang et al. 2011; Val 2006)). This latter *a priori* influences $p_f^{ELS}$ (Frederiksen et al. 2008; Kenshel 2009).

$C_{sa}$ and $D_a$ are functions of aging factors $\beta$ and $\alpha$ which modulate reference values $C_{sar}$ and $D_{ar}$. These are in practice estimated at a time $t_r$ from chloride profiles measured on concrete cores or specimens (Figure 3).

Given these assumptions, this model allows to model $C(z,t)$ satisfactorily for existing massive maritime structures for which we only have access to data on the average diffusion parameters.

### *1.3. (Geo)statistical models of parameters*

In order to render uncertainties on the corrosion parameters, we model $\alpha$ and $\beta$ as random variables, (no spatial variability) and $C_{cr}$, $d$, $C_{sar}$ and $D_{ar}$ as RF. Considering a single reinforcement bar for simplicity, $p_f^{ELS}$ is then written according to Eq. (3). We note that the study of APOS data shows that $\ln(C_{sar})$ and $\ln(D_{ar})$ are linearly correlated (coefficient $\rho_{C_{sar},D_{ar}}$) and that the distributions of $\ln(C_{sar})$ and $\beta$ depend of the exposure to the humidification-drying cycles (EXT side exposed, INT side sheltered). This is not the case for $\ln(D_{ar})$ and $\alpha$.

## 2. METHOD: CONFIGURATION OF THE SENSITIVITY ANALYSIS

### *2.1. Main hypothesis*

The main assumption of the conducted SA is the fine description of the uncertainties on the RFs marginal law's parameters and the coarse description of both HP $\theta$ and $\rho_{C_{sar},D_{ar}}$. We thus consider in the case favorable to the sensitivity of the unknown HP, knowing that the latter are expensive to characterize. Concretely, we consider the following distribution models:

- **marginal distribution parameters:** distribution models of their estimators (Maximum Likelihood), evaluated from the APOS data. Only the estimation error is taken into account and the measurement protocol errors are not included in the SA;
- **distribution models of $\boldsymbol{\theta}$ and $\boldsymbol{\rho_{C_{sar},D_{ar}}}$ :** uniform distributions between the largest possible bounds.

### *2.2. Sampling of hyper-parameters*

The 12 HP of the SA are the set of parameters of the marginal laws of the previously defined RFs as well as the ratios $r_\theta = \theta/L$. This choice allows to generalize the results obtained to any beam dimension. Their estimation (not detailed here) required several developments. We underline in particular:

$$C(z,t) = \underbrace{C_{sar}\left(\frac{t}{t_r}\right)^{\beta}}_{C_{sa}(t,t_r)} \times \mathrm{erfc}\left(\frac{z}{2\sqrt{(t-t_r)\,\underbrace{D_{ar}\left(\frac{t_r}{t}\right)^{\alpha}}_{D_a(t,t_r)}}}\right) \tag{2}$$

$$p_f^{ELS}(t) = \frac{1}{NoS}\sum_{i=1}^{NoS} I\left\{Q_f - \frac{1}{N_{el}}\sum_{j=1}^{N_{el}} I\left[C_{cr,j}^{i}(t) - C_{sar,j}^{i}\left(\frac{t}{t_r}\right)^{\beta^i} \times \mathrm{erfc}\left(\frac{d_j^i}{2\sqrt{(t-t_{ex})D_{ar,j}^{i}\left(\frac{t_r}{t}\right)^{\alpha^i}}}\right) \leq 0\right] \leq 0\right\} \tag{3}$$

- the consideration of the definition domain of the $r_\theta$ ratio: the lower bound is imposed by the dimensions of a volume within which the properties are supposed to be homogeneous; the upper bound is imposed by the imperative of non-regionalization of the HPs, necessary for the validity of the SA and concomitant to the ergodicity properties of the trajectories[2].
- the implementation of an extension of the circulant matrix method of RF simulation to take into account the spatial correlation between the diffusion parameters while respecting the mathematical constraints imposed by the need for positivity of their cross-correlation function (Helgason et al. 2011; Pichot 2018).
- the assessment of the distributions of diffusion parameters' aging factors from the joint APOS data ($t_r = 28$ years) and two other *in situ* research programs, for which $C_{sa}$ and $D_a$ were routinely measured from early age to 10 years of exposure for similar concrete and exposure conditions (Skjølsvold 2011; Tang 2003).

### *2.3. Variants, Sensitivity Analysis methods and convergence study*

Additionally, we investigate the combined influence on the HP sensitivity repartition of (i) the exposure face (INT or EXT), (ii) the CoV value of $C_{sar}$ (variable according to the exposure), (iii) the consideration of the correlation of $C_{sar}$ and $D_{ar}$ (uncorrelated or correlated RF, respectively noted URF and XRF), (iv) the consideration of the aging (ageing or no ageing), and (v) the model of $\rho(\theta)$ (exponential or Gaussian). This latter option is only available for URF, whereas $\rho(\theta)$ is exponential for XRF. This leads us to consider 24 SA variants: $2^4$ for URF and $2^3$ for XRF.

Moreover, 7 calculation times (from 10 to 70 years), 6 deterministic values of $Q_f$ (from 5 to 30%) and 2 AAT methods are considered (standard rank regression coefficient indices - SRRC - and Borgonovo indices, respectively noted $\hat{\gamma}$ and $\hat{\delta}$ (Pianosi et al. 2016)). This allows to respectively cover the common project service life of marine structures, cover the set of DLS threshold values encountered in the literature and industry (between 5 and 25%, from an expert knowledge), and verify and ensure the robustness of the SA results. Thus, 2016 SA of $p_f^{ELS}$ are carried out on a desktop computer for a calculation time of 35 hours ($24 \times 7 \times 6 \times 2$ variants). Their robustness is validated by verifying the convergence and consistency of the SIs computed with both of the methods (Figure 4).

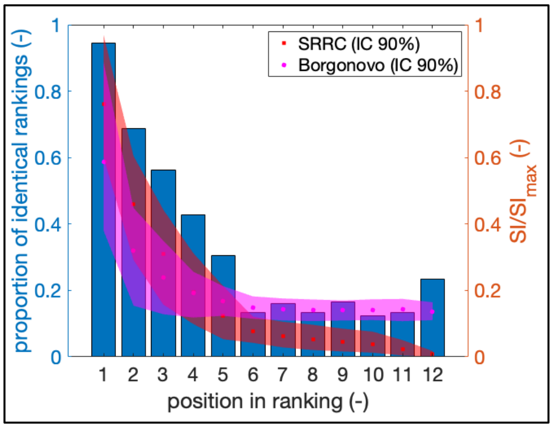

*Figure 4 - Evolution of the proportion of HPs with identical SI rankings for the SRRC and Borgonovo methods, as a function of the rank*

## 3. RESULTS

In order to answer the problem of identifying the parameters whose fluctuation scales (grouped in the vector $X_{\theta,i=1..M_\theta}$) are particularly sensitive compared to the marginal laws' parameters (grouped in the vector $X_{p,i=1..M_p}$), the results are expressed in the form of sensitivity index ratios $R_{\hat{\gamma}}$ and $R_{\delta}$ defined on the model of Eq. (4). $M_p$ and $M_\theta$ are the total numbers of respectively fluctuation scales, and marginal law's parameters.

---

[2] regionalization: distinct trends in the evolution of the probability of failure at a fixed value of a hyper-parameter on its definition domain

$$R_{\hat{\gamma}}\left(Q_f,t\right)=\frac{\max\limits_{\mathrm{i=1..M_\theta}}\left(\left|\hat{\gamma}\left(X_{\theta,i},Q_f,t\right)\right|\right)}{\max\limits_{\mathrm{i=1..M_p}}\left(\left|\hat{\gamma}\left(X_{p,i},Q_f,t\right)\right|\right)} \qquad (4)$$

We only consider the SA configurations (SA variant + calculation time + Limit State threshold value) for which the standard deviation and the mean of $p_f^{ELS}$ are respectively greater than 1% and less than 50%, and for which $R > 50\%$. We qualify these configurations as *of interest*. Indeed, this filtering process guarantees (i) the robustness of SA, (ii) spread out values of $p_f^{ELS}$ (uncertainty on its evaluation), and (iii) the sensitivity of at least one $r_\theta$.

The results obtained are presented on Table 1. They show that in the case of independent diffusion parameters, the most sensitive HP is the mean of $C_{sar}$ ($\mu_{C_{sar}}$) or the mean and standard deviation of $D_{ar}$ ($\mu_{D_{ar}}, \sigma_{D_{ar}}$). In the case of correlation of the diffusion parameters, $\rho_{C_{sar},D_{ar}}$ is the most sensitive HP for the configurations *of interest* and $p_f^{ELS}$ increases. Finally, the only sensitive $r_\theta$ for the configurations *of interest* is that of $D_{ar}$.

Considering the current means of measurement and in a case of fine investigation, representative of this study-case, these results indicate that the reduction of uncertainties on the DLS failure probability of chloride-induced corrosion initiation passes initially by the characterization of the correlation of the diffusion parameters. The errors are in a second time primarily due to the too few measurements of the diffusion parameters as well as to their limited precision. The only geostatistical characterization of spatial variability that finally makes sense for structure managers is that of $D_{ar}$. Eventually, although the error of the measurement protocols is not integrated, its consideration does not affect the $r_\theta$ values according to the modeling choices we made. It therefore does not modify the rankings and accentuates the sensitivity of the other HP.

## 4. CONCLUSIONS

This work studies the sensitivity of the Durability Limit State (DLS) failure probability of chloride-induced corrosion initiation for a reinforced concrete structure in a marine environment.

It is based on the implementation of a global all-at-time sensitivity analysis of the failure

*Table 1 - proportion of Sensitivity Analyses corresponding to a given ranking for each sensitive Hyper-Parameter according to the configuration sets of interest*

| | variants with independent $C_{sar}$ and $D_{ar}$ | | | | |
|---|---|---|---|---|---|
| HP\ranking | 1 | 2 | 3 | 4 | other |
| $r_{\theta,D_{ar}}$ | 0% | 23% | 36% | 41% | 0% |
| $\mu_{C_{sar}}$ | 0% | 18% | 27% | 27% | 27% |
| $\mu_{D_{ar}}$ | 82% | 18% | 0% | 0% | 0% |
| $\sigma_{D_{ar}}$ | 18% | 41% | 36% | 5% | 0% |
| other | 0% | 0% | 0% | 27% | 73% |
| | variants with correlated $C_{sar}$ and $D_{ar}$ correlated | | | | |
| HP\ranking | 1 | 2 | 3 | 4 | other |
| $r_{\theta,D_{ar}}$ | 0% | 0% | 13% | 31% | 56% |
| $\mu_{C_{sar}}$ | 0% | 0% | 31% | 31% | 38% |
| $\mu_{D_{ar}}$ | 0% | 88% | 6% | 6% | 0% |
| $\sigma_{D_{ar}}$ | 0% | 13% | 50% | 31% | 6% |
| $\rho_{C_{sar},D_{ar}}$ | 100% | 0% | 0% | 0% | 0% |
| other | 0% | 0% | 0% | 0% | 100% |

probability computed from false-erfc model of chlorides penetration, with correlated mean diffusion parameters, function of time and exposure, within the framework of a geostatistical modeling. The objective is to characterize the relative sensitivity of their fluctuation scales with respect to their statistical moments in order to decide on the need for their fine geostatistical definition. This approach is unprecedented: the only study dealing with this issue employs a qualitative One-At-Time SA methodology, serial DLS modeling, and parameter definition ranges far from the reality of uncertainties on a given structure (Kenshel 2009).

The results obtained indicate that the characterization of the spatial variability of the degradation parameters is globally superfluous to characterize the failure probability, and it emerges that the characterization of the correlation between the average surface concentration and the average diffusivity of chlorides is necessary in order not to underestimate it.

## 5. REFERENCES

Bastidas-Arteaga, E., and F. Schoefs. 2015. “Sustainable maintenance and repair of RC coastal structures.” *Proceedings of the ICE-Maritime Engineering*, 168 (4): 162–173.

Chilès, J.-P., and P. Delfiner. 2012. *Geostatistics: modeling spatial uncertainty*. Wiley series in probability and statistics. Hoboken, NJ: Wiley.

Desbois, T., S. Pasquiet, R. Queguiner, and B. Thauvin. 2012. *Poste charbonnnier Grand Port Maritime de Nantes Saint-Nazaire Montoir de Bretagne : Synthèse des mesures réalisées sur site avant réparation de l’ouvrage par protection cathodique*. CETE de l’Ouest.

Frederiksen, J. M., L. Mejlbro, and L.-O. Nilsson. 2008. *Fick’s 2nd law - Complete solutions for chloride ingress into concrete– with focus on time dependent diffusivity and boundary condition*. 112. Lund, Sweden: Lund Institute of Technology.

Helgason, H., V. Pipiras, and P. Abry. 2011. “Fast and exact synthesis of stationary multivariate Gaussian time series using circulant embedding.” *Signal Processing*, 91 (5): 1123–1133. https://doi.org/10.1016/j.sigpro.2010.10.014.

IFSTTAR & CEREMA. 2016. *Opération de recherche APOS (Auscultation Pour des Ouvrages Sûrs)*. 2. IFFSTAR.

Karimi, A. R. 2002. “Probabilistic assessment of deterioration and strength of concrete bridge beams and slabs.” London: Imperial College London.

Kenshel, O. 2009. “Influence on spatial variability on whole life management of reinforced concrete bridges.” Dublin, Ireland: Trinity College.

Li, Y. 2004. “Effect of spatial variability on maintenance and repair decisions for concrete structures.” Delft, Netherlands: Delft University Press.

Othmen, I., S. Bonnet, and F. Schoefs. 2018. “Statistical investigation of different analysis methods for chloride profiles within a real structure in a marine environment.” *Ocean Engineering*, 157: 96–107. https://doi.org/10.1016/j.oceaneng.2018.03.040

Pianosi, F., K. Beven, J. Freer, J. W. Hall, J. Rougier, D. B. Stephenson, and T. Wagener. 2016. “Sensitivity analysis of environmental models: A systematic review with practical workflow.” *Environmental Modelling & Software*, 79: 214–232. https://doi.org/10.1016/j.envsoft.2016.02.008.

Pichot, G. 2018. *Algorithms for Gaussian random field generation*. Technical Report. INRIA Paris.

Skjølsvold, O. 2011. *Kloriddiffusjon i betong. Vurdering av aldringseffekten ved felteksponering COIN P4 Operational service life design SP 4.1 Service life modelling*. COIN P4 Operational service life design - SP 4.1 Service life modelling, 51. COIN Projet Report. Oslo, Norway: SINTEF.

Stewart, M. G. 2009. “Mechanical behaviour of pitting corrosion of flexural and shear reinforcement and its effect on structural reliability of corroding RC beams.” *Structural Safety*, 31 (1): 19–30. https://doi.org/10.1016/j.strusafe.2007.12.001.

Tang, L. 2003. *Chloride ingress in concrete exposed to marine environment - field data up to 10 years exposure*. Natural Sciences, 61. SP Bygg och Mekanik.

Tang, L., L.-O. Nilsson, and P. A. M. Basheer. 2011. *Resistance of Concrete to Chloride Ingress: Testing and modelling*. CRC Press.

Val, D. 2006. “Service-life performance of RC structures made with supplementary cementitious materials in chloride-contaminated environments.” *ConcreteLife’06 - International RILEM-JCI Seminar on Concrete Durability and Service Life Planning: Curing, Crack Control, Performance in Harsh Environments*, 363–373. Dead Sea, Israel: RILEM Publications SARL.