Multi-agent Value of Information for components' inspections

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ABSTRACT: We assess the Value of Information (VoI) for inspecting components in systems managed by multiple agents, using game theory and Nash equilibrium analysis. We focus on binary systems made up by binary components which can be either intact or damaged. Agents taking maintenance actions are responsible for the repair costs of their own components, and the penalty for system failure is shared among all agents. The precision of inspection is also considered, and we identify the prior and posterior Nash equilibrium with perfect or imperfect inspections. The VoI is assessed for the individual agents as well as for the whole set of agents, and the analysis consider series, parallel and general systems. A negative VoI can trigger the phenomenon of Information Avoidance (IA), where rational agents prefer not to collect free information. We discuss whether it is possible that the VoI is negative for one or for all agents, for the agents with inspected or uninspected components, and for the total sum of VoIs.

1. Introduction

Many infrastructure systems (e.g. transportation and gas pipeline networks) consist of multiple binary components, arranged in a system to fulfill various functions [Song and Der Kiureghian (2003), Song and Ok (2010), Tien and Der Kiureghian (2013), Malings and Pozzi (2016)]. The system condition is determined by the states of the components, which can be binary, either intact or damaged. The maintenance and inspection actions can be optimized to trade the risk of system malfunctioning for the cost of maintenance. The belief of the agents controlling the maintenance process is described by a probabilistic distribution on the possible states of the components. Thus, the information on components' states collected via inspections can improve decision making and reduce the uncertainty and the maintenance cost. Many factors can affect the inspection preferences, such as the probabilities of failure events, the maintenance costs and the role of each component in the system. These factors can be integrated in the Value of Information (VoI) analysis to summarize the benefit of observing the state of a component. VoI assessment is based on Bayesian pre-posterior analysis, as introduced by [Howard (1966)] and investigated for binary systems by [Lin et al. (2022)].

The traditional framework for optimizing static and sequential system maintenance assumes that there is a central authority with full control of all components, and it can gather information and facilitate maintenance actions. Such assumption may not hold, especially for large-scale infrastructure systems which are often managed by multiple holders. In this paper, we develop a framework inspired by cooperative game theory, as introduced to describe how individuals maximize expected utility in economic settings [Neumann et al. (1947)].

In a context of system management, components have different functions and need to cooperate with each other to make the system working and generate profit. In the system reliability problem, the network connectivity is studied under the presence of an intruder (or attack on certain links and components) with a game-theoretic formulation [Bhattacharya and Başar (2010)], modeling the network with a dynamic graph where the existence of an edge between two components depends on the state of the components as well as the attack. The network connectivity is also analyzed in degradable transportation networks [Du and Nicholson (1997), Iida and Wakabayashi (1989)], by estimating statistical distributions of link performance such as travel time and capacity of the link and calculating the impact of link performance variations on network performance. The game theory model for system maintenance has also been proposed for sequential decision making problems, when the interaction among several players spans a certain period of time [Oviedo (2000)].

When the system is managed by multiple agents, the VoI may become negative for some players. This means that some agents may prefer avoid collecting information, even if free of cost. This is the so called phenomenon of "Information Avoidance" (IA), studied in the field of psychology and public health. For example, people tend to seek information that is consistent with their benefits and decisions and, after the decision is made, to neglect information about the chosen alternative [Mills et al. (1959)]. In the medical context, it is revealed that people delay visiting physicians about suspicious symptoms of diseases, hoping they can avoid undergoing serious medical exams and operations [Ajekigbe (1991)]. It is also shown that with external constraints, the agents may wish to avoid collecting information even if it is free, but by enforcing epistemic constraints in the regulations, the policy-maker can induce a range of behaviors in the agents obeying the regulations, from information avoidance to over-evaluation of barely relevant information [Balcan et al. (2015), Pozzi et al. (2020)].

In this paper, we adopt a game-theoretic framework to assess VoI and identify cases of IA in multi-agents system maintenance.

2. METHODOLOGY

We consider a binary system composed of binary components: each component can be in one of two states: either intact or damaged. The system is represented as a block diagram, as that shown in Figure 1, where the system works properly only if there is an intact path from origin (o) to destination (d) (non functioning components cut the path). Each component is managed by one agent (say the owner of that component), who decides whether to repair it or not. The cost for each agent is the sum of two terms: the repair cost, if she takes the repair action, and a penalty if the system is not working. Agents select actions minimizing the expected cost. In this context, we assess the impact of inspections, i.e. of available information on the components' state.

n components $\{c_1, c_2, ..., c_n\}$ are arranged in a system, and the set of n agents $\{h_1, h_2, ..., h_n\}$ is such that agent h_i manages component c_i. Let $x_i \in \mathbb{B} = \{0, 1\}$ define the state of component c_i, with $x_i = 1$ indicating a functioning component, and $x_i = 0$ a failed one. Binary vector $\mathbf{x} = [x_1, x_2, ..., x_n] \in \Omega = \mathbb{B}^n$ lists the state of all components. Let $a_i \in \mathbb{B}$ define the action taken by agent h_i on c_i, with $a_i = 1$ meaning repairing (RE), and $a_i = 0$ doing nothing (DN). Binary vector $\mathbf{a} = [a_1, a_2, ..., a_n] \in \Omega$ lists all actions. The state of c_i after action a_i is $x'_i = \max(x_i, a_i) : \mathbb{B}^2 \to \mathbb{B}$, so that the state is intact if it was so, or if it is repaired, while it is failed if it was so and it is not repaired. Updated states are listed in vector $\mathbf{x}' = [x'_1, x'_2, ..., x'_n] \in \Omega$. The binary system state is $u = \phi(\mathbf{x}) : \Omega \to \mathbb{B}$, before the maintenance actions, and it is updated to system state: $u' = \phi(\mathbf{x}')$ after maintenance. ϕ is the components-system function. Cost for agent h_i is $L_i = C_i a_i + 1 - u'$ where C_i is the repair cost of component c_i, and the cost for system failure is unitary. As noted before, u' is a function of $\mathbf{x}'$, which depends on $\mathbf{x}$ and $\mathbf{a}$. After an inspection is performed, the information collected is shared among all agents, so they can optimize their maintenance action with the updated belief. If component c_i is inspected, information $y_i \in \mathbb{B}$ is collected, where $y_i = 0$ indicates that the component is not working, and $y_i = 1$ that is working: value $y_i = z \in \mathbb{B}$ is emitted with probability $e(i,z,p)$.

2.1. *Inspections*

For general systems with imperfect inspections, we define:

$$\varepsilon_i = \mathbb{P}[y_i = 0 | s_i = 1]$$
$$\delta_i = \mathbb{P}[y_i = 1 | s_i = 0]$$

to identify the inspection error rate for component i. By Bayes rule, we define the posterior failure probabilities for component i as:

$$k_i = \mathbb{P}[s_i = 0 | y_i = 1] = \frac{\delta_i p_i}{(1-\varepsilon_i)(1-p_i) + \delta_i p_i}$$
$$q_i = \mathbb{P}[s_i = 0 | y_i = 0] = \frac{(1-\delta_i) p_i}{\varepsilon_i (1-p_i) + (1-\delta_i) p_i}$$

Variable $p_{a_1,a_2,\cdots,a_n,y_i}$ denotes the posterior failure probability of the system given that the inspection on component c_i yields outcome y_i and the repair actions of agent $h_1, h_2, \cdots, h_n$ are $a_1, a_2, \cdots, a_n$ respectively. For example, if there are only two agents, then $p_{0,0,1}$ is the system failure probability when we discover that c_i is functioning and no component is repaired. Intuitively, it is equal to $p_{1,0,0}$ (indicating that c_i is damaged, c_1 is repaired and c_2 is not) when inspections are perfect and to $p_{1,0,1}$ (indicating that c_i is functioning, c_1 is repaired and c_2 is not). There are $2^3 = 8$ possible values for p_{a_1,a_2,y_i}. We use 6 values $\{\alpha_1, \alpha_2, ..., \alpha_6\}$ to identify these posterior probabilities, with $\alpha_3 < \alpha_1, \alpha_2$ and $\alpha_6 < \alpha_4, \alpha_5$. (When the inspection error rate is limited to a small value, more inequalities can be yield.) If the inspection is perfect, i.e. $k_i = 0$ and $q_i = 1$, we have $\alpha_2 = \alpha_3$ and $\alpha_5 = \alpha_6$, the number of necessary variables to identify the posterior probabilities will be reduced to 4. We define $p_{\pi,i} = \mathbb{P}[y_i = 0] = \varepsilon_i(1-p_i) + (1-\delta_i)p_i$. The prior and posterior cost matrices are shown in Table 4,

$p_{\omega,i}$	$p_{\omega,j}$	p_{a_i,a_j,y_i}	α
q_i	p_j	$p_{0,0,0}$	α_1
k_i	p_j	$p_{0,0,1}$	α_2
0	p_j	$p_{1,0,0}$	α_3
0	p_j	$p_{1,0,1}$	α_3
q_i	0	$p_{0,1,0}$	α_4
k_i	0	$p_{0,1,1}$	α_5
0	0	$p_{1,1,0}$	α_6
0	0	$p_{1,1,1}$	α_6

Table 1: Posterior system failure probability p_{a_i,a_j,y_i}

Table 2 and Table 3 respectively. We note here that some equilibria (formally defined in the next Section) are only possible when the inspection is imperfect, such as joint action $\{1,0\}$ (that is RE-DN) when c_i is functioning (since $\alpha_2 < \alpha_3 + C_1$ when inspection is perfect).

a_i/a_j	DN	RE
DN	α_2, α_2	α_5, $\alpha_5 + C_2$
RE	$\alpha_3 + C_1$, α_3	$\alpha_6 + C_1$, $\alpha_6 + C_2$

Table 2: Posterior cost matrix when c_i is functioning

a_i/a_j	DN	RE
DN	α_1, α_1	α_4, $\alpha_4 + C_2$
RE	$\alpha_3 + C_1$, α_3	$\alpha_6 + C_1$, $\alpha_6 + C_2$

Table 3: Posterior cost matrix when c_i is damaged

2.2. *Posterior costs*

The belief about components' state $\mathbf{x}$ is modeled by distribution p, and we assume that all agents share the same belief p. The corresponding belief on $\mathbf{x}'$ is $p'(p,\mathbf{a})$. The system failure probability after action $\mathbf{a}$ is:

$$P_\phi(p,\mathbf{a}) = 1 - \mathbb{E}_{\mathbf{x}\sim p}\left[\phi\left(\mathbf{x}'(\mathbf{x},\mathbf{a})\right)\right] \tag{1}$$

The expected loss for agent h_i is:

$$l_i(\mathbf{a},p) = \mathbb{E}_{\mathbf{x}\sim p}[L_i] = C_i a_i + P_\phi(p,\mathbf{a}) \tag{2}$$

a_i/a_j	DN
DN	$p_{\pi,i}\alpha_1 + (1-p_{\pi,i})\alpha_2$
RE	$\alpha_3 + C_1,\ \alpha_3$
a_i/a_j	RE
DN	$p_{\pi,i}\alpha_4 + (1-p_{\pi,i})\alpha_5 + \{0, C_2\}$
RE	$\alpha_6 + C_1,\ \alpha_6 + C_2$

Table 4: Prior cost matrix

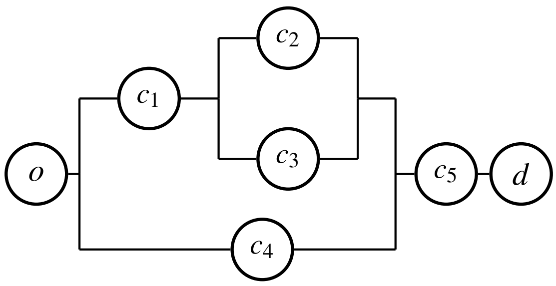

Figure 1: Figure 1. A general system with 5 components

To differentiate between the action taken by one agent and those by all others, we rewrite the loss function with three arguments where $\mathbf{a}_{\setminus i}$ refers to actions of agents other than h_i: $l_i(\mathbf{a}, p) = l_i(a_i, \mathbf{a}_{\setminus i}, p)$.

2.3. Optimal decision making

Among the 2^n possible actions in Ω, some of them are pure Nash Equilibria (NAs). Action $\mathbf{a} \in \Omega$ is a Nash Equilibrium under belief p if, for each agent h_i, her response is optimal, that is: $\forall i:\ a_i \in \mathscr{R}_i(\mathbf{a}_{\setminus i}, p)$, where the set of optimal response is:

$$\begin{aligned} &a \in \mathscr{R}_i(\mathbf{a}_{\setminus i}, p) \subseteq \mathbb{B} \\ \Leftrightarrow &\forall b \in \mathbb{B} : l_i(a, \mathbf{a}_{\setminus i}, p) \leq l_i(b, \mathbf{a}_{\setminus i}, p) \end{aligned}$$

Let $\mathscr{E}(p) \subseteq \Omega$ be the set of all NAs, when the belief is p. We define the global loss as the sum of losses for all agents:

$$g(\mathbf{a}, p) = \sum_i l_i(\mathbf{a}, p) \tag{3}$$

A NA is "global" if no NA with lower global loss exists. The set $\mathscr{E}_G(p) \subseteq \mathscr{E}(p)$ of Global NAs (GNAs) is defined as:

$$\begin{aligned} &\mathbf{a} \in \mathscr{E}_G(p) \\ \Leftrightarrow &\{\mathbf{a} \in \mathscr{E}(p), \forall \mathbf{b} \in \mathscr{E}(p) : g(\mathbf{a}, p) \leq g(\mathbf{b}, p)\} \end{aligned}$$

On the contrary, a NA is "local" if it is not global. Sets $\mathscr{E}(p)$ and $\mathscr{E}_G(p)$ can be identified by total enumeration, in set Ω.

To predict the agents' behaviour, we assume they select an action which is a NA. The behaviour of agent h_i is defined by a policy, π_i, function of current belief: $a_i = \pi_i(p)$. The corresponding policy for all agents is:

$$\mathbf{a} = \pi(p) = \{\pi_1(p), \pi_2(p), ..., \pi_n(p)\}$$

To define π, we refer to the set of NAs. If there is a unique NA (which is trivially global), we assume that all agents select the corresponding action. In case of multiple NAs, we investigate scenarios where the selected NA is global and where it is not. We use notation π_G to indicate a policy relating to a GNA.

When policy π is followed, the shortcut notation for the loss of agent i and for the global loss is:

$$l_i^\pi(p) = l_i\big(\pi(p), p\big)\ ;\ g^\pi(p) = g\big(\pi(p), p\big) \tag{4}$$

2.4. Information, inference and posterior actions

We assume that the same policy π is followed in the prior and in the posterior setting. Following Bayes' formula, the prior belief p becomes posterior belief $\omega(i, z, p)$ when, after inspecting component c_i, the inspection outcome is $y_i = z$.

The expected posterior loss for agent h_i, when component c_j is inspected and prior belief is p, is:

$$\mathscr{L}_{i,j}^\pi(p) = \mathbb{E}_Z\big[l_i^\pi\big(\omega(j, z, p)\big)\big] \tag{5}$$

$$= \sum_{z=0}^{1} e(j, z, p)\ l_i^\pi\big(\omega(j, z, p)\big) \tag{6}$$

The VoI for agent h_i, when component c_j is inspected, is:

$$\mathrm{VoI}_{i,j}^\pi(p) = l_i^\pi(p) - \mathscr{L}_{i,j}^\pi(p) \tag{7}$$

The overall VoI is the sum of VoIs for all agents:

$$\overline{\mathrm{VoI}}_j^\pi(p) = \sum_i \mathrm{VoI}_{i,j}^\pi(p) \tag{8}$$

Hence, this overall value can be related to the global loss:

$$\overline{\mathrm{VoI}}_j^{\pi}(p) = g^{\pi}(p) - \mathbb{E}_Z\left[g^{\pi}\left(\omega(j,z,p)\right)\right] \tag{9}$$

with expected value

$$\mathbb{E}_Z\left[g^{\pi}\left(\omega(j,z,p)\right)\right] = \sum_{z=0}^{1} e(j,z,p)\, g^{\pi}\left(\omega(j,z,p)\right)$$

3. NEGATIVE VOI AND PRISONER'S DILEMMA IN MULTI-AGENT SYSTEMS

We analyze cases of negative VoI, for various types of networks and under different settings, such as whether the inspection is perfect and whether the agents select the global or local NE. During the maintenance decision process, the information collected through inspection can change the optimal repair actions and the expected posterior cost for the agents, which create a potential incentive for the agents to avoid the information, even if it is free to collect. IA in a multi-agent maintenance system can happen in various scenarios. For example, with two agents, it is intuitive that a higher posterior cost may happen when a local equilibrium is selected, resulting in negative VoI for both agents. Another scenario is when one agent prefers to avoid information that may be beneficial to the other, because the inspection of one component may force the other agent to skip the repair of her component, even under the global equilibrium. However, can the VoI be negative for both agents when the global equilibrium is selected?

To answer this question, we first look at the following example. Suppose that in a series system with two agents, $p_1 = 0.538$, $p_2 = 0.619$, $c_1 = 0.350$, $c_2 = 0.491$. Parameters of the inspection of c_1 are $\delta_1 = 0.795$ and $\varepsilon_1 = 0.501$. The cost matrix for the two agents under the prior and posterior situations is given in Table 5. The prior global equilibrium action is repairing both components. When the inspection of c_1 shows that it is functioning, the global equilibrium remains the same (because of the error rate). So far, both equilibria are globally optimal in their respective settings.

But when the inspection shows that c_1 is damaged, the global equilibrium becomes doing nothing for both components. If we consider repairing both components, it is obvious that that action has the lower cost for each agent. But when a_2 is fixed as repairing, the cost for agent h_1 will be reduced if h_1 does nothing instead of repairing. Therefore, the action pair of repairing both is not a posterior equilibrium, and the VoI is negative for both agents. When the inspection shows that c_1 is damaged, suppose that agent h_2 repairs. Then h_1 needs to pay 0.324 for doing nothing, and 0.350 for repairing, and so is better off doing nothing. Likewise, suppose h_2 does nothing. Then h_1 needs to pay 0.742 for doing nothing, and 0.969 for repairing, and so doing nothing is again better. Doing nothing for h_1 now strictly dominates the alternative of repairing: whatever action h_2 takes, h_1 is always better off choosing to do nothing. By symmetry, doing nothing also strictly dominates repairing for h_2. Thus, two rational agents will both do nothing and pay 0.742 each, which is also the only strict NE, while two "irrational" agents can cooperate and achieve the much lower cost of 0.350 and 0.491, respectively, by repairing both components. This is a case of "prisoner dilemma", which describes the possible discrepancy of decisions made under collective rationality and those made under individual rationality and was discussed by Nash (1953), Hardin (1968), Rapoport (1974).

Table 5: Cost matrix when global equilibrium is chosen, c_1 inspected, and VoI is negative for both agents

Cond- -itions	Cost (agent h_1-agent h_2) DN-DN	DN-RE
Prior	0.824, 0.824	0.538, 1.029
$y_i = 1$	0.867, 0.867	0.650, 1.141
$y_i = 0$	0.742, 0.742	0.324, 0.814
Cond- -itions	**RE-DN**	**RE-RE**
Prior	0.969, 0.619	0.350, 0.491
$y_i = 1$	0.969, 0.619	0.350, 0.491
$y_i = 0$	0.969, 0.619	0.350, 0.491

4. Equilibrium and VoI analysis with perfect inspection

In this section we discuss the case of n independent components (and hence the random variables in vector $\mathbf{x}$ are independent), and the inspections are perfect. Now belief $p = [p_1, p_2, ..., p_n]$ can be seen as a n-entry vector, and $p_i \in [0,1]$ is the probability that component c_i fails, so that the component state is a Bernoulli random variable: $x_i \sim \text{Ber}(1-p_i)$. The corresponding conditions after maintenance, in vector $\mathbf{x}'$, are still independent Bernoulli random variables, conditional to actions $\mathbf{a}$, and vector p' is updated such that $p'_i = \min\{p_i, 1-a_i\}$. Table 6 shows an example of cost matrix for two agents: h_1 and h_2. In that table, we use a shortcut notation for system failure probability $P_\phi(p, \mathbf{a})$. $p_{\mathscr{F}}$ indicates the system probability of failure when all components whose indexes are in set $\mathscr{F}$ are repaired. Hence, if no component is repaired, that probability is $p_\emptyset$. The column defines the action of agent h_j, and the row that of h_i. For perfect inspection where $x_i = y_i$, posterior vector $\omega(i, y_i, p)$ is identical to prior vector p in all entries, except for entry i, which becomes $1 - y_i$.

Table 6: Prior cost matrix for general two-agent systems

a_i/a_j	DN	RE
DN	$p_\emptyset$, $p_\emptyset$	$p_{\{j\}}$, $C_j + p_{\{j\}}$
RE	$C_i + p_{\{i\}}$, $p_{\{i\}}$	$C_i + p_{\{i,j\}}$, $C_j + p_{\{i,j\}}$

Based on the assumption that inspection and repair are perfect, the components are independent, the prior system failure probabilities are shared by all agents, and the information of inspection outcomes and repair actions are available to all agents, we have summarized the possible situations of negative VoI for series, parallel and general systems with perfect information under global equilibrium in Table 7, and under local equilibrium in Table 8. Specifically, the tables are listing answers to the question whether it is possible to have negative VoI for any agent, for the agent owning the inspected components, for the agent owning the uninspected components, for all agents, and for the total sum of VoI. In these tables, positive answers are marked with ✔ and negative ones with ✖. Negative answers marked with ⊗ are inferred from numerical simulations without a strict analytical proof (yet).

As anticipated above, in the case of two agents, it is possible that a joint action, e.g. repairing both components, yields less expected cost for both agents, but it is not an equilibrium. Instead, another joint action, e.g. doing nothing, may become an equilibrium with higher cost for both agents. This "prisoner dilemma" can generate a negative VoI for each agent.

For global equilibrium, we have proved that it is impossible to have negative VoI in parallel systems with any number of agents and in series systems with two agents. For series systems with any number of agents, we have not found negative VoI for any agent through numerical simulations, but the strict analytical proof has not been discovered yet. For general systems, we will show examples where the negative VoI happens for the owners of the inspected or uninspected component, and the sum of VoI for all agents is negative. We have also proved that it is impossible to have negative VoI for every agent in a general system with two agents, but this conclusion still needs analytical proof for multiple agents.

For local equilibrium, we have shown that it is possible to have negative VoI for agents managing either the inspected or uninspected components, and the sum of VoI of all agents can be negative too. For parallel systems with any number of agents, it is impossible to have negative VoI for all.

Table 7: Possible negative VoI for general systems with perfect information and global equilibrium

configure		**negative VoI for $\cdots$ agent?**				
system	**agent**	≥ 1	**ins.**	**unin.**	**sum**	**all**
series	two	✖				
parallel	two	✖				
series	any	⊗				
parallel	any	✖				
general	two	✔	✔	✔	✔	✖
general	any	✔	✔	✔	✔	⊗

Table 8: VoI for general systems with perfect information and local equilibrium

configure		**negative VoI for ⋯ agent?**				
system	**agent**	≥ 1	**ins.**	**unin.**	**sum**	**all**
series	any			✔		
parallel	two	✔	✔	✔	✔	✖
general	any			✔		

Due to the limitation on the length of this conference paper, the proofs and examples to validate each conclusion in the table are not included, but we can share here some brief discoveries regarding the negative VoI. Firstly, if the prior and posterior equilibrium actions are the same, then obviously the VoI is nil. Secondly, if there is only one different action, then it is also impossible to have negative VoI for both agents. For example, suppose we have a case where prior equilibrium is (a_1, a_2), and two posterior cases are (a'_1, a'_2) and (a''_1, a''_2) where $a_1 = a'_1 = a''_1$, and $a_2 = a'_2 \neq a''_2$. Because we know that if we take (a_1, a_2) in the second posterior case, then the VoI must be 0. Since (a''_1, a''_2) is the equilibrium, we have the cost for the second agent (it doesn't matter which agent we choose) must be lower than that in (a_1, a_2) (which may not be an equilibrium in this posterior case). So the VoI for that agent must be non-negative. Thirdly, it is impossible to completely flip the actions in the posterior equilibrium when the inspected component is functioning or damaged. For example, it is impossible to have two posterior cases like (a'_1, a'_2) and (a''_1, a''_2), and $a'_1 \neq a''_1, a'_2 \neq a''_2$. For agent h_2, we know that $c^1_{(a'_1,a'_2),2} < c^1_{(a'_1,a''_2),2}$ and $c^0_{(a''_1,a''_2),2} < c^0_{(a''_1,a'_2),2}$. But when c_1 is functioning, then the optimal action for h_1 in the posterior equilibrium must be doing nothing, so we also have: $c^1_{(a'_1,a''_2),2} = c^0_{(a''_1,a''_2),2} < c^0_{(a''_1,a'_2),2} = c^1_{(a'_1,a'_2),2}$, which causes a contradictory.

5. EQUILIBRIUM AND VOI ANALYSIS WITH IMPERFECT INSPECTION

If the inspection is imperfect, i.e. $\varepsilon_i, \delta_i > 0$, the negative VoI can happen for the agents managing the inspected and uninspected components and the sum of VoI can be negative under global and local equilibrium in series, parallel and general systems. For parallel systems under global metric, it is impossible to have negative VoI for all agents. For the rest of the system configurations, it is possible that information hurt all agents. The detailed conclusions are listed in Table 9 and Table 10.

Table 9: VoI with imperfect information and global equilibrium

configure		**negative VoI for ⋯ agent?**				
system	**agent**	≥ 1	**ins.**	**unin.**	**sum**	**all**
series	any			✔		
parallel	any	✔	✔	✔	✔	✖
general	any			✔		

Table 10: VoI with imperfect information and local equilibrium

configure		**negative VoI for ⋯ agent?**				
system	**agent**	≥ 1	**ins.**	**unin.**	**sum**	**all**
series	any			✔		
parallel	any			✔		
general	any			✔		

6. CONCLUSIONS AND FUTURE WORKS

We have illustrated the assessment of VoI for a network system managed by multiple agents following a Nash equilibrium analysis. When multiple equilibria exist, we define the global Nash equilibrium as the one with minimum sum of all the agents' costs.

When the information is perfect, for simple systems such as series and parallel systems, under the assumption that the global Nash equilibrium is selected, we prove that the VoI of revealing one component's status always has always non-negative VoI. When local equilibrium is selected, we illustrate that the VoI can be negative. For general systems, we have shown that the VoI can be negative, even when global equilibrium is selected. Information avoidance can happen to the agent managing the inspected component, but also to the others.

When the information is imperfect, however, the VoI can be negative for all agents under global or local equilibrium in series and global systems, except for parallel systems. The VoI can be negative (when the information is imperfect) for the agents managing the inspected or the uninspected components, and the sum of VoI can be negative under global and local equilibrium in all systems.

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