

THE BARRINGTON PRIZE LECTURE, 1989/90
THE DEVELOPMENT OF YIELD CURVES FOR THE IRISH
SECURITIES MARKET

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1. INTRODUCTION

This paper outlines the results of work carried out for an M.Sc. Thesis in Trinity College Dublin. This work involved the investigation of the development of Yield curves for the Irish securities Market during the period 1987 to 1988.

A Yield curve is a fitted relationship between time and the yield to maturity on fixed interest stocks with varying maturity dates. Gilt edged securities are securities which carry a promise to pay their registered owner a fixed sum known as the Coupon at a regular frequency up to the date of maturity when another fixed sum known as the Capital is paid terminating the contract. It is generally observed that securities with identical coupon and capital values and comparable risk of default will command different prices if their dates of maturity differ. It is this dependence of the price on maturity that is the focus of yield curve analysis.

Before 1972 the yield curve was derived by fitting an N order polynomial where N was determined by the extent to which the addition of a

higher order polynomial would improve the goodness of fit. A compromise between some low degree curve which was simple and informative and a high order more flexible curve which fits better should have been used. However, usually the curve was chosen on ad hoc grounds with the result that the yield curve never fully measured up to its potential (in Section 3 this is examined in greater detail).

Around 1972 it became apparent that factors other than a security's maturity affects its yield in particular it is generally believed that the size of the coupon affects the yield. The Bank of England developed a model in 1972¹ which was amended in 1973², 1976³ and 1982⁴ for estimating the yield curve for British government securities taking account of the coupon effect and other factors (see Section 4 for a detailed description of this model). The authors of this model regarded it as an attempt to model the components that determine the yield for Government stocks rather than just an exercise in curve fitting.

Up to 1978 a separate yield curve of Ireland was not required since the yields on Irish securities were very close, at all maturities, to yields on British Securities. In 1978 however, Ireland joined the E.M.S.(European Monetary System) and from then on there was a divergence in yields between the two markets thus making the estimating of a separate yield curve for Ireland necessary.

The only work done on yield curves for the Irish securities market was by Patrick Honohan in 1980⁵ and by Noel Brennan in 1981⁶. Both adapted the Bank of England model for the Irish securities market and found that the model seemed to be applicable to the Irish Market giving errors slightly higher than those experienced by the Bank of England. Noel Brennan also investigated the use of different decision periods for the Irish Market.

This paper is divided into six sections. Section 2 introduces the topic of fixed interest securities, it gives examples of different securities on the Irish Gilts market and describes the operation of the Irish securities market. This Section also explains some of the differences between the Irish and British securities markets. This section finishes by explaining what is meant by the yield of a stock.

Section 3 introduces the theory of yield curves and describes some of

the uses made of the yield curve including Switching, and how market expectations about inflation and interest rates can be assessed from the curve.

Section 4 outlines the Bank of England Model. This section contains a description of how the model has been developed over the past 16 years and contains a description of the underlying assumptions of this model. It also describes the results obtained from applying this model to the Irish securities market.

Section 5 contains a description of how the method of locally weighted regression (LOWESS) could be used as an alternative to the curve fitting section of the Bank of England Model.

Section 6 contains the conclusions of the study.

2. FIXED INTEREST SECURITIES

A government, local authority, private company or other body may raise money by floating a loan on a stock exchange. The terms of the issue are set out by the borrower and investors may be invited to subscribe to the loan at a given price (called the issue price), or the issue may be by tender, in which case investors are invited to nominate the price that they are prepared to pay and the loan is then issued to the highest bidders, subject to certain rules of allocation. In either case the loan may be underwritten by a financial institution, which thereby agrees to purchase, at a certain price, any of the issue which is not subscribed by other investors.

Description of fixed interest securities

Fixed interest securities normally include in their title the rate of interest payable, e.g. $7\frac{1}{4}$ % exchequer bond 1992, which is an Irish government stock. The annual interest payable to each holder, which is often but not always paid half yearly (some British stocks pay interest quarterly), is found by multiplying the nominal amount of his holding by the rate of interest per annum which is generally called the coupon rate⁷.

The money payable at redemption is usually but not always equal to the nominal amount. If the redemption price is equal to the nominal amount then the stock is said to be redeemable at par, if the redemption price is less than the nominal amount then the stock is said to be redeemable below par or at a discount and if the redemption price is greater than the nominal amount then the stock is said to be redeemable above par or at a premium. Some securities have varying coupon rates or varying redemption prices; these will be discussed later. The redemption date is the date on which the redemption money is due to be paid. Some bonds have variable redemption dates, in which case the redemption date may be chosen by the borrower (or perhaps the lender) as any interest date, within a certain period, or any interest date on or after a given date. In the latter case the stock is said to have no final redemption date or to be undated. At present there are no undated stocks on the Irish securities market but these are available on the British securities market.

The Irish Gilts Market

In the Irish government securities market all stocks are redeemable at par and all pay interest half yearly. On the British securities market there are some stocks that pay interest quarterly, for example the $2\frac{1}{2}\%$ Consols pay interest on the 5th of January, the 5th of April, the 5th of July and on the 5th of October.

The operation of the Gilts Market

There are in fact two different Gilt markets, the official market and the unofficial market. The official market is one where the major broking firms trade with the Government and the unofficial market is where the broking firms trade with each other. There are two different prices quoted on each market, the bid and the offer prices. The offer price is the price for which the Gilts can be bought on the market. The bid price is the price for which the Gilts can be sold on the market. At present the official market is not operating since the Government broker no longer bids for the Gilts.

The issue price and subsequent market prices of any stock are usually quoted in terms of a certain nominal amount, e.g. £100 or £1 nominal. The statement that an investor owns a bond of £100 nominal does not in general imply that his holding of the stock in question is worth £100. If

it is worth £105, it is said to be above par or at a premium; if it is worth £90, say, it is said to be below par or at a discount; and if it is worth £100 the stock is said to be at par.

Irish government securities exist in large volumes, and may be considered to offer perfect security against default by the borrower. Gilts are usually classified according to the term to redemption. The usual classifications used are Exchequer Bills (three months), short dated (up to five years), medium dated (up to ten years) and long dated.

Examples of different Securities

In this section the difference between different securities is explained by considering some examples of Irish government stocks

1. The 8% Capital stock 1993. This stock bears interest at 8% per annum, payable half yearly on the first of November and the first of May. The stock is redeemable at par on the first of November 1993.
2. The $6\frac{1}{4}\%$ Exchequer stock 2005. This stock bears interest at $6\frac{1}{4}\%$ per annum payable half yearly on the 27th of June and on the 27th of December, and is redeemable at par on any date between the 27th of June 2000 and the 27th of June 2005 (inclusive) at the option of the Government, the only restriction being that the Government must give 3 months notice. Since the precise redemption date is not predetermined this stock is said to have optional redemption (or maturity) dates.

Example of an undated stock is the $3\frac{1}{2}\%$ British War Loan. This stock was issued in 1932 by the British government as a conversion of an earlier stock, issued during the First world war. Interest is at $3\frac{1}{2}\%$ per annum and is paid half yearly on the first of June and on the first of December. This stock is redeemable at par on any interest date that the British government chooses, there being no final redemption date. The stock may therefore be considered as having optional redemption dates, the second of them being infinity.

The Yield of a Security

The yield of a bond is the rate of return that it will give the investor. There are five ways of calculating yields on bonds, nominal yield, current yield, yield to maturity, yield to call, and the realized yield. The important one when dealing with yield curves is the yield to maturity and this is the only one which will be discussed here (for a description of the other yields see Griffith and Chandy, 1986⁸).

The yield to maturity is the coupon rate of return the investor would earn buying the bond at the prevailing price and holding it to maturity. The yield to maturity is determined by finding the rate of interest at which the present value of the future cash flows is equal to the current market price. The cash flows from a stock are the coupon payments it will make and the face value it will pay at maturity. Thus yield to maturity is r in the following formula:

$$P_p = \sum_{i=1}^n \frac{I_t}{(1+r)^i} + \frac{P_m}{(1+r)^n} \quad (2.6.1)$$

where

I_t = annual coupon value,

P_m = Maturity value of the stock,

P_p = Market price of the stock,

and

n = years to maturity

Depending on the accuracy desired several procedures can be used to compute yield to maturity. The most precise procedure (the one used in the market place) involves calculating the yield using semiannual compounding and explicit present value calculations. Several approximate yield to maturity formulas are described by Griffith and Chandy ⁸. The most common yield to maturity approximation formula is

$$\text{Approximate Yield} = \frac{\frac{P_m - P_p}{I_t + n}}{\frac{P_m + P_p}{2}} \quad (2.6.2)$$

Griffith and Chandy also conducted an analysis of the various formulas used to calculate the yield to maturity. They found that no single formula was consistently superior to the others, but they found that the Henderson formula ⁹

$$\text{Approximate Yield} = \frac{c + a}{1000 + 0.6(P - 1000)} \quad (2.6.3)$$

- where c = annual coupon on the bond
- a = the annual amortization = $(1000P)n$
- P = the market price
- n = number of years to maturity

could be used in all cases with a reasonable degree of accuracy. Griffith and Chandy analyzed all formulas for short dated securities, medium dated securities and for long dated securities. They also grouped securities into segments according to their coupon. They found that the Henderson formula produced the lowest average absolute error in every segment of the short-term maturities. For medium maturities they found that the Henderson formula had the lowest error in more cases than any other formula. For long dated securities the Henderson formula was also one of the best.

3. THE YIELD CURVE

The yield curve is essentially concerned with the prices of marketable fixed interest securities (a description of which was given in the last section). It is generally observed that securities with identical coupon and capital and comparable risk of default, traded in a market where they can be regarded as having comparable liquidity will command different prices if their date of maturity differs. It is this dependence of price on maturity that is the focus of yield curve analysis.

Since yield curves are normally drawn on a cartesian diagram (with yield as the abscissa and maturity as the ordinate), i.e, they are two dimensional, the implication is that the relationship portrayed in one such curve is solely one between yield and maturity. No other variables are supposed to enter the relationship between a group of stocks yields from which the yield curve is derived. Ideally this would mean that bonds were alike in all respects except only their time to maturity. But in the real world stocks are different in many respects. In fact, when yield is plotted against maturity, the resulting relationship is not usually a smooth curve. In other words there are many deviations, occasionally significant, from the best fitting smooth curve. This suggests that other characteristics of securities, apart from maturity, affect their yield. In particular, it is generally believed that coupon affects yield at given maturity. This coupon effect is normally attributed to differential tax treatment of coupon and capital payments. For a given yield and maturity the after tax return to some taxpayers may depend on coupon and that effect might be enough to influence the price of securities.

Some researchers in the past have not bothered to take account of the coupon effect, risk aversion and other factors and by doing this have assumed that bonds are alike in all respects, except for time to maturity. Other researchers have taken account of coupon but the methods they used have been vastly different. The following section outlines some of this research and some of the different methods used in fitting the yield curves to a set of data.

The Development of the Yield Curve

The usual way of fitting a yield curve to a set of yields on stocks on a particular day is by using a Least squares method. In previous research when Least Squares was considered for fitting the data the problem was set out as follows; given n observed points

$$(M_1, Y_1), (M_2, Y_2), \dots, (M_n, Y_n)$$

where M is the maturity and Y is the yield. The problem was then regarded as determining an analytic function which could be used to obtain for

any given value of the independent variable (Maturity), the corresponding value of the yield. The first decision that had to be taken in this case was whether or not there was reason to assume that the yields of the points were true values, not effected by any error term. If this assumption is made, then this requires that the estimated and observed points are the same. So that interpolation of the data is required, i.e., a means of connecting up the observed data points relating yield to maturity had to be found. In early research, the simplest way to do this was to plot the observed data (yield on the abscissa, maturity on the ordinate) and then fit a curve to them by hand.¹⁰

This method was used by David Durand¹¹ for securities in the U.S.A. He fitted curves using this method on an annual basis for the period 1900 to 1942. He omitted some bonds that had special redemption features. The problem with this is that not only was he reducing his sample size but he was also leaving out a very important part of the market, the yields of which also need explaining. Of course, the fitting of such curves depends heavily on the assessment of the drawer and thus are subject to human error. Even more errors are introduced by the fact that Durand did not consider coupon effects.

Another crude procedure of this type was used by Grant¹². He collected quarterly yield data for the British Government securities from 1924 to 1962. Like Durand he excluded stocks with special redemption features. He also omitted stocks where coupon rates were markedly higher or lower than the majority of stocks outstanding at the time. He also ignored tax considerations. He basically used linear interpolation between successive points.

In the late sixties and early seventies regression analysis was used to construct yield curves, both in the United States and in Great Britain. In Great Britain the Bank of England¹³ used regression to fit the curve from 1967 to 1972. They also excluded stocks with coupons less than 5%. At the time this was their way of excluding the coupon effect.

Also in Britain D.Fisher used the following regression model on yields from the British securities Market¹⁴.

$$Y = a + b_1M + b_2M + b_3M + b_4\log M + b_5C + b_6C^2 + b_7\log C \quad (3.2.1)$$

where Y stands for yield, M for the term to Maturity, and C for the coupon. The variables were introduced in a step-wise fashion in the order of their significance, and accepted if they reached the 0.10 level of significance in a two-tailed test. Fisher was the first person to try and identify, and hence remove, the coupon effects from the yields. However Fisher reported that in 10 of the 52 estimated yield curves the coupon effect did not reach the 0.10 level of significance and so was dropped from the equation. Masera¹⁵ gives a criticism of this approach and explains why he believes it was wrong.

A regression model was also used by Cohen, Kramer, and Waugh to estimate yield curves for U.S. government securities¹⁶. The model they used was

$$Y = aM + b(\log M)^2 + d \quad (3.2.2)$$

where a , b and d are regression coefficients.

They also allowed for differential tax treatment on different stocks by using before tax yields and after tax yields. However, the problem with this model is that it does not consider other factors which influence yield, like the coupon effect.

In 1971 Masera¹⁵ examined the construction of yield curves and gives a very good description and criticism of the various methods used up to 1970. Masera also fitted yield curves to the Italian Securities market for data from the period 1957 to 1967. Masera points out that the Italian securities market is entirely different from the British and American markets. However he was able to study a section of the Italian market, B.T.P securities (Buoni Tesoro Poliennali), all of which have equal coupon of 5%. However for each time period there were only 5 to 9 data points. Masera fitted three equations to each month's data.

$$Y = \alpha_1 + \beta_1 M \quad (3.2.3)$$

$$Y = \alpha_2 + \beta_2 M + g_2(\log M)^2 \quad (3.2.4)$$

$$Y = \alpha_3 + \beta_3 M + g_3(\log M)^2 + \delta_3 M^3 \quad (3.2.5)$$

where Y is the Yield and M the Maturity. The best equation was chosen out of these three for each month, the choice being made in terms of the performance of the equation as measured by their goodness of fit (the coefficient of determination adjusted for degrees of freedom, R) and the significance (judged at the 10% level) of the individual regression coefficients. Thus for a consecutive 3 month period the yield curve could have been determined by three different equations.

The major problem with this method is the large amount of error introduced because of the very small sample size. This method can not be applied to the Irish securities market, since there is no group of securities similar to the Italian B.T.P's (i.e. there does not exist a large group of securities with exactly the same coupon).

In 1972 the Bank of England changed their method from the fitting of a simple regression model (in terms of maturity) to a model which was derived from the theory of the market¹. This model takes account of the coupon effect, risk aversion, tax considerations and the segmentation of the market. The Bank of England's model takes account of coupon by considering the yield curve as a three-dimensional non-linear relationship between yield, coupon and maturity. So for depicting a yield curve in two dimensions, maturity is plotted against the yield which would according to the estimated relationship, exist for a security whose coupon was equal to that yield (a detailed description of the model is given in Section 4).

In Britain another method of curve fitting used, is the one adopted by the Financial Times-Actuaries (F.T.A) introduced in 1977. The curve fitting is done by a mathematical formula. The coefficients of the formula are calculated daily by the minimisation of a weighted sum of squares of the difference between the value of the yield given by the formula, and the observed yield. This method is nearly exactly the same as the Bank of England model except that the F.T.A account for the coupon effect in a different way. The F.T.A yield curves are found for each of three coupon bands, consisting of low, medium and high coupon stocks (the

irredeemable stocks are included in each band). This method cannot be used on the Irish securities market, since the size of the market is very small and thus the splitting of it into three sections would mean that the sample size for each of the three yield curves would be so small that large errors could arise.

In 1981 S.M Schaefer¹⁸ used a completely different approach to determine the yield curve. He made the assumption that short sales were not permitted. Instead of a single set of rates that equates price and present value for all bonds, he assumes that the term structure is specific to a particular tax bracket and the present value of each bond is less than or equal to its price. Under this assumption he determines the par yield curve by finding the solution to a linear programme. In his article Schaefer also compares his approach to that of the Bank of England. In his comparison he found that there are large differences between the two models but does not give a full explanation of why these have arisen. Schaefer seems to concentrate mostly on the effects of tax on the yield curve and does not consider some of the other factors that are believed to influence the curve and which are considered in the Bank of England Model.

In Ireland the Central Bank used a polynomial (in terms of Maturity only) up to 1981 to fit the yield curve. From then on the adopted the Bank of England model. In his article P.Hohonan describes some of the results he obtained from the Bank of England's model applied to the Irish Market. The major problem with fitting a polynomial as the yield curve is that the nature of the curve fitted, for example 2nd and 3rd order polynomials in maturity, will mean that there is high multicollinearity in the statistical equations. So it may not be possible to use the Yield curve to predict the unobserved yields that well (which is required in assessing the terms of new issues in the bond market) and neither may it be possible to single out such things as the coupon effect.

The Types of Yield Curve

The yield curve generally assumes four different shapes, as illustrated in Figure 3.1.

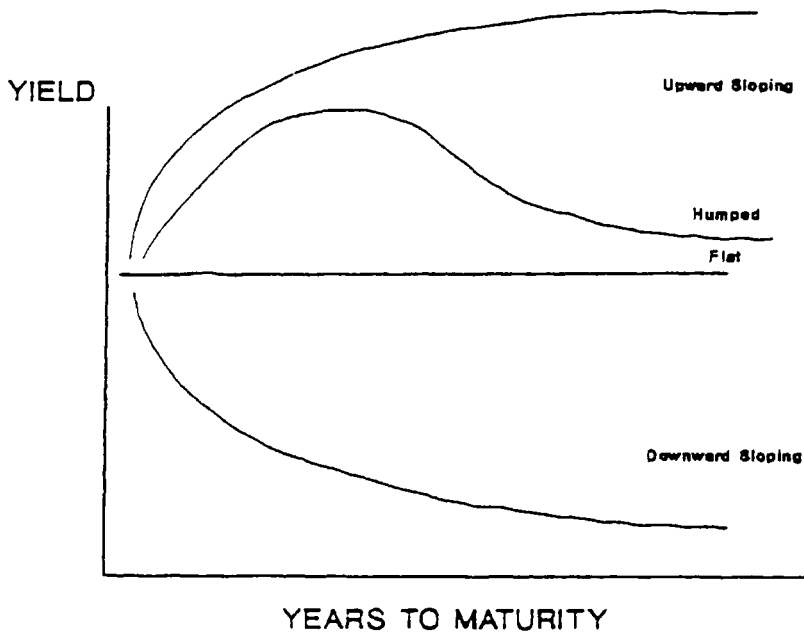


FIGURE 3.1
Different types of Yield Curve

The upward-sloping yield curve is characterised by lower yields in the shorter-term stocks and higher yields, as the length of time to maturity increases. The downward-sloping yield curve is characterised by higher yields in the short-term maturities and lower yields as the length of time to maturity increases. The flat yield curve is characterized by the same yield throughout the spectrum of maturities, so that the short-term and the long-term maturities have the same yield. The fourth yield curve is the humped curve so called because the curve first rises and, after reaching an apex, falls as maturities lengthen. The appearance of the curve is such that there is a hump either shortly after the most immediate maturities or towards the middle of intermediate-term maturities.

Each of the theories of the market (i.e. the theories that lie behind yield curves) discussed below offers an explanation of the various shapes of the yield curve. Each shape may also have different implications for interest rates and financial market behaviour.

Theory behind the Yield Curve

There are three different theories about the structure of the market, each giving explanations about the different shapes obtained by the Yield curve. These theories are described below.

Segmented Market Theory

This theory suggest that the market for bonds is segmented into separate sub-markets according to maturity of the stocks and that the participants in each sub-market are locked into and unwilling to move into another sub-market. The participants who buy stocks in each maturity range do not move to another maturity range because of possible legal restrictions and portfolio requirements, regardless of expected interest rate fluctuations. For example, the commercial banks have traditionally confined their activity to the more liquid shorter-term securities, because they must be in a position to meet the demands of their depositors almost immediately. The Central Bank has to ensure that they keep a certain percent of deposits in the form of liquid assets and they have to do this by law. Unlike the banks, life insurance companies do not experience sudden, perhaps unexpected, demands. They, therefore, confine their investment to those stocks with the longer, less liquid maturities.

Similarly, the supply of stocks in each maturity depends on the time period for which the funds are needed by the government or the time that the government believes it will need to pay back the loan. Therefore the supply of bonds to the market is also restricted to various submarkets. With both the supply and demand for bonds restricted to submarkets, the interest rate in each submarket is determined by the supply and demand in this particular submarket, independently of the other submarkets. This also implies that bonds in one market range are not substitutes for those in another. In this case, an oversupply in the long-term maturities would depress demand and raise yields in the longer-term submarket without affecting the yields and prices of the short-term stocks. The empirical evidence^{20,21} does not support the rigid inflexibility of the market segmentation theory. Although it is found that certain institutions concentrate their portfolios in either the long or short-term maturities, none of the institutions is so inflexible that it does not substitute among securities in a relatively wide range of maturities. On average, the maturity compo-

sition of institutional portfolios is found to be variable. The supply of bonds is also found to be relatively variable in relation to interest rates. At the higher levels on interest rates the sale of securities tends to be deferred and the government sells short-term obligations to meet their immediate needs. An examination of individual commercial bank portfolio procedures also suggests that there is more flexibility in purchasing among the various maturities than is maintained in the rigid segmentation theory. All this suggests a less rigid segmentation in which long and short-term participants move to the intermediate-term segment if it is attractive.

The less rigid segmented market theory aids in explaining the various shapes of the yield curve. The upward-sloping yield curve, which is characteristic of a recession in the economy, is associated with

1. an increased demand for short-term securities;
2. a decreased supply of short-term securities and
3. an increased supply of longer-term securities.

During the economic downturn commercial banks are experiencing an accelerated inflow of demand deposits and have excess funds that must be invested. This creates an increased demand from the banks for the shorter-term securities that commercial banks generally hold for liquidity reasons. Thus the prices of securities are bid up and the yield falls in the short-maturities market.

In the long-term segment of the market, meanwhile, the demand for securities remains relatively constant but the supply increases. The Government tries to exchange its short-term debt for longer-term debt, increasing the supply of long-term securities during the downturn. This increases the supply of long-term securities and, with the constant demand, forces prices to fall and yields to rise. The downward-sloping yield curve may also be explained by the segmented market theory. In the expansion phase of an economy, the supply and demand conditions in each segment of the market reverse themselves from the declining phase. The supply of securities in the short-term segment increases as commercial banks lose demand deposits and sell off their excess holdings of short-term liquid assets. These

actions lead to an increase in the supply of securities and a decreased demand for securities in the shorter-term segment, which forces down prices and forces up yields. In the long-term segment the supply of securities decreases as the government stops selling long-term stocks in an effort to minimise cost. This forces the long term yields to decline and hence the yield curve slopes downward.

The humped yield curve is also explained by the segmented markets theory. In this theory the long-term lenders and the short-term lenders operate strictly within their own segment; they do not bother to shorten or lengthen their relevant maturity ranges. The intermediate maturities' supply and demand determines their yield independently of the other two segments. If the supply of intermediate maturities is greater than the demand, the yield must rise and may very well rise to a level above the other two market segments. Although this seems to be a reasonable explanation within the very rigid, inflexible segmented theory the empirical evidence of substantial flexibility, particularly between the intermediate and the other two maturity ranges, detracts from this explanation.

The Expectation Theory

In direct opposition to the segmented markets theory, the expectation theory maintains that the maturity composition of the securities has no effect on the yield curve. According to the theory, the long-term rate is equal to the sum of the expected short-term rates so that the investor is indifferent between buying one long-term stock or a series of short-term bonds in the same maturity range. For example, the yield on a 3 year stock should be equal to the yield on three 1 year stocks, one of which was purchased at the beginning of each of the 3 years.

This theory says that the shape of the yield curve is determined by investors' expectations of short-term yields within the maturity of the competing long-term bond. When investors expect interest rates to be higher in time, we have an upward-sloping yield curve. If they expect interest rates to be lower, the yield curve slopes downward. The flat yield curve indicates that they expect them to remain the same. This theory could also explain the humped yield curve. Investors must hold a series of varying expectations. First, investors must assume that yields will rise quite sharply from their present levels for a few years. Then rates must be

expected to decline rapidly and sharply to lower levels and then rise again to a new plateau on which they are expected to remain. This is a very awkward explanation for the humped yield curve and thus forces us to look elsewhere.

A more complex version of the theory is the Liquid premium version. The liquid premium version of the expectation theory maintains that there is a natural tendency for yields to rise as the maturity lengthens because of risk aversion. Because there is greater potential price fluctuation in the longer maturities, the long-term yields have to be higher to induce the investors, regardless of their expectation, to purchase the long-term bonds. Therefore, the natural tendency is for the long-term yields to be higher than the short-term yields. This liquidity premium causes an upward push on all the yield curves. The upward sloping yield curve tends to be steeper, the downward-sloping yield curve tends to be less steep, and the flat curve tends to have a slight upward slope.

The empirical evidence^{20,21} on the expectations theory has been divided. Several researchers have found that the expected yields implied in the term structure prevailing at a particular time did not occur at the future date so they concluded that the expectations theory must be empirically invalid. Mieselman²² found that when forecasting error (i.e. incorrect expectations) was taken into consideration, the expectations theory had empirical validity.

The Eclectic Theory or the Preferred Habitat Theory

This theory was put forward by Medigliani and Such²⁵. This is the theory that the Bank of England Model is based on. The eclectic theory draws on both the expectations and the segmented markets theories. As in the expectations framework, it assumes that if investors start expecting higher short-term interest rates, they shift out of the long-term bonds into the short-term bonds, forcing up the long-term yields. With diverse expectations among investors some expect rates to rise and others expect rates to fall. As in the segmented markets theory, it assumes that various stock buyers have preferred maturity ranges and require a yield premium to switch from their preferred positions to another maturity range.

We are now able to explain the humped yield curve. Starting with the

expectations theory, we may assume that the term structure is settled into a posture in which all the short-term and long-term alternative yields are equal. All investors who believe that yields are going to rise invest in short-term securities, and all investors that believe that yields are going to fall invest in long term securities. An increase in the supply of intermediate-term bonds necessitates a higher yield than previously existed in order to sell these bonds. This makes the yields on the intermediate bonds higher than before when the expectations on yields were formed. The alternative of switching to these intermediate-term bonds instead of staying with the short-term bonds and reinvesting is more attractive and the short-term holders switch to the intermediate-terms bonds. By the same token, those investors who are expecting declining yields and invest in long-term bonds also have their expectations altered and the intermediate-term bonds are more attractive than remaining with the long-term securities. They sell their long-term bonds and buy the intermediate-term bonds. If, at the same time, the short-term holders demand a premium to switch from their preferred short-term position to the intermediate-term position and the long term holders demand a premium to switch from their preferred position to an intermediate-term position, the yields on the intermediate-term bonds must be higher than either the long- or the short-term bonds. Thus the Eclectic theory seems the best because it has least difficulty in explaining all forms of the yield curve.

Application of the Yield Curve

As shown earlier the yield curve can take on a variety of different shapes. The shape the yield curve takes on contains a large amount of informational content. It can indicate market expectations about future interest rates and inflation^{10,23}.

The shape of the yield curve which is historically the most common comprises a relatively low exchequer bill rate with correspondingly low yields on very short dated stocks, rising upwards to yields on stocks with terms to redemption of perhaps 10 to 15 years, and thereafter levelling out so that the yield on stocks between 15 or 20 years to redemption are fairly close to the yields on very long dated stocks. Although this general shape is statistically the most common, it has happened in Britain in times of financial crisis that the yields on very short term securities have been significantly higher than the yields on long term securities, so that the

traditional pattern is reversed around a horizontal axis. This happened in Britain in 1932 and 1937. If the yield curve takes this form (as shown in figure 3.2) this indicates that investors do not believe that interest rates will be high for very much longer, so that they prefer to secure a relatively high yield on a longer dated stock which they can enjoy for some time in the future rather than a very high yield for what they feel will be a very short time with the need for reinvestment at a lower yield later²³.

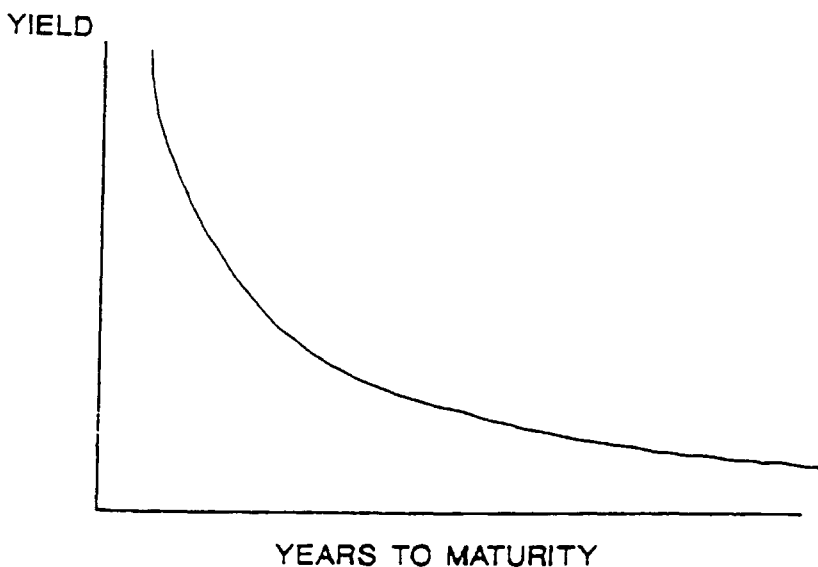


FIGURE 3.2
The shape of the yield curve

A further influence in determining the shape of the yield curve is the market's opinion of the likelihood of future inflation. If investors expect inflation to continue for some time in the future they make allowance for this by insisting on a higher redemption yield for long dated stocks. Of course fears of inflation may also give rise to suspicions that a Government will be obliged to adopt unpleasant measures to contain the threat to the balance of payments position and, if these measures involve higher Bank rates and the restriction of bank credit, the repercussions could be felt

on the short term market, perhaps with more violent fluctuations in yield than may take place at the long end of the market. This would mean that investors would not invest in short-dated securities, with the result that the yield of the short-dated securities would increase. Thus if the market believed that inflation was going to increase the overall effect would be for the yield curve to move up¹⁰.

Switching from one Stock to Another

The yield curve also gives an indication of which stocks an investor should switch out of and in to. Consider for example the following situation; suppose the yield curve is shaped as shown in Figure 3.3.

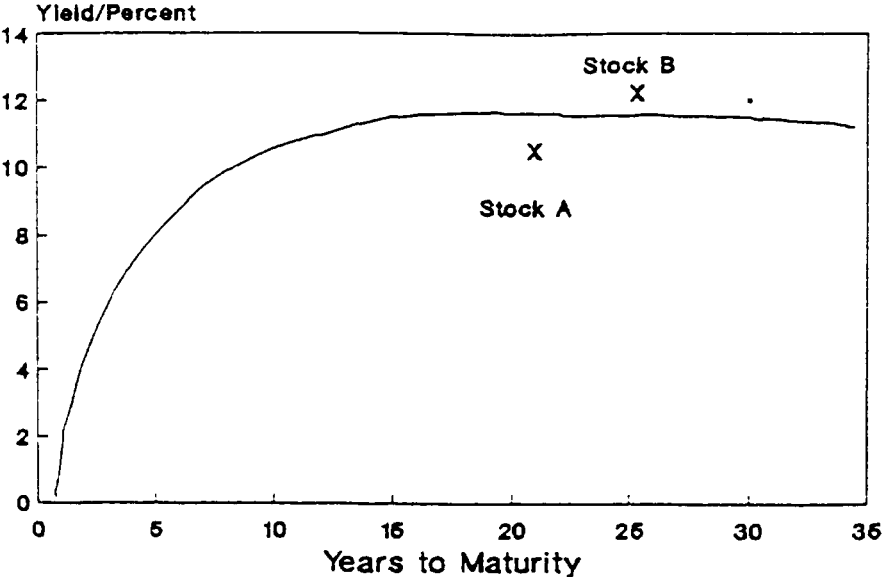


FIGURE 3.3
Switching from one stock to another

Suppose A and B are Government Stocks with yields shown in figure 3.3 and an investor is currently holding some of the A stocks. Stock A has a lower yield than the yield curve predicts and thus it has a higher price.

Stock B has a higher yield than the curve predicts and thus has a lower price than it should have.

Therefore, the investor should switch out of A into B.

Riding the Yield Curve

If the yield curve is shaped as shown in Figure 3.4, the investor may be able to increase his profit if he undertakes a strategy known as riding the yield curve¹⁹.

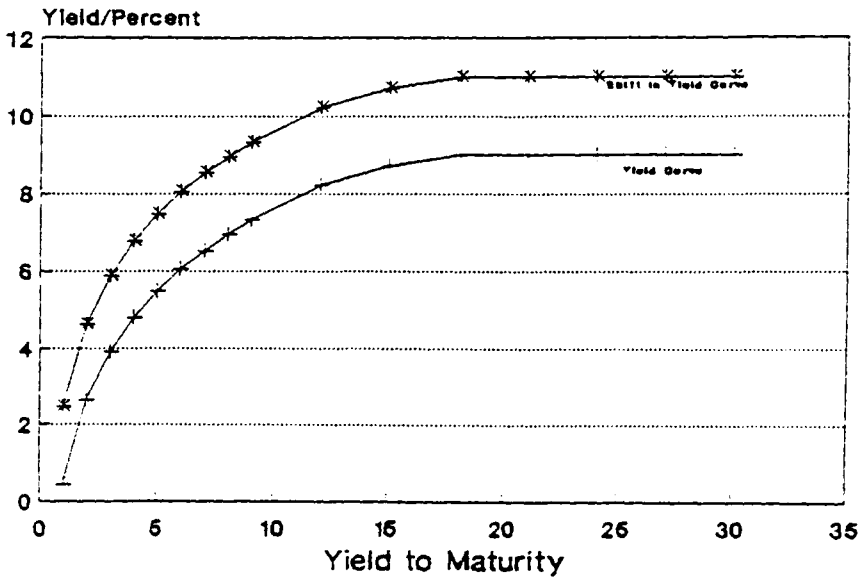


FIGURE 3.4
Riding the yield curve

If, for example, the investor feels that the yield curve is to remain in a similar configuration for two years, he can purchase the bonds that lie less than two years to the right of the break in the yield curve, that is, the point at which the yield curve slopes downward. The investor sacrifices little yield by buying these stocks because the yield is about the same as

stocks of longer maturity. If he holds these stocks past the breaking point, however, they start to appreciate in price because of the lower yield on the short-term bonds. Simply by waiting for the stocks to pass from the higher yields of the longer term to the lower yields of the shorter term (after the breaking point), the investor has earned substantial capital gains. Caution must be taken, however, that the entire yield curve does not shift upward during the period the investor holds the bonds. If the curve does shift upward, the stock price may not have declined at the end of the holding period; it may in fact decrease.

The final use of the yield curve is in assessing the issue price of a new security.

Assessing the appropriate issue price of a security

The yield curve can give an indication of how much the market will be prepared to pay for a new security with a given coupon and maturity⁵. so that when a new security is issued at a fixed price, it can be used to assess whether that price is too high or too low and thus help investors maximise their profits. Alternatively, the government could use the yield curve to determine the price that the market is willing to pay for a new issue.

4. THE BANK OF ENGLAND'S MODEL

Up to 1972 the Bank of England obtained yield curves by fitting a polynomial in terms of maturity to stocks on the British securities market. The polynomial used was chosen as a compromise between some low order curve which is simple and informative and a high order more flexible curve which fitted better. The main reason for the Bank of England not going for a high statistical fit by using a high order curve was that they felt that this may obscure the true relationship between yield and maturity.

In 1972 Burman¹ stated that the form of the curve chosen has underlying implications about the nature and workings of the market. By using a low order curve, the assumption being made is that the market brings about continuous smooth adjustment in the prices and yields of neighbouring stocks throughout the whole range of maturities since the curve has no

sharp bends. Thus the choice of a low order curve assumes that the market operates in a way explained by the expectation theory (see Section 3). However, as pointed out in the last chapter, this theory has some flaws and most market operators believe in some form of segmentation of the market.

The Bank of England felt that curves up to 1972, including theirs, were usually chosen on statistical grounds and not on the workings of the market. They felt that the workings of the market should be the used as a basis for choosing the method of curve fitting and they developed their model in that way.

The Workings of the Market

The first thing that the Bank of England assumed about the workings of the market is that different types of investor have different preferred habitats (or maturity ranges) in which they like to operate. For example discount houses deal mainly in stocks under five years from maturity, banks in those below fifteen years, and insurance companies and pension funds with long- term liabilities tend to concentrate on stocks with lives over fifteen years.

In Ireland the commercial Banks are required to keep a certain percentage of their deposits in the form of liquid assets. Also discount houses, pension funds and insurance companies operate in a similar manner to those in the U.K. so this assumption about preferred habitats would also apply to the Irish Market.

These habitats arise because of the desire to match the maturity of assets and liabilities in order to minimise risk. At the same time investors have expectations about future interest rates usually from 2 to 3 years. The furthest point at which an investor's expectations are fairly well formed is called his planning horizon and the time from now till then is his decision period. The investor therefore has expectations about the yields on his horizon on his stock and other stocks which imply expected capital gains or losses. Thus if an alternative stock in the planning horizon period is expected to give an improvement in return then the investor should switch. However there is a risk that the actual return on the alternative stock may be lower rather than higher than the original stock, so that

the investor can only be tempted away from his preferred habitat if the expected improvement in return from investing in the alternative stock exceeds some minimum amount. This is called the risk premium.

This is basically how the Bank of England's model assumes investors will operate. The next section outlines some of the factors that this model says influences investors in their choice of investing in a stock.

Influences on the Yield Curve

The construction of a yield curve from the analysis of the market meant that the Bank of England had to specify all factors that might be expected to influence prices and yields of stocks.

Two important effects on yield curves which up till this time had not been considered were the effect of difference in coupon rate and the tax position of various investors. These are not important when the main purpose of the analysis is to identify stocks out of line. However these factors are important in judging appropriate terms of new government issues. The Bank of England model considers these along with the influence of risk aversion. It takes coupon into consideration by using the par yield curve. This type of yield curve shows the nominal rate of interest which a stock of each maturity should bear in order to be issued at a price of 100.

The influence of risk aversion

If there were no risks to investors they would put all their funds into one stock which offered the highest expected return. However there is a desire to diversify portfolios; the main reason for this is risk aversion. An investor cannot forecast with certainty what yields will be at their planning horizons. The effect of this uncertainty increases with the length of the period to maturity. The risk free return expected over the decision period is the (certain) yield of a stock maturing at the planning horizon. But, for stocks maturing beyond this horizon, risk is taken into account by assuming that the expected return over the decision period must exceed the (certain) yield by the risk premium. The risk premium is measured as the percentage change in the price of a stock caused by a 1% change in the yield. The risk premium is in fact proportional to the volatility of a

stock. Volatility is the interest elasticity of price. It is zero when a stock is due to mature and reaches a maximum for irredeemable stocks.

Irish investors would also be influenced by risk aversion and there is no reason to believe that they would act differently to their counterparts in the U.K.. So the way the Bank of England take this into account would also apply to the Irish Market. However, since there are less stocks, and since the amount issued of each stock is less than on the British securities market, the stock may be more volatile and thus the risk premium may have to be higher.

Influence of taxation

Tax has an effect on yield curves due to the fact that the investment return obtained as income (dividends) is liable to tax and the return received as capital gains is tax free if a stock is held for more than one year. By considering this fact two main types of investors can be identified. Net investors are investors for whom the above is important. Gross investors are investors for whom dividends and capital gains are equivalent. Low coupon stock has a greater appeal to net investors because they offer a larger capital gain at redemption. The Bank of England found that the simplest way to incorporate the tax effect into the model was to create an 'effective tax rate'; this is used to calculate an effective yield representing the average of the net returns for net investors and gross return for gross investors.

In order to clarify the link between coupon, yield and tax, it is helpful to consider the yields on two stocks with different coupons. At low tax rates, the higher coupon will have the higher net yield but, as the tax rate increases, the yields come closer together and eventually cross over so that the higher coupon then has the lower yield. The tax rate at the crossover point defines an 'effective tax rate' for this pair of stocks.

In Ireland there are also Net investors and Gross investors, so the way the Bank of England handles this would also apply to the Irish Market. The rate of income tax which is applicable to the Irish securities market is the flat rate of 30%. On the British securities market the rate is around 23%. Thus the effective tax rates for the Irish securities market should be higher.

The model developed so far assumes that all investors consider the total range of stocks maturing beyond their planning horizons, notwithstanding their preferred habitats (i.e. the expectation theory). However, as mentioned in section 3, such a theory remains incapable of explaining the occasional humps and troughs which are sometimes evident in the pattern of yields. To explain these the Bank of England model basically uses the eclectic theory (i.e. a combination of the expectation and segment theories). The segmentation introduced into this model is explained in the next section.

The Segmentation of the Market

The Model believes in the segmentation of the Market. It assumes that two distinct markets exist, one in short dated stocks, the other in medium Long dated stocks. Each group of investors, according to the segment of the market in which it operates, is assumed to have its own average planning horizon and decision period with associated expectations about future yields and returns. For each group, the effective tax rates are also likely to differ because of the varying importance of gross and net investors.

The model also assumes it would be most unlikely that investors' preferences would give rise to two sharply distinct segments in the market, for there will always be some investors prepared to operate in both and so contemplate switching between the two segments; and in any event the statistical identification of the different planning horizons associated with each segment cannot be very precise. As the observed shape of the yield curve tends to be fairly smooth the model supposes that the relative importance of investors with shorter and longer planning horizons changes gradually with maturity, so that the final curve can be constructed by splicing the two separate curves together. This means that in the case where the shorter planning horizon is one year and the longer three years, the average expectations of investors operating in the short segment will determine a smooth yield curve starting from the one year point, whereas the yield curve relating to the long segment will begin at the three year point. For the stocks maturing more than three years hence, the curve generated by the short segment of the market is given gradually decreasing weight and the curve associated with the long segment increasing weight in the composition of the final curve. The Bank of England found that

a smooth combined curve can be obtained with a splicing band of four years or more.

Thus the original model was defined in terms of the following nine parameters

- (1) and (2) Short and long planning horizons
- (3) and (4) Expected level of net yields at the short and long horizons
- (5) and (6) Risk-free expected net returns over the decision periods up to the short and long horizons.
- (7) and (8) Effective tax rates for the two segments of the market
- (9) Maximum risk premium

The Bank of England model estimates the par yield curve as follows: first the values of the nine parameters above were varied to find the combination which gave rise to estimated gross redemption yields most nearly approximating to actual gross redemption yields; then from this best set of parameters it was possible to estimate the par yield curve, a continuous curve tracing out the gross yield on a hypothetical stock standing at 100 (net of accrued interest).

The Bank of England found that not every one of the nine parameters could be estimated unambiguously. They found that when the length of either of the planning horizons was varied while holding fixed the values of the other eight parameters, the statistical fit of the theoretical curve changed significantly. They also found that when both the long planning horizon and the corresponding expected return were varied together, theoretical curves of almost equally good fit could be obtained and this was also true of the short planning horizon. Some experts suggest that the most likely lengths of the short and long planning horizons are of the order of six months and two to three years respectively, but the Bank of England found that more plausible values for the expected returns over the decision period were obtained with somewhat longer planning horizons. As a compromise, they arbitrarily assigned values of one year and four years to the planning horizons.

Apart from this problem with the planning horizon, the model seemed to be an excellent fit to market operations. However in 1973, 1976 and in 1982 some problems arose in the curve and the Bank of England had to make alterations in the model. These alterations and the reasons behind them are given in the next section.

The Development of the Bank of England Model

In 1973 the Bank of England modified their model². They felt that their model did not represent the true relationship between coupons and yields of stocks with equal maturity. The model up till this time implied a linear relationship between coupon and price at the same maturity. But in reality this was found to be untrue and meant that the model had to be altered.

In the new model the observed price of a stock is assumed to be a weighted average of the prices that are necessary to attract gross and net investors, the weights depending on coupon and maturity. The relative weight of net investors decreases steadily as the coupon increases up to a certain level and thereafter remains steady. For coupons higher than this level (which will vary with maturity), differences in gross redemption yield are small (even at times zero) and the region where this occurs is called the 'gross zone', because gross investors dominate in setting market prices. In the gross zone at a given maturity, price is usually linearly related to coupon. Below the gross zone, the price coupon relationship is curvilinear, and is described as the coupon curvature effect²⁴.

In January 1975³, a dip appeared in the yield curve at the maturity range between 4 and 6 years. The dip did not appear to be reflected in the yields of actual gilt edged stocks on the British market. To fix this problem the Bank of England decided to anchor the long curve more firmly at four years by using the estimate from the short curve of the yield at four years to fix long investors' expected return to the long horizon. This was achieved by including an extra term in the estimation procedure which progressively penalises the divergence between long investors' expected return over the four years to their planning horizon and the estimate of the four year yield for the short curve. The underlying assumption is that the short curve at four years is a good and stable estimate of yields at that maturity and that it should therefore be reasonably close to the long investors' views of the expected return up to their horizon. The result of this alteration was

to eliminate the dip in the curve.

In September and October 1981⁴ the peak in the par yield curve at around five years on the British securities market began to display signs of instability. At this time, yields were well above the highest coupons in the market, so there were no stocks standing close to par. Consequently the yield on a par stock could only be estimated by extrapolation. It was decided that the upper limit for one of the model parameters should be reduced in order to stabilise the peak of the curve at a slightly lower level than would otherwise have been estimated. This change was accomplished with, on average, little change in the goodness of fit of the model, suggesting that it had too many parameters.

The previous model had nine variable parameters; four to describe interest rate expectations and four for relative weights of gross and net investors in the two segments. There was also a ninth parameter, representing the risk premium. However, it has never been possible to measure the risk premium separately; a higher premium is offset by a lower value of another parameter giving virtually identical calculated stock yields and par curve. The Bank of England decided to drop this variable parameter. The previous model assumed that net investors have been paying income tax at the standard rate. In 1982 this was changed so that the model contains three effective tax rates as parameters instead of the four relative weights of gross and net investors. These are:

1. the effective tax rate on low coupon stocks in the short dated market;
2. the effective tax rate on low coupon stocks in the long dated market;
3. the effective tax rate on high coupon stocks ('the gross zone') in both segments of the market.

In 1982 the Bank of England also changed the splicing band from being at the maturity range 4 to 8 years to the range 5 to 10 years. They also altered the long planning horizon from four to five years. These are all of the alterations made to the Bank of England model.

Results Obtained for the Irish Securities Market.

Figures A.1 and A.2 display the yield curves produced by the old Bank of England program (i.e. the version obtained from the Central Bank of Ireland) for various dates in 1988. Every single one of these curves has a kink starting at 3 years and ending at 8 years maturity. The curves rise steadily up to 3 years then level off up to 4 years then rise again until 5 years maturity where they again level off and finally there is a rise. All of the curves have a distinct maximum at about 8 years.

Notice that in these plots the trend over time is very clear. The highest yield was obtained in January and for every month since yields for every maturity have decreased.

Figure A.3 displays yield curves produced by the new Bank of England program. These curves, like those from the old version all predict a distinct maximum at about 8 years maturity. These curves however do not display the kink that the old version displays. The new curves also display the trend seen with the old ones, i.e. a decrease in yield at all maturities over time. However the new version consistently predicts a lower yield than the old version. For example for February 1988 the old curve predicts a maximum yield of 10.5% at about 8 years maturity. The new version predicts a maximum yield of 10.1% at about 8 years maturity.

5. THE USE OF LOWESS FOR CURVE FITTING

Introduction

In this chapter the use of locally weighted regression (LOWESS)²⁴ is considered as an alternative to the Bank of England curve fitting method. Locally weighted regression works as follows. Each point in the data set is considered separately; the fraction of points (f) required in a band centred around the point of interest is chosen by the user. All points in that band are then given weights based on their distance from the central point of interest. A weighted regression line is then fitted to the points within the band. For each of these points a fitted value is obtained using the regression line and then the residual of the fitted value from the actual value is calculated. Additional weights are calculated for each point these

weights are based on the size of the residual so that extremes have small weights.

A combined weight is calculated using the two different weights described above. A new weighted regression line is then fitted using these combined weights. The X value of the point of interest is substituted into the equation for this line giving a new smoothed Y . This entire method is repeated for all points in the data set, (see Appendix for a detailed mathematical description).

Results Obtained from Lowess

LOWESS curves were fitted to yields from the 9th of March and from the 1st of June 1988 using stocks on the Irish securities Market. Different curves were fitted using different values of the parameter f (the fraction of points used in the computation of each fitted value).

The effect of using different f

Figures F.1 to F.3 show the LOWESS curves fitted to the data set for March 9th 1988 for various f values (note: all Figures are given in pages 150 to 177).

For March 1988

The curve with $f=0.1$ has large sharp bends this is because only 10% of the points in the data set are used in fitting each smoothed value and so it is affected more by extremes. This 'curve' is not really suitable for yield curve analysis as it is approaching some of the old methods used, i.e interpolation, which proved unsuccessful in the past (see chapter 3). For the curve with $f=0.25$ most of the large bends in the curve have been smoothed out. This is because more points (25% of points) are used in the calculation of each smoothed value and thus extremes are given small weights. This curve has a slight kink starting at around 3 years going down to a minimum at 5 years and then there is a sharp rise until it reaches a distinct maximum at 10 years. In Chapter 5 it was shown that a similar kink was characteristic of some of the curves produced by the Bank of

England model which uses a splicing band between 5 and 10 years.

For the curve with $f=0.5$ there is no longer a kink, but this curve still just has the distinct maximum at 10 years.

Figure F.4 shows a comparison of the LOWESS curves with $f=0.25$ to the curve with $f=0.75$. This shows that the 0.75 curve does not contain the kink that the 0.25 curve does, nor does it have the distinct maximum at 10 years. This is again due to the fact that the 0.75 curve uses more points in the calculation of each smoothed value and is therefore less affected by extremes than the 0.25 curve.

Figure F.5 shows the curves for a series of f values. This shows how erratic the curve with $f=0.1$ is and it also indicates the following :

1. For low maturity between 0 and 5 years, the curves with low f ($f=0.1$ and $f=0.25$) predict a lower yield than the other curves. In fact as f increases so does the predicted yield on the curves. This is due to the fact that for low f the curves are effected more by 'extremes', because fewer points are used in the calculation of each smoothed value. Also the 0.25 curve predicts a kink starting at around 3 years that none of the other curves predict.
2. Between 5 and 10 years the curves with high f predict lower yields than those with low f . As f increases so do the predicted yields.
3. About 10 years maturity the 0.25 curve predicts a distinct maximum. The 0.5 curve predicts a smaller maximum at 10 years. But the 0.75 curve predicts that the maximum will be reached just after 10 years, and that this maximum will be maintained for all maturities greater than 10 years, i.e. it predicts no distinct maximum.
4. Over 10 maturity the 0.25 curve predicts a lower yield than the rest. All the other curves predict approximately the same yield.

The effects of using different f for June 1988

Figures F.6 to F.10 show the LOWESS curves fitted to the data set for

June 1st 1988 for various f values. The results obtained here are nearly exactly the same as for March. The main differences are outlined below:

1. With $f=0.1$, the curve is a great deal less erratic than for March.
2. For the curve with $f=0.25$ the kink that was just evident in March has now become more extreme.
This time it starts at 4 years, achieves a minimum just after 5 years and then increases to a distinct maximum at 10 years, similar to the kink obtained in March. In this case however the decrease from 4 years to 5 years is much greater than before.
3. With $f=0.5$ the slight kink is still evident. In March no kink was evident.
4. Figure F.10 shows a comparison of the LOWESS curves with $f=0.25$ to the curve with $f=0.75$. The difference between the 0.75 curve and the 0.25 curve is much more distinct than it was in March; the main reason for this is that the kink in the 0.25 curve is more extreme. The 0.25 curve starts off below the 0.75 curve. From 2 to 4 years maturity the 0.25 curve predicts a higher yield than the 0.75 curve. But at about 4 to 6 years maturity it predicts a lower yield which did not occur in March.

Comparison with the Bank of England Model

In this section the Bank of England Model is compared to the different curves produced using LOWESS for March and June 1988.

for March 1988

LOWESS with $f=0.1$ for March 9th 1988

Figure F.11 displays a plot of the two curves together. This plot shows the following:

1. At Low maturity there is some correlation between the LOWESS curve and the Bank of England curve.

2. The LOWESS curve with $f=0.1$ is very erratic between 2 and 7 years (because only a small number of points are used in the calculation of each smoothed value) and as a result is very different to the Bank of England model during this maturity range.
3. At maturities greater than 7 years the yield predicted by both curves is almost exactly the same.

Figure F.12 shows the yields predicted by the Bank of England model plotted against those predicted by LOWESS with $f=0.1$. If there were perfect correlation between the two curves this would be a straight 45 degree line through the origin. Thus we can see from this figure that for low yields there is little correlation between the two curves and for higher yields the correlation is slightly better.

LOWESS with $f=0.25$ for March 9th 1988

Figure F.13 shows the Bank of England curve plotted against the LOWESS curve with $f=0.25$. This plot shows the following

1. At Low maturity this LOWESS curve is not as good as the LOWESS curve with $f=0.1$. It predicts higher yields than the Bank of England yield curve does.
2. This LOWESS curve predicts the slight kink in the curve between 5 and 10 years that is evident in the Bank of England curve. There is a slight difference between the two at 3 to 4 years maturity. But for the kink and the early years there is almost perfect correlation between the two curves.
3. This LOWESS curve predicts the distinct maximum at 10 years predicted by the Bank of England Model.
4. At High maturities the LOWESS curve predicts a slightly lower yield than the Bank of England model.

Figure F.14 backs this up, with most of the points on or close to a 45 degree straight line through the origin, except at low maturities and at the kink where there is a divergence from the line.

LOWESS with $f=0.5$ and $f=0.75$ for March 9th 1988

Figures F.15 to F.18 display a comparison of these curves with the Bank of England curve. These display the following points for both LOWESS curves:

1. They do not predict the low yields at low maturities that the Bank of England curve does.
2. As f increases the kink in the LOWESS curve seems to straighten out, giving a greater departure from the Bank of England curve.
3. The distinct maximum at 10 years is still just there for $f=0.5$ but disappears with $f=0.75$.
4. At high maturities all the curves predict approximately the same yields.

This concludes the comparison of the Bank of England curves with does produced by LOWESS for March 1988.

For June 1988

For June 1988 the kink in the Bank of England Model is a great deal more extreme than for the month of March.

LOWESS with $f=0.1$ for June 1st 1988

Figure F.19 displays a plot of the Bank of England curve and the LOWESS curve with $f=0.1$ together. The only differences between this and the March analysis are:

1. At low maturities there is a bigger divergence from the Bank of England model than was evident in March.
2. The LOWESS curve predicts a higher maximum at 10 years than the Bank of England model does. In March the maximum predicted by both curves was the same.

3. For maturities greater than 15 years the yield predicted by both curves are almost exactly the same. Which was also the case in March.

Figure F.20 shows the yields predicted by the Bank of England model plotted against those predicted by LOWESS with $f=0.1$. We can see for this figure that for low yields there is little correlation between the two curves for low maturity, but as maturity increases the correlation improves.

LOWESS with $f=0.25$ for June 1st 1988

Figure F.21 shows the two curves plotted together. Most of the points made in the analysis of the March curves can also be made here. The main differences between this and the March analysis are

1. At low maturities, less than one year, this lowess curve is much closer to the Bank of England curve than it was in March.
2. This LOWESS curve predicts a slightly higher yield at 3 years than the Bank of England curve. It also predicts a slightly lower yield at 5 years. But for the rest of the kink between 5 and 10 years maturity there is almost perfect correlation between the two curves. In March a similar thing happened. However, in this case, it is much more extreme.
3. This LOWESS predicts the distinct maximum at 10 years which is slightly lower than the maximum predicted by the Bank of England Model. In March the Bank of England and LOWESS curves predicted equal values for the maximum.

This shows that overall this LOWESS curve is a very good approximation to the Bank of England curve. Figure F.22 backs this up, with most of the points on or close to a 45 degree straight line through the origin, except for a small band of points in the centre which correspond to the peak in the curve.

LOWESS with $f=0.5$ June 1st 1988

Figures F.23 to F.24 display a comparison of this curve with the Bank of England curve. These display the following points:

1. This LOWESS curve does not predict the low yields at low maturities that the Bank of England curve does. It predicts a slightly higher yield than the 0.25 curve, i.e. it is not as good an approximation to the Bank of England model at low maturities as the 0.25 curve.
2. This curve is closer in the 3 to 7 year range to the Bank of England curve than the 0.25 curve.
3. This curve predicts the distinct maximum at 10 years, but it is lower than the maximum predicted by the LOWESS curve with $f=0.25$ and the maximum predicted by the Bank of England.
4. At high maturities this LOWESS curve predicts a slightly higher yield than the Bank of England curve.

LOWESS with $f=0.75$ for June 1st 1988

Figures F.25 to F.26 display a comparison of this curve with the Bank of England curve. These display the following points:

1. This LOWESS curve is the worse one for predicting the low yields at low maturities that the Bank of England curve does. It predicts a higher yield than all the other LOWESS yield curves.
2. This curve does not predict the kink in the yield curve that the Bank of England curve does.
3. This curve does not predict the distinct maximum at 10 years that the Bank of England curve does.
4. At high maturities this LOWESS curve predicts a slightly higher yield than Bank of England curve.

Lowess fitted to the Actual Yields

All the LOWESS curves presented in this chapter so far have been fitted to the calculated yields produced by the Bank of England model. In this section an examination of the fitting of LOWESS to the actual yields is considered. The main reason for doing this was that the client was unhappy, theoretically, with the use of the planning horizons in the Bank of England model. They saw them as basically a device for obtaining better fits. They said that theoretically they would be happier with a fit to the actual yields using LOWESS. The advantage of using this method over normal regression is that the dealer can use his knowledge of the workings of the market to determine the type of curve fitted to the data. For example if the dealer believed in the segmentation of the market and that investors only considered a group of say 10 stocks with approximately the same maturity, he could set his f value in LOWESS accordingly and his curve would be fitted on the workings of the market rather than purely on a statistical basis. However, even though this would be an improvement on normal regression, this type of fit is still inappropriate. Thus this analysis was undertaken to show why such a fit was still inappropriate and why the Bank of England model was the best method for producing yield curves on the Irish securities market.

Figure F.27 shows LOWESS fitted to actual yields from the 1st of January 1988. This shows that the curve produced using LOWESS with $f=0.25$ is very smooth. But some points, like the one marked A, have a yield far below the rest of the other stocks and as a result are far away from the curve because LOWESS with $f=0.25$ (and higher f) regards them as extremes and gives them low weight. However, the reason why this stock has a low yield is because of its coupon. So by using LOWESS with an f value of 0.25 , the stock marked A and stocks like it are in fact not being used in the calculation of the fitted curve (because LOWESS would give them zero or close to zero weight).

The way to overcome this is to reduce the band size (i.e. decrease f) so that these points will no longer be regarded as extremes by LOWESS. Figure F.28 shows the LOWESS curve on the same data set with $f=0.1$. This shows that now the curve deviates down to the point A and to points like it. The trouble with this is that now the curve fitting method is approaching interpolation, which as mentioned before proved unsuccessful

in the past.

Thus by using LOWESS with high f fitted to the actual yields, stocks with special coupon features are not being considered in the calculation of the curve. By using LOWESS with low f the method of curve fitting is in fact approaching interpolation which proved unsuccessful in the past. This shows that some method for accounting for the coupon effect is required, which is what the Bank of England Model does.

6. CONCLUSIONS

The use of LOWESS fitted to the actual yield was shown to be inappropriate as LOWESS fitted in this way does not consider the coupon effect. This indicates that some method of accounting for coupon before the curve is fitted is required. This is how the Bank of England model works. Also, even though the client was happier theoretically with LOWESS fitted to the actual yields, he preferred the Bank of England curves on a practical basis. The main reason for this is that the Bank of England model produces curves the shape of which he expected from the Irish securities Market.

The following conclusions can be drawn from the analysis into LOWESS for curve fitting to the calculated yields from the Bank of England model:

1. The LOWESS curves that gave the best correlation with the Bank of England Model were with $f=0.25$ and $f=0.5$
2. As f increased from $f=0.25$ so did the departure from the Bank of England Model
3. For high f there was no kink in the curve.
4. For high f there was no distinct maximum.

It was also shown that even for an extreme kink in the curve (as occurred in June 1988), the Bank of England curve could be replicated by using LOWESS.

As pointed out in section 4, the Bank of England made a number of adjustments to their program in order to get rid of a kink in the curve that started to appear in the mid seventies. Here we have shown that by using LOWESS with high enough f (f greater than 0.5) as the curve fitting method we can get rid of this kink. This was true of both months data, even in June, when the kink was at its extreme.

However, even though the Bank of England found the kink to be unrealistic of the British Market, the client for this project considered the following characteristics true of the Irish Market:

- (a) The kink in the curve between 5 and 10 years caused by the use of the splicing band.
- (b) The distinct maximum at 10 years.

The analysis above showed that by using a LOWESS curve with $f=0.25$ the Bank of England curve can be replicated and if at a later date the client became unhappy with the kink in the curve he could by using a high enough f , eliminate these to produce a smoother flatter curve.

YIELD CURVE FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE
OLD BANK OF ENGLAND MODEL FOR 9th OF MARCH 1988

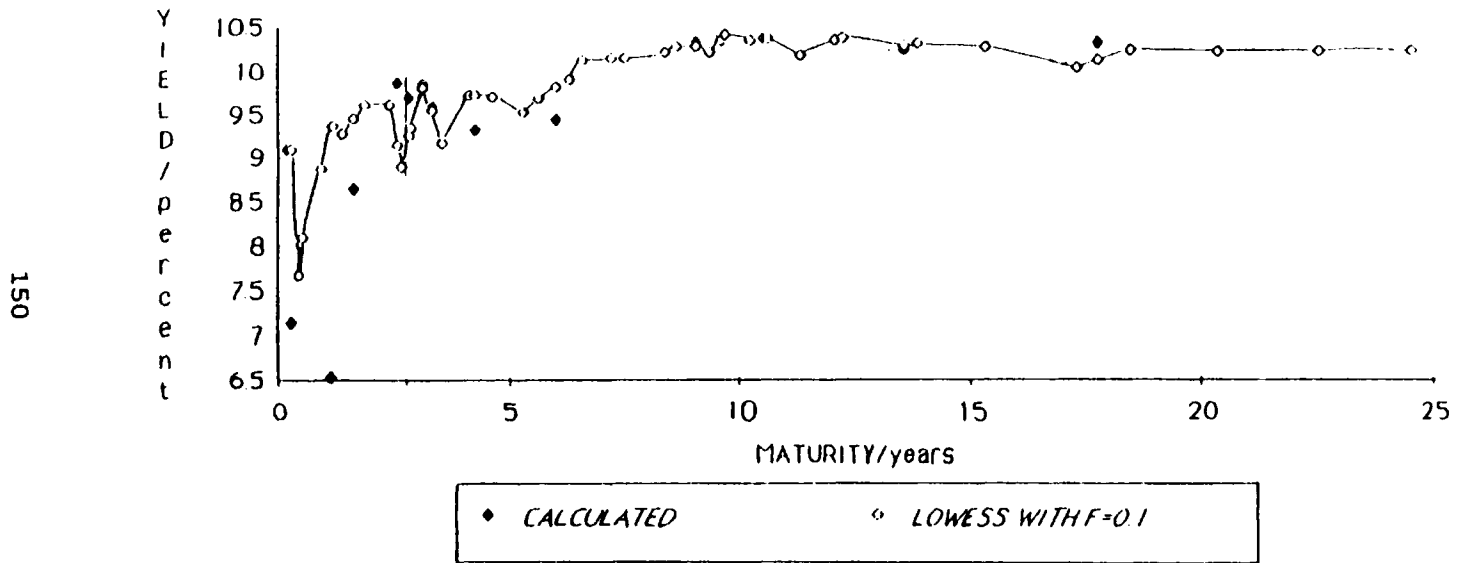


FIGURE F.1

YIELD CURVE FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE OLD BANK OF ENGLAND MODEL FOR 9th OF MARCH 1988

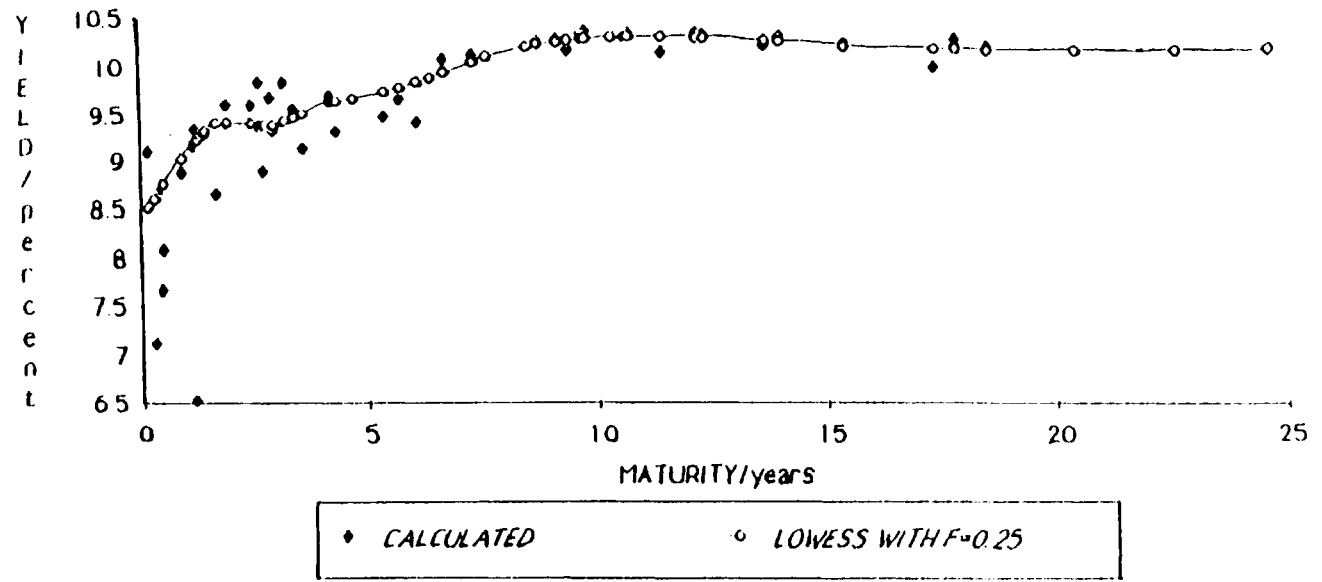


FIGURE F.2

YIELD CURVE FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE OLD BANK OF ENGLAND MODEL FOR 9th OF MARCH 1988

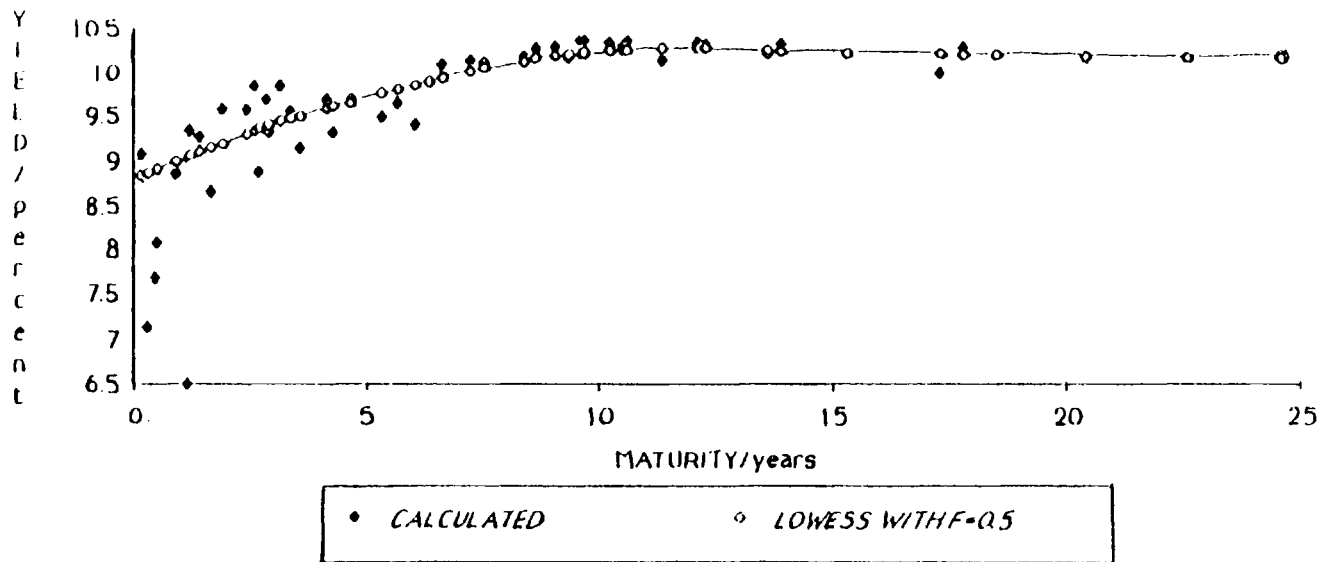


FIGURE F.3

YIELD CURVE FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE
OLD BANK OF ENGLAND MODEL FOR 9th OF MARCH 1988

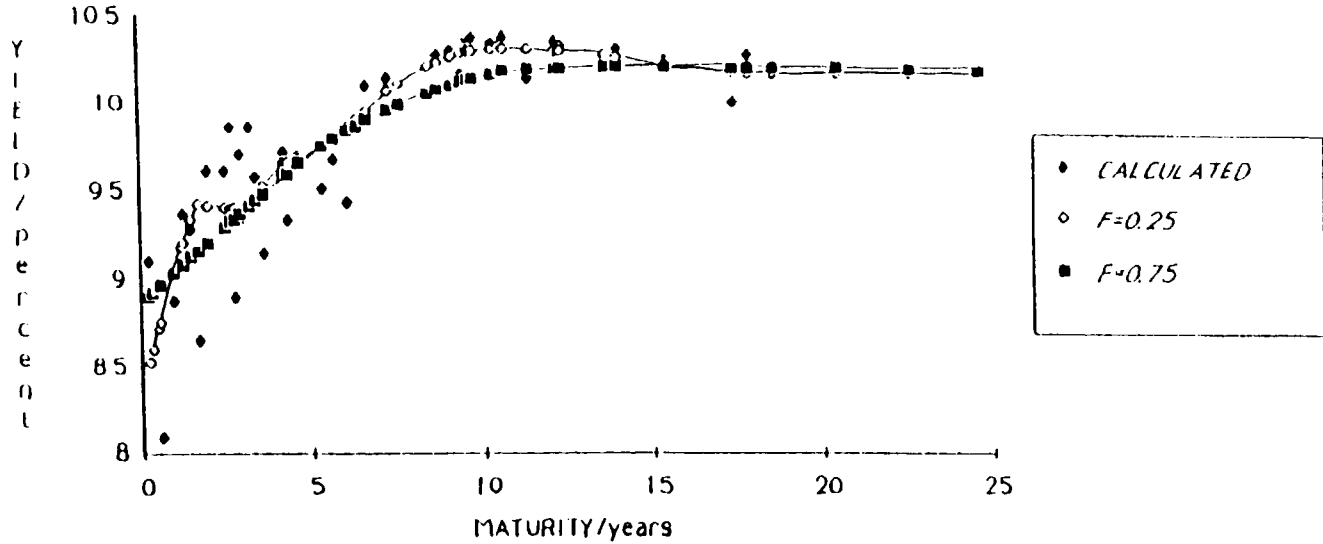


FIGURE F.4.

YIELD CURVES FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE
OLD BANK OF ENGLAND MODEL FOR 9th OF MARCH 1988

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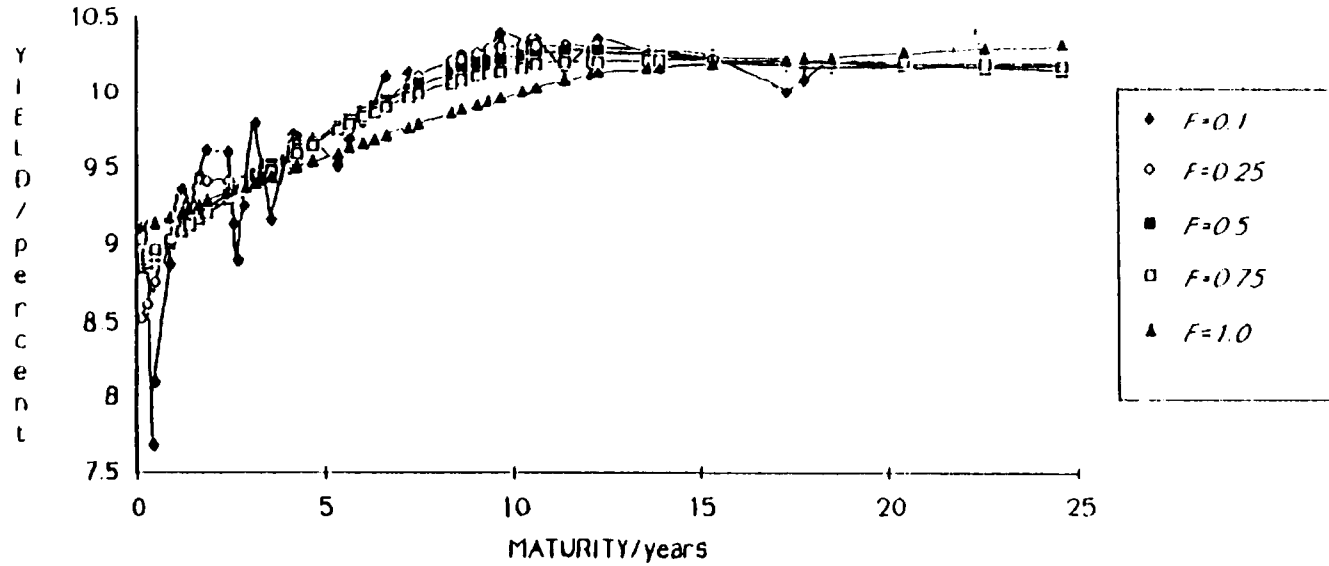


FIGURE F.5

YIELD CURVE FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE
OLD BANK OF ENGLAND MODEL FOR 1st OF JUNE 1988

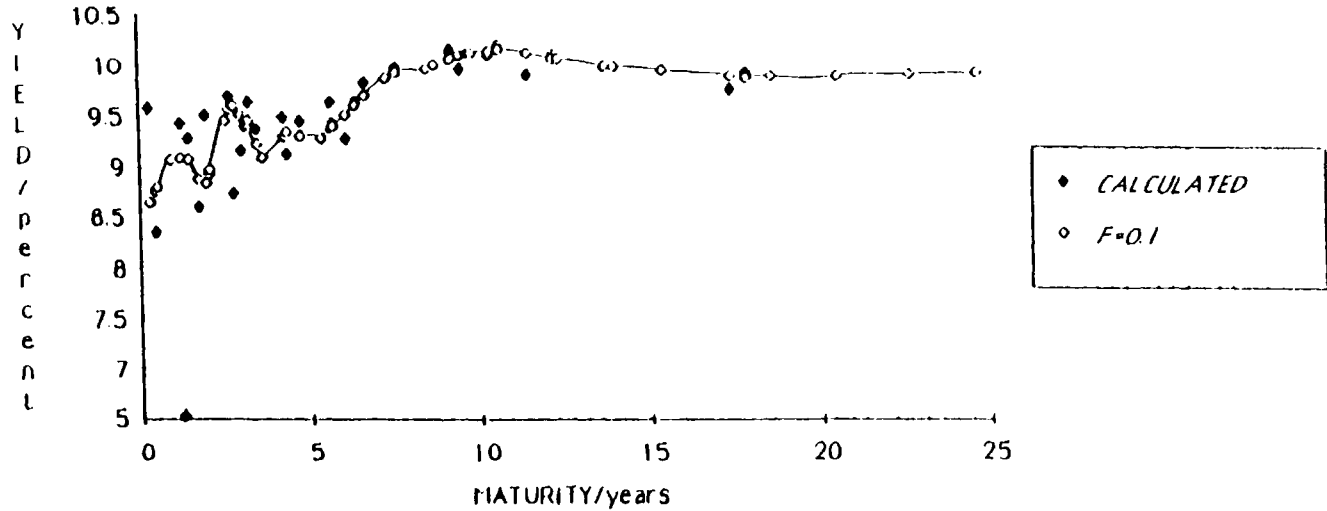


FIGURE F.6

YIELD CURVE FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE
OLD BANK OF ENGLAND MODEL FOR 1st OF JUNE 1988

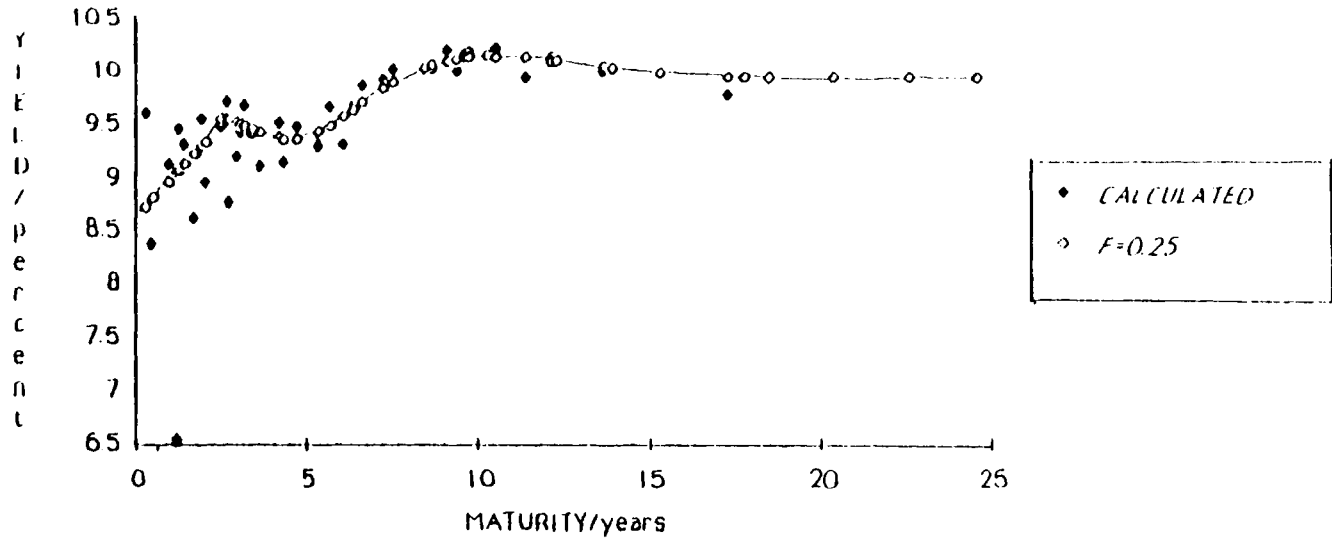


FIGURE F.7

YIELD CURVE FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE
OLD BANK OF ENGLAND MODEL FOR 1st OF JUNE 1988

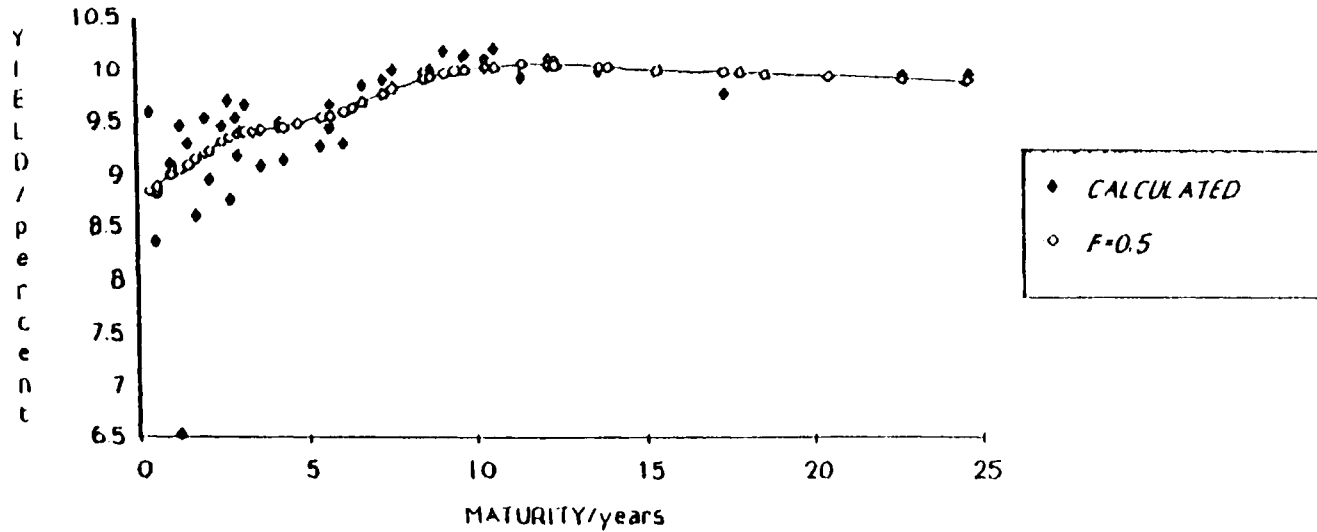


FIGURE F.8

YIELD CURVES FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE OLD BANK OF ENGLAND MODEL FOR 1st OF JUNE 1988

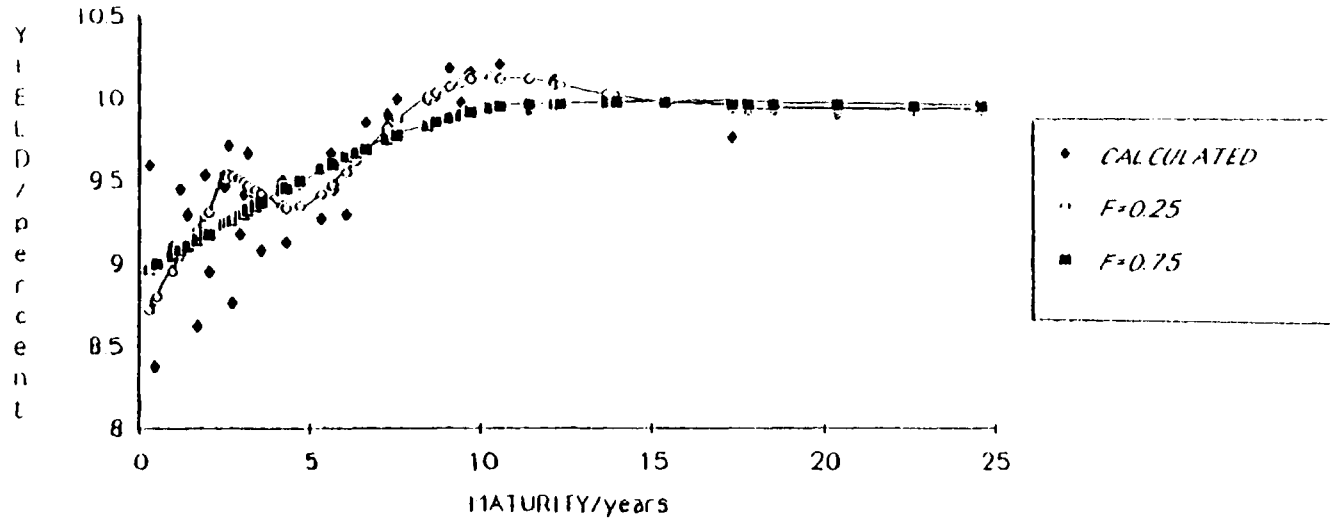


FIGURE F.9

YIELD CURVES FITTED USING LOWESS TO THE CALCULATED YIELDS FROM THE
OLD BANK OF ENGLAND MODEL FOR 1st OF JUNE 1988

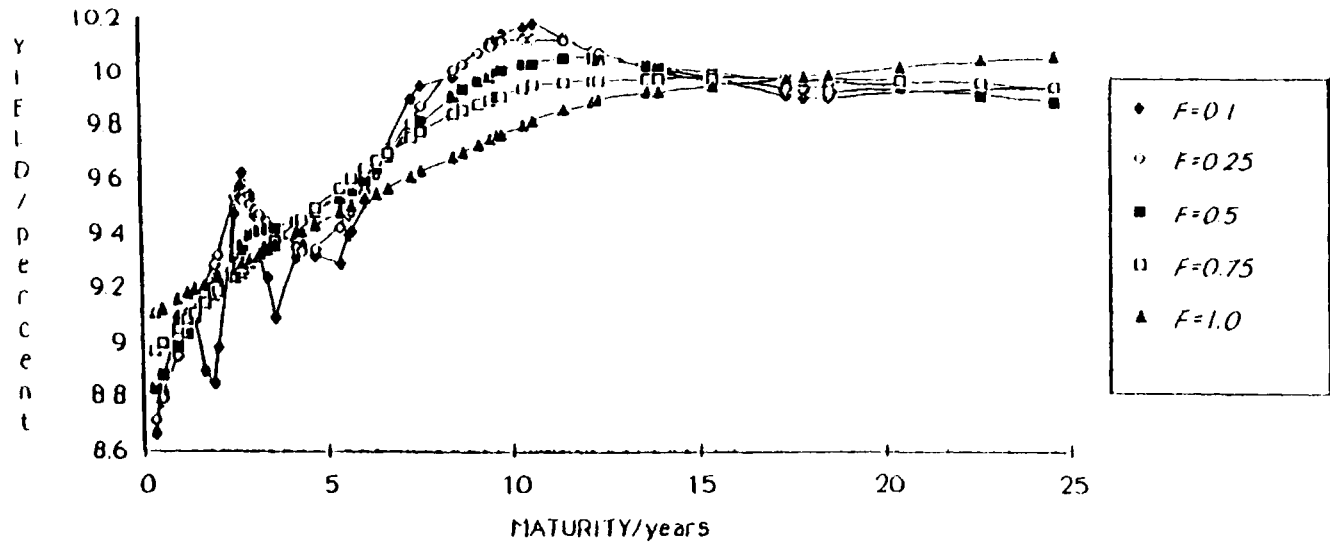


FIGURE F.10

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR 9th OF MARCH 1988

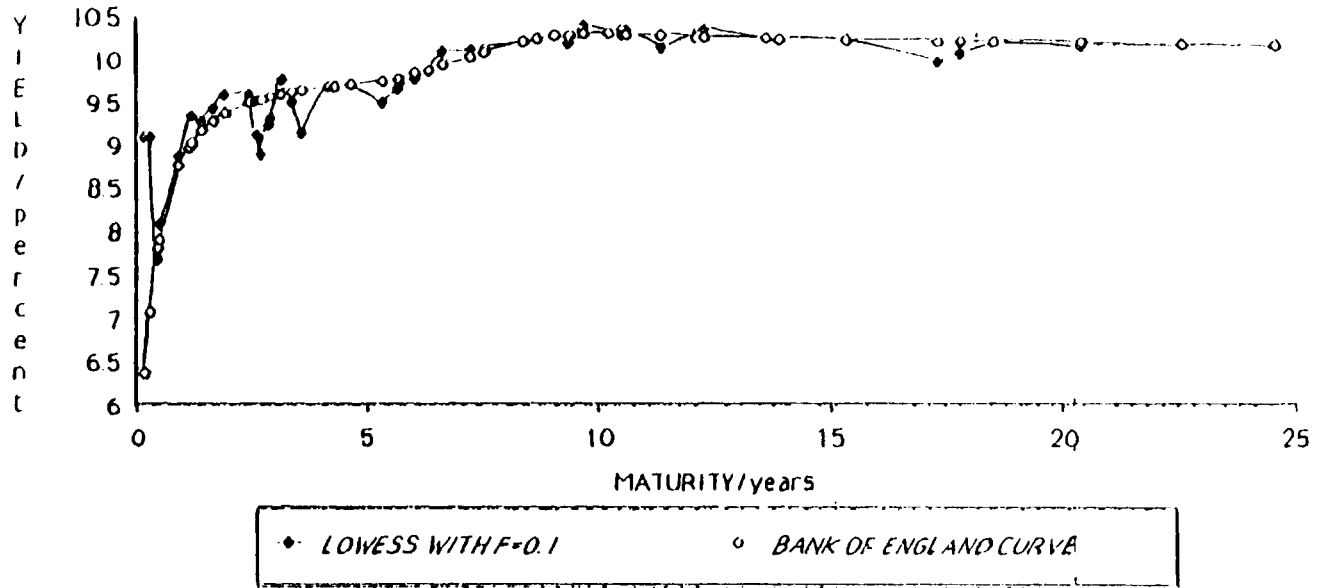


FIGURE F.11

A PLOT OF THE BANK OF ENGLAND CURVE AGAINST THE CURVE FITTED BY LOWESS

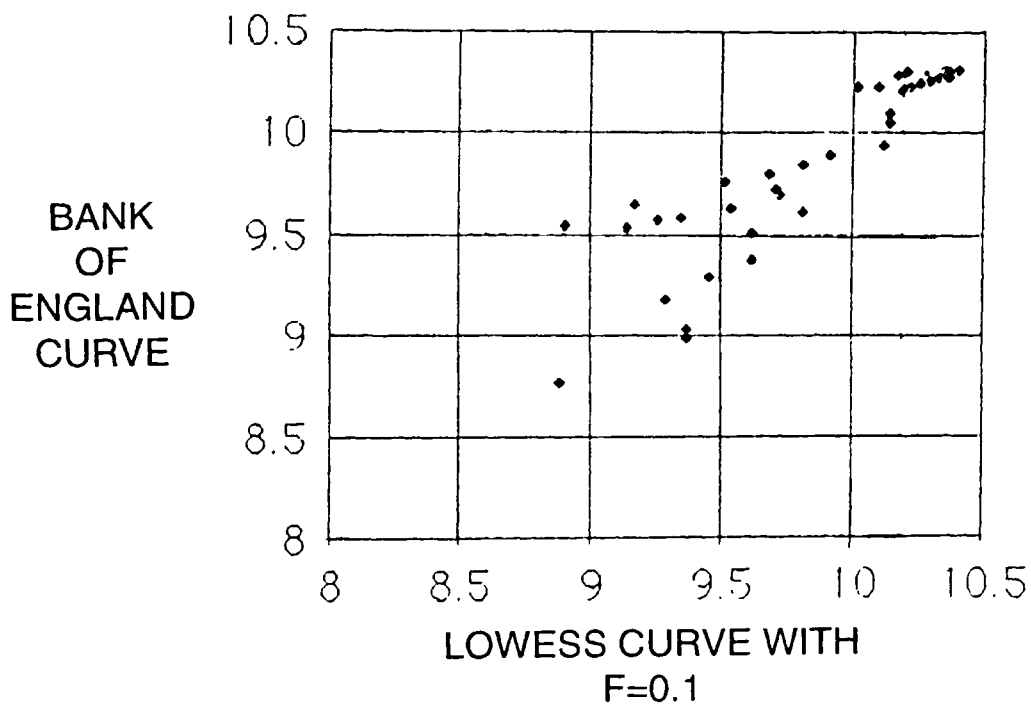


FIGURE F.12

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR 9th OF MARCH 1988

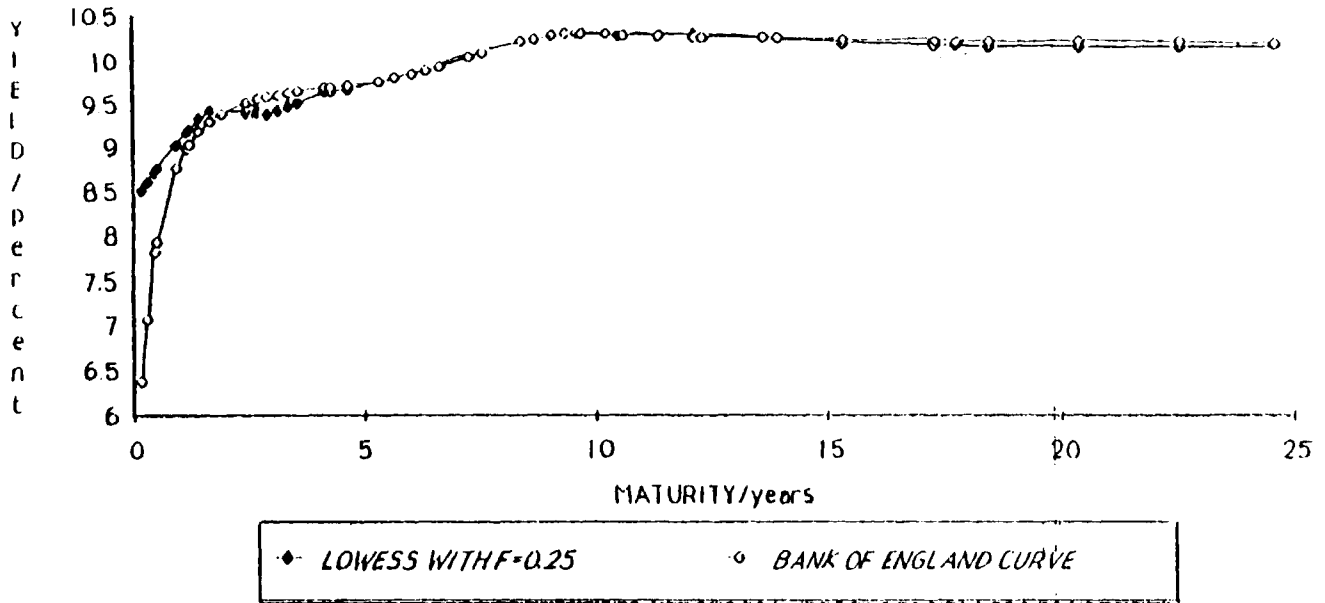


FIGURE F.13

A PLOT OF THE BANK OF ENGLAND CURVE AGAINST THE CURVE FITTED BY LOWESS

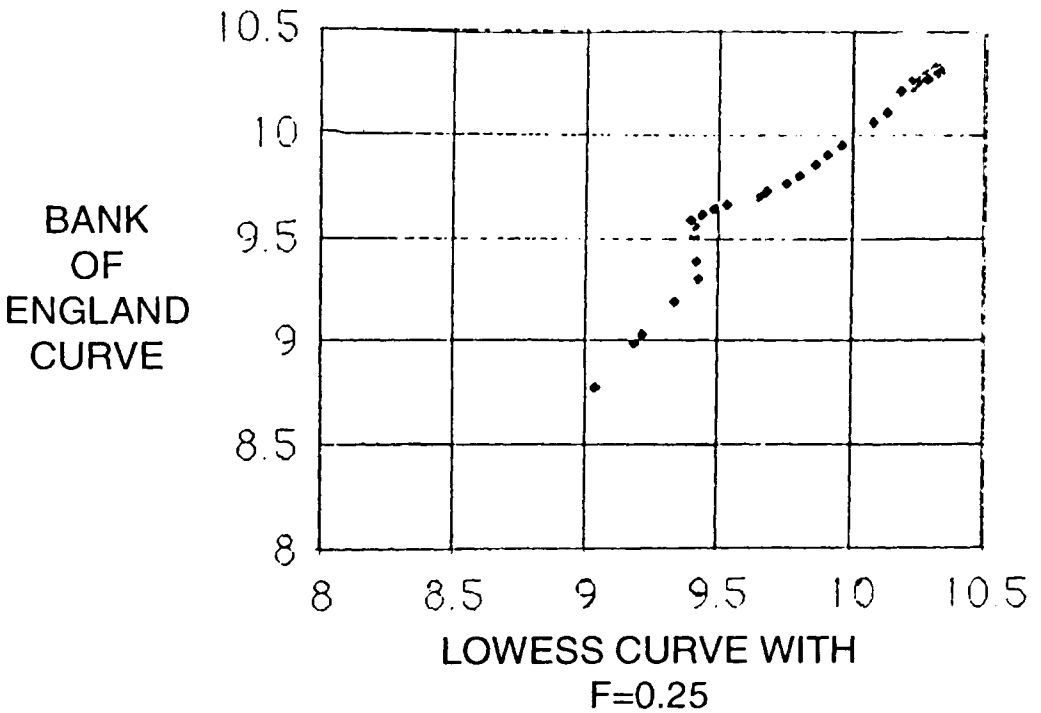


FIGURE F.14

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR 9th OF MARCH 1988

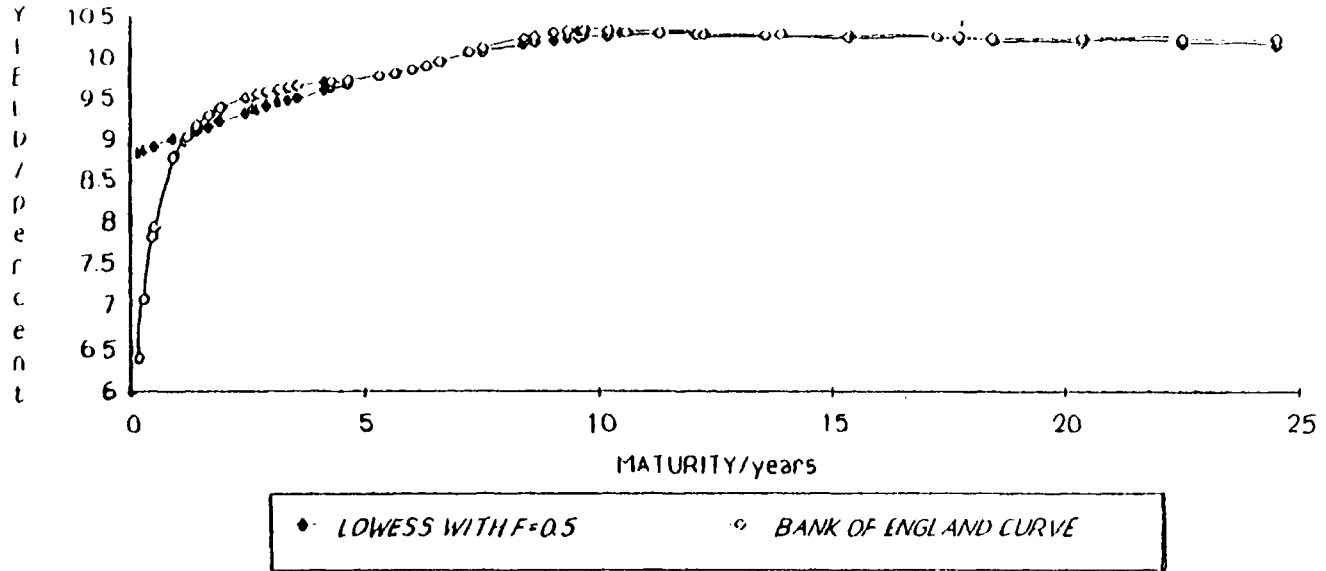


FIGURE F.15

A PLOT OF THE BANK OF ENGLAND CURVE
AGAINST THE CURVE FITTED BY LOWESS

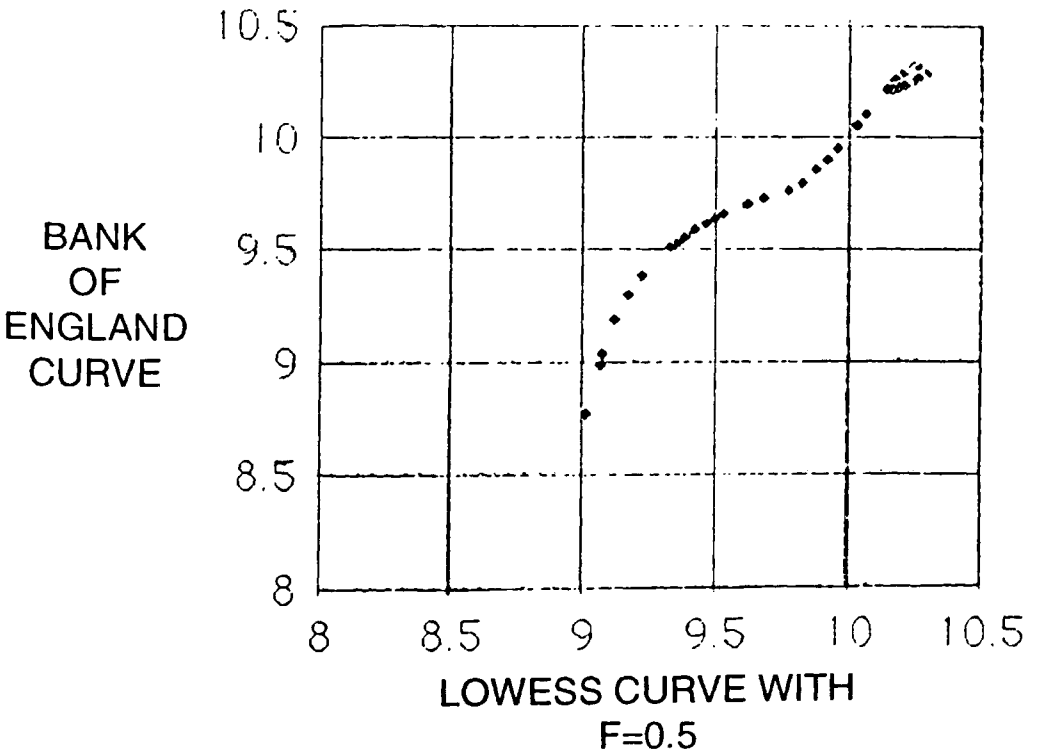


FIGURE F.16

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR 9th OF MARCH 1988

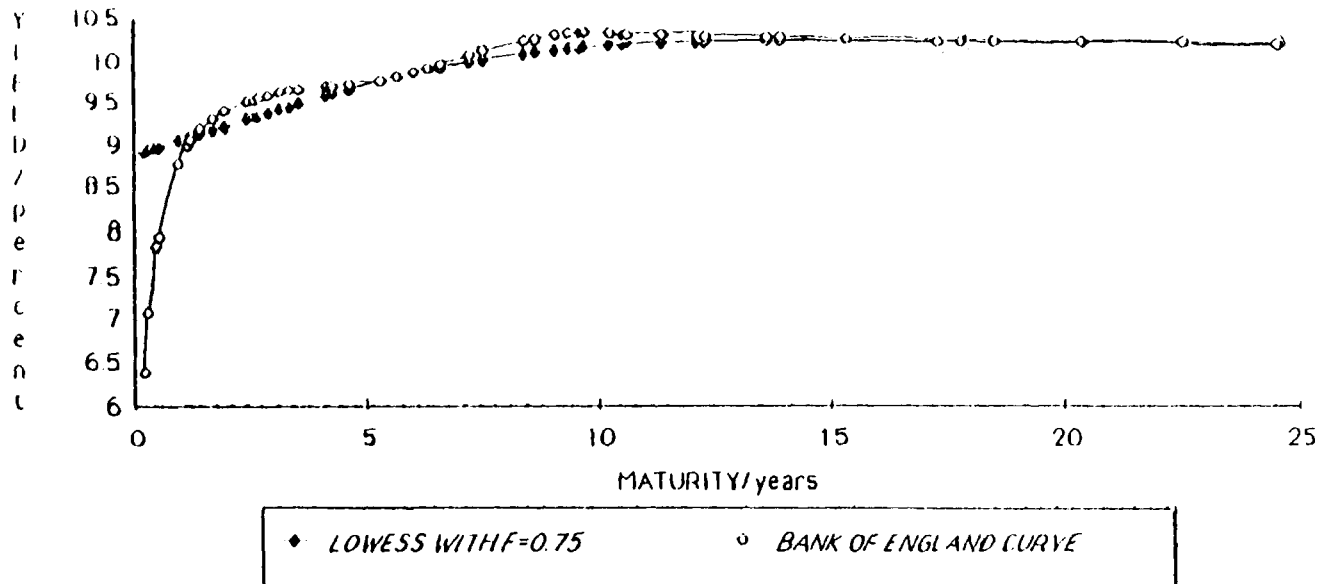


FIGURE F.17

A PLOT OF THE BANK OF ENGLAND CURVE
AGAINST THE CURVE FITTED BY LOWESS

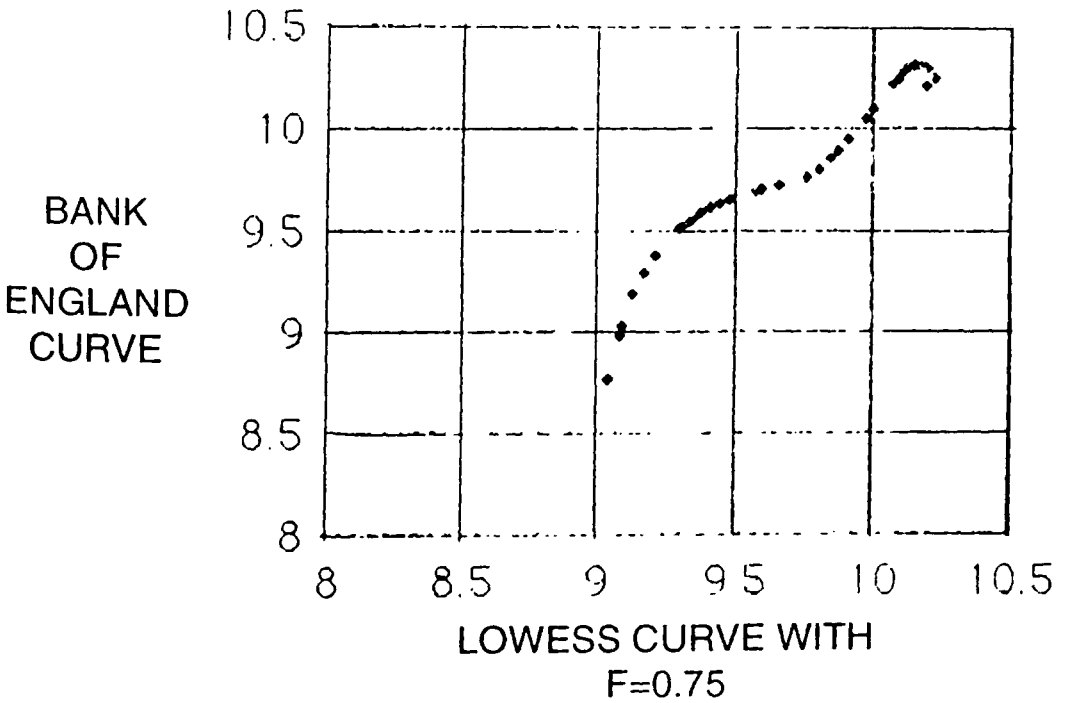


FIGURE F.10

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR 1st OF JUNE 1988

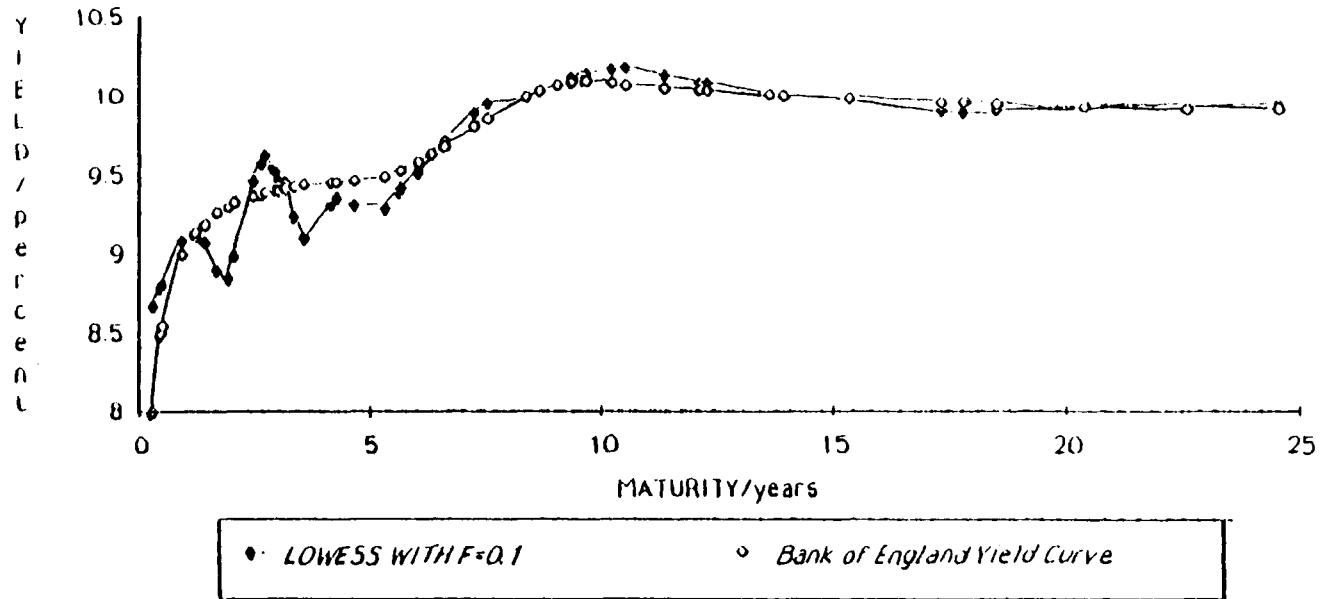


FIGURE F.19

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR 1st OF JUNE 1988

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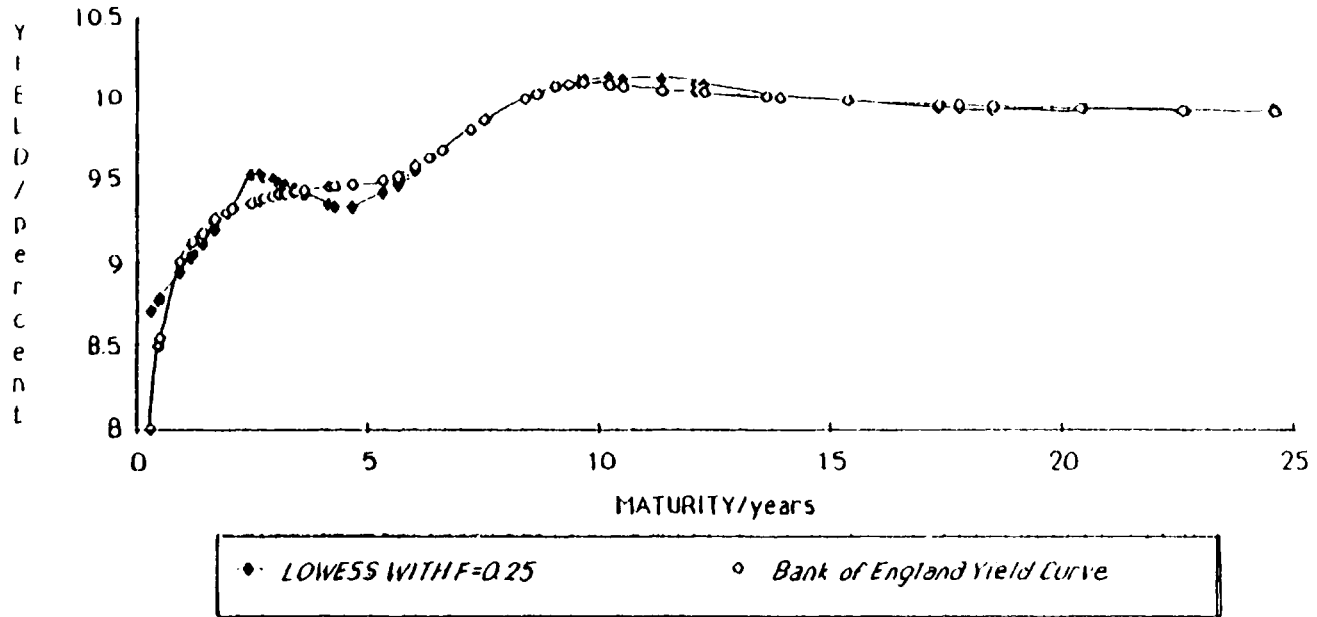


FIGURE F.21

A PLOT OF THE BANK OF ENGLAND CURVE
AGAINST THE CURVE FITTED BY LOWESS

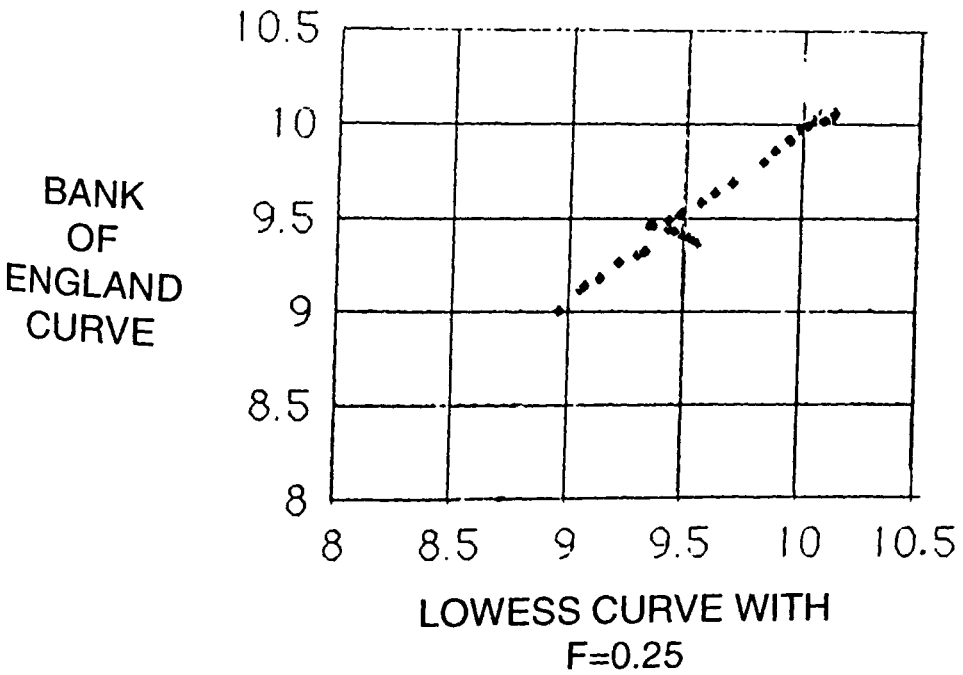


FIGURE F.22

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR JUNE 1st 1988

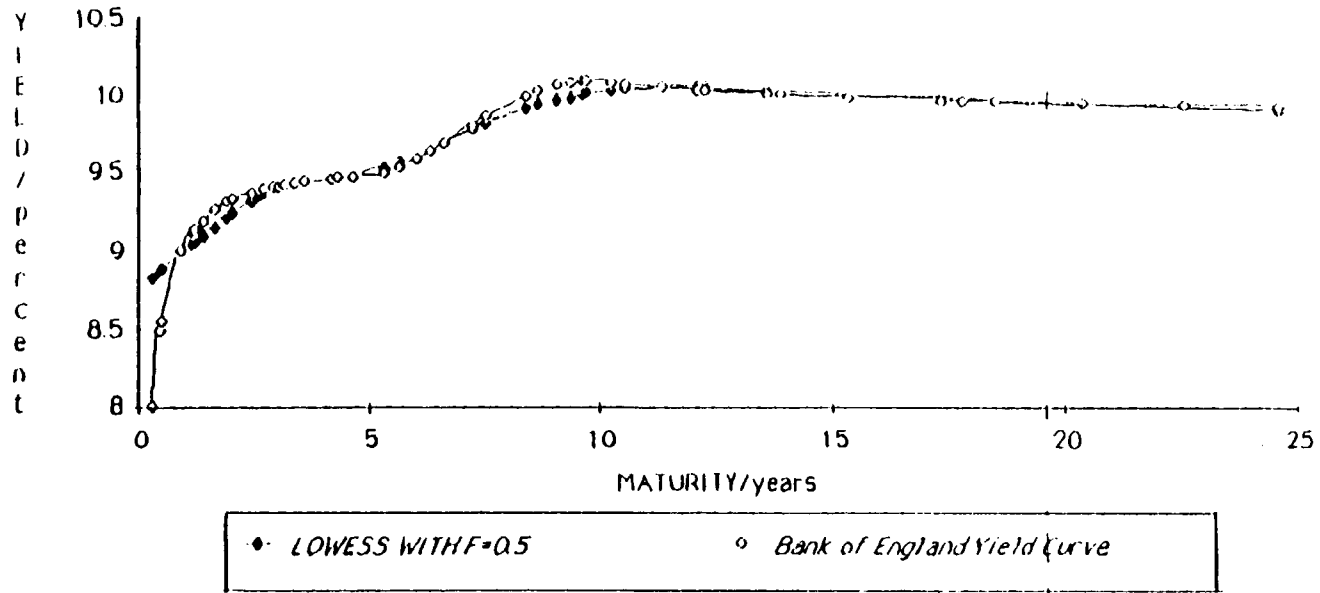


FIGURE F.28

A PLOT OF THE BANK OF ENGLAND CURVE
AGAINST THE CURVE FITTED BY LOWESS

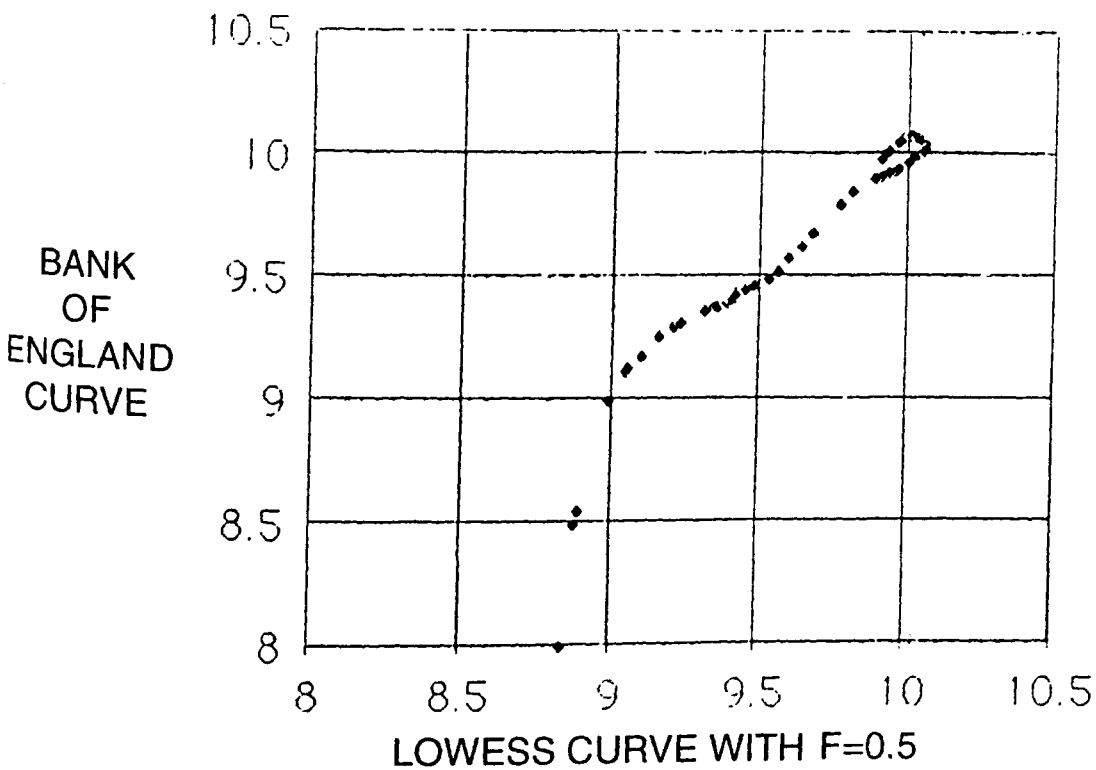


FIGURE F.24

YIELD CURVES FITTED BY USING LOWESS AND THE BANK OF ENGLAND MODEL
FOR JUNE 1st 1988

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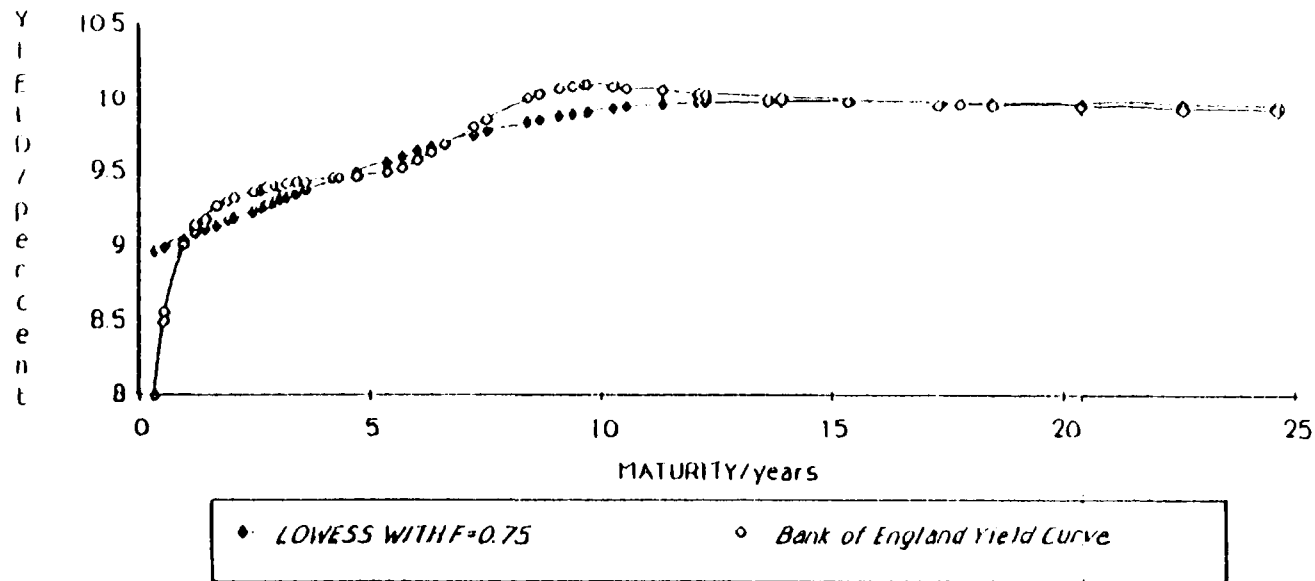


FIGURE F.29

A PLOT OF THE BANK OF ENGLAND CURVE
AGAINST THE CURVE FITTED BY LOWESS

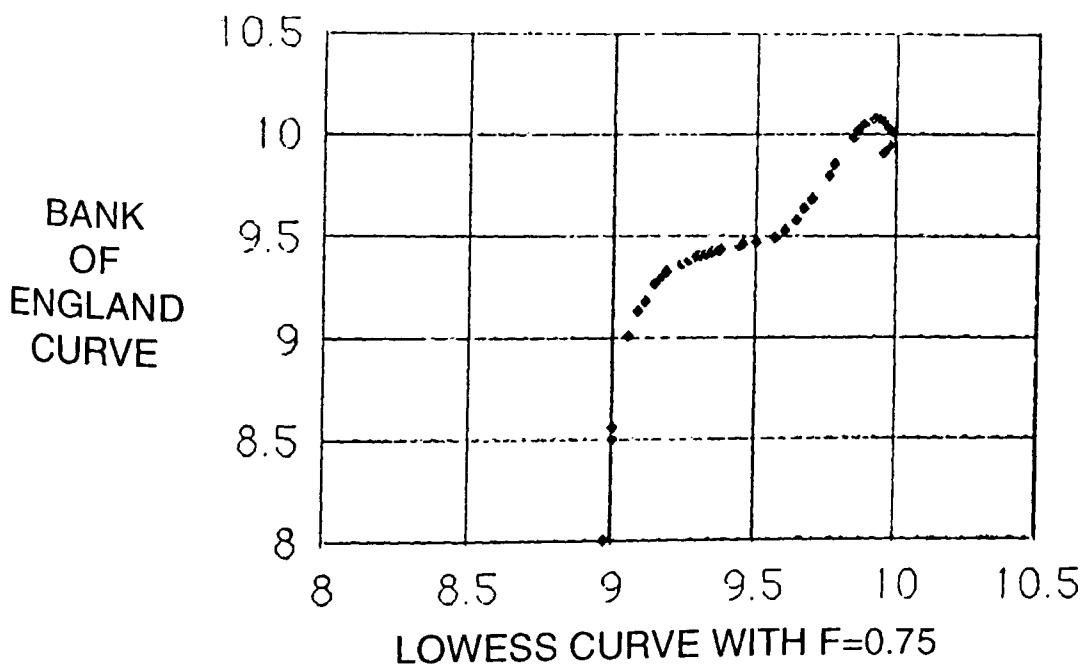


FIGURE F.26

LOWESS YIELD CURVE FOR JANUARY 1988

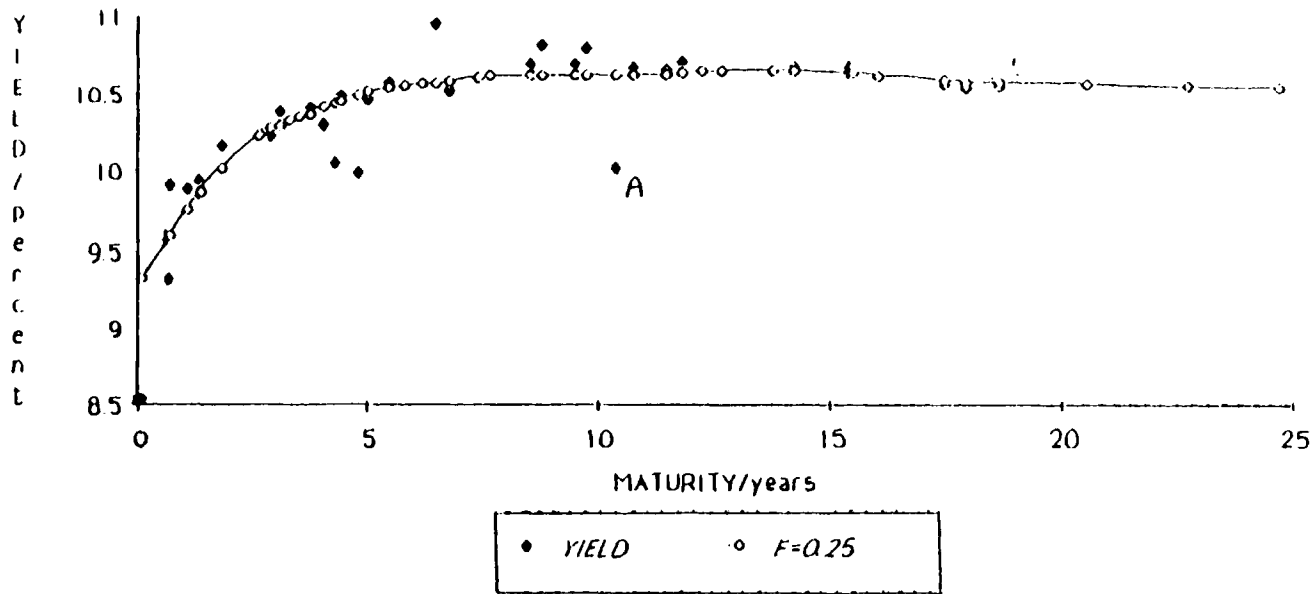


FIGURE F.27

LOWESS YIELD CURVE FOR JANUARY 1988

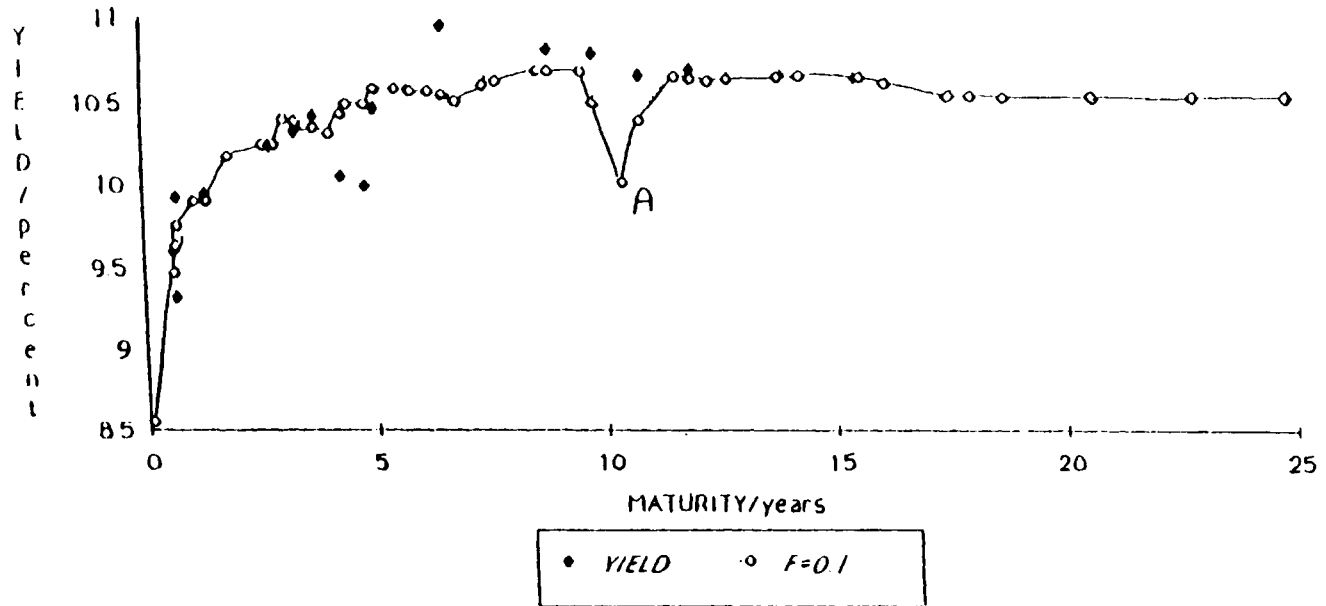


FIGURE F.28

YIELD CURVE FOR VARIOUS DATES

USING THE OLD BANK OF ENGLAND MODEL

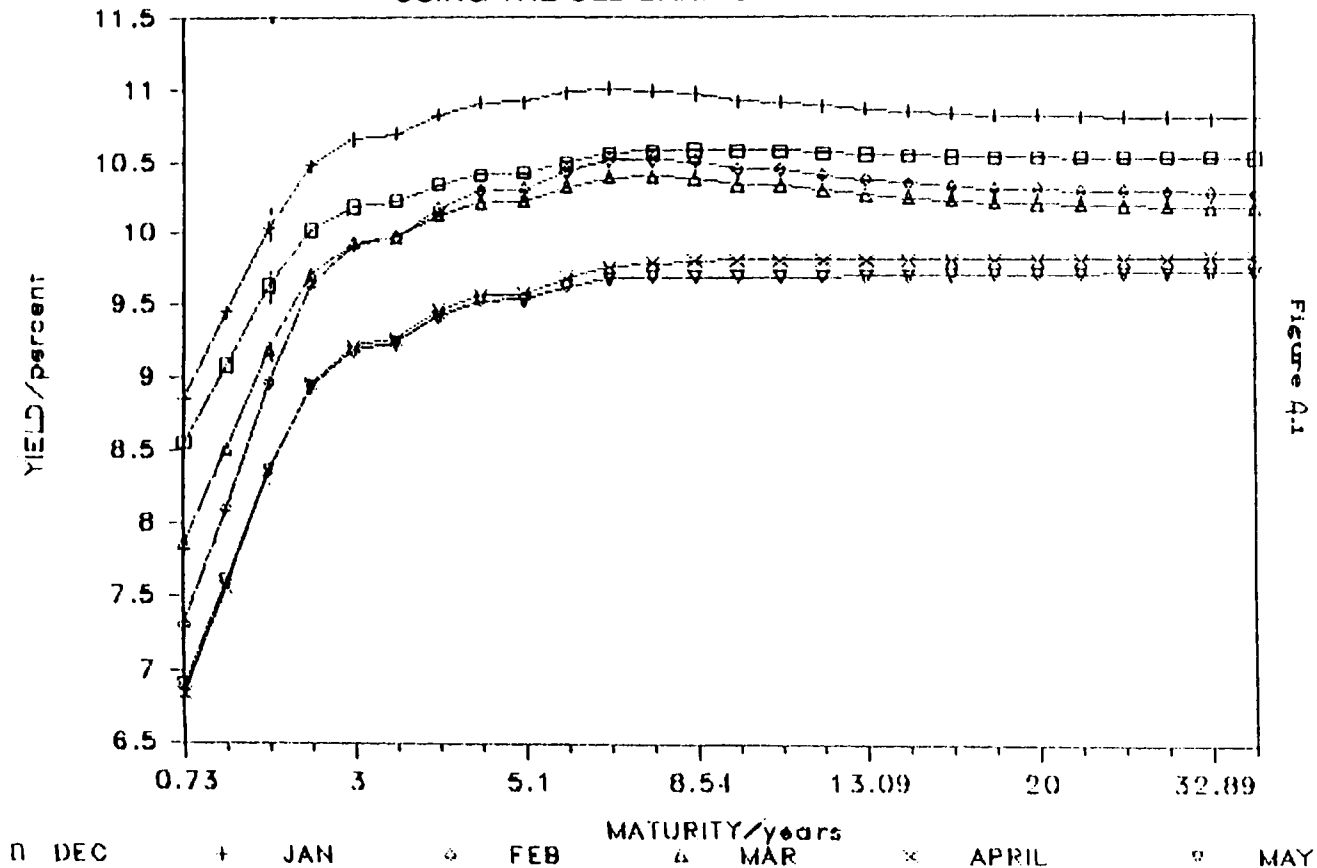


Figure A.1

YIELD CURVE FOR VARIOUS DATES IN 1988

USING THE OLD BANK OF ENGLAND MODEL

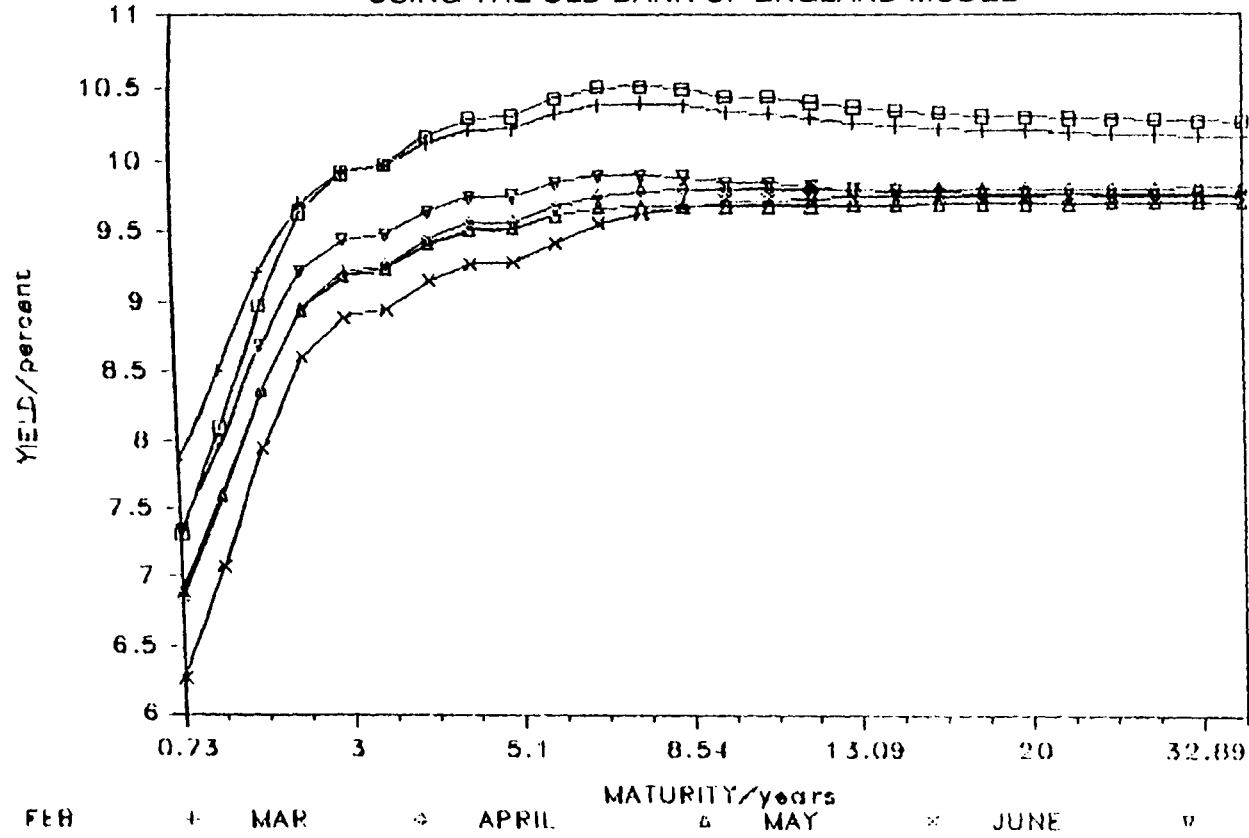
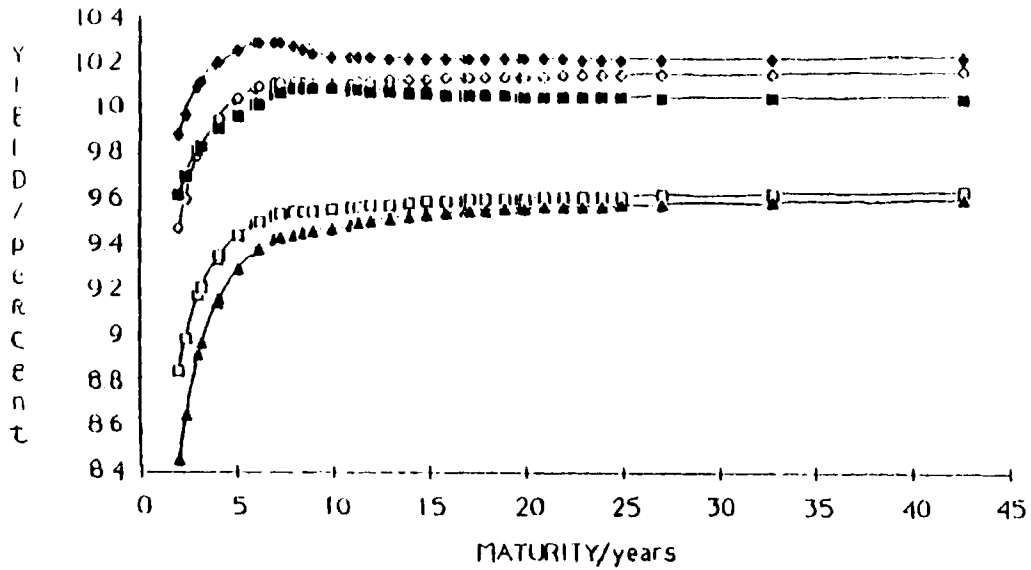


Figure A.2

YIELD CURVES PRODUCED BY THE NEW BANK OF ENGLAND
MODEL FOR VARIOUS DATES



- ◆ DEC '87
- ◇ FEB '88
- MARCH '88
- APRIL '88
- ▲ JUNI '88

Figure A.3

Appendix: The Detailed Mathematical Description of Lowess

LOcally Weigthed rEgression Scatter plot Smoothing (LOWESS) (25) employs weighted least squares, which is a statistical method that can be used to fit a line to a set of points on a scatter plot. The method used of LOWESS is described in the following steps.

1. First f is chosen which is approximately the fraction of points to be used in the computation of each fitted value. A vertical strip is then found which is just big enough to cover Nf points (where N is the total number of points) and is centred at the point of interest.
2. Neighbourhood weights are defined for all points within the strip using the weight function. (which is described below)

Let q be Nf rounded to the nearest integer. Let $d(i)$ be the distance from $x(i)$ to its q th nearest neighbour along the x -axis. Let $T(u)$ be the tricubic weight function

$$T(u) = (1 - u^3)^3 \text{ for } |u| < 1 \text{ otherwise } T(u) = 0$$

Then the weight given to a point $(x(k), y(k))$ when computing a smoothed value at $x(i)$ is defined to be

$$t_i(x(k)) = T\left(\frac{x(i) - x(k)}{d(i)}\right)$$

3. A line is fitted to the points of the scatter plot that lie within the strip, using weighted least squares.
4. The fitted value \bar{Y} is defined to be the Y value of the fitted line at $x = x(i)$. That is, if the fitted line is

$$Y = \bar{a} + \bar{b}x(i)$$

then,

$$\bar{Y}(i) = \bar{a} + \bar{b}x(i)$$

The steps above are carried out for each point in the scatter plot.

To compute a fitted value at $x(i)$, a and b are found which minimise

$$\sum_{k=1}^n t_i(x(k)) \cdot (y(k) - \bar{a} - \bar{b}x(k))^2$$

If a and b are values which achieve the minimum then the initial fitted value at $x(i)$ is defined to be

$$y(i) = \bar{a} + \bar{b}x(i)$$

residuals are computed

$$r(i) = y(i) - \bar{y}(i)$$

and robustness weights are computed from them as follows:

Let $B(u)$ be the bisquare weight function,

$$B(u) = (1 - u^2)^2 \text{ for } |u| < 1 \text{ otherwise } B(u) = 0$$

Let m be the median of the absolute values of the residuals,

$$m = \text{median } |r(k)|$$

The robustness weight for the point $(x(k), y(k))$ is defined to be,

$$W(x(k)) = \frac{B(|r(k)|/m)}{6m}$$

We now get updated fitted values by fitting lines again. In the weighted linear regression for refitting $y(i)$ the point $(x(k), y(k))$ is given a weight

$$W(x(k)).t_i(x(k))$$

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DISCUSSION

Richard Breen: Mr. Chairman, Ladies and Gentlemen: I am very pleased to have been asked to thank Alan Joyce for presenting his very interesting paper to the Society tonight, and I would like to congratulate him on winning the Barrington Prize. Considering how much has been said and written about the Irish national debt over the past decade it is, I think, a little surprising that we have not seen more published papers dealing with the workings of the gilts market and issues such as the pricing of gilts. Indeed, the only published work on an Irish yield curve is my colleague Patrick Honohan's paper which appeared in the Central Bank Quarterly some years ago. Alan Joyce's paper is therefore welcome, not least because, at a stroke, it has doubled the number of papers published on the Irish yield curve.

Having said that, I must admit to entertaining a number of misgivings about the concept of a yield curve. These are misgivings which, over the past 15 years, have increasingly appeared in research in financial economics. It is noticeable, I think, that the references in Alan Joyce's paper are somewhat elderly: there is very little there from the 1980s. This could be because financial economists and financial analysts have nothing more to say about matters such as measuring the returns to gilts or pricing new issues: but of course, this is not the reason. The reason is that researchers in this area have, in the main, moved away from the yield curve concept and towards a slightly different way of examining the relationship between the prices of, and returns to gilts, based on the idea of an implicit term structure underlying the prices of all government issued securities. I will say a little more about this later, but let me try first to explain some of what Stephen Schaefer, in a justly celebrated paper in 1976, called 'The Problems with Redemption Yields'.

The yield to maturity or redemption yield of a gilt is simply an internal rate of return, a statistic much used in project evaluation and investment decision making. But whereas we normally find an internal return which sets the total value of a project's future cash flows to zero, the redemption yield is that internal rate of return on a gilt which sets its estimated value equal to its observed market price. However, all the problems that attend IRR when used to evaluate projects carry over directly to its use in calculating redemption yields. So the first point to note is that the yield to maturity will be unique to each gilt: if there are 40 gilts in the market there will be 40 different redemption yields. More specifically, the redemption

yield is a function of four things: the gilt's price, the gilt's maturity date, the gilt's coupon payments, and the way the coupon payments are spaced through time. This means that two gilts with the same maturity will not to indeed should not to have the same yield to maturity if they have different coupons. And this result is not simply due to taxation effects: coupon differences determine redemption yields even in a world without taxes. This has immediate consequences when we come to use the yield curve to price new issues. If the government wished to issue a new gilt to mature on the first of September 2006, then it should not price it to have the same yield as the gilt which is currently in the market and expires on that day, unless the new gilt carries the same nine per cent coupon as the existing one. Similarly, the fact that two gilts which have approximately the same maturity but different coupons have different redemption yields need not tell us anything about which of the two we should switch our investments into, since, unless their redemption yields are very much out of line, this difference is simply an artefact of the way redemption yields are calculated. The dependence of redemption yield on coupon spacing is a trivial issue if all gilts pay semi-annual coupons: but imagine that the issuing authorities decided to market long maturity zerotocoupon bonds or gilts paying annual or quarterly coupons. In such cases, and ignoring, for a moment, the different tax implications of instruments like zero coupon bonds, current yields on semi-annual gilts would not be a good guide as to the pricing of such instruments.

A point which is of interest to those who hold portfolios of gilts is that, like any IRR based measure, redemption yields are not additive. In other words, the yield on a portfolio of gilts is not equal to the weighted sum of the yields on the gilts making it up. Hence the effect that adding more of a given gilt will have on the yield profile of a portfolio of gilts will depend upon what gilts are already in the portfolio. Clearly this is not a desirable property, since it implies that the marginal effect on the yield of a portfolio of a given gilt is different for all investors who hold different portfolios. And finally, redemption yield is not what it claims to be. The redemption yield appears to be the return that an investor would get from holding a gilt from today until maturity, conditional on current interest rates. Unfortunately, any investor who does this will almost certainly not realize the redemption yield. This is because, as with an IRR measure, the redemption yield is calculated on the implicit assumption that any dividend payments made by the gilt prior to maturity can be reinvested at

the gilt's redemption yield. But redemption yield is not a rate available in the market: indeed, how could it be, since if there are 40 gilts there are 40 redemption yields, and they cannot all be rates available for reinvestment of gilt coupon payments.

It is problems such as these which have led most academic researchers, and many analysts employed by the world's large securities firms, to move away from the yield curve concept and towards the modelling of the term structure. The difference between the two is simple: in the case of redemption yields, every gilt has its own yield and all its payments (coupons and redemption of principal) are discounted by this yield. In the term structure approach, there is one set of rates which are used to discount the payouts from all gilts, but these rates differ according to how far in the future each payment is made. In other words, there is an implicit single term structure of interest rates which underlies the prices of all government securities. To extract such a term structure is a fairly straightforward non-linear regression problem. The first attempt to do this was made by Huston McCullough in 1972 using US data, and since then the method has been widely adopted. In last July's issue of *The Economic and Social Review*, Jim Steeley described the fitting of just such a term structure model to UK gilts data. The 1981 *Economic Journal* paper by Stephen Schaefer which Alan Joyce refers to in his paper was, in fact, an attempt to fit a term structure to UK gilts prices, incorporating some very detailed corrections for differential taxation effects. Alan Joyce notes that Schaefer's model is very different from the Bank of England model: but this is because the latter is a yield curve model, while Schaefer's is a term structure model.

The Bank of England model is a complicated attempt to use the yield curve approach and to overcome the problems with redemption yields which I talked about earlier. My impression is, however, that when compared with term structure models it is less parsimonious and does not fit the data as well as term structure models. Perhaps the most crucial issue in modelling gilts' prices or yields is how well the model's output actually fits the data. In their papers in the *Bank of England Quarterly*, Burman and his associates usually report an R^2 measure of the goodness of fit of the model's estimated yields with the observed yields in the market. I think it is a pity that, in comparing the models in his paper, Alan Joyce did not provide some such measure by which we could evaluate the alternatives.

Burman and his co-workers generally report R^2 values between estimated

and observed yields of around .90. This does not seem to me to be a particularly good fit to this sort of data. Our experience with term structure models applied to Irish and UK data to and this is typical of experience elsewhere to suggests that the predicted gilts' prices from such a model are within plus or minus thirty pence per £100 nominal or one third of one per cent of the observed price (Breen and Keogh, forthcoming). This translates into an R^2 between observed and fitted prices in excess of .98.

Carrying out research into yield curves or term structures in Ireland, however, runs up against one very difficult problem. Given that, on any one day, most of the gilts listed will not trade, what do closing gilts' prices actually mean? Even if two gilts have traded on a given day, one may have traded at ten in the morning, the other at four in the afternoon: hence if we use actual traded prices they are not likely to be comparable. Alternatively we might use what brokers sometimes call 'guide prices' to the prices as published in the *Irish Times*, for example. But since no broker, so far as I am aware, acts as a market maker in gilts, publishing firm bid and offer prices, it is difficult to know quite what on-logical status these guide prices have. Over the past decade financial economists have become increasingly sensitive to questions such as the non-synchronicity of price data and other problems. It is an issue which might usefully be investigated in the Irish context.

Let me conclude by saying that, although I have misgivings about the premises on which Alan Joyce's paper is based, I nevertheless found it an extremely interesting piece of work, reporting the results of careful and thorough-going analysis. I sincerely hope that it will stimulate a good deal more research in this area. It gives me great pleasure, therefore, to once again congratulate Alan Joyce on winning the Barrington prize and to propose the vote of thanks.

References

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M. Grace: First of all, I would like to congratulate Alan Joyce on his work in fitting yield curves to Irish data and, for one such as myself who is prone to rustiness on the subject, his excellent summary of the theoretical background.

This, to my knowledge, is only the second serious attempt at fitting yield curves to Irish data - the first being the work carried out in the Central Bank by Patrick Honohan - and while it is relatively easy, on a number of grounds, to undermine the validity of the straight forward yield/maturity approach, attempts at greater sophistication generally tend to flounder on the complexity either generated or involved. It is not that complexity is a problem in itself but rather that its addition with some exceptions with which I will deal later seldom seems to add significantly to the broad information available from the analysis of the unvarnished yield/maturity relationship.

Before commenting further, I think it is worthwhile reminding ourselves of the value of yield curve analysis, however derived statistically. Chief amongst these is the facility to price existing untraded and future issues with a reasonable degree of accuracy. The degree of protection this offers to both issuers and investors in a market, such as Irish gilts, where the bulk of issues only trade irregularly, is fairly clearcut even if only a good approximation of the would be trading level is supplied. This, admittedly less than perfect, degree of transparency is of special value and protection to the smaller and/or occasional investor in gilts.

The benefit to the issuer has only increased in recent months with the arrival of the National Treasury Management Agency whose special emphasis, apart from borrowing on behalf of Exchequer, is to minimise the cost of servicing the National Debt. Work such as Alan Joyce's would help through more thorough pricing analysis.

The statistical path finders in yield curve analysis have of course been the Bank of England in the various contributions it has made since its seminal article of December 1972. The various refinements of the model, particularly those on the investor time horizons, I have found to be somewhat unsatisfactory from the intellectual view point. The assumption of the time horizons that give the best results from the model seems to me to state that because these time horizons provide the best fit for the curve then these must be the investor time horizons. This would seem to assume away the possible existence of other determinants of yield curves. This arbitrariness might conceal the relative importance of, for example, duration instead of maturity in the market and in particular segments of it. This would tend to show up in Mr. Joyce's approach in the positioning of high and low coupon stocks relative to the yield curve.

I would like here to digress somewhat into the difficulties in fitting yield curve to shorter-dated Government stock - in particular stocks under two year to maturity. Mr. Joyce has, I know, encountered the problem and my advice was to ignore it for the purpose of the present exercise. The gremlin is, of course, the taxation regime and I can think of at least three ways it distorts stock prices particularly in that maturity range and to a lesser extent out to five year maturities. The nature of the tax effects would not appear to be easily amendable to yield curve analysis but it might be an area for refinement in future work.

Dr. Antony Unwin: Alan Joyce has kindly thanked me for my role in supervising this research while he was an M.Sc. student in the Department of Statistics at Trinity College. It was a pleasure to work with him and I am delighted to see his research honoured by SSISI in this way.

The approach taken in using LOWESS is that of a statistician not an economist. There is no attempt to model processes, the aim is to summarise data effectively. That LOWESS is successful in this should not be surprising, a similar situation may be found in short-term forecasting where it is commonplace that simple univariate forecasting outperforms sophisticated econometric models. The lack of assumptions in LOWESS

is a positive benefit as earlier discussion has shown, but there is the further advantage of speed. LOWESS produces yield curves extremely quickly, the Bank of England model does not. If tools are to be useful to analysts and dealers they must be available fast. This leads on to thoughts of what developments might come next. I predict yield curves which may be interrogated, edited, adjusted and analysed interactively with a mouse-driven system. This is feasible now and would encourage analysts to see the yield curve as a flexible tool and not as a rigid structure.

Patrick Honohan: At a time when representative yields on government stocks published by the Central Bank for different maturities were each based simply on the redemption yield of a single stock with a maturity close to the specified date, it seemed desirable to look at more sophisticated approaches to the yield curve. The immediate applicability of the Bank of England model was one reason for trying it; another was the apparent interpretability of the fitted parameter values in terms of such variables as effective tax rates. In practice, however, although the Bank of England curves fitted reasonably well, the estimated parameter values from the Irish data were not plausible. In the light of that, I have no difficulty in agreeing with the author that a simplification was in order.

However, I do not find myself in sympathy with the direction in which he has led the debate. I believe that there are sufficiently strong reasons to believe that coupon effects are important in influencing yields to make it vital to include some allowance for these effects in plotting the link between yields and maturity. I would be prepared to sacrifice the sophistication of the curve fitting process in order to build-in coupon effects.

It is true that low-order polynomials do not do a good job of fitting the typical yield plot which curves at short maturities but is very flat at the longer end. In practice, however, most of these weaknesses can be eliminated by suitable transformation (for example through log or double log) of the maturity scale. It is hard to be convinced that very complex curves (such as the jagged LOWESS plots shown by Mr. Joyce for low values of f) are appropriate solutions to the curve-fitting exercise.