

**AN EXPERIMENTAL INDICATOR TO FORECAST TURNING POINTS
IN THE IRISH BUSINESS CYCLE**

PADRAIG DALTON and GERARD KEOGH*
Central Statistics Office

(read before the Society, 9 March 2000)

Abstract: The Business Cycle in Gross Domestic Product (GDP) describes the recurring cycles of expansion and slowdown in overall economic well being. A Composite Leading Indicator (CLI) is a small group of important economic indicators that can be used to predict likely future phases, or turning points, of the business cycle. The business cycle in the Irish economy is extracted in this paper using a combination of symmetric Henderson Moving Average filters. These filters are also applied to a broad range of economic indicator series that are then combined together, on the basis of cyclical conformity and lead time, to form an Experimental Composite Leading Indicator (XCLI). This exercise is undertaken for both annual and interpolated quarterly GDP data. The ability of the XCLI to predict future phases in the Irish business cycle in GDP is examined.

Keywords: Economic Time Series, Henderson Moving Averages, Business Cycle, Turning Points, Cyclical Conformity, Leading Indicator.
JEL Classifications: C600, E300

1. INTRODUCTION

The analysis of Business Cycle in a given reference series such as Gross Domestic Product (GDP) is a well-established practice in many countries. It endeavours to identify the recurring cycles of expansion and slow down in aggregate business activity over time. The generally accepted definition of the business cycle, given by Burns and Mitchell (1946) from the United States' National Bureau of Economic Research (NBER) is:

“Business Cycles are a type of fluctuation found in the aggregate economic activity of nations that organise their work mainly in business enterprises: a cycle consists of expansions occurring at the same time in many economic activities followed by general recession, contraction and revival which merge into the expansion phase of the next cycle: this sequence of changes is recurrent but not periodic...”

* The views expressed in this article are not necessarily those held by the CSO and are the personal responsibility of the authors.

It is clear from this definition that the term “cycle” which has traditionally been used to describe these fluctuations, is somewhat misleading as neither the period nor amplitude is regular. Short-term irregular movements including measurement errors, socio-economic shocks and long-term shifts due to structural factors tend to act independently of the rest of the economy. These therefore should be removed from the reference series so as to identify the underlying business cycle.

It is generally accepted that where the reference series involves quarterly data that the business cycle varies in length between two and eight years, see Salou and Kim (1993). Methodological constraints associated with using annual data forces us in this paper to assume the business cycle varies in length from four to ten years. In later sections we interpolate to get quarterly data estimates and in this case we will revert to the two to eight year definition.

An issue closely associated with the determination of the business cycle, is the ability to predict its emerging phases by identifying its turning points. This is accomplished using an appropriate mix of economic indicators such as investment, interest rates etc. that can be expected to lead the business cycle. This mix is called a *Composite Leading Indicator (CLI)*.

The main purpose of this paper is to identify an experimental CLI from statistics available in Ireland. We generate an experimental CLI largely based on methodology used by *the Australian Bureau of Statistics (ABS)*. A similar methodology is used in a number of other countries, see OECD (1987). Usually a CLI is based on quarterly GDP data. The Irish *Central Statistics Office (CSO)* has for the first time released quarterly national accounts covering the period 1997 to the first quarter of 1999. A secondary purpose of this article is to highlight an economically relevant application of this quarterly GDP data.

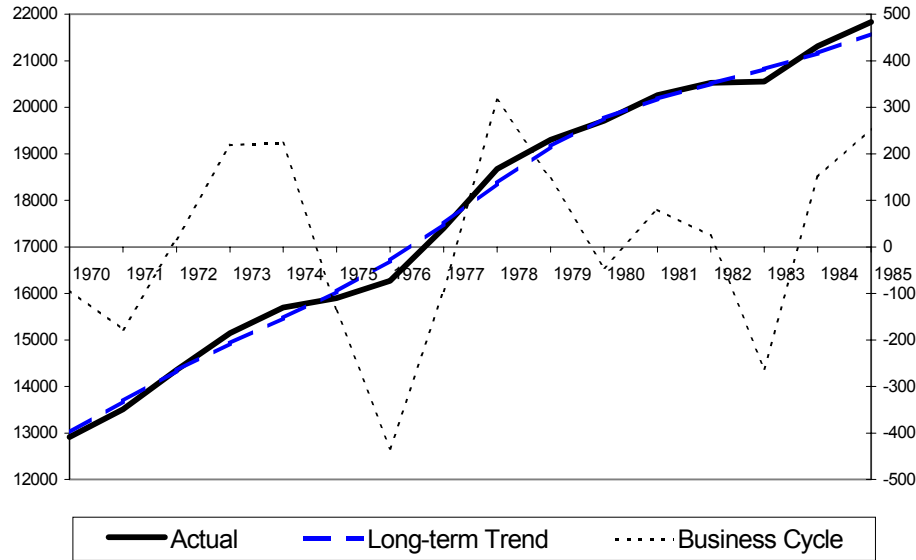
2. BACKGROUND AND OVERVIEW

The methodology adopted in this article uses the so called ‘growth cycles’ approach to identify the Irish business cycle in GDP; that is the average of income and expenditure measures of GDP at constant 1990 prices. Growth cycles generally manifest themselves as periods of strong growth followed by periods of slower growth or, at worst, a small absolute decline in activity. This type of fluctuation portrays aggregate economic activity in Ireland since the 1960’s better than the traditional ‘classical cycles’. These, by contrast first show a rise in activity followed by a definite fall in activity. We note that that the oil-price rises of the 1970’s were classical cycles in many countries, see OECD (1987).

In order to analyse growth cycles it is necessary to estimate the underlying trend and then remove it from the series under consideration. The method adopted in this article to estimate the trend is based on smoothing out short-term fluctuations using a moving average. The theory underlying this is explored in Section 3. When the

trend is eliminated in this way from a series such as GDP the residual component reflects the cyclical behaviour of the economy about its long-term growth pattern.

Figure 1: Actual and Long-term Trend in GDP (LHS) and Business Cycle (RHS) (£million)



In Figure 1 it is clear that the graphs of actual GDP and also its long-term trend are continuously growing over the period 1970 to 1985. This indicates that a growth cycle is the dominant behaviour. At no stage do we see a sustained fall in the value of GDP as would be associated with a classical cycle. The business cycle which is obtained by subtracting (or dividing in case of multiplicative time series) the long-term trend from the actual GDP series shows a cycle with period varying between three and six years

As stated in Section 1, the methodology adopted here to generate a CLI closely follows that used by the ABS. The steps we follow for the construction of a CLI for a reference series such as GDP are:

1. Extract the business cycle in the reference series and identify its turning points; this procedure is fully described in Section 4.
2. For each economically relevant indicator time series extract its business cycle. For annual data extract cycles of between four and ten years while for quarterly data extract cycles of between two and eight years.

3. Assesses the temporal relationships at turning points between the business cycle in GDP and the business cycle in each individual indicator series; this is outlined in Section 5.
4. Select a small group (seven in our case) of indicator time series whose business cycles have appropriate temporal relationships. Combine these to generate a CLI having the same oscillatory behaviour as the GDP business cycle but with turning points that lead those of GDP. This process is given and results assessed in Sections 6, 7 and 8.

Up to now two important studies of indicators for the Irish business cycle have been undertaken. The first of these, conducted by the OECD (1987) derived a CLI for the business cycle in the Monthly Industrial Production Index reference series. The OECD chose this series because of its broad economic coverage and consistency of treatment throughout OECD countries. Up to the mid-1980s monthly production provided a reasonable proxy for aggregate economic activity. However, this is no longer the case due mainly to the growth in the services sector. This fact renders industrial production inadequate as a comprehensive measure of aggregate economic activity and so it is unsuitable as a reference series for the whole Irish economy.

More recently Fagan and Fell (1992, 1994) have also studied Irish business cycle indicators. Their approach which follows that of Stock and Watson (1991), was to initially find a composite index that would be coincident with and act as a 'proxy' reference series for aggregate economic activity. They call this proxy a composite coincident index; it is adopted as the reference series in place of GDP. They then constructed their CLI by producing vector auto-regressive time series forecasts of the composite coincident index.

Both studies mentioned avoid using GDP or Gross National Product (GNP) as the business cycle reference series. The main reasons for this are:

1. Both GDP and GNP are only available annually and as a consequence may hide certain shorter cycles.
2. GDP and GNP are questioned as measures of aggregate economic activity due to the influence of multinational companies.
3. Fagan and Fell (1994) express concern over the effects of "*certain sector specific developments that impact on GDP but do not impact on the business cycle*".

Taking each of these issues in turn, when quarterly GDP data is available it is preferred over annual data because it locates the timing of turning points in the business cycle more precisely. This fact is accepted without question. The methodology and analysis for this article focuses primarily on annual data because it is both readily available and reliable. Also, to demonstrate the wide applicability of our approach and ascertain the extent to which annual data might hide cycles in quarterly data, we also derive a CLI based on quarterly GDP data obtained using interpolation.

In relation to the impact of multinationals, we demonstrate (see Section 4) that the choice of GDP or GNP as the reference series appears irrelevant, as the timing of turning points in both are largely identical. Also, we mention that arguments regarding the use of GDP or GNP as a reference series appear moot, since methods such as those of Fagan and Fell (1992) adopt GDP as a first approximation to their composite index. In this sense they to are indirectly using GDP as a reference series.

It is unlikely that sector specific shocks of between four and ten years will exist independently and therefore not influence other sectors of the economy. In this situation, GDP will pick up any resulting medium-to-longer term influences. GDP we believe provides the best aggregate to base our reference business cycle.

3. METHODOLOGY

In some situations it is informative to view a time series as being composed of cycles of different frequencies (or period lengths). Using the growth cycles methodology, short-term effects can be filtered (i.e. extracted leaving behind an underlying signal) from a time series such as GDP. Similarly long-term effects can also be identified using an appropriate filter. The business cycle is obtained by combining these two filtered series.

Smoothing filters generally take the form of a (weighted) moving average. A moving average filter is normally characterised by the number of weights (or points/terms) required for computation. Also, a moving average is symmetric when the weights are symmetric about the central weight (e.g. the 5-term moving average with weights: 1, 2, 5, 2, 1, is symmetric about the third weight).

Moving averages have two other important properties. First is their time shift or phase (angle) shift. This occurs where the timing of turning points is misrepresented in the smoothed series. Second, to assess the ability of a filter to smooth out the effect of certain (usually high frequency) cycles it is necessary to plot the associated squared gain (or filter/transfer) function (see Wei, 1990). For a symmetric moving average filter having $2k+1$ weights w_j , the squared gain (or gain for short) function is defined as:

$$gain = \left| \sum_{j=-k}^k w_j e^{-i2\pi j \lambda} \right|^2 = \left(w_0 + 2 \sum_{j=1}^k w_j \cos 2\pi j \lambda \right)^2, (0 \leq \lambda \leq 1/2)$$

where λ is the frequency.

The gain function defined above describes the degree of smoothing that results from the application of a moving average filter to an arbitrary sinusoidal input time series. A plot of the gain against the frequency shows whether the shorter cycles are being sufficiently damped and the extent to which the strength of longer cycles are retained. In this sense the gain function describes the frequency domain behaviour of the filter.¹

Among the most widely used polynomial based moving average filters are the so-called Symmetric Henderson Moving Average filters or Henderson filters for short (Henderson, 1916). These filters possess properties desirable for trend extraction purposes. First, as with trend estimation filters (including that of Spencer², see Kendall and Stuart, 1968), the Henderson filter is designed to follow a cubic polynomial trend without distortion. In addition to this Henderson filters constrain trend smoothness. This is accomplished by minimising the variance of the third differences of the series defined by the application of the moving average (see Kenny and Durbin, 1982).

Second, the weights required in any Henderson filter are readily computed using a methodology outlined by Grey and Thomson (1996). The formulae for the calculation of these weights are given in the Appendix.

Third, they do not induce appreciable phase shifts at interior points of a time series. However, as with all symmetric filters there is some phase shift associated with the end points (the end point problem) because there are insufficient observations available for computation³. To overcome this problem we apply the, so-called, Henderson surrogate filters. These approximate the Henderson symmetric filter at the end points; full details are given in the Appendix.

Fourth, Henderson filters do not amplify any cycles. So, if the user is interested in extracting cycles below a specified frequency then a Henderson Filter of suitable length can be chosen to dampen the unwanted cycles⁴. Unfortunately Henderson filters do not entirely remove the unwanted cycles.

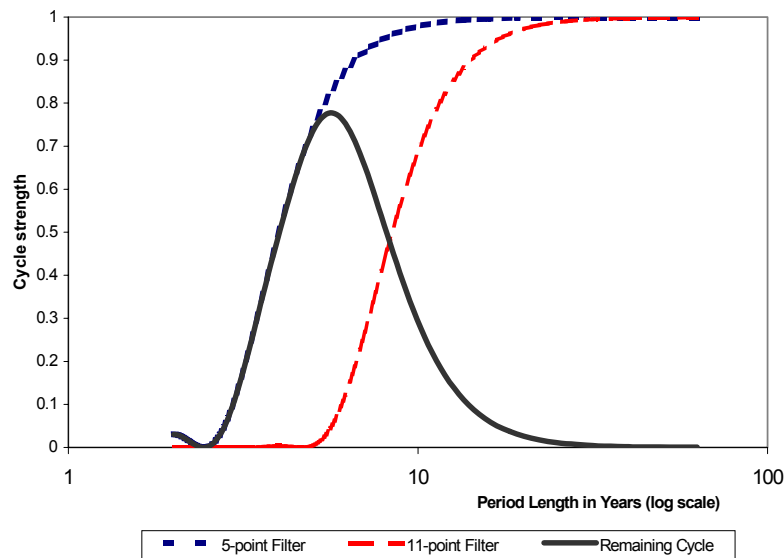
The Business Cycle in annual GDP data comprises cycles of between four and ten years in duration. To extract these cycles we apply Henderson moving average filters. These were chosen both for their filtering characteristics outlined above and because they are widely used for business cycle extraction.

Using the growth cycles method the business cycle is extracted by applying 5-point and 11-point⁵ Henderson moving average filters in the following four-step procedure.⁶

- We applied a 5-point Henderson moving average to the GDP series to output a trend that comprised cycles of four or more year's duration.

- We applied an 11-point Henderson moving average to the GDP series to output a trend comprising cycles of ten or more year's duration.
- By dividing⁷ the output from the first (i.e. 5-point) filter step by the second (11-point) we obtain cycles in the desired range.
- The resulting business cycle is scaled to have maximum amplitude of one.

Figure 2: Gain Functions for Henderson Moving Averages



To assess the ability of the above procedure in identifying the relevant range of cycles the associated gain functions have been plotted on the log scale in Figure 2. The gain functions demonstrate that the filters only allow approximate extraction of the four to ten year cycles. The 'Remaining Cycle' curve shows that at most 50 percent of the strength of these cycles are damped. This still leaves a sufficient level of useful cycle strength. For cycles under three years and over eleven years the plot shows that at least 80 percent of their strength is damped. This indicates that the combination of 5-point and 11-point Henderson filters is likely to be effective in extracting the business cycle in GDP.

4. THE BUSINESS CYCLE

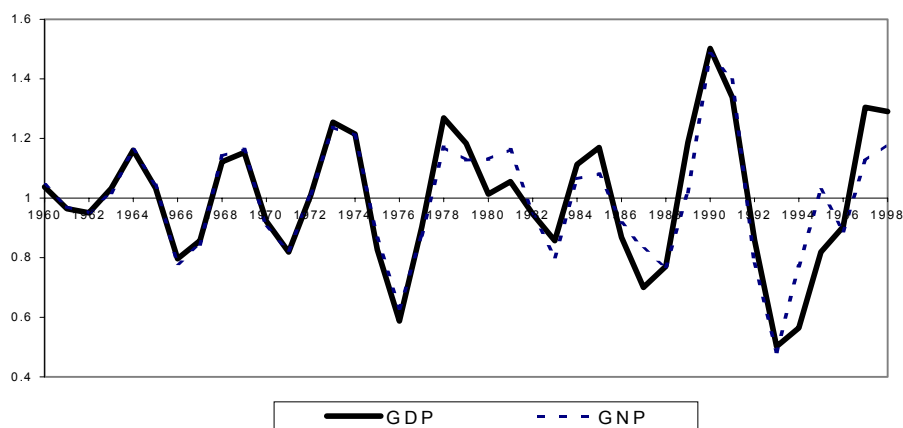
In order to select turning points in the business cycle the Bureau of Economic Analysis (1987) in the US provides the following guidelines:

- peaks and troughs must alternate
- a turning point is clearly identified when either the next turning point has been identified or the corresponding phase has an amplitude greater than the smallest clearly recognised phase
- the last value is chosen as the turning point in the case of equal values.

The internationally recognised measure of economic activity is GDP. However, due primarily to the influence of multinational companies GNP is viewed as a better measure of national economic well being in Ireland. The economic arguments surrounding this topic concentrate on issues such as transfer pricing, profit repatriation etc. Given this, we were left with a decision as to whether to base our reference cycle on GDP or GNP.

The business cycle for both GDP and GNP was extracted for the period 1960 to 1998 using the methodology of the previous section. The resulting two business cycles are displayed in Figure 3.

Figure 3: Business Cycles in Irish GDP and GNP 1960 - 1998



It is clear from the Figure 3 and Table 1, which summarises the timing of turning points, that there is no significant difference in the timing or amplitude of turning points in GDP or GNP. In almost all cases turning points are coincident, with the exception of the last trough in the 1980's. The GNP business cycle does have a kink occurring between 1993 and 1996. This is reflected in the GDP business cycle as a point of inflexion. Based on these observations we decided to follow practice elsewhere and adopt GDP as our reference series.

Looking once again at Figure 3 the amplitude of the cycles appears to double over the period. This would indicate that booms and recessions are becoming more pronounced.

Table 1: Timing of turning points for Business Cycles in Irish GDP and GNP

	GDP	GNP
Trough	1962	1962
Peak	1964	1964
Trough	1966	1966
Peak	1969	1969
Trough	1971	1971
Peak	1973	1973
Trough	1976	1976
Peak	1978	1978
Trough	1983	1983
Peak	1985	1985
Trough	1987	1988
Peak	1990	1990
Trough	1993	1993
Peak	1997	1995

In Table 2 we summarise the expansions and contractions of the business cycle. The average cycle length is 5 years. The longest cycle runs from 1976 to 1983 and coincides with the second oil price shock. The 5-year contraction, starting in 1978 is the longest period of contraction in the business cycle. The duration of this contraction could be related to the impact of the oil price shock and the inability to counteract it with an expansionary fiscal policy.

Table 2: The Irish Business Cycle

Dates of Turning points			Duration (years)		
<i>Trough</i>	<i>Peak</i>	<i>Trough</i>	<i>Expansions</i>	<i>Contractions</i>	<i>Total</i>
1962	1964	1966	2	2	4
1966	1969	1971	3	2	5
1971	1973	1976	2	3	5
1976	1978	1983	2	5	7
1983	1985	1987	2	2	4
1987	1990	1993	3	3	6
1993	1997		4		
Mean			3	3	5

One reviewer has pointed out that the business cycle we have observed may not be real, for when a moving average is applied to a time series it will induce a cycle in the irregular (i.e. random) component of the residual series. This is called the Slutsky-Yule effect, (see Kendall and Stuart, 1968). This induced cycle can be shown to mimic the business cycle quite closely. However, using time-trend regression, which is not based on moving averages we have observed almost identical cycles. Also, other countries using this methodology do not appear to

stress the impact of the Slutsky-Yule effect. This, it appears is mainly due to the absence of a robust alternative methodology. Therefore, while some doubt is cast over the truth of the cycles observed, it would appear, in the absence of further evidence that the Slutsky-Yule does not invalidate our findings.

5. ECONOMIC INDICATORS OF THE BUSINESS CYCLE

In this section we assess the temporal relationships of several economic indicators with those of the business cycle. This assessment is based on the following criteria:

- economic relevance
- length of the series
- timeliness of the series

The decision as to what indicator series to examine has been largely subjective. The majority of series analysed have been taken from the Central Statistics Office databank with some others coming from the Economic and Social Research Institute, Office of National Statistics, United States Bureau of Commerce and Reuters.

Data availability is an important issue. Many economic series were excluded because they are only available from the mid-1970s onwards. However, economically relevant indicators were assessed regardless of length. Series with discontinuities were not assessed. Timeliness of indicators is also vital, for in order to be useful in predicting the timing of future turning points these indicators must lead the business cycle sufficiently.

Finding leading indicator series is not a trivial task; this is especially true when using annual data. To overcome this problem we followed the ABS and inverted (take the reciprocal of the series) a number of the indicator series. Many of these inverted series were found to have temporal behaviour similar to the reference cycle.

The analysis of possible indicator series in this methodology concentrates on two issues:

- cyclical conformity
- lead/lag analysis

Salou and Kim (1993) state that a series is said to cyclically conform to the reference series if it contains one and only one cycle per cycle in the reference series. To check for cyclically conformity the cycle within the indicator series is graphed with the business cycle and a decision is based on a visual examination of both plots.

The study of the temporal relationships of turning points between the indicator and reference cycle is referred to as a lead/lag analysis. This analysis is completed both

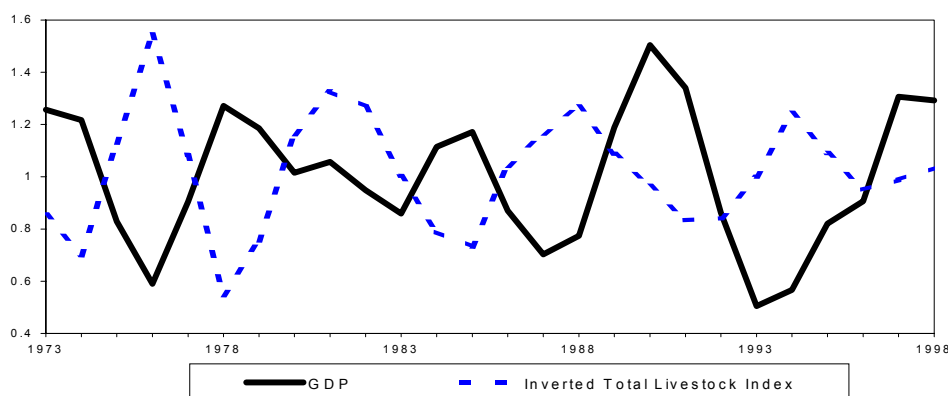
by visual means and an assessment of the (lag) correlation between both cycles.

Table 3: Lead-in Years at Turning Points in the GDP Business Cycle

	T	P	T	P	T	P	T	P
GDP	1976	1978	1983	1985	1987	1990	1993	1997
Inverted Total Livestock Index	2	2	5	4	2	2	2	3

Identifying whether or not an indicator's turning points are leading or lagging the business cycle turning points is not always clear. We based our judgements on a visual examination. For example, Figure 4 graphs the business cycle in GDP and the cycle extracted from the Inverted Total Livestock Index (indicator cycle). The indicator cyclically conforms to the reference cycle, even though the cycles are moving in different directions. In Table 3 we summarise the lead/lag at turning points. This shows that while the cycle in the indicator always leads the reference cycle, it does so with significant variation. This lack of consistency in the lead-time at turning points is extremely important as it renders the indicator unreliable in forecasting the timing of future turning points.

Figure 4: Business Cycles in GDP and Inverted Total Livestock Index



The cyclical conformity test was carried out on 60 economic indicator series. In all 28 series conformed to the business cycle and were put through a lead/lag analysis. Based on this and timeliness as well as economic relevance considerations we chose the 14 'best' as candidates for inclusion in an *Experimental Composite Leading Indicator* (XCLI).

The results of the lead/lag analysis for each of those 14 series are presented in Table 4. The first row of figures highlighted in bold in the table dates the turning points in the business cycle. Each subsequent row gives the lead/lag period at each turning

point with regard to the business cycle turning point. A negative figure in any of these rows implies that the indicator series is lagging the business cycle, a positive figure implies the indicator is leading, a zero implies it is coincident. A blank indicates that the indicator misses a cycle or that no data is available for the period concerned.

On examination Table 4 shows that only five series led the Business cycle by an average of more than 1 year. Experience gained elsewhere (see OECD 1987) suggests that an XCLI should normally contain at least seven indicator series that lead the reference cycle by at least 1 period. To identify more leading indicator series we inverted 11 of the 14 series. The notion behind using inverted series centres on the fact that the cyclical behaviour of series values that are relatively small are swamped by the moving average. By inverting the series any cyclical component in the smaller values is identifiable. The results of the lead/lag analysis on the 11 inverted series are presented in Table 5.

Looking at Table 5, 10 of the series, i.e. all excluding *Inverted Terms of Trade Index* which misses one cycle and also has an extra cycle, cyclically conform to the Business cycle. All 10 exhibited an average lead period greater than 1 year with 9 series having an average lead-time of two or more years.

6. COMPONENTS OF THE EXPERIMENTAL COMPOSITE LEADING INDICATOR

Indicator Selection Methodology

In order to construct a leading indicator we follow the practice adopted elsewhere (see OECD 1987) and consider a combination of the individual indicators. Such an indicator is referred to as a composite indicator. Clearly the identification of the best composite indicator based on a combination of up to 25 individual series is not trivial. The following empirical approach, which in effect is a data mining exercise, was used to identify candidate XCLIs.

We restricted candidate experimental indicators to be comprised of between 7 and 14 indicator series. Next, we randomly selected about 30,000 different possible XCLI combinations from the 25 candidates. While, based on the above analysis we had a preference for leading series, we initially included all series to ensure broad economic coverage. Their correlation coefficient (lagged two years) with the Business cycle was computed. A subset of about 500 of these was then chosen as candidate XCLIs on the basis of largest correlation coefficient. Each of these candidates was then graphed for visual inspection against the business cycle. Finally, some indicator series were lagged by one or two years to see if the fit to the business cycle could be improved.

Based on this search and examination procedure, an XCLI was generated from the following 7 short-term economic indicators:

- Inverted Total Livestock Index
- Inverted New Private Cars
- Terms of Trade Index (External Trade) - lagged 1 year
- Inverted Interest Rate on Long-term Government Securities
- Inverted Total Investment - lagged 1 year
- UK Gross Domestic Product at constant prices
- Inverted US Gross Domestic Product at constant prices

These series cover a broad range of economic factors such as external demand, monetary policy, competitiveness and consumption. The combination of these series provides us with an XCLI that predicts turning points on average two years in advance of those occurring in the business cycle. Table 6 provides a summary of the lead/lag analysis for the seven component series.

In Table 7 we provide a correlation analysis for each of the seven component series against the business cycle. The correlation coefficient was calculated at leads of between 0 and 6 years with only the highest correlation coefficient (ρ) given in Table 7. It is clear from the table that all correlation coefficients lie approximately in the range 0.5 – 0.6 with a confidence coefficient above 0.99 based on a t_{n-2} distribution. Overall the lead times found in the correlation analysis are similar to those obtained from the lead/lag analysis. Note however that the correlation is calculated based on all points and therefore is a different measure to the lead/lag at turning points measure. In fact the mean lead/lag at turning points is a correlation measure based solely on the turning points.

Table 4: Results of Lead/lag Analysis for the 14 Short-listed Series

	T	P	T	P	T	P	T	P	T	P	T	P	T	P	Average Lead/Lag			
GDP Business Cycle	62	64	66	69	71	73	76	78	83	85	87	90	93	97				
Total Livestock Index									-1	0	0	1	0	-1	-2	-1	1	-0.3
CPI All Items, Base 1968 = 100									3	2	4	3	-1	-1	-1	2		1.4
ISEQ Index												2	2	0	1	3		1.4
Industrial Disputes Begun					-1	-1	0	0	2	1	1	3	4	6				1.5
Total Live Register, Seasonally Adjusted					2	2	2	2	4	2	2	3	3	1				2.3
Production Volume Index, Manufacturing. Industry					2	2	2	2	4	2	2	3	3	1				2.3
Retail Sales Index All Excluding Garages, Seasonally Adjusted					0	0	0	-2	0	-1	-1	-1	-3					-0.9
New Private Cars			-1	0	0	0	1	0	0	0	0	0	1	0				0.1
Terms of Trade Index (Trade)	0	0	0	1	0	0	1	0	2	2	2	1	2	4				1.1
Long-term Interest Rate on Government Securities	-1	-1	-1	-1	-1	-1	-2	-2	0	-1	-1	0	0	2				-0.7
Total Investment	0	-1	-1	0	0	0	0	-1	-1	-1	-1	0	0					-0.5
Passenger Movements, Air and Sea Outwards	-1	-1	-1	-1	-1	-1	0	-1	2	1	1	1	1	2				0.1
UK GDP	0	0	0	0	0	0	1	-1	2	2	1	1	1	3				0.7
US GDP	-2			0	0	0	0	-3	0	0	0	1	1	3				0.0

Table 5: Results of Lead/lag Analysis for the 11 Inverted Series

	T	P	T	P	T	P	T	P	T	P	T	P	T	P	Average Lead/Lag
GDP Business Cycle	62	64	66	69	71	73	76	78	83	85	87	90	93	97	
Total Livestock Index							2	2	5	4	2	2	2	3	2.8
Industrial Disputes Begun						1	2	2	5	4	3	1	2	3	2.6
Production Volume Index, Manufacturing Industry									4	3	3	4	4	6	4
Retail Sales Index, All excl. Garages, Seasonally Adjusted						2	3	2	4	2	1	2	2	1	2.1
New Private Cars				3	3	3	3	3	2	1	2	3	4	3	2.7
Terms of Trade Index (Trade)				Misses one cycle and also has one extra cycle											
Long-term Interest Rate on Government Securities		1	1	2	2	1	2	1	3	3	1	2	2	4	1.9
Total Investment			2	2	2	2	3	3	4	1	1	2	3	4	2.4
Passenger Movements, Air and Sea Outwards		1	1	2	1	1	2	2	4			4	4	5	2.5
UK GDP		2	2	3	2	2	3	3	4	4	4	4	4	4	3.2
US GDP		0	0	2	2	2	3	3	2	2	2	3	4	5	2.3

Table 6: Results of Lead/lag Analysis on the Component Series

	T	P	T	P	T	P	T	P	T	P	T	P	T	P	Average Lead/Lag
GDP Business Cycle	62	64	66	69	71	73	76	78	83	85	87	90	93	97	
Inverted Total Livestock Index							2	2	5	4	2	2	2	3	2.8
Inverted New Private Cars				3	3	3	3	3	2	1	2	3	4	3	2.7
Terms of Trade Index - lagged 1 year	-1	-1	-1	0	-1	-1	0	-1	1	1	1	0	1	3	0.1
Inverted Interest Rate on Long-term Government Securities		1	1	2	2	1	2	1	3	3	1	2	2	4	1.9
Inverted Total Investment-lagged 1 year			1	1	1	1	2	2	3	0	0	1	2	3	1.4
UK GDP	0	0	0	0	0	0	1	-1	2	2	1	1	1	3	0.7
Inverted US GDP	-1	0	0	2	2	2	3	3	2	2	2	3	4	5	2.1

Table 7: Mean lead/lag and Correlation (ρ) Analysis

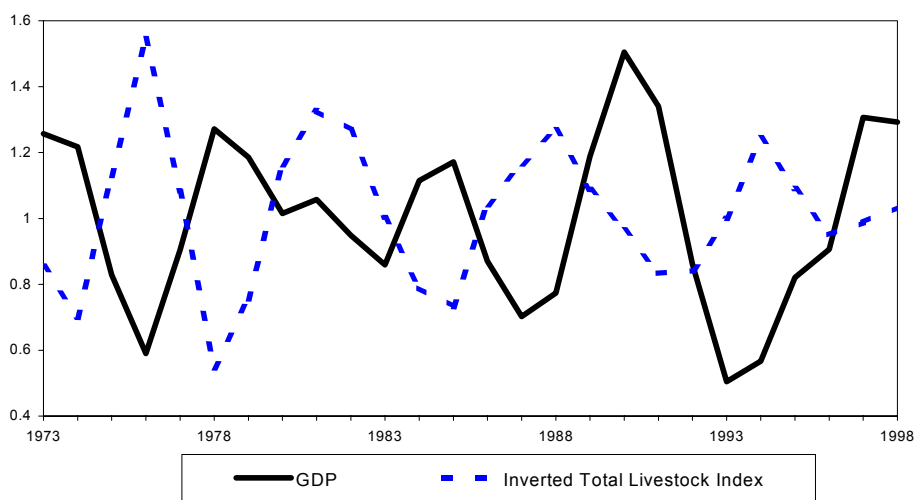
	Mean lead/lag			Correlation			
	All	Peaks	Troughs	Lead	ρ	Obs	Confidence coefficient
Inverted Total Livestock Index	2.8	2.8	2.8	3	0.61	23	0.998
Inverted New Private Cars	2.7	2.7	2.8	3	0.60	31	0.991
Terms of Trade Index - lagged 1 year	0.1	0.1	0.0	0	0.50	38	0.998
Inverted Interest Rate on Long term Government Securities	1.9	2	1.8	2	0.54	37	0.999
Inverted Total Investment - lagged 1 year	1.4	1.3	1.5	2	0.50	36	0.998
UK GDP	0.7	0.7	0.7	0	0.59	39	0.991
Inverted US GDP	2.1	2.4	1.7	3	0.47	36	0.996

Component Indicator Series

Total Livestock Index

Agriculture has been a key sector of the Irish economy and we consider its inclusion in the XCLI is therefore warranted. Figure 5 graphs the business cycle in both the Inverted Total Livestock Index and GDP.

Figure 5: Business Cycles in GDP and Inverted Total Livestock Index



The Inverted Total Livestock Index has no missing or extra cycles and leads the business cycle by 2.8 years on average. On its own this is not a good predictor of the reference cycle as the length of the lead varies considerably from between 2 and 5 years.

New Private Cars

This series is sometimes viewed as an early indicator of changes in personal consumption patterns. The cycle graphed in Figure 6 in the inverted indicator leads the reference cycle by an average of 2.7 years.

Terms of Trade Index – Lagged 1 year

The Terms of Trade Index has always been seen as an indicator of the competitiveness of the economy. The indicator lagged by 1 year was found to be coincident with the business cycle.

Rate of Interest on Long-term Government Securities

Interest rates are seen as a driving force in the economy and their impact on real activity is widely debated. We examined the cycle within the series and found it

lagged the reference cycle by an average of 0.7 years. The inverted cycle graphed in Figure 8 leads the reference cycle by an average of 2.4 years. The graph of the indicator cycle shows no extra or missing cycles. However, there is considerable variability in lead-time.

Figure 6: Business Cycles in GDP and Inverted New Private Cars

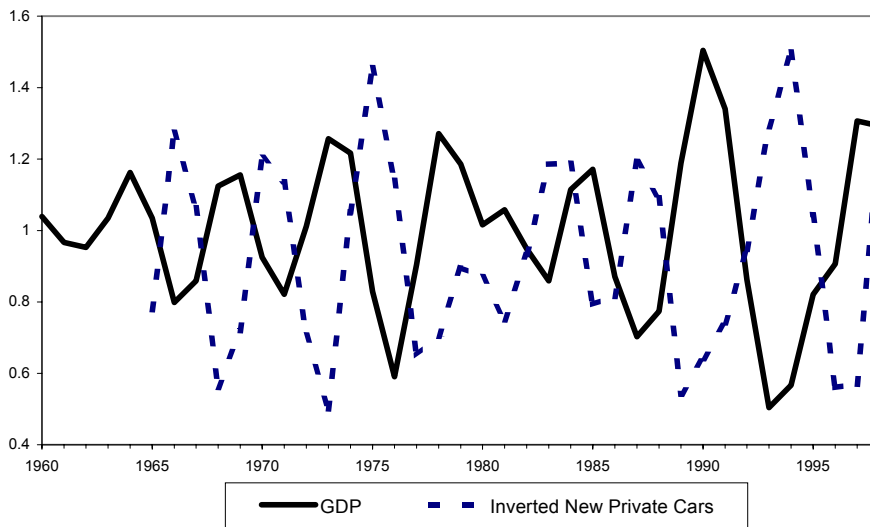


Figure 7: Business Cycles in GDP and Terms of Trade Index - lagged 1 year

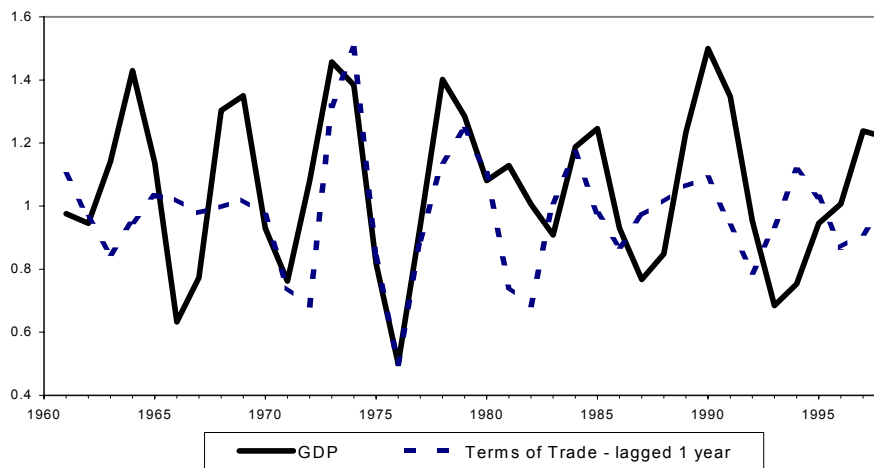
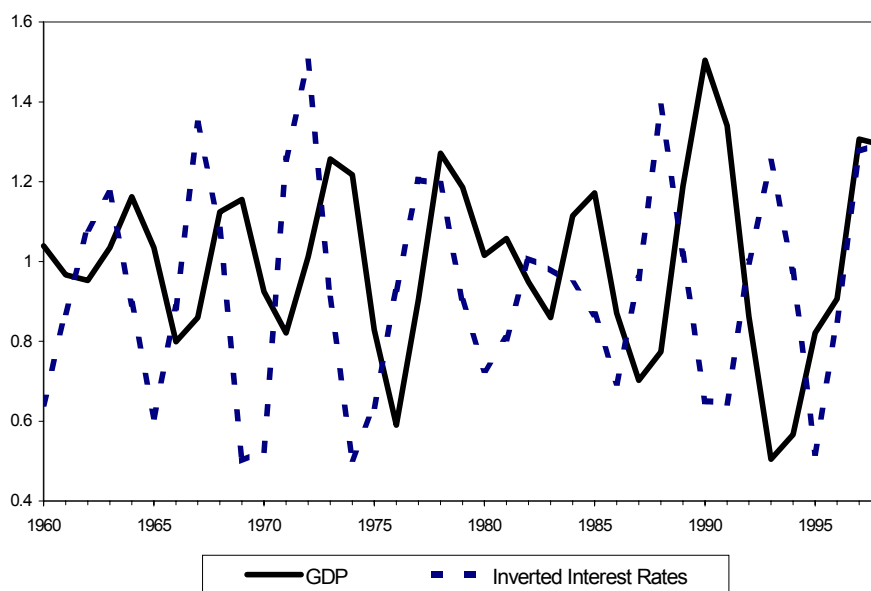


Figure 8: Business Cycles in GDP and Inverted Interest Rates



Total Investment

Investment to some extent reflects confidence in the economy. In Figure 9 Inverted Total Investment⁸ lagged by 1 year is graphed for visual comparison with the Business cycle. The indicator cycle leads the reference cycle by 1.4 years on average. There are no missing or extra cycles.

UK GDP

The UK has traditionally been our largest market, though over the years our dependence on it has diminished somewhat. Figure 10 graphs the business cycles in UK GDP and Irish GDP. It clearly illustrates the cyclical conformity that exists between the two series. There are no extra or missing cycles. Up to 1973 cycles were coincident, however after 1973 the UK GDP cycle led the Irish business cycle by an average of 1.25 years.

Figure 9: Business Cycles in GDP and Inverted Total Investment lagged 1 year

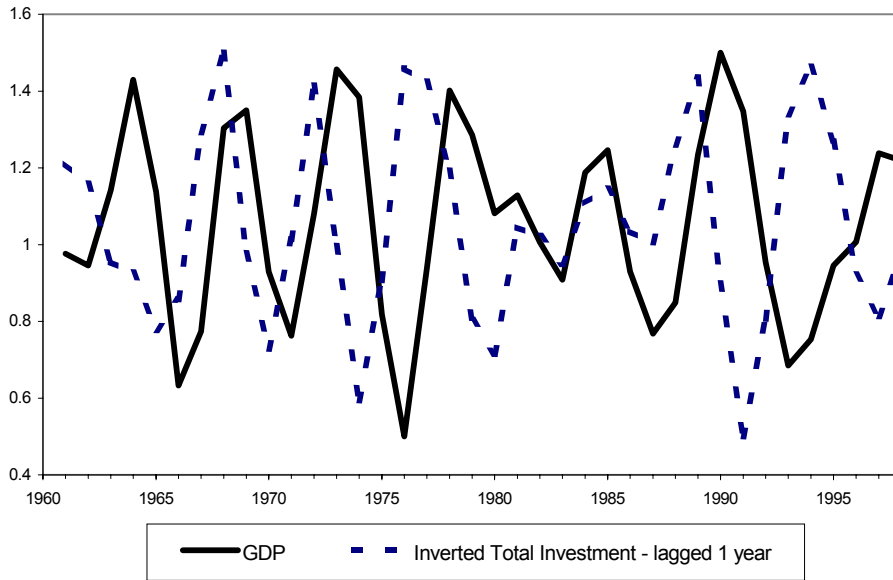
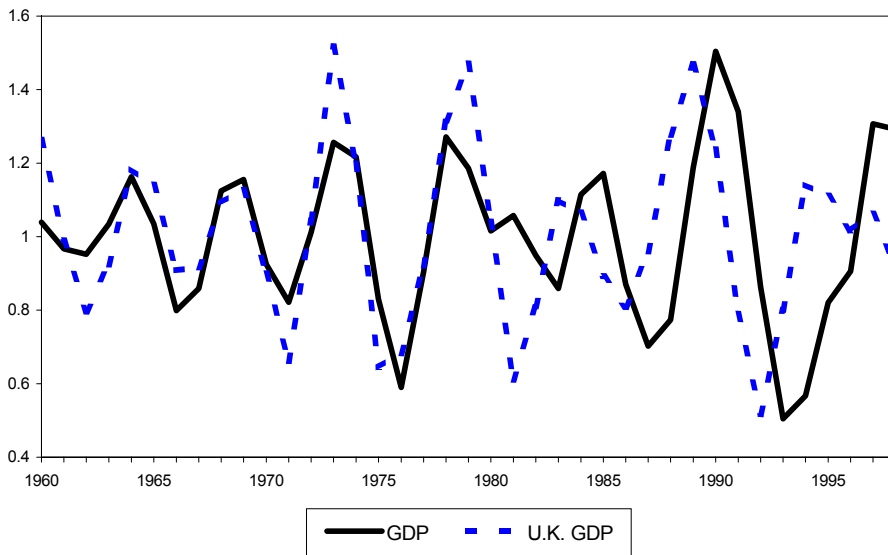


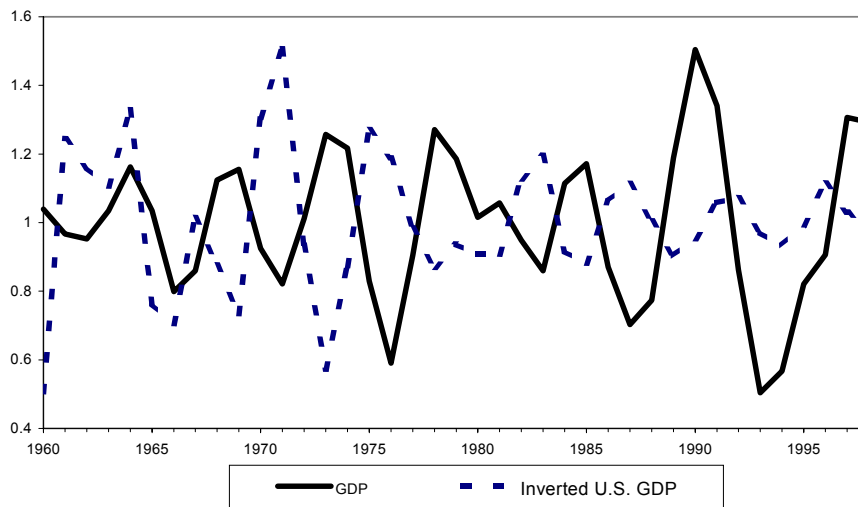
Figure 10: Business Cycles in GDP and U.K. GDP



United States GDP at constant market prices

Like the UK the United States is an important trading partner and reflects to some extent external demand for Irish products. The United States is the largest economy in the world and as such its GDP figure reflects cyclical movements in the world wide economy.

Figure 11: Business Cycles in GDP and Inverted U.S. GDP



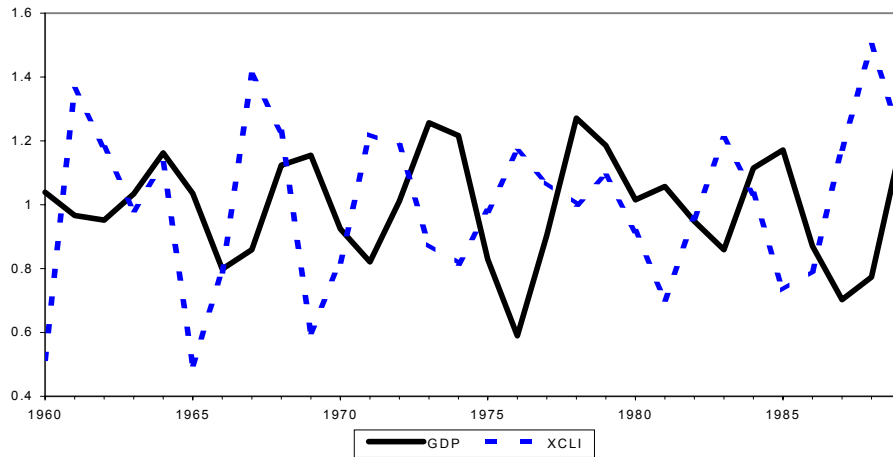
The cycle within US GDP was found to be coincident with the Irish business cycle although the 1964 to 1969 cycle was missing. The inverted US cycle led the Irish cycle by an average of 2.1 years.

7. THE EXPERIMENTAL COMPOSITE LEADING INDICATOR (XCLI)

Salou and Kim (1993) noted that while economic indicators contained some information relevant to movements in aggregate economic activity they could also show extra or missing cycles and produce false signals. Aggregating the individual indicators into an XCLI broadens the coverage and hopefully includes all relevant information while diluting the effects of any extra cycles introduced by individual indicators.

Each individual indicator contains its own early signal of economic activity and the aggregation of these in theory should improve the reliability of predicting turning points in the business cycle. In this section we graph both the XCLI and the business cycle.

Figure 12: Business Cycle for GDP and XCLI



The XCLI was calculated as the average of the seven component indicators for all years from 1960 to 1989. No weighting system was applied to the average used to compute the XCLI. Experience elsewhere (see for example Salou and Kim 1993) has shown that weighting the components adds little to the usefulness of an XCLI.

Table 8: Timing of XCLI turning points against Business Cycle turning points

	GDP	XCLI
T	1962	
P	1964	3
T	1966	1
P	1969	2
T	1971	2
P	1973	2
T	1976	2
P	1978	2
T	1983	2
P	1985	2
T	1987	2

The main reason for this is that it is equivalent to allocating different degrees of importance to the selected causes, early manifestations and expectations related to the business cycle. Also, due to the fact that all cycles are different in their causes and effects it is probable that a weighting system applying to the current cycle would be unsuitable for a subsequent cycle.

In Table 8 we display the temporal relationships between the XCLI and the Business cycle. The XCLI leads the Business cycle by an average of two years. A correlation

analysis reinforces the finding of the lead/lag analysis; that is, that the XCLI leads the business cycle by 2 years with a correlation of 0.7 and confidence coefficient of 0.999.

Clearly the worth of the XCLI can only be measured by its ability to forecast turning points in the business cycle. Section 9 looks at the performance of the XCLI in predicting the turning points in the 1990's.

8. THE QUARTERLY BUSINESS CYCLE AND QUARTERLY EXPERIMENTAL LEADING INDICATOR

The emphasis of this article has so far been on developing an XCLI for the Business cycle in annual GDP data. This is not wholly satisfactory because annual data may hide cycles that are visible in quarterly data and also the timing of turning points in a particular year is uncertain. Our preference would be to follow the ABS (see Salou and Kim, 1993) and focus on Quarterly Gross Domestic Product (QGDP) data and cycles from two to eight years. However, at present no official set of QGDP figures going back to 1960 is available. In this situation we decided to generate quarterly estimated values via interpolation.

Recently CSO has examined some of the issues relating to generating quarterly interpolated values for certain CSO economic time series⁹. Among the univariate interpolating methods assessed the Spline method was the most robust; that is, most accurate, reliable and straightforward to implement across a wide variety of economic time series such as those encountered in this article.

To generate the quarterly business cycle in GDP we adopted the following procedure:

- Fit a smooth cubic¹⁰ spline curve through the annual GDP data and use it to estimate quarterly data, subject to the constraint that within each year the quarterly data sum to the annual total.
- We applied 7-point and 33-point Henderson moving averages to QGDP to separate out cycles of interest.
- By dividing the output from the 7-point filter by the 33-point we obtain cycles in the desired range.
- The resulting business cycle is standardised to have maximum amplitude of one.

The quarterly business cycle in GDP is plotted in Figure 13. Also plotted with a dotted line is the annual business cycle identified in Section 4. Comparing the plots it is clear that from 1960 to 1979 both the number and timing of turning points is similar. The quarterly curve is, as expected, more jagged. However, from 1980 onward a very different picture emerges. The timing of turning points from 1980

onward is given in Table 9. This shows the extent to which cycles might be hidden as a consequence of using annual data and relying on annual filters. In each of the periods 1980-83, 1987-90, 1990-93 and 1993-97, the business cycle derived from quarterly data shows an extra cycle. On the basis of interpolated quarterly data it appears that the business cycle has been much more volatile than might be indicated based only on annual data.

From the timing of quarterly turning points in Table 9 it appears the period length of the cycles is about 3 years. This contrasts with a period of about 5 or 6 years for the cycles in annual data. In Figure 14 we plot the associated periodograms (or spectral density functions¹¹ when the horizontal axis is expressed in units of frequency) of the annual and quarterly business cycles. A periodogram identifies the relative importance of the different periods appearing in a time series. Peaks at a particular period indicate that cycles having this period are dominant in the time series. As expected both periodograms show cycles of about 5 years (20 quarters) are present in the business cycles. However, the periodogram for the quarterly data also shows, as expected from the period lengths observed in Table 9, that a weaker peak at 2.5 years (10 quarters) also exists.

To construct a *Quarterly Experimental Composite Leading Indicator* (QXCLI) we took the XCLI as a starting point. Where available we initially replaced all annual series with their quarterly counterparts. In situations where only annual series were available we used spline interpolation to obtain quarterly estimates

Table 9: Turning points for Annual and Quarterly Business Cycles in GDP 1980-97

Peak or Trough	GDP	QGDP
Trough		1980
Peak	1981	
Trough	1983	1983
Peak	1985	1985
Trough	1987	1986
Peak		1987
Trough		1989
Peak	1990	1990
Trough		1991
Peak		1992
Trough	1993	1994
Peak		1995
Trough		1997
Peak	1997	1997

Figure 13: Quarterly and Annual Business Cycles in GDP

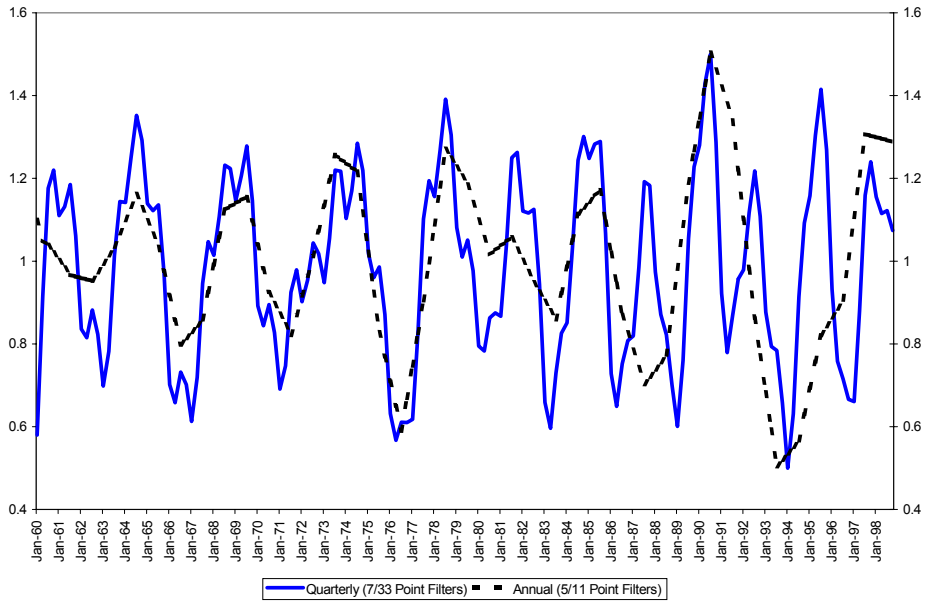
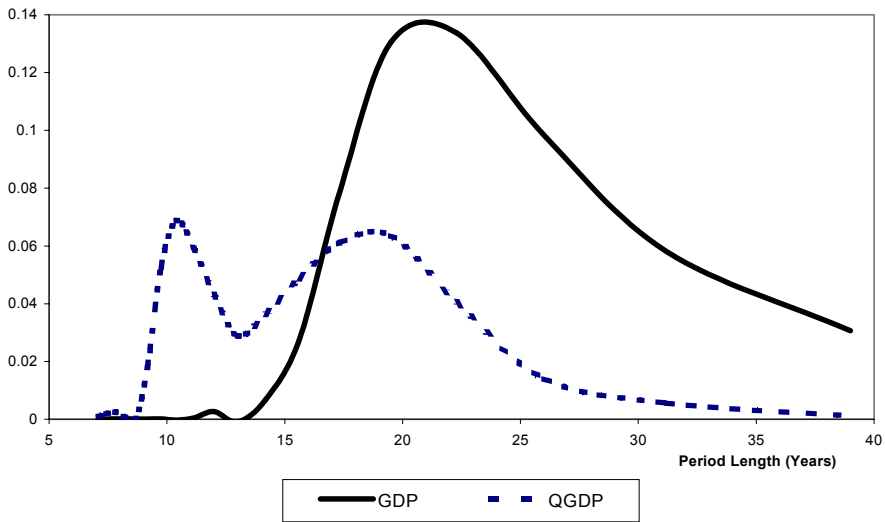


Figure 14: Periodograms of Business Cycles in Annual and Quarterly GDP



Next, the business cycle was extracted from each quarterly series. Here we applied the combination of 7 and 33 point Henderson filters, as used above for QGDP. To generate a first approximation to the QXCLI we calculated the average of the business cycles for the seven indicators listed for the XCLI in Section 6. This was found to provide a good first approximation. However, this candidate QXCLI had problems in the period 1968-73 where it appeared to have a new cycle. By experimentation with various combinations of five or six of the seven indicators, we found that by excluding the investment series and lagging two of the component series we improved the fit of both the business cycle and periodogram. The average of the following six quarterly component series was found to be the ‘best’ QXCLI.

Series Name	Quarters lagged
Inverted Total Livestock Index	0
Inverted New Private Cars	0
Terms of Trade Index (External Trade)	1
Inverted Interest Rates on Long term Government Securities	2
UK GDP	0
Inverted US GDP	0

The indicator resulting from the combination of these components is plotted in Figure 15. This indicator cyclically conforms to the business cycle in QGDP at all times except for the appearance of an extra peak at about October 1970. However, close examination of the small double peak occurring in QGDP in 1971-72 indicates that the peak in QXCLI is leading the small double peak.

Table 10: Turning points and leads for Business Cycle in QGDP and QXCLI

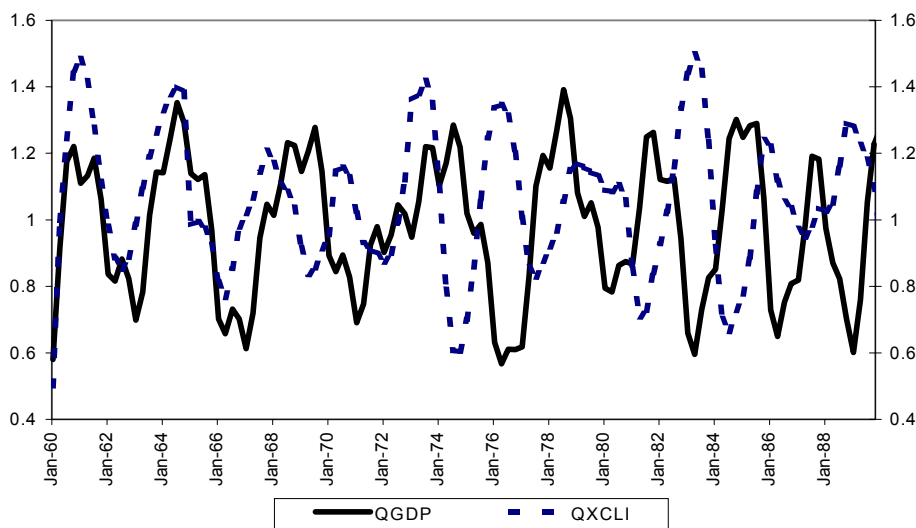
	QGDP	QMCLI	Lead/Lag	QGDP	QXCLI	Lead/Lag
P	3-61	1-61	2	3-78	2-76	9
T	1-63	3-62	2	2-80	3-77	11
P	3-64	3-64	0	4-81	1-79	11
T	1-67	2-66	3	2-83	2-81	8
P	3-69	4-67	3	3-85	2-83	9
T	1-71	2-69	4	2-86	3-84	7
P	3-72	3-70	8	3-87	4-85	7
T	1-73	1-72	4	1-89	2-87	7
P	3-74	3-73	4	3-90	4-88	7
T	2-76	4-74	6			

In Table 10 we provide a lead/lag analysis based on turning points for the QXCLI from 1960 to 1989. The lead/lag pattern displays considerable variation throughout. The average lead of the QXCLI at the turning points is calculated at about 5.9 quarters. The average lead at peaks is 6.0 quarters while that of troughs is 5.8 quarters. The variation of the lead over time is a cause for concern, but from 1980 onward it is settling down to about seven quarters. This is in line with the lead of

two years found for the XCLI based on annual data. We also computed the correlation coefficient between the business cycle in QGDP and the QXCLI at quarterly lags 0-12. The largest positive correlation was 0.41 with confidence coefficient of 0.999 and it occurred at a lag period of seven quarters. This strengthens the belief that the QXCLI leads the Business cycle in QGDP by about seven quarters.

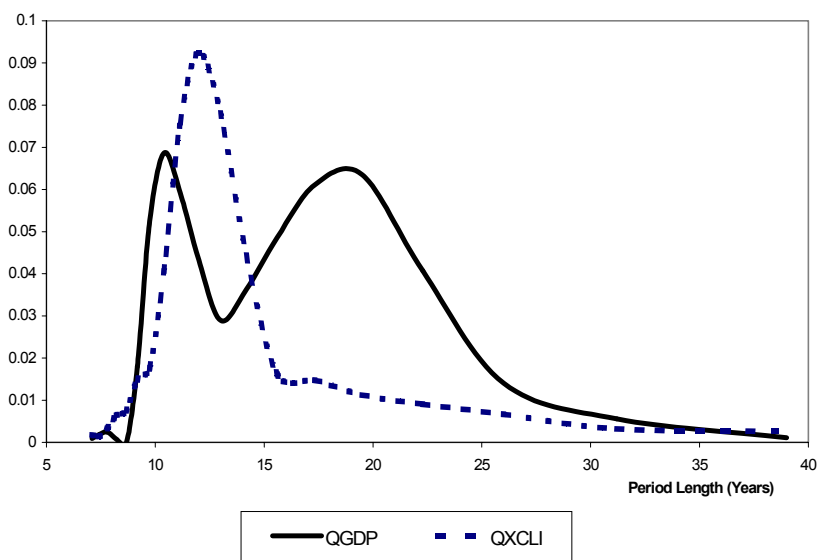
It is worthwhile to momentarily compare the XCLI and QXCLI with the results of Fagan and Fell (1992) for the period 1966 to 1989. The XCLI found 4 troughs while the QXCLI found 7 and these largely coincided except for the extra quarterly cycles. Fagan and Fell (1992) by contrast found 6 troughs at 1970, 1971, 1974, 1980, 1982 and 1986. Close comparison shows that there is good agreement between the indicators up to about 1975. From that point onward there is less agreement and this may be attributed to differing emphasis on what is or is not a trough.

Figure 15: Business Cycle in QGDP and QXCLI



In Figure 16 we plot the periodograms for the business cycles in both QGDP and the QXCLI. The signature for the QXCLI is clearly comprised of cycles of 3 years in length. Examining the signature closely there is evidence, though it is quite faint, of cycles of about 4-5 years.

Figure 16: Periodograms for Business Cycle in QGDP and QXCLI



9. FORECASTS WITH THE EXPERIMENTAL LEADING INDICATORS

Annual Forecasts

In this section we look at the forecasting abilities of both the XCLI and the QXCLI. Two types of forecasts are given. The first is ‘in-sample’ where data from 1990 to 1998 are used. The main function of in-sample forecast is to see how well the XCLI has been able to predict known turning points. Computing the XCLI as the average of the seven indicator series for the period 1990 to 1998 generates in-sample forecasts.

The second type of forecast is ‘out-of-sample’ (i.e. pure forecast). These relate to the period 1999 and beyond where no data currently exists. The approach adopted here involves testing several ARIMA¹² (see Box and Jenkins, 1976) models to identify the one providing the best fit to the XCLI (or QXCLI). This is measured in terms of smallest coefficient of variation (CV)¹³, that is, residual sum of squares error divided by the mean value, for the period 1960 - 1989. This model is used to predict future out-of-sample values. ARIMA modelling, which may also involve taking differences (denoted by d) of data values, attempts to model both auto-regressive (denoted by p) and moving average (denoted by q) patterns in a time series.

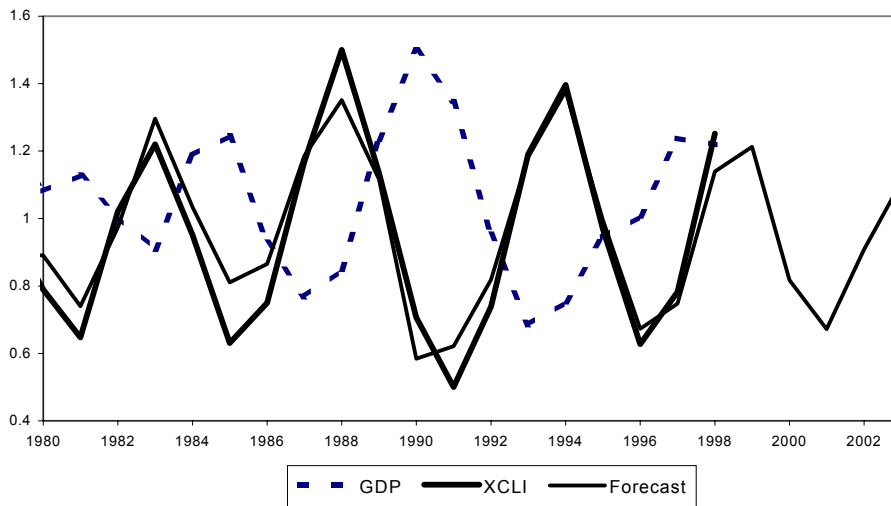
Table 11: Percentage CV Error for ARIMA models fitted to XCLI and QXCLI

% CV	XCLI		QXCLI	
	Actual Series	Log(Series)	Actual Series	Log(Series)
<5	14	18	0	0
5 - <10	133	121	0	0
10 - <20	129	106	295	232
20 - <50	74	69	403	369
>= 50	20	19	21	35
Total	370	333	719	636

In Table 11 we summarise the accuracy of the fit obtained by all ARIMA models tested. For the XCLI series it is clear that there are a large number of models fitting the data with less than 10 percent error. The model chosen from among these and used to produce the out-of-sample forecasts for the period 1999 to 2003 was the $p = 4, q = 3, d = 0$ ARIMA model. It had a CV of about 4.4 percent.

In Figure 17, the business cycle in GDP, the XCLI and ARIMA Model Forecast are plotted from 1980 onward. The period 1990 to 1998 represents the in-sample forecast while 1999 and beyond is the pure out-of-sample forecast.

Figure 17: The XCLI Forecasts



Using the XCLI first, the in-sample predictions show a trough occurs in 1990 followed by a peak in 1994. These two turning points lead corresponding turning

points in 1993 and 1997 respectively in the business cycle. A trough also occurs in the XCLI in 1996. This would indicate, given a cycle length of about five to six years, that the business cycle is going to reach a trough in or about the year 2000.

The ARIMA forecast of the XCLI has a good fit to the actual XCLI. In particular turning points match turning points in the XCLI in all cases except for a slight difference at 1990. The Forecast has a peak in 1999 and trough in 2001. On this basis a peak in the business cycle could be expected in 2002 followed by a trough in 2004 or 2005.

Quarterly Forecasts

Looking once again at Table 11 for the QXCLI series no model had an error less than 10 percent. The best fitting model used to produce the out-of-sample forecasts had $p = 9$, $q = 9$, $d = 1$ with a CV of about 10.7 percent. In Figure 18 the business cycle in QGDP, QXCLI and Forecast of QXCLI are plotted from 1980 onward. Once again the period 1990 to 1998 represents the in-sample forecast while 1999 to 2001 is the out-of-sample forecast. It is clear from the graph that the selected ARIMA model does fit the XCLI values very well.

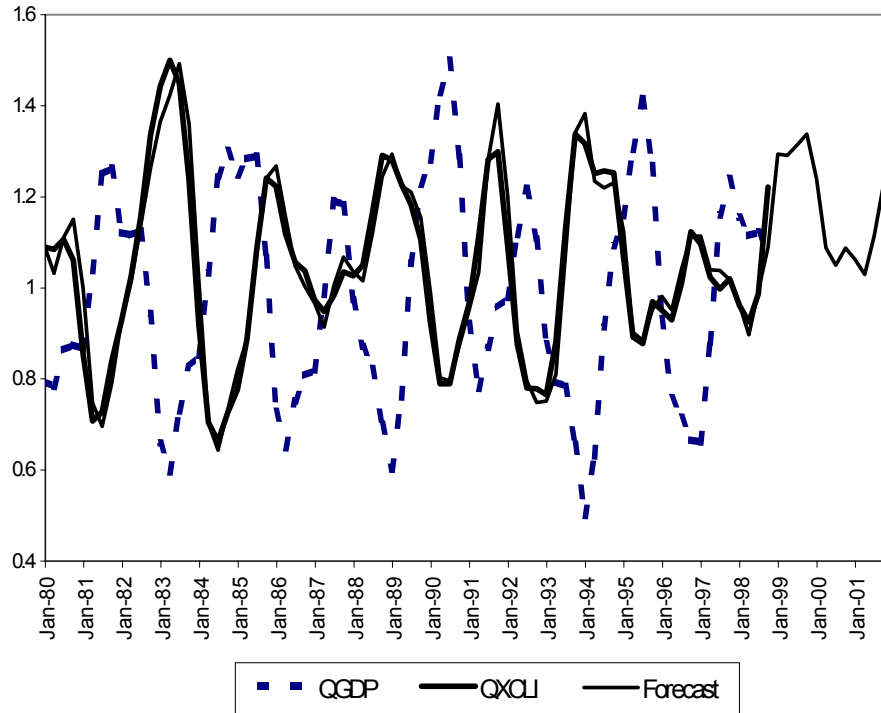
Table 12: Turning points for the Business Cycle in QGDP, QXCLI and Forecast and Quarterly Leads of QXCLI on QGDP

	QGDP	QXCLI	Forecast	Lead/Lag
T	2-91	2-90	2-90	4
P	3-92	4-91	4-91	3
T	1-94	1-93	1-93	4
P	3-95	4-93	1-94	7
T	1-97	3-95	3-95	6
P	4-97	4-96	4-96	4
T		2-98	2-98	
P			4-99	
T			2-01	

The turning points for the Business Cycle in QGDP, QXCLI and Forecast plots are also given in Table 12. Turning points in the QXCLI match those of the Forecast in all but one case. The in-sample predictions of turning points in the QXCLI show a good degree of consistency with those of QGDP. The average lead of the QXCLI at turning points is 4.7 quarters; this figure is slightly lower than the figure of 5.9 derived for the period 1960 to 1990.

On the basis of the trough in QXCLI in the 2nd quarter of 1998, it is expected that a trough will occur in the business cycle of QGDP in 3rd quarter of 1999. From the Forecast it is possible that a peak will be reached in the business cycle in the 1st quarter of 2001 followed by a trough in the 3rd quarter of 2002.

Figure 18: QXCLI Forecasts



10. SUMMARY OF FINDINGS

In this article we have demonstrated that a business cycle with a fairly regular period of five to six years exists in annual GDP data. We have identified an XCLI that cyclically conforms with and leads turning points in the business cycle by an average of two years over the historical period 1960 to 1989. Also, this XCLI has two turning points in 1990 and 1993. These correctly predict actual turning points in the reference series.

In order to generate a quarterly business cycle it was necessary to interpolate quarterly values from the annual GDP series. When the business cycles in GDP and QGDP were compared extra cycles were found in QGDP that were not present in GDP; this was as expected.

A QXCLI based on the components of XCLI was constructed. This led the turning points in QGDP over the historical period 1960 to 1989 though with some variation

in lead-time. The QXCLI also successfully predicted actual turning points in the quarterly reference cycle in the period 1990 to 1998. Additionally, using ARIMA modelling of the QXCLI we showed the possibility that a peak will be reached in the business cycle in the 1st quarter of 2001.

The results we have obtained should be treated cautiously. In particular the Slutsky-Yule effect is a point for concern with any moving average methodology. Our view is that while this may cause problems it does not appear to invalidate our findings. Also, it is worth noting that very different cycles were found in many of the 60 series initially examined. This fact would seem to diminish to some extent the relevance of the Slutsky-Yule effect in this study.

It is important to emphasise the limited nature of this exercise. The reliance of the XCLI on inverted series is a worry as their economic relevance can be hard to justify. Also, the attention has been on trying to forecast turning points that occur once every 2 or 3 years. Therefore, for the majority of the time the only useful information that can be gleaned from the business cycle is that no turn is predicted. Furthermore, the temporal relationships are derived from data analysis rather than econometric modelling (i.e. using structural economic factors) and therefore it is only justifiable to regard a turn in the XCLI as an indication that a turn may be going to occur.

The CLI that has been generated is experimental. Further analysis of the performance of the indicator as new data becomes available will be required before any concrete comment on its usefulness can be made.

Endnotes

1. For a stationary input series, the gain function is in fact the filter's multiplicative effect on the input's spectral density, which describes the contributions of its different frequency components to its variance (see Koopmans 1974).
2. Henderson filters replaced Spencer moving averages for trend extraction in the first release US Bureau of the Census X11 Seasonal Adjustment Program (see Shiskin *et. al.* 1965)
3. In fact with every $M (=2k+1)$ -point symmetric moving average there are k -points at the beginning and end of the time series where a moving average value cannot be calculated. So, for example, smoothing GDP from 1960 to 1998 with an 11-point moving average would result in a smoothed series only covering the years 1965 to 1993.
4. In this sense Henderson filters are low-pass filters.
5. In the Irish case GDP data is only available annually. This constrains significantly the choice of Henderson filters. Henderson filters fit a cubic polynomial exactly and so the shortest possible filter is 5-point. Consequently cycles of less than four years duration in annual data will be significantly damped. To mitigate this drawback we therefore have decided to restrict our attention to cycles of between four and ten years for annual GDP data.
6. A reviewer suggested we base this procedure on the 'Band-Pass Filters' used by Baxter and King. We, however, avoided a frequency domain approach to filter design due both to the relatively short length of the GDP reference series and the impact of Gibbs oscillations (see Oppenheim and Schafer, 1975). The presence of Gibbs oscillations means that a filter based on $n+1$ terms will not necessarily have the same or better gain function characteristics as one based on n terms.
7. The output from the 5-point Henderson filter is divided by the output of the 11-point Henderson filter because the GDP series is assumed multiplicative. This assumption is reasonable given the growth pattern in the series. However, we note that even when the series is treated as additive and the output from the 11-point Henderson filter is subtracted from the output of the 5-point Henderson filter, the resulting business cycle is very similar to that obtained under the multiplicative assumption.
8. Also, as pointed out by one reviewer, to be consistent we should have assumed from the outset that the series was multiplicative and used the $\log(\text{GDP})$. However, even if this change is made it does not significantly alter the timing of turning points in the annual GDP business cycle.
9. Total Investment is derived from Table 17 (Gross Domestic Physical Capital Formation at constant 1995 market prices) and from Table 25 (Central and Local Government – Details of Gross Physical Capital Formation) of the CSO National Income and Expenditure release.
10. The SMD-33 study showed that all series with similar smoothness characteristics to those of GDP could be very accurately interpolated, that is to within a couple of percentage points of error.

11. In fact we constructed business cycles using both linear and cubic spline methods. As both methods gave turning points at the same points in time we adopted the cubic spline method.
12. The spectral density function (or spectrum) is the Fourier transform of the autocovariance function of a time series. It can be interpreted as the decomposition of the variance of a time series. A peak in the spectrum indicates an important contribution to the variance in a small frequency interval.
13. One reviewer has mentioned that the ARIMA models used to generate out-of-sample forecasts are in fact parsimonious models of the respective filters. In fact the combination of the 5 and 11 point Henderson filters used for annual data can be expressed as a 7-term symmetric moving average of the fourth difference of the time series values. Thus the option to forecast by simply projecting forward solely using the moving average is available though we have not used it here.
14. Clearly, AIC or BIC tests could have also been used to select the model. However, our data mining approach to selecting the best model and the quality of fit obtained (see Figure 17) shows that the use of these criteria to select a model is unlikely to improve the result much further.

References

Baxter, M. and R. King, 1995. “Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series”, *NBER 5022*, New York.

Box, G. E. P. and G. M. Jenkins, 1976. *Time Series Analysis: Forecasting and Control*, Revised Edition, Holden Day, San Francisco,.

Bureau of Economic Analysis, 1987. “Programmed Selection of Cyclical Turning Points” in *Handbook of Cyclical Indicators*, US Government Printing Office: Washington D.C..

Burns, A. F. and W. C. Mitchell, 1938. “Statistical Indicators of Economic Revivals”, NBER, New York, 1938.

Burns, A. F. and W. C. Mitchell, 1946. *Measuring Business Cycles*, NBER, New York.

Carlson, G. E., 1992. *Signal and Linear Systems Analysis*, Houghton Mifflin Company: Boston.

Dagum, E. B., 1980. “The X-11-ARIMA/88 Seasonal Adjustment Method”, *Technical Report No. 12-564E*, Statistics Canada: Ottawa.

Doherty, M., 1992. “The surrogate Henderson Filters in X-11”, *Technical Report*, Statistics New Zealand: Wellington.

Fagan, G. and J. Fell, 1994. “Techniques for Forecasting the Irish Business Cycle”, *Technical Paper 2/RT/94*, Central Bank of Ireland.

Fagan, G. and J. Fell, 1992. “Business Cycle Indicators for the Irish Economy”, *Quarterly Bulletin*, Central Bank of Ireland, Winter 1992.

Findley, D. F., B. C. Monsell, W. R. Bell, M. C. Otto and B. C. Chen, 1996. “New Capabilities and Methods of the X12-Arima Seasonal Adjustment Program,” US Census Bureau, Washington D.C..

Frain, J., 2000. “The Slutsky –Yule Effect”, Personal Communication.

Gray, A. and P. Thompson, 1996. “Design of Moving Average Trend Filters using Fidelity and Smoothness Criteria”, *Research Report 96-01*, Statistical Research Division, US Bureau of Census, Washington D.C.

Henderson, R., 1916. “Notes on Graduation by Adjusted Average”, *Transactions (Actuarial Society of America)*, Volume 17.

- Keating, W., 1995.** “Measuring Growth”, *Proceedings of Conference on Measuring Economic Growth*, CSO.
- Kendall, M. G. and A. Stuart, 1968.** “The Advanced Theory of Statistics, Volume 3”, Griffin and Company: London.
- Kenny, P. B. and J. Durbin, 1982.** “Local Trend Estimation and Seasonal Adjustment of Economic and Social Time Series”, *J. R. Stat. Soc. (A)*, Volume 145.
- Koopmans, L. H., 1974.** “The Spectral Analysis of Time Series”, Academic Press: New York.
- Musgrave, J. C., 1964.** “A set of End Weights to End All End Weights”, *Working Paper*, US Bureau of the Census, Washington D. C..
- OECD, 1987.** “OECD Leading Indicators and Business Cycles in Member Countries 1960-1985”, *OECD Sources and Methods*, No. 39.
- Oppenheim, A. V. and R. W. Schaffer, 1975.** “Digital Signal Processing”, Prentice-Hall International (UK).
- Salou, G. and C. Kim, 1992.** “The Business Cycle in Australia: 1952 to 1992”, *Australian Economic Indicators*, Australian Bureau of Statistics.
- Salou, G. and C. Kim, 1993.** “Generating an Experimental CLI”, *Australian Economic Indicators*, Australian Bureau of Statistics.
- SAS, 1990.** *SAS Economic Time Series Manual*, SAS Institute, USA.
- Shiskin, J., A. H. Young and J. C. Musgrave, 1967.** “The X-11 variant of the Census Method II Seasonal Adjustment Program”, *Technical Paper 15*, US Census Bureau, Washington D. C..
- SMD-33, 1999.** “Interpolating, Calibrating and Forecasting”, *Internal Technical Report*, CSO.
- Stock, J. H. and M. W. Watson, 1991.** “A Probability Model of Coincident Economic Indicators”, in K. Lahiri. and G. H. Moore, (eds), *Leading Economic Indicators: New Approaches and Forecasting Records*, Cambridge University Press.
- Sutcliffe, A., 1995.** “Sifting the Signals from the Noise”, in *Australian Economic Indicators*, Australian Bureau of Statistics.
- Wei, W. W. S., 1990.** *Time Series Analysis – Univariate and Multivariate Methods*, Addison-Wesley (UK).

APPENDIX:

Formulae and Gain Functions for Henderson Symmetric and Surrogate Filters
Henderson symmetric filter coefficients h_j ($j = 1 \dots 2k+1$) are calculated as follows:

Define $h_j = 0$ for $j = \pm(k+1), \pm(k+2), \pm(k+3)$.

With

$$q_j(k) = \left[(k+1)^2 - j^2 \right] \left[(k+2)^2 - j^2 \right] \left[(k+3)^2 - j^2 \right]$$

and with the coefficients a and b determined by

$$a \sum_{j=-k}^k q_j(k) + b \sum_{j=-k}^k q_j(k) j^2$$

$$a \sum_{j=-k}^k q_j(k) j^2 + b \sum_{j=-k}^k q_j(k) j^4$$

the Henderson coefficients h_j ($-k \leq j \leq k$) are given by:

$$h_j = q_j(k) \left(a + b j^2 \right)$$

To overcome the end-point problem and provide smoothed values over the whole length of a time series; Musgrave (1964) found the asymmetric filter closest to the corresponding Henderson filter. That is, for an $N (=2k+1)$ -point Henderson filter with weights w_j , he successively identified filters of length $2k, 2k-1, \dots, 2k-d, \dots, k+1$ points having coefficients $v_j, 1 \leq j \leq M = N-d$ ($k+1 \leq d \leq 2k+1$) that sum to one and minimised:

$$E \left(\sum_{j=1}^N w_j x_j - \sum_{j=1}^M v_j x_j \right)^2$$

Using this criterion M. Doherty, in an unpublished report (Doherty 1992, see also Findley *et. al.*, 1996) derived the following explicit formula for the coefficients of these filters:

$$v_j = w_j + \frac{1}{M} \sum_{i=M+1}^N w_i + \frac{\left(j - \left(\frac{M+1}{2} \right) \right) \frac{4}{\pi} R^2}{1 + \frac{M(M-1)(M+1)}{12} \frac{4}{\pi} R^2} \sum_{i=M+1}^N \left(i - \frac{M+1}{2} \right) w_i$$

Here the parameter R describes the ratio of an assumed linear signal trend to noise and is set to 0.333. This choice is similar to that adopted by Findley (see Findley *et al.* 1996) and used in X12-RegARIMA, the latest update of the X11 Seasonal Adjustment program. To extend the business cycle into the period 1994 to 1998 we compute the necessary end point asymmetric moving averages according to the above formula. The resulting moving average filters are normally referred to as Henderson filter surrogates. These are used as substitutes for the symmetric Henderson filter at the end points of time series.

Henderson symmetric filter weights and their surrogate weights (at 3 decimal places) are given in Table A1 for the 5-point moving average filter. Additionally, we plot the gain functions for the 4 and 3-point surrogates in Figure A1 to illustrate the closeness of these as approximations to the Henderson symmetric 5-point filter (also plotted). It is clear from these plots that the quality of moving average resulting from the application of these surrogates at the end points, will be almost as good as that provided by the Henderson 5-point moving average at interior points.

In Table A2 we also provide the weights rounded to three decimal places for the Henderson 11-point symmetric filter and its surrogates. Figure A2 shows the corresponding gain functions. These illustrate that the surrogates provide reasonably close approximations to the symmetric filter. Only the 6-point filter displays slightly different characteristics with some overshoot occurring. This effect will amplify certain cycles slightly and therefore may induce some error. This filter is only used to compute the moving average value at the first and last observation in the time series. So, small errors may be introduced into the business cycles at these points.

Table A1: Henderson 5-point moving average weights

Symmetric 5-point filter	-0.073	0.294	0.558	0.294	-0.073
4-point surrogate filter	-0.069	0.283	0.534	0.252	
3-point surrogate filter	-0.041	0.367	0.674		

Figure A1: Gain function for Henderson Surrogate Filters

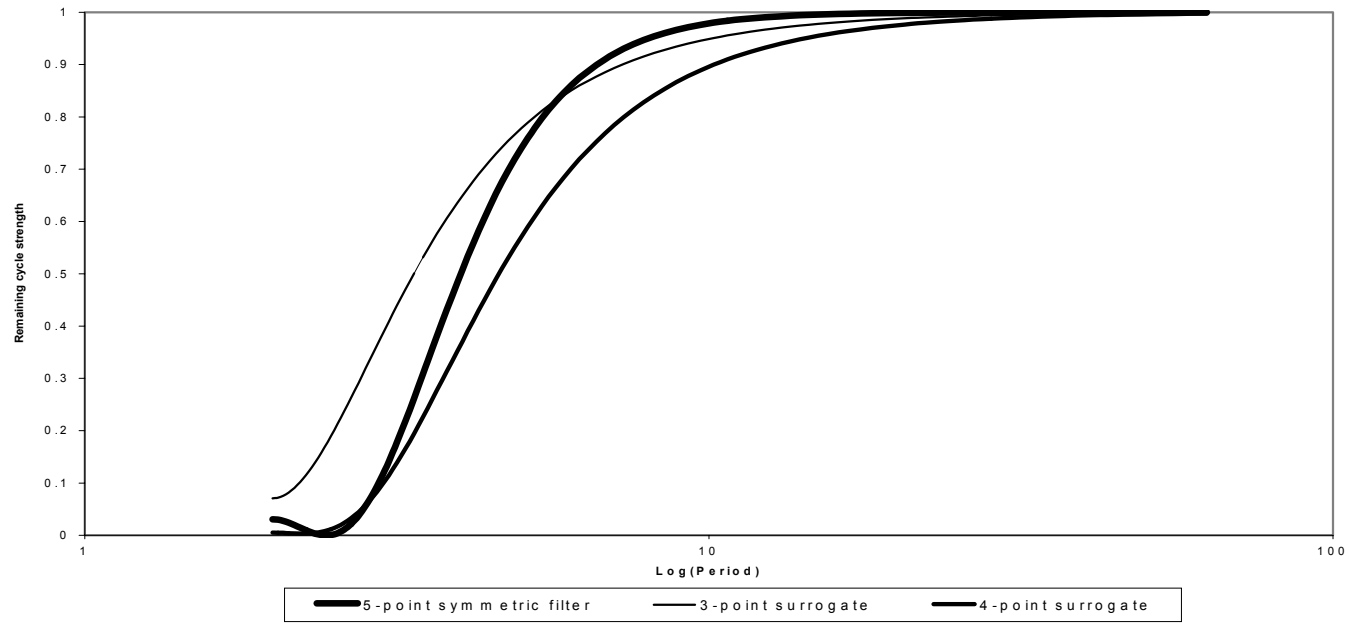
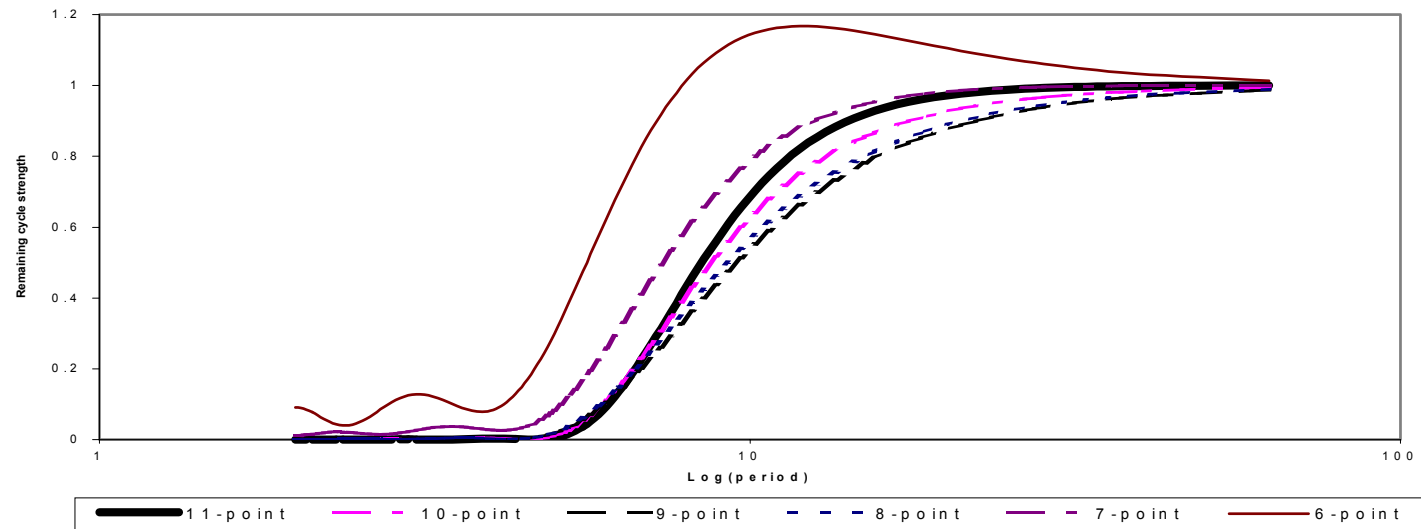


Table A2: Henderson 11-point moving average weights

Symmetric 11-point	-0.028	-0.027	0.036	0.141	0.239	0.278	0.239	0.141	0.036	-0.027	-0.028
10-point surrogate	-0.023	-0.023	0.037	0.141	0.237	0.274	0.233	0.134	0.027	-0.037	
9-point surrogate	-0.016	-0.019	0.039	0.140	0.232	0.267	0.224	0.121	0.012		
8-point surrogate	-0.018	-0.021	0.038	0.140	0.235	0.270	0.227	0.129			
7-point surrogate	-0.043	-0.032	0.042	0.159	0.267	0.318	0.289				
6-point surrogate	-0.098	-0.045	0.070	0.227	0.377	0.469					

Figure A2: Gain function for Henderson 11-point filter and surrogates



DISCUSSION

Mr. Jim O’Leary: This paper records the authors’ progress in their attempt to construct a leading indicator of economic activity in Ireland. It is one of only two such attempts that have been made and reported in the public arena. The other was that of Fagan and Fell in 1992, of which more anon. The authors also make passing reference to an OECD exercise in the same vein in 1987, but I believe that it was too slight to be comparable with the present one.

One of the first questions that must be posed before embarking on this sort of exercise is of what precisely does one want to construct a leading indicator? The answer provided will hugely influence the course of the subsequent work, including the methodology adopted, and the usefulness of the results. Dalton and Keogh have decided that the focal point should be GDP. Fagan and Fell, by contrast, judged GDP to be a seriously flawed measure of the Irish business cycle, and constructed a coincident cyclical indicator as an alternative.

It is worth recalling the principal objections to using GDP. They are as follows.

- GDP is available only on an annual basis, at least up to 1997. The GDP series, therefore, may hide cycles of relatively short duration.
- Sector-specific shocks may impact on GDP but the result may not be in keeping with the standard definition of a cycle, which emphasises the simultaneity of expansions and contractions across many economic activities
- GDP is not strongly correlated with economic aggregates, like employment, which are widely and reasonably held to be important cyclical indicators.
- Estimates of GDP are liable to substantial routine revisions as well as to major methodological change from time to time. Amongst the undesirable consequences of this is the emergence of significant discontinuities in the series.

It seems to me that these points together amount to a powerful case against using GDP, one that I don’t believe is materially weakened by the authors’ counter-arguments. Moreover, I am entirely unconvinced that Dalton and Keogh’s interpolation of the annual GDP data to produce a quarterly series is an adequate way of dealing with the periodicity problem. The quarterly results reported in their paper look quite odd, a point that I will elaborate on later.

One of the consequences of focussing on GDP is that the authors are constrained to adopt a ‘growth cycle’ approach rather than a classical cycle approach to the exercise. Why? Well, the classical cycle is predicated on the existence of periods of absolute decline in economic activity; the ‘growth cycle’ approach requires only that significant differences in the (positive) rate of growth occur over time. According to the national accounts, there has never been a year-on-year decline in Irish GDP, at least over the period since 1960. So, a GDP-based investigation of the Irish business cycle, in the classical sense, would be a nugatory exercise. Fagan and Fell’s work,

on the other hand, generated a coincident indicator of economic activity that evinced periods of absolute decline.

In order to identify the cycles in GDP, the authors adopt a methodology that involves the application of symmetric weighted moving average filters, or Henderson filters. The use of this methodology has a number of less than desirable implications. One of these is that the cycles extracted from the annual GDP series must be at least four years long. To put this in perspective, Fagan and Fell's work identified seven cycles in Irish economic activity between 1965 and 1990, of which four were of less than four years duration.

Other consequences of the methodology emerge in the results. The authors' representation of business cycles in Figure 3 of their paper suggests that the amplitude of cycles has doubled over the last forty years. The 1990-1993 downswing, as captured in this graph, conveys the impression that the economy fell off a cliff. From this they infer that booms and recessions are becoming more pronounced.

I have two points to make about this. The first is that this inference, though reasonable on the face of it, is not valid. The second is that I have misgivings about a methodology that invites such inferences to be drawn.

Table 1: Irish Economic Performance in Downswings

(% p.a. average)	GDP	GNP	Employment
1990-1993	2.6	2.3	0.7
1985-1987	2.4	1.8	0.5
1978-1983	1.9	0.7	0.3
1973-1976	2.4	1.9	-0.1
1969-1971	3.9	3.6	-0.8
1964-1966	1.3	1.5	-0.2

What actually happened to GDP in the 1990-93 period, and how does this compare with its peak-to-trough behaviour in earlier cycles identified by Dalton and Keogh? The accompanying table provides the answers. GDP increased at an annual average rate of 2.6 percent between 1990 and 1993. This was faster than its rate of growth in all but one of the five downswings over the previous thirty years. The same is true of GNP. A similar conclusion also holds in respect of other cyclical indicators, such as economy-wide employment. Indeed, in the case of employment, the rate of increase in the 1990-93 downswing was faster than in all previous downswings, three of which produced absolute reductions in the total number at work.

So what are we to make of the fact that Dalton and Keogh's methodology represents the 1990-93 downswing as the steepest by far in the post-war era? Not that it was the most pronounced recession, because recession it wasn't. Rather, that it was a period during which the rate of growth in GDP was particularly low relative to the

years either side of it, namely 1987-90 and 1994-98. This is clearly not as apocalyptic a conclusion as the authors paint.

So far my remarks have concerned the measurement of the business cycle. Let me now turn to the Experimental Composite Leading Indicator (XCLI) itself. Following a truly exhaustive selection process, which involved 30,000 different combinations from 25 data series, the XCLI was generated from seven short-term economic indicators. I list them, together with some other relevant information, in the accompanying table.

As for the indicators selected, they are not especially remarkable, at least in their raw or untransformed state. Variables reflecting the terms of trade, long-term interest rates and the state of the US and UK economies also featured in Fagan and Fell's Leading Indicator. New car sales were also incorporated by Fagan and Fell, but in their Coincident Indicator. The Livestock variable is an interesting inclusion by the present authors, though not one that would have struck me as obvious, or as especially easy to rationalise on intuitive grounds.

Table 2: Components of XCLI

Indicator	Transformation	Average Lead (+)/Lag (-)	
		Raw Data	Transformed
Livestock index	Inverted	-0.3	+2.8
New car sales	Inverted	+0.1	+2.7
Terms of trade	Lagged	+1.1	+0.1
Long-term interest rate	Inverted	-0.7	+1.9
Total investment	Lagged & inverted	-0.5	+1.4
UK GDP	None	+0.7	+0.7
US GDP	Inverted	+0.0	+2.1

What troubles me rather more than the inclusion of a livestock variable is the way in which the data have been transformed. For example, no fewer than five of the seven series have been inverted. I know this practice has been followed elsewhere. But it has typically been done more sparingly. In the US Dept of Commerce Leading Indicator, for example, just two out of eleven of the components were inverted series.

In any event, despite inversion (and, in most cases, the resultant transformation of the series concerned from a lagging to a leading indicator), some of the indicator components evince highly variable lead lengths. This is especially true of US GDP, where the lead length range is no less than 6 years.

Two of the seven components are lagged: Terms of Trade and Total Investment (which is also inverted). I have great difficulty getting to grips with the rationale for this. Take the terms of trade, for example. Table 4 of the paper indicates that the terms of trade on average lead GDP by about one year. In other words, a turning

point in the terms of trade in year t is in loose terms indicative of a turning point in GDP in year $t+1$. If that is so, what sense does it make to lag the terms of trade?

One of the most heroic aspects of the paper is the authors' attempt to identify a quarterly business cycle. This is clearly an exercise where the construction of an index of economic activity along the lines of Fagan and Fell's Coincident Indicator, commends itself as the way to proceed, particularly in the absence of quarterly national accounts.

Instead, Dalton and Keogh generate a quarterly GDP series from the available annual data. They do so by fitting a smooth cubic spline curve through the annual data, constraining the sum of the quarterly estimates to equal the annual total. No doubt, the resultant quarterly series is consistent with the annual series. However, the quarterly series cannot in my view be regarded as a true reflection of the actual quarterly pattern of GDP, since there is no increment of information about GDP over and above that contained in the annual series incorporated in it. Much less can it be taken as a true measure of the quarterly pattern of overall economic activity.

Figure 13 in the paper portrays the estimated quarterly business cycles in GDP. To people acquainted with the behaviour of the Irish economy, especially over the past decade or so, it paints an unfamiliar picture. The period from 1987 to 1997 is represented as containing no fewer than four cycles with peaks in 1990, 1992, 1995 and 1997. This is not so much a case of the Celtic Tiger as the Celtic Kangaroo!

My intuition is that this pattern of apparently frequent and high amplitude cycles is a result of using Henderson filters on quarterly data that has been interpolated from an annual series. The effect of using this methodology is, I believe, to greatly amplify year to year variations in the rate of increase in GDP.

While the results may provide an interesting statistical description of what might be called 'gear changes' in the rate of GDP growth, the economic significance of these changes is questionable. As far as GDP (or GNP) growth since 1993 is concerned, it is overwhelmingly more interesting, to an economist at least, that the average rate has been almost 9 percent than that there have been year-to-year variations of 2-3 percentage points around this average. I thank the authors for stimulating paper.

Mr. John Frain: This paper deals with the identification and forecasting of turning points in the Irish business cycle, a topic that apart from Fagan and Fell (1990, 1992), has been much neglected. The importance of this kind of business cycle analysis can not be understated. It is a basic input to both theory and practice in economics. Any satisfactory economic theory must be able to simulate the basic fluctuations of the business cycle. Politicians, entrepreneurs, employers, employees and most other persons all appear to have an built-in understanding of the business cycle. However, if you asked them to explain any definitions they might provide might not be of any use to the statistician or economist in any analysis of the cycle. For a government an understanding of the business cycle is of the great importance

as the appropriate stance of fiscal policy depends on the stage of the cycle. Such data and an indicator that can predict movements in the data are a valuable tool both for the making of policy and in explaining its relevance and effectiveness.

Internationally there has been an immense volume of research completed on the business cycle. The work of the National Bureau of Economic Research in (NBER) the US is a prime example of what can be done if one had the data and the resources to analyze them. Many methodologies that do not require the NBER resources have been proposed but none has been found to be uniformly best. Padraig Dalton and Gerard Keogh present a business cycle analysis based on Henderson filters. They are to be congratulated on an excellent paper that gives a full account of this method and of its implementation for Ireland. This is a valuable addition to the existing work in Ireland and I would appreciate further work on this topic.

In my comment here I will deal mainly with the statistical fundamentals of business cycles and highlight the difficulties that arise from the use of linear filters in the identification of the cycle. My aim is twofold. First I wish to identify the various hurdles that must be tackled before the turning points in the cycle can be identified. I hope in this way to indicate some of the directions that further work may take. Secondly I would like to warn users about the problems that may arise from the inappropriate use of linear filters to extract a business cycle signal. Much of what I will be saying also applies to the Hodrick Prescott (HP) filter (1997) and the band-pass filters of Baxter and King (1999). Linear filters of this kind do have the considerable advantage that they are objective, can be replicated and do not require any element of subjective judgment

The history of the business cycle is long and has generated a considerable amount of controversy over the years. There is no universally accepted method of measuring and predicting the cycle. We can profitably spend some time looking at the history of the identification of the business cycle paying particular attention to the various problems that have been encountered. This will also help to put my later comments in context. I would tend to view the present analysis as providing initial estimates of the cycle but I think that further analysis is required before it is truly relevant for policy making.

- William Stanley Jevons is well known for his work in economic theory, and statistics. His empirical work on index numbers, the price of gold, time-series of meteorological and economic data etc. were of particular importance. Jevons was the most successful “social physicist” of his day. He believed in the application of probability theory to uncertainty in economics and tried to establish a mathematical/probabilistic way of thinking about the social sciences and economics, in particular. In the period 1875 to 1882 he expounded a considerable amount of energy in the pursuit of a relationship between business cycles and sunspots. It is extraordinary that someone with his talent and ability should have spent such a period in pursuit of a theory that, today appears to be

ridiculous and, even at the time, was a subject of considerable amusement to many of his contemporaries.

The editor of the Journal of the Statistical Society (Robert Giffen of Giffen good fame, March 1879) published an anonymous satirical note entitled "University Boat Races and Sun-spot cycles" that showed that sunspots favoured Cambridge over Oxford in the annual boat race. The two explanations for Jevons' persistence are as important today as they were then and are, even today often forgotten. First unlike his other empirical work which was based on sound economic theory his sunspot analyses were not. It is likely that without the anchor of theory he allowed his own prejudices and imagination to guide his modelling work. Second it must be remembered that the methods of measuring the correlation between time series were not well understood at the time. The methods in use would often lead to spurious results. In some respect Jevons was in advance of his time and would have been more at home sixty years later working with the likes of Frisch, Haavelmo, Koopmans, Tinbergen and similar spirits.

- Clement Juglar (1862, 1889) looked at financial crises in France, England and the USA. He found that they occurred at similar dates in all three countries in the nineteenth century. The cycles he found were different in length, in amplitude but the sequence of events that lead up to each crisis was similar. He could not find any regularity in the period between crises observing that they occurred once every five to ten years. He rejected Edgeworth's theories of regular cycles and their association with a sunspot cycle as they did not conform to the data as he observed them. He believed that if there was a cycle then changing conditions of trade must effect its periodicity and, thus, a regular cycle such as the sunspot cycle could not be the main determinant of the business cycle. Juglar's work was descriptive rather than statistical. He differed from Jevon's in that he sought an explanation of the cycle from within the economic cycle. It appears that his work was well received at the time.
- Henry Ludwell Moore (1914, 1923) was another person who sought an explanation from outside the economic cycle. One of his later theories related the cycle to movements in the planet Venus which passes between the earth and the sun at eight yearly intervals. Moore's analysis was better received than Jevons'. Compared to Jevons he used more elaborate (but not necessarily better) statistical methods that were then better known and appreciated by a larger number of his contemporaries. Perhaps he was also working somewhat before his time. Looking back, it is hard to understand how one could accept such unrealistic conclusions. Perhaps his contemporaries were blinded by his statistical arguments. To-day one would realise that either the theory or the statistical method or both were wrong.

- George. Undy Yule and Eugen E. Slutsky independently made a considerable contribution to our understanding of the business cycle. Yule (1927) showed how cycles similar to the sunspot cycle could be generated by a second order difference equation subject to random shocks. Slutsky (1927) showed that if a shock to a system extended over more than one period then business type cycles would be generated. In simple terms they showed that business cycle behaviour could arise from either or both random shocks impinging on the economy and the mechanisms which transmitted these shocks to the economy. This reasoning is the origin of the Slutsky-Yule effect described below. Yule (1926) contained a series of simulation exercises which demonstrated clearly how spurious regressions can arise in the type of correlation analyses carried out by those who sought to explain the business cycle. It should be noted that appropriate theoretic foundations (cointegration etc.) to deal with these problems have only been put in place in the last twenty or so years.
- W. C. Mitchell (1913, 1927) and A. F Burns and Mitchell (1949) have had probably the greatest influence on business cycle measurement this century. In 1913 Burns argued that each cycle was different and worthy of analysis on its own merits or demerits. Statistical averaging was therefor not appropriate. There are considerable differences between Moore's careless but imaginative use of economic and statistical theory and Burns caution.
- The work of Frisch, Haavelmo, Koopmans, Tinbergen lead to the specification of to-days macro-econometric models which seek to explain and forecast the business cycle as a process generated from within the economy. Such econometric models are, in effect stochastic multivariate difference equations and are elaborate versions of the systems considered by Yule.
- Luckily modern business cycle analysts are unlikely to suffer the fate of Nicolai Dimetrieievich Kondrattieff of long cycle fame. As a direct result of his analyses he was banished to Siberia and the Moscow Institute which he had founded was closed. His long wave theory was dismissed as wrong and reactionary.

Today the measurement of business cycles is well established in most western economies. While the business cycle chronology of the National Bureau of Economic Research is accepted as a standard in the United States their methodology is not feasible in many other countries. In most countries there are simply not sufficient quarterly or monthly time series of sufficient quality. For example Burns and Mitchell (1946) used 199 individual quarterly or monthly series for dates after 1890 and 655 after 1920. Thus an alternative methodology must be found. The Henderson linear filters used in this analysis are only one way of addressing the problems. Economists more often use the linear filters of Hodrick and Prescott (1981, 1997). Alternatively the filters proposed by Baxter and King (1995, 1999) may be theoretically better than either of the alternatives of Henderson or Hodrick-Prescott but as yet are not as widely used as those of Hodrick-Prescott. Analyses of

comovement of time series in dynamic settings can also make use of the autocorrelation function and the spectral density function.

The vector autoregression (VAR) methodology introduced by Sims (1980) is another approach but, in my opinion, this approach is probably better directed at specific problems rather than the general assessment of the cycle. Another approach uses a technique known as dynamic factor analysis. This approach is based on an assumption that many macroeconomic variables are driven by a common set of shocks known as factors. This analysis is used in asset markets where asset prices are determined by fundamentals which themselves may have factor structure. The dynamic factor approach can also be extended to account for the fact that the behaviour of the economy is different in the recession and expansion phases, Hamilton (1994). The asymmetry in the recession and expansion phases business cycle is a basic feature of the Burns–Mitchell-NBER approach that can not be accommodated in a linear filter approach. The filtering methodology has the considerable advantage that it is objective and its result can be easily replicated. There are, however, a number of problems in using a linear filter approach. Below I give more details on these problems. I should point out that this work is very time consuming and that the amount of resources put into this work in other countries is a large multiple of those devoted to it in Ireland

One may ask why so little attention has been paid to the subject in Ireland. Perhaps it was thought that because of the openness of the Irish economy the Irish business cycle might follow that of its main trading partner(s) and that there was no need for a separate determination of an Irish cycle. This is now, and probably always was, an oversimplification of the true situation. I would suggest that the determination of the cycle is more difficult than is often thought. It requires a considerable amount of data and calculations to be completed. These problems would have been accentuated before the current availability of relatively cheap computer resources.

I have several more specific points to make about the methodology, contents and results contained in the paper. These are given below under the following headings

- Additive or Multiplicative Adjustments
- Slutsky-Yule effect
- End Point Problems
- Comparisons with Fagan and Fell (1992)
- Indicators
- Use of Box-Jenkins Methodology

Additive or Multiplicative Adjustments

In an analysis such as this a time-series (e.g. GDP) is considered as being generated by either adding or multiplying together the trend, cycle and irregular components (and any other relevant components). If the series is additive we may write the series in the following form

$$y_t = \tau_t + \gamma_t + \varepsilon_t$$

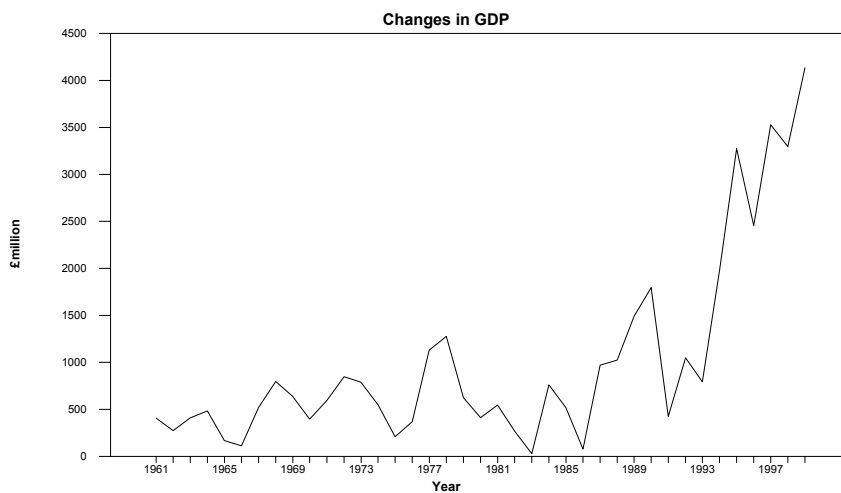
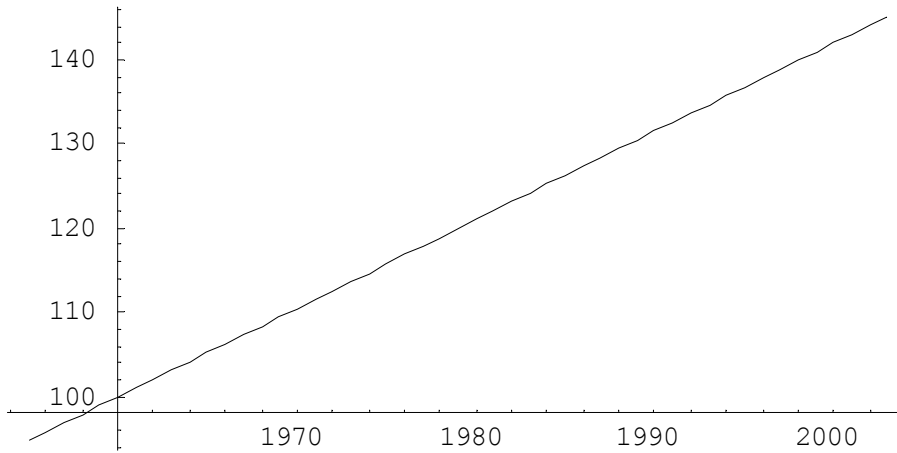
If the series is multiplicative it may be written

$$y_t = \tau_t \cdot \gamma_t \cdot \varepsilon_t$$

where τ_t , γ_t and ε_t are, respectively the trend, cycle and irregular components

The use of Henderson filters in the first two steps of the procedure set out in the paper require that the process be additive. This is not a great disadvantage as a multiplicative process can be transformed to an additive one by applying a log transformation to it. In the X12 and TRAMO/SEATS seasonal adjustment programs multiplicative series data are transformed, in this way, before analysis to make the process additive. The third step in the procedure in the paper assumes that the process is multiplicative. One can not assume that the process is both additive and multiplicative. If the series is multiplicative the assumptions that underlie the application of the Henderson filters to their levels do not hold.

The graph below shows year on year changes in GDP from 1961 to 1999. In an additive process the value of the changes are of similar magnitude as the value of the series grows. In a multiplicative process the changes in value will tend to increase as the value of the series increases. The multiplicative effect is most obvious when one looks at the later values in the series. As one adds further data the multiplicative effect will become more obvious and will have a more marked effect on the results. Dalton and Keogh (endnote 9) argue that the use of log transformed data gives similar results and does not significantly effect the timing of turning points. Certainly the sequence of expansions in early years followed by a prolonged depression may to some extent have hidden the true multiplicative nature of the GDP series. Clearly the analysis can be affected as the series is extended and the multiplicative effects become more important. If the assumption of linearity is lost there is no body of mathematical theory to act as a foundation for the analysis.



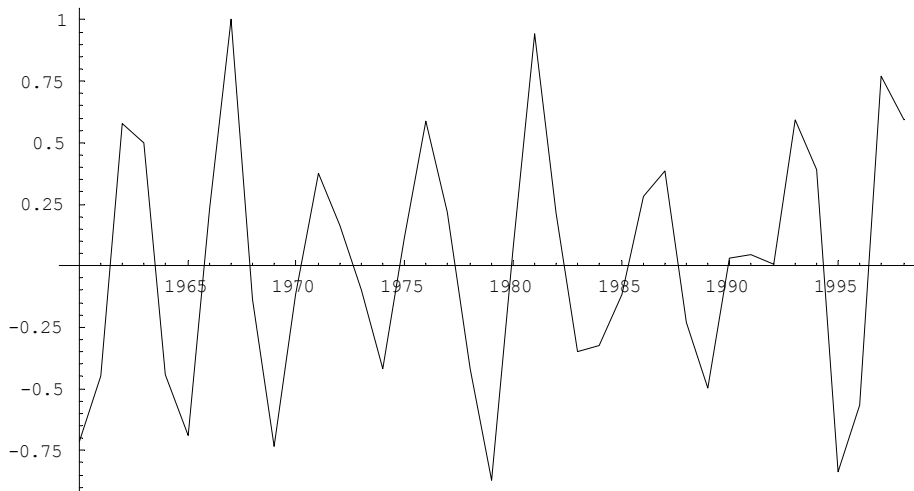
Slutsky-Yule Effect

When a linear filter derived from the 5-point (H5) or 11-point (H11) Henderson, as used in the paper, is applied to a time series which consists of uncorrelated normal white noise ε_t , the output of the filter ε_t^* will contain more or less regular oscillations of a form similar to those of business cycles. This effect is due to the non-zero correlations induced by the filter in ε_t^* . This effect is known as the Slutsky-Yule effect and may be illustrated as follows. Use a random number generator to simulate the series

$$y_t = 100 + 1.05 (t-1960) + \varepsilon_t, \quad t = 1950, \dots, 2002, \quad \varepsilon_t \text{ are IIN}(0,0.1)$$

This series consists of a simple trend and a random disturbance term. A sample realisation of this series is given in the graph below. A linear trend is used in this example but this could be replaced by a quadratic or cubic trend with the same result as the filter removes all such trends. There is no cyclical component in this series.

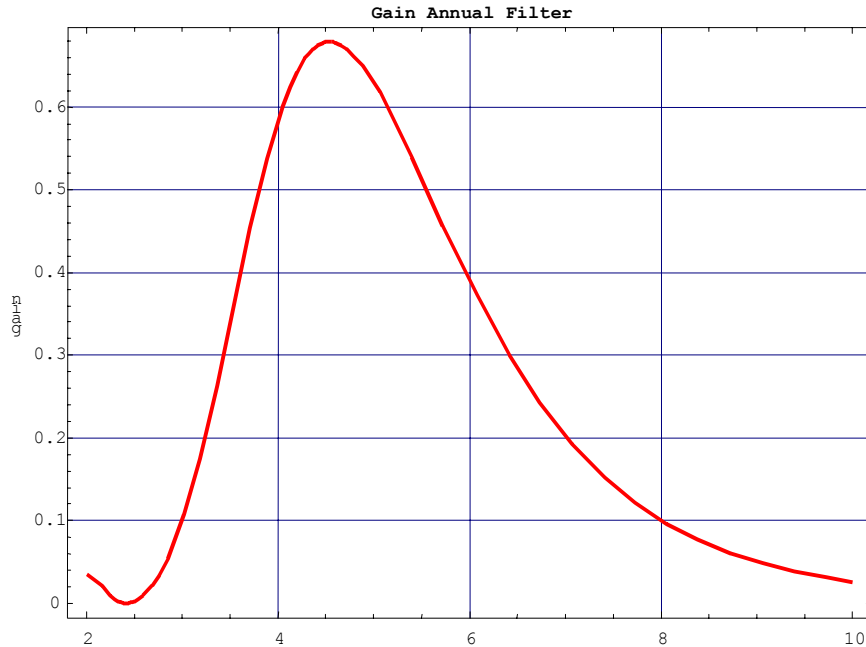
A filter H5-H11 is applied to this series (all effects treated as additive). The resulting filtered series is given in the following graph



This graph is somewhat similar to the GDP graph in the paper. A form of ‘business cycle’ has been introduced into the filtered series even though there is none in the original series. This experiment can be repeated and will give consistent results on each replication.

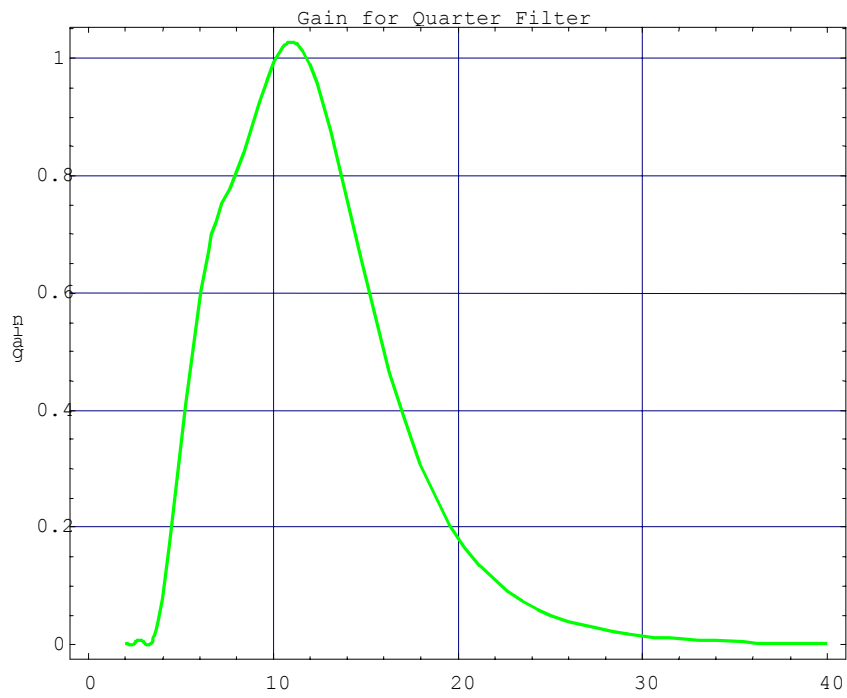
The realisation in the graph is similar, apart from a phase effect, to those in the paper. It leads each turning point in the GDP series by almost 3 years. One might argue that it would make an excellent leading indicator and I will return to this point later.

The gain function of a linear filter shows the effect of the filter at different periodicities. The graph below sets out the gain function of the H5-H11 filter used. Recalling that a white noise term, such as ε_t above, can be thought of as a uniform mixture of all frequencies one can see that the output of the filter applied to a white noise term will have greatest content at a periodicity of 4.54 years. This is very close to the average frequency of the cycle found in the paper for annual GDP data.

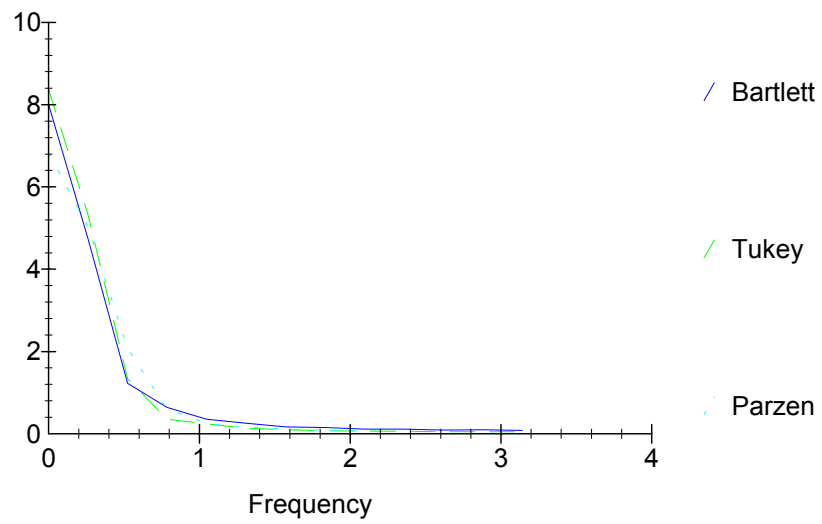


The gain function of the H7-H33 filter is given on the diagram below. The maximum gain is at 11 quarters which corresponds to the first peak of the spectrum of the filtered quarterly series as given in figure 16 in the paper.

Guay and St-Amant (1996) examine the effects of mechanical filters such as the Hodrick-Pescott (1981) filter and the optimum filters of Baxter and King (1995, 1999) when used as approximations of the business cycle. They show that these filters perform adequately when the spectrum of the original series has a peak at business cycle frequency. When the spectrum is dominated by low frequencies the filters provide a distorted business cycle.



Various estimates of standardized spectral density of Log(GDP)



The diagram above shows three estimates of the pseudo-spectrum of the log of the GDP series for Ireland. Regardless of the kernel used in estimating the pseudo-spectrum it is dominated by low frequency noise and there is no peak at the business cycle frequency. Similar estimates of the spectra of GDP, the first difference of GDP and of the first difference of log(GDP) do not show a peak at a business cycle frequency. While Guay and St-Amant (1996) do not analyse the Henderson filters used in this analysis their results are likely to apply to similar band-pass filters. The lack of peak in the graph at a business cycle frequency implies that the output of the filter gives a distorted measure of the cycle.

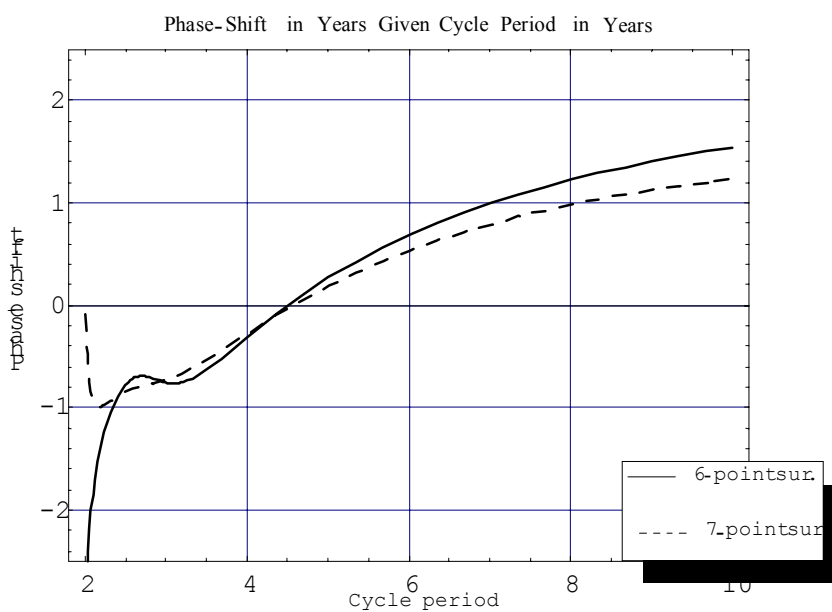
End Point Problems

All symmetric moving average filters of order $2r+1$ require r observations before and after the observation being filtered. Thus when such a symmetric moving average filter is applied to a series the final r observations are lost. This is known as the end-point problem. The solution proposed in Dalton and Keogh is to use asymmetric surrogate Henderson filters for the final values. This was the approach adopted in the X11 seasonal adjustment program. There are two problems with this approach.

- First, as new data become available a different filter is applied to the later points in the series and this will be a source of revisions in the data. The current best practice in seasonal adjustment is to use an optimal forecast to extend the data-series and then use the symmetric filters on the extended data-series. This is the practice followed in the newer seasonal adjustment programs such as X11-ARIMA, X12 and in TRAMO/SEATS.
- Second, as these filters are asymmetric they may advance or delay the observation of turning points in the cycle. This advance or delay is known as the phase shift of the filter. The diagram below shows the phase shift for two of the surrogate filters used in the annual analysis namely:
 1. the 6-point surrogate filter which is the effect of applying the Henderson 5point 3-point surrogate filter in step 1, the Henderson 11 point 6-point surrogate in step 2 and then subtracting the results in step 3 and
 2. the 7-point surrogate filter which is the effect of applying the Henderson 5point 2-point surrogate filter in step 1, the Henderson 11 point 5-point surrogate in step 2 and then subtracting the results in step 3.

If the period of the latest cycle is just over four years in length the surrogate filters measure it accurately. If the cycle is a long and about six years in length it will be missed by the surrogate filters and they will observe it more that a year after it has occurred. If the cycle is short and about two years in length the surrogate filters will give a false signal at one year in length. The diagram shows that the addition of an extra data item does not significantly reduce the phase shift. This property of the

surrogate filters makes them unsuitable as indicators or early warnings of turning points of the cycle



Comparisons with Fagan and Fell (1992)

As the Fagan and Fell analysis is based on monthly data it is somewhat difficult to compare the results of both analyses. In the 33 years from 1961 to 1993 Fagan and Fell find 15 turning points in the cycle. They find two turning points in each of the years 1970 and 1971. Thus they find turning points in 13 years in the period.

Dalton and Keogh also find turning points in 13 years in the period. To this extent the studies agree. However there is considerable divergence in the timing of the turning points. Of the 13 years turning points found in this paper, Fagan and Fell have similar tuning points in 5 years, 7 turning points in different years and in 1990 Dalton and Keogh have a peak while Fagan and Fell have trough.

Some explanation or comment on the divergence would be welcome

Indicators

If the filtering of the basic GDP series can insert a cycle into the filtered series it is also likely that applying the same filter to the indicator series can insert a similar cycle into the indicator series. In the analysis of a series, consisting of a pure linear

trend and a normal white noise term, we were able to generate what appeared to be a good leading indicator of the cycle. This result was, of course, completely spurious. If we apply the filter to two series that have trend and significant random components we are likely to generate cycles of similar periodicity in both series but they will be out of phase. Thus one will be found to lead or lag the other. There is a grave danger that the application of the same filter to GDP and the indicators has inserted a cycle in both series. It would have been preferable if unfiltered indicator series were used as components of the XCLI.

I would be concerned at the quantity of data mining that went into finding the XCLI. A similar approach in econometrics without the guidance of a sound theory would be lead to over-fitting and bad forecasts. The inverted livestock index is said to lead the business cycle by about a half a cycle. This means that a trough in the inverted livestock index will predict a trough in GDP. Equivalently a peak in the livestock index will predict a trough in GDP in over 2 years time. Alternatively the livestock index and GDP are coincident. It would be my understanding that the livestock index depends on agricultural input and output prices and weather over a number of previous years. While it contributes to GDP on a contemporaneous basis the different causes of changes livestock output and GDP would incline me to think that one would not be a good predictor of the other. Is this a practical example of a Slutsky-Yule effect? If one inverts any coincident indicator variable the inverted indicator will appear to lead (or lag) by half a cycle. How does one decide when a variable is coincident or its inverted form is leading or lagging. This is particularly difficult when the cycles are as regular as those found in this analysis

Use of Box-Jenkins Methodology

When expressed as polynomial operators in the lag operator both the H3-H11 annual and H7-H33 filters contain a factor of $(1-L)^4$, where L is the lag operator. Thus the filtering operation can be completed in two steps. First the series is differenced four times. Then a linear filter is applied to the result of this first step. For example the H3-H11 filter may be written

$$\frac{3(-1+L)^4(78+387L+980L^2+1309L^3+980L^4+387L^5+78L^6)L^{-5}}{8398}$$

The result of the application of this filter to an economic time series will make it non-invertible. The application of the Box-Jenkins methodology is problematic in such cases.

An ARIMA(4,0,3) as found for the XCLI series is over-parametrised. This is particularly so for a series with less than 40 observations. I would think that the model chosen is derived from the filter and not the data. The same can be said about the ARIMA(9,1,9) model found for the quarterly QXCLI.

Perhaps the authors could clarify their way of identifying the appropriate Box-Jenkins models. I would be concerned about the use of the sample coefficient of variation for that purpose. In a model fitted by least squares the sum of the residuals will be zero and dividing by zero (or by a very small number caused by computer rounding errors) will eliminate such a model even if it is correct. Even if the residual mean value is taken over some sample sub-period or forecast period its expected value is zero and a similar problem will occur. As the expected value is the same in all cases there is no need to standardise the residual sum of squares error to compare models. The use of the residual mean square error is also inappropriate as this will decrease as extra coefficients are added to the system. Information criteria such as AIC or BIC are widely used for such identification and are more appropriate.

The analysis of the XCLI suggests that the Irish economy peaked in 1997 and that the decline following that peak will end in 2000. The quarterly analysis times the peak in the 4th quarter of 1997 and the decline ending in the 3rd quarter of 1999 (see Section 9). I would be very concerned about these results.

Conclusion

The measurement of business cycles is very difficult. I have commented at length on this paper with a view to encouraging further work in this important field in Ireland and I would encourage the authors to continue their work. I am particularly pleased to see CSO staff completing and publishing analyses of this kind. Both users and producers of data can benefit from the resulting debate on the topic. Finally I will close by congratulating Padraig Dalton and Gerard Keogh on their paper and on behalf of the society it gives me great pleasure to propose a vote of thanks to them on behalf of its members.

References

Baxter, M. and R. G. King, 1999. "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series", *The Review of Economics and Statistics*, Vol. 81(4): 575-593

Burns, A. F. and W. C. Mitchell, 1913. *Measuring Business Cycles*, New York, NBER.

Hamilton, J. D. 1994. "State-Space Models", in R. F. Engel and D. McFadden (eds), *Handbook of Econometrics, Vol. 4*, Amsterdam, Elsevier Science.

Hodrick, R. J. and E. C. Prescott 1997. "Postwar U.S. Business Cycles: An Empirical Investigation", *Journal of Money Credit and Banking*, Vol. 29.1.

Juglar, C. 1862. *Des crises commerciales et leur retour periodique en France, en Angleterre et aux Etats-Unis* (2nd edn 1889), Paris: Guillaumin

Mitchell, W. C., 1913. *Business Cycles and their Causes*, Berkeley, California University Memoirs, Vol III.

Mitchell, W. C., 1928. *Business Cycles: The Problem and its Setting*, New York, NBER.

Moore, H. L., 1914. *Economic Cycles – their Lay and Cause*, New York, Macmillan.

Moore, H. L., 1923. *Generating Economic Cycles*, New York, Macmillan

Sims, C., 1980. “Macroeconomics and Reality”, *Econometrica*, Vol. 48, pp1-48.

Slutsky, E. E., 1927. “The summation of random causes as a source of cyclic processes”, Conjecture Institute, Moscow, (also (1937) *Econometrica*, 5, 105-146

Yule, G. U. 1926. “Why do we sometimes get nonsense correlations between time-series? – A study in sampling and the nature of time series”, *Journal of the Royal Statistical Society*, 89, 1-64

Yule, G. U. 1927. “On a method of investigating periodicities in disturbed series, with particular reference to Wolfer’s sunspot numbers”, *Philosophical Transactions of the Royal Society of London, Series A*, 226, 267-98

Ms. Anne Marie McKiernan: While noting that the approach used in this paper is one of data examination, unfettered by prior beliefs regarding economic structure, one would imagine that issues of economic structure would nevertheless have an important bearing on the predictive value of the Experimental Indicator. In this light, the following points are worth noting :

1. From the results of the cyclical conformity tests (Table 4, for 14 out of 60 series tested), one can see that the “lead period” (the period by which each series leads the turning points in the business cycle) has, for a number of series, changed over different decades. This is consistent with what one would expect from changing economic structures. It would seem, therefore, that the predictive power of an indicator would be enhanced if, in selecting the component series, a higher value was placed on a series performing well in terms of “lead period” in the recent past (which encompasses the changing economic structure), even if the series had poor average “lead period” properties over the full period reviewed.

What methodology was used by the authors to deal with series which have

become more important in recent years in terms of their “lead period” properties, but which were relatively unimportant in earlier decades?

2. In the same vein, it is notable that no series relating to the European economy is mentioned as having been tested, although the importance of the EU economy for Ireland would have been increasing in importance in recent years (even if from a low base). Were any series, which are indicative of European economic activity, tested for cyclical conformity?
3. It would be useful to have the list of 60 series, on which cyclical conformity tests were carried out, included as an appendix to the paper.
4. Regarding the inclusion of long-term interest rates on government securities, it is worth noting that, in the second half of the 1990s, structural factors came into play in the government bond market which will affect the evolution of this series going forward. Initially, preparation for EMU led to a decline in long term government bond yields of prospective members, reflecting a fall in the inflation risk premium and the effects of “convergence trades”. Following the establishment of the euro area on 1 January 1999, the National Treasury Management Agency undertook organisational changes in the Irish government bond market to improve liquidity, leading to a reduction in the liquidity risk premium on Irish yields. Since around mid-1999, Irish government bond yields have traded in a relatively fixed differential of between 20-30 basis points over German yields. Thus, this series will, in the future, reflect developments in euro area long-term interest rates, rather than domestic economic activity.

Fr. John Brady SJ: I enjoyed the paper. A feature of the Irish economy is that there are a considerable number of multinational companies, which have large plants here, e.g. Intel and Hewlett Packard. If one took the 20 largest of these they would constitute a substantial part of the Irish economy. These organisations have a global perspective in the decisions they take about expanding or contracting output. The business cycle of the Irish economy is unlikely to influence them in any way. I think this factor has to be taken into account in any theoretical understanding of the Irish business cycle.