# An Extended Stratified Sampling Approach for Probabilistic Performance Assessment of Structures under Wind and Seismic Hazards

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ABSTRACT: Risk-based decision-making for the assessment and design of the built environment against natural hazards is fast gaining recognition not just for critical structures but also for engineered structures in general. Robust frameworks formalized within the setting of performance-based engineering typically involve high-fidelity numerical computations and need to account for the uncertainties in responses. However, the computational demand associated with uncertainty propagation can be large which is further exacerbated when dealing with expensive numerical model evaluations creating a need for efficient simulation schemes. In this study, a two-phase sampling approach is presented as an extension of stratified sampling, for the simultaneous estimation of multiple failure probabilities and Bayesian updating of fragility functions. Unlike traditional stratified sampling, any stratification variable can be selected, not necessarily belonging to the input set of random variables, with the first phase of sampling efficiently generating samples in each stratum through subset simulation when Monte Carlo simulation is considered infeasible. Based on user-defined targets on accuracy for the failure probabilities, an optimization routine is introduced to minimize the number of model evaluations in the second phase. The strata-wise sampling approach directly allows for the Bayesian updating of fragility functions which can be attractive for incorporating any prior beliefs and knowledge on uncertainty in the resulting fragility functions. A case study is presented to illustrate both the efficient estimation of multiple small failure probabilities and the Bayesian approach to constructing fragility functions with a limited number of analyses.

## 1. INTRODUCTION

The rigorous consideration of uncertainty and the explicit evaluation of performance at different load levels is the hallmark of performance-based engineering (PBE) for the rational design and appraisal of structures subject to natural hazards. By qualifying the performance of engineered systems through probabilistic system-level metrics that can be understood by a wide range of technical and non-technical stakeholders, PBE frameworks enable risk-based decision-making. Such frameworks include approaches to characterize the natural hazard, the loads on the structural and the non-structural components, the resulting system response, and subsequent damage and losses (e.g. Yang et al., 2009; Chuang and Spence, 2017; Arunachalam and Spence, 2022). This is often realized through a sequence of high-fidelity numerical models. The need for stochastic simulation schemes arises from the need to propagate uncertainty through the numerical models in order to quantify the uncertainty in the outputs. The associated problems can take the form of an estimation of responses associated with a given annual exceedance probability, reliabilities associated with a given set of limit states, sensitivity analyses for highly nonlinear responses, construction of fragility functions, and finally the study of highly-nonlinear behavior or collapse under uncertainty (e.g., the relative frequency of different collapse modes or the response of energy-dissipative devices under extreme stochastic loads). Whereas these are closely related problems associated with the stochastic simulation of rare events, this work is presented within the context of reliability analysis and efficient fragility construction.

The objective of this paper is to develop an efficient two-phase sampling approach as an extension of stratified sampling, for the simultaneous estimation of multiple failure probabilities and Bayesian updating of fragility functions. The literature is replete with a wide spectrum of reliability estimation methods ranging from classic techniques, such as first/second-order reliability methods; methods that fall under the umbrella of Monte Carlo (MC) methods, including importance sampling (Schueller et al., 2004), control variates, stratified sampling, subset simulation (SuS) (Au and Beck, 2001); and finally coupled approaches, e.g., adaptive strategies for coupling sampling-based methods with metamodeling (Cadini et al., 2014). However, for the problems of interest to this work, namely, problems involving high-dimensional input spaces (in the order of several thousand), estimation of multiple small failure probabilities (e.g., $\leq 10^{-4}$), and highly nonlinear and expensive performance functions (i.e., limit state functions (LSFs)), it is either ineffective or infeasible to apply many of the above techniques. The proposed scheme has the same spirit as double sampling (Cochran, 2007), but exploits the merits of SuS and stratified sampling, and is shown to be practical and efficient for the performance assessment of structures under wind and seismic hazards. Additionally, it is illustrated that the simulation scheme facilitates the integration of prior information to update fragility functions using a Bayesian approach when the hazard intensity measure (IM) is chosen as the stratification variable. This is extremely beneficial when explicit hazard modeling is involved wherein it is not straightforward to conduct nonlinear dynamic analyses at desired IM levels since the IM is not a basic random variable. Moreover, the fragility functions are estimated efficiently with a small set of analyses and the statistical uncertainty can be expressed by samples drawn from the posterior distribution.

## 2. EXTENDED STRATIFIED SAMPLING FOR PBE

### 2.1. *Overview*

Stratified-sampling-based MC simulation is attractive to tackle the estimation of multiple failure probabilities given a limited computational budget. In addition, the procedure can quantify the uncertainty in the estimates using coefficients of variation (Arunachalam and Spence, 2023a). However, it suffers from limited flexibility in the choice of the stratification variable, $\chi$. Stratification is required to be performed directly in the input probability space meaning that the strata need to be defined by specifying bounds for the components of $\mathbf{X}$, the vector of basic random variables, which can limit the applicability of the approach. The proposed scheme overcomes this limitation by the use of SuS to generate strata-wise samples, by which the possible choice of $\chi$ is expanded. This enables the consideration of both conventional IMs (e.g., wind speed and spectral acceleration) as well as less conventional IMs (e.g., peak elastic base moment or shear force) that are often an output of a numerical model (e.g., hazard model or elastic structural model) with an unknown probability distribution. Obviously, if $\chi$, a potentially complex function of two or more random variables in $\mathbf{X}$, is not expensive to evaluate, then a MC simulation can be used to generate many realizations of $\chi$ such that adequate samples (i.e., a set of realizations of $\mathbf{X}$) are available in the strata corresponding to large $\chi$ values. Although such samples will be independent, unlike the samples generated by Markov chains, SuS ensures that Phase-I sampling, i.e., generation of strata-wise samples, does not, in itself, represent a substantial computational burden. It should be mentioned that the adaptive generation of samples by SuS implies adaptively defined strata bounds and therefore results in uncertainty in the strata probabilities. That is, at the $k$th simulation level in the SuS procedure, if $\chi_k$ is defined as the $(1-p)$th quantile of the conditional samples generated, then it is important to recognize that $p^{k-1}(1-p)$ is only a sample estimate for $P(\mathbb{S}_k) = P(\mathbf{X} : \chi \in (\chi_{k-1}, \chi_k])$ for $1 < k < m$, with special considerations at the boundaries (i.e., $k = 1, m$).

Given $N$ samples in each stratum as a result of SuS with $\chi$ as the driving variable, Phase-II sampling concerns the selection of $n_i \ll N$ samples from the $i$th stratum, $\mathbb{S}_i$, so that the conditional failure probabilities, $P_f^{(i)} = P(\text{failure} \mid \chi \in \mathbb{S}_i)$ can be estimated. Subsequently, by the application of the total probability theorem, the estimates of $P_f^{(i)}$ and the stratum probability, $P(\mathbb{S}_i)$, are combined to provide an estimate of the overall failure probability, $P_f$, as shown below:

$$\tilde{P}_f = \sum_{i=1}^{m} \tilde{P}_f^{(i)} \tilde{P}(\mathbb{S}_i) \tag{1}$$

where $\sim$ denotes estimated values and $m$ the number of strata. The estimate, $\tilde{P}_f$, can be shown to be asymptotically unbiased and convergent to $P_f$ as $n_i \to \infty$, the derivations of which are not presented here due to space constraints. The key requirement of efficiently performing Phase-II sampling is realized using an optimization routine to minimize $n = \sum_{i=1}^{m} n_i$ subject to constraints ensuring a desired accuracy is met.

### 2.2. *Estimation of Multiple Failure Probabilities*

The proposed simulation scheme is capable of estimating failure probabilities associated with multiple LSFs, $\mathbb{G}_l(\mathbf{X})$ for $l = 1, 2, \ldots, N_L$, with a single run of the entire procedure, a meritorious feature not easily achieved using other popular variance reduction methods, e.g., SuS. The capability to tackle multiple limit states stems from how the stratification is carried out in the space of the random variables and is therefore independent of the limit states. That is, for every limit state, the strata-wise failure probabilities are evaluated using the same set of strata-wise samples followed by the application of Eq. (1). The coefficient of variation (COV) associated with failure probability estimate for the $l$th LSF denoted by $\tilde{P}_{f,l}$ is given by (Arunachalam and Spence, 2023b):

$$\kappa_l \approx \frac{\sqrt{\sum_{i=1}^{m} \tilde{\vartheta}_{i,l}^2 \left( \tilde{\vartheta}_{\mathbb{S}_{ii}}^2 + \tilde{P}^2(\mathbb{S}_i) \right) + \omega^2}}{\tilde{P}_{f,l}} \tag{2}$$

where $\tilde{\vartheta}_{i,l}^2$ is the estimate of the variance, $\mathbb{V}(\tilde{P}_{f,l}^{(i)})$ and $\tilde{P}_{f,l}^{(i)}$ denotes $\tilde{P}_f^{(i)}$ for $\mathbb{G}_l$; $\tilde{\vartheta}_{\mathbb{S}_{ij}}^2$ is the estimate of the covariance, $\mathbb{CV}(\tilde{P}(\mathbb{S}_i), \tilde{P}(\mathbb{S}_j))$; $\omega^2$ denotes the component of the variance that quantifies the quality of the Phase-I sampling and is given by:

$$\omega^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{P}_{f,l}^{(i)} \tilde{P}_{f,l}^{(j)} \tilde{\vartheta}_{\mathbb{S}_{ij}}^2 \tag{3}$$

Essentially, $\kappa_l$ accounts for the sample correlations induced by the Markov chains (within the SuS procedure) and the uncertainty in the estimated strata probabilities. The estimate $\tilde{\vartheta}_{i,l}^2$ depends on the $i$th stratum samples chosen to evaluate $\mathbb{G}_l$ and the variability of $\mathbb{G}_l$ within the stratum. In summary, the $\tilde{\vartheta}^2$ terms can be obtained from the simulated Markov chains and by evaluating the intra-chain correlation between the states of the stationary Markov chains with respect to the occurrence of failure (i.e., $\mathbb{G}_l < 0$). A detailed discussion including the derivations and the expressions for evaluating the $\tilde{\vartheta}^2$ terms can be found in Arunachalam and Spence (2023b).

The sample distribution $n_i/n$ affects $\kappa_l$ and the optimal distribution that minimizes $\kappa_l$ varies for different limit states. To this end, the following nonlinear convex optimization problem is formulated to account for user-defined constraints on the target accuracy:

$$\text{minimize} \quad n = \sum_{i=1}^{m} n_i$$

subject to:

$$\kappa_l^2(n_1, n_2, \ldots, n_m) \le \Delta_l^2 \quad l = 1, 2, \ldots, N_L \tag{4}$$

where $\Delta_l$ is the user-defined COV threshold for $\mathbb{G}_l$. Clearly, the optimal solution represents the sample sizes to be considered in Phase-II sampling for the efficient estimation of multiple failure probabilities. However, to perform the above optimization, one requires the knowledge of $\kappa_l, \forall l$ which can be estimated using a test set of samples, e.g, $n_p$ samples in each stratum, leading to an approximately optimal sample distribution which is acceptable for practical problems.

### 2.3. *Bayesian Updating of Fragility Functions*

#### 2.3.1. *Preamble*

The fragility function specifies the probability of attaining a damage state (e.g., earthquake-induced collapse) or violating a performance objective (e.g., occupant comfort under wind loads) as a function of the hazard IM. Fragility functions are critical components of risk assessment and life cycle cost analyses to help analyze system vulnerabilities and develop cost-effective management solutions (Darestani and Shafieezadeh, 2019). This section describes the applicability of a Bayesian approach for fragility estimation within the framework of the proposed extended stratified sampling scheme. The essential motivation for a Bayesian approach is the integration of prior information, and the ability to describe uncertainty in the model parameters using a distribution rather than point estimates. Through a full probabilistic description of the parameters of the fragility function, samples can be drawn from the posterior distribution to infer confidence in the fragility estimation as well as to propagate uncertainty to other functions (e.g., collapse risk by combining the collapse fragility with a site-specific hazard curve).

In the present context, the fragility, $\theta_{k,l} = P(\mathbb{G}_l < 0 \mid \chi = x_k)$ is desired to be estimated corresponding to a target $\mathbb{G}_l$ and the IM given by $\chi$. The data, $\mathfrak{D}$, comprises outcomes from Bernoulli experiments (i.e., categorical data with a binary outcome) conducted at $n_B$ different values of $\chi$. Among many representations of the fragility functions, including theoretical forms and metamodels (e.g., kriging (Bhat et al., 2018)), the most common are the logit regression model (Ghosh et al., 2013; Reed et al., 2016) and the lognormal cumulative distribution function (CDF) (Ellingwood et al., 2004). Since both are two-parameter models, let the parameters be denoted by $(\alpha, \beta)$. For example, the lognormal CDF expresses the relation between $\theta_{k,l}$ and the model parameters at a given value of $\chi = x_k$ as:

$$\theta_{k,l} = \Phi\left(\frac{\ln(x_k/\alpha)}{\beta}\right) \tag{5}$$

where $\Phi(\cdot)$ is the standard normal CDF, $\alpha$ represents the median of the fragility function (the IM level associated with 50% failure probability) and $\beta$ represents the dispersion value. Using Bayes rule, the posterior distribution, $f(\alpha, \beta \mid \mathfrak{D})$, is defined as follows:

$$f(\alpha, \beta \mid \mathfrak{D}) \propto P(\mathfrak{D} \mid \alpha, \beta) f(\alpha, \beta) \tag{6}$$

where $L = P(\mathfrak{D} \mid \alpha, \beta)$ is the likelihood function and $P(\alpha, \beta)$ is the prior. The likelihood, regarded as a function of $\alpha$ and $\beta$ for a fixed $\mathfrak{D}$, is not a probability distribution but the conditional probability of observing the data given the parameters (Gelman et al., 2013). For the estimation of fragility with limited data, it is proposed to use a preliminary study, henceforth referred to as Step-I (to differentiate from the preliminary study of the previous section), e.g., $n_{B,\mathrm{I}}$ samples in each stratum, to obtain first-level estimates of $P_{f,l}^{(i)}$, denoted by $\tilde{P}_{f,l,\mathrm{I}}^{(i)}$. The next set of samples (i.e., Step-II) to refine the fragility estimate will be selected according to $n_{i,B,\mathrm{II}} \propto \sqrt{\tilde{P}_{f,l,\mathrm{I}}^{(i)}(1 - \tilde{P}_{f,l,\mathrm{I}}^{(i)})}$. Clearly, the adaptive sampling procedure identifies the strata where prediction uncertainty is high (through Step-I) followed by aggressive sampling at such locations, such that the total number of limit state evaluations will be $n_B = m n_{B,\mathrm{I}} + \sum_{i=1}^{m} n_{i,B,\mathrm{II}}$.

#### 2.3.2. *Prior selection*

The prior information corresponding to $\mathbb{G}_l$ can be sought from a variety of sources including engineering judgment based on previous analyses/experience, simplified analysis procedures (e.g., using low-fidelity models, pushover analysis for seismic fragilities), publicly available datasets provided in the literature (e.g., hurricane-induced power outage data (Reed et al., 2016)), and data on the performance of electrical substation equipment in past earthquakes (Straub and Der Kiureghian, 2008). The alternative is to use non-informative or weakly informative priors. It is imperative to point out that while conjugacy provides significant mathematical convenience in obtaining analytical posteriors, it is not necessary as drawing samples from an unnormalized posterior has become computationally cheap in the last decade. Gokkaya et al. (2015) illustrated a Bayesian approach to estimate the seismic collapse fragility function, albeit performing inference on $\theta_{k,l}$ rather than the fragility model parameters so that beta-binomial conjugacy could be exploited. However, the approach reduced the posterior fragility model parameters to point estimates.

### *2.3.3. Likelihood model*

The data, $\mathfrak{D}$, consists of binary outcomes, $\{y_k\}$, corresponding to $\{\chi = x_k\}$ for $k = 1, \dots, n_B$. Since $P(\mathbb{1}_{\mathbb{G}_l<0}(x_k) = y_k \mid \theta_{k,l})$ is given by the Bernoulli likelihood function (where $\mathbb{1}(\cdot)$ is an indicator function), and if the lognormal link function of Eq. (5) is adopted, then the likelihood for the entire data set can be written as:

$$L = \prod_{k=1}^{n_B} \Phi\left(\frac{\ln(x_k/\alpha)}{\beta}\right)^{y_k} \left[1 - \Phi\left(\frac{\ln(x_k/\alpha)}{\beta}\right)\right]^{1-y_k} \tag{7}$$

The above equation assumes the independence of the outcomes at the different $\chi$ values which is true when MC-based Phase-I sampling is carried out. However, it is not strictly valid when SuS-based Phase-I sampling is adopted due to the correlation between Markov chain samples. This issue could potentially be alleviated by choosing samples that are not consecutive but well-spaced, although this aspect is not investigated in the present study. It is noteworthy that the above likelihood is unaffected by the uncertainty in the estimation of the strata probabilities.

### *2.3.4. Sampling from the posterior distribution*

Posterior sampling entails drawing samples of $\alpha$ and $\beta$ from the unnormalized posterior density given by Eq. (6). This can be achieved numerically using the Metropolis-Hastings algorithm (a family of Markov chain simulation methods), rejection sampling, and importance sampling (Gelman et al., 2013) when conjugacy is inapplicable or simple grid-based approaches are infeasible. With access to the posterior samples of the model parameters, not only any posterior statistic of the parameters can be obtained, but also samples of $g(\alpha, \beta)$ can be simulated for any function $g(\cdot)$ to describe its uncertainty, unlike the substitution of point estimates (e.g., the maximum likelihood estimates of the model parameters). From Eq. (7), it is easy to note that, for reasonably large $n_B$ (e.g., $\geq 30$), the likelihood values can be extremely small. As such, to avoid computational underflows, one should work with the logarithm of $L$ (Gelman et al., 2013).

## 3. DEMONSTRATION PROBLEM

### *3.1. Problem Setup*

The objective of this demonstration example is two-fold: (i) To highlight the capability to simultaneously estimate three failure probabilities associated with the highly nonlinear response of an archetype 45-story reinforced concrete building subjected to extreme hurricane events; (ii) To illustrate the Bayesian fragility estimation procedure for a chosen LSF. The structure is assumed to be located in New York City, and the hazard modeling entails the simulation of site-specific hurricane tracks from which time-varying wind speed and direction outputs are obtained (Ouyang and Spence, 2021). The annual occurrence rate of the hurricanes is $\lambda_H = 0.67$. A non-stationary/-Gaussian wind load model calibrated to building-specific wind tunnel data is adopted to convert the wind speed and direction time histories to aerodynamic loads (Ouyang and Spence, 2021). A stress-resultant plasticity-based structural model is adopted to perform nonlinear dynamic analyses using the adaptive fast nonlinear analysis (AFNA) technique (Li et al., 2021). A modal damping ratio of 2% was considered. The basic random variables include the parameters of the hurricane track generation model and the filling rate model (Vickery and Twisdale, 1995) besides the high-dimensional white noise sequence within the stochastic wind load model (Suksuwan and Spence, 2018). It should be noted that the time-varying wind loads, which comply with the full evolution of a stochastic hurricane event in the proximity (a circular sub-region) of the building site, span several hours in duration leading to significant computational cost for each time-domain nonlinear analysis.

The SuS-based Phase-I sampling was implemented with nine strata, 1300 samples in each subset, and $p = 0.2$ (within the SuS procedure). The LSFs of interest were the peak roof drift ratios in the X and Y directions, $\delta_{X,roof}$ and $\delta_{Y,roof}$, and the Y-direction residual inter-story drift ratio at the 45th floor level, $\delta_{Y,45}$. The response thresholds considered for the failure probability estimations were 1/400 for the peak roof drifts and 1/1000 for the residual. The case study is representative of modern performance-based reliability assessment problems for wind-excited structures and the response thresholds are associated with the operational and continuous occupancy performance objectives (American Society of Civil Engineers, 2019). To demonstrate the mechanism to approximately control the estimator accuracy, different COV targets, $\Delta_l$, of 15% for LS1 (i.e., $\tilde{P}(\delta_{X,roof} > 1/400)$), 10% for LS2 (i.e., $\tilde{P}(\delta_{Y,roof} > 1/400)$), and 20% for LS3 (i.e., $\tilde{P}(\delta_{Y,45} > 1/1000)$) were specified. The peak mean hourly wind speed, $v_H$, at the building height, $H = 180.6$ m, was chosen as the stratification variable as wind-induced responses are the most sensitive to this quantity.

### 3.2. *Results*

#### 3.2.1. *Estimation of multiple failure probabilities*

For the preliminary study, $n_p = 75$, i.e., the number of structural analyses in each stratum, was considered and it was observed that the $n_i$-independent COV contribution, $\omega/\tilde{P}_{f,l}$, was larger than 19% for all the three limit states, indicating large uncertainty in the strata probabilities due to insufficient samples in Phase-I sampling. This was rectified by revising Phase-I sampling with $10^4$ samples in each subset so that $\omega$ could be reduced roughly by a factor of $\sqrt{1300/10^4}$. The wind speed hazard curve which represents the annual exceedance rate, $\lambda(v_H)$, as a function of $v_H$ can be constructed from the Phase-I samples. Figure 1 shows the hazard curve along with the strata thresholds to illustrate the rapidly decaying strata probabilities that the stratification achieves. Based on the results from the preliminary study, the constrained optimization of Eq. (4) was solved to obtain the number of additional simulations, $(n_i - 75)$, in the $i$th stratum, required to attain the target accuracy. While $mn_p = 675$ nonlinear analyses were conducted as a part of the preliminary study, based on the optimization routine, an additional 330 analyses were conducted. The estimated annual failure rates (i.e., the annual failure probabilities times $\lambda_H$) are reported in Table 1 along with the estimates of the COV, $\kappa_l$.

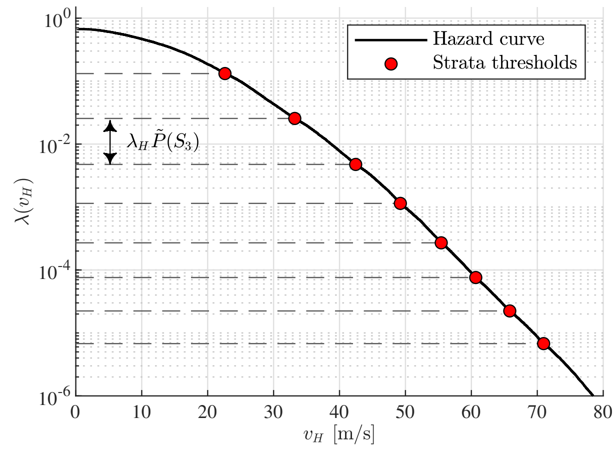


*Figure 1: Wind speed hazard curve.*

It is notable that the attained COV values approximately meet the set target values even for small failure probabilities/rates. For LS1, the COV has slightly exceeded the target which can be attributed to the sensitivity of the optimal sample allocation to the information obtained from the preliminary study. It is interesting to observe that the quality of Phase-I sampling (i.e., associated with the use of $10^4$ samples in each subset) which affects the uncertainty in the strata probabilities has contributed significantly (about 40% for LS1 and LS3, and about 70% for LS2) towards the overall COV values.

*Table 1: Estimated annual failure rates and margin of error measured by the COV.*

| Limit state | LS1 | LS2 | LS3 |
|---|---|---|---|
| $\Delta_l$ | 15.0% | 10.0% | 20.0% |
| $\omega/\tilde{P}_{f,l}$ | 7.9% | 7.0% | 7.9% |
| $\kappa_l$ | 17.8% | 10.0% | 18.8% |
| $\lambda_H \tilde{P}_{f,l}$ $(\times 10^{-6})$ | 1.05 | 142.00 | 0.99 |

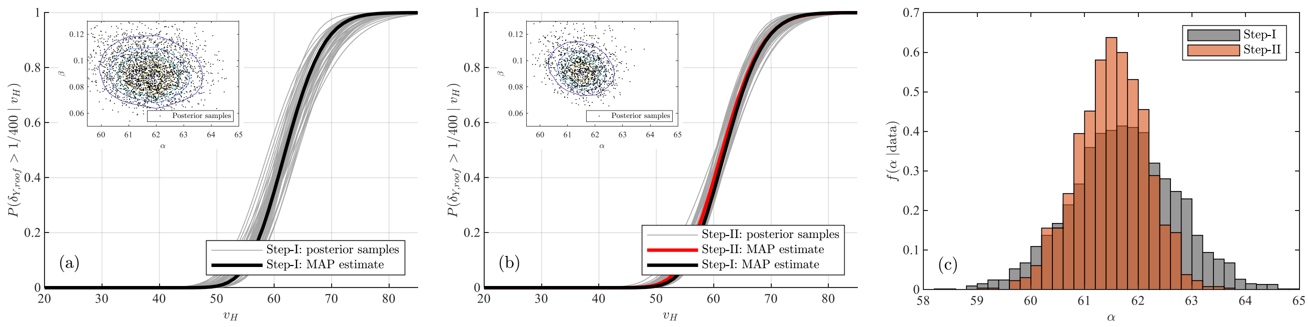


*Figure 2: Bayesian fragility estimation: (a) Posterior samples and MAP after Step-I; (b) Posterior samples and MAP after Step-II; (c) Marginal posterior distribution of $\alpha$.*

### 3.2.2. *Bayesian fragility estimation*

The aim of this section is to illustrate the Bayesian fragility estimation procedure for LS2. For Step-I, $n_{B,\text{I}} = 20$ was considered. The likelihood function of Eq. (7) was constructed by assuming a lognormal CDF form for the fragility function. A non-informative prior is assumed for the model parameters, i.e., $f(\alpha, \beta) = 1$. Rejection sampling with uniform proposal distribution was performed to sample from the posterior distribution. The resulting samples of $(\alpha, \beta)$ and the fragility functions along with the maximum *a posteriori* (MAP) estimate of the fragility function are shown in Figure 2(a). The MAP estimate of the fragility function is obtained by the substitution of the model parameters which are taken equal to the mode of the posterior distribution. Clearly, from the posterior samples drawn at the end of Step-I, the large uncertainty in the fragility fit can be observed, and subsequently, to reduce the variance an additional 100 sample points were considered. As discussed in Section 2.3.1, the proportions of samples from each stratum to be considered for Step-II analysis were optimally obtained. Figure 2(b) reports the refined MAP estimate after Step-II. Although there is not a huge difference from the Step-I MAP estimate, the posterior draws illustrate the improved confidence in the fragility estimate. Finally, Figure 2(c) shows the marginal posterior distribution of $\alpha$, the IM level associated with 50% probability of failure, and how a small variance is achieved with only $n_B = 280$ analyses. It should be noted that $n_B$ could have been much smaller if an informative prior, perhaps from the fragility of a different but related limit state, was available.

## 4. DISCUSSION

An extended stratified sampling scheme was outlined that integrates SuS and stratified sampling for the estimation of multiple failure probabilities with user-specified accuracy targets as well as the efficient Bayesian estimation of fragility functions. The scheme targets problems where the desired stratification variable is an intermediate model output, a situation commonly encountered when explicit hazard modeling is involved. In the case study, the proposed scheme was successfully used to estimate rare event probabilities/rates with specified accuracy limits using limited sample sets. In comparison, the use of standard MC simulation would have required a 10000-fold increase in samples. The Bayesian approach was shown to facilitate the integration of prior knowledge while effectively communicating the uncertainty in the resulting fragilities.

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