ECONOMIES OF SCALE AND TECHNICAL CHANGE
IN ELECTRICITY PRODUCTION IN IRELAND

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1. INTRODUCTION

Knowledge of the structure of production in public-utility enterprises is of considerable economic importance. Such things as the nature and extent of returns to scale and returns to individual factor imputs, the degree of substitutability between inputs, and the extent of allocative inefficiency and X-inefficiency may have bearings on various issues at the plant, firm and industry levels. For example, the nature and extent of returns to scale are known to have important implications for, amongst other things, investment policies in growing industries and the institutional arrangements necessary to achieve an optimal allocation of resources (Nerlove 1963, p. 167). But of course, whatever the particular concern, the formulation of precise policy prescriptions or appraisals requires suitable quantification of the production structure at the appropriate level.

The main purpose of this paper is to analyse the structure of production in the Irish electricity supply industry at the level of the individual thermal generating station. Such an analysis has not previously been carried out for Ireland, despite the abundance and availability of good-quality data; but various econometric studies of the electric-power industry have been reported for several other countries, and most of these have made use of some explicit production function. This production function approach is adopted here.

More specifically, the study is based on the use of the so-called non-homothetic Leontief production function which was developed recently by Lau and Tamura (1972). The prime objective is to estimate the parameters of this function using pooled cross-section and time-series data, and to assess its adequacy as a description of the thermal electricity generation process at the plant level. A second objective is to use the estimated parameters to investigate certain matters which may be of relevance and interest to policy-makers, such as how the scale of the plant affects the quantities used of each of four major inputs, namely, capital, fuel, energy and labour; how variations in the intensity of the use of the plant affect the quantities of inputs used; and whether and how the pattern of technical change in plants has affected the use of individual inputs. The analysis includes explicit testing of hypotheses concerning the homotheticity of the production structure, the constancy of returns to scale, and the existence and Hicksian neutrality of embodied and disembodied technical progress. It is hoped that such an econometric analysis will provide a useful supplement to the considerable amount of engineering and other information on electricity generation at plant level.

In addition to the econometric analysis, the paper contains a small amount of background material. Section 2 contains a brief sketch of the Irish generation system and the electricity production process; the latter may be viewed as providing the essential rationale for the choice of production function. Section 3 contains a brief outline of the relevant earlier econometric research on electricity production, and a general account of the non-homothetic Leontief production function and its properties. Section 4 reports the

econometric analysis proper. It includes a detailed account of the specification and methodology for estimation of the particular model used, comments on the data available for individual generating stations and the derived data used for the analysis, and a discussion of the results.[1] As will be seen, the main findings to emerge are that the chosen production function describes the data well; that there are substantial economies in the use of labour and energy in electricity production, and smaller economies in the use of capital and fuel; and that the impact of embodied and disembodied technical change on the use of factors has been slight. A summary and conclusion are given in Section 5. The conclusion includes comments on the value for policy purposes of the main findings of the analysis.

2. PRODUCTION OF ELECTRICITY IN IRELAND

The ESB Generation System[2]

The generation of electricity in Ireland — and its transmission, distribution and sale — is undertaken by the Electricity Supply Board (ESB), a corporation established under statute in 1927 and now the largest of the state-sponsored bodies and, in terms of capital assets, probably the largest industrial concern in the country. However, due mainly to Government policy to utilise indigenous resources as far as possible, the generation system of the ESB is essentially a small unit system. It currently comprises 28 generating stations — 18 thermal and 10 hydro — and has a total installed capacity of about 2,000 megawatts (MW), roughly four times more than what total capacity was in 1960.[3] Since the mid-1960s, when the scope for further feasible development of peat and conventional hydro generation diminished, the system has made increasing proportional use of thermal plants, especially oil-fired plants, and thermal installations now account for about 80 per cent of the total capacity and about 92 per cent of actual electricity production. Of the remaining capacity accounted for by hydro stations, over half is provided by the new 292 MW pumped storage station at Turlough Hill. Immediately prior to the commissioning of the Turlough Hill station the share of thermal capacity was about 87.5 per cent of the total. By comparison, the corresponding shares in 1950 and 1960 were about 53 per cent and 69 per cent, respectively.

The present study is concerned solely with the production of electricity from thermal energy sources, of which there are three in wide use by the ESB, namely, peat, coal and oil. While gas is expected to become increasingly important in the near future, only very small amounts have been used to date in the Poolbeg station in Dublin. Of the 18 thermal stations in the system, six are designed to burn sod peat, four to burn milled peat, one to burn sod peat and milled peat, one to burn Irish semi-bituminous coal, and the remaining six to burn oil.[4]

In addition to differences in the type of fuel used, there are wide variations amongst these plants with respect to age of equipment, capacity, and the number and sizes of generating sets. Plant capacity, for example, varies from 5 MW, as in the case of each of the four sod peat stations in the West of Ireland, to 620 MW, the capacity of the Tarbert oil-fired station in County Kerry with an average capacity of approximately 112 MW per station; the capacity of individual generating sets varies from 5 MW to 270 MW, the size of the recently commissioned set in the Poolbeg oil-fired station in Dublin, although 31 out of a total of 48 sets have a capacity of less than 20 MW. Only six sets have a capacity greater than 100 MW: one in the Great Island station in County Wexford, two in the Tarbert station, and three in the Poolbeg Station.

There are, therefore, likely to be differences in the efficiencies of individual stations, even for a given type of fuel. Naturally, it is the older (pre-1964) stations, with their smaller sets, lower steam pressures, and higher labour requirements, that are the least efficient. The more modern peat, and particularly oil, installations probably have significantly higher efficiencies, largely by virtue of their greater size and technically more advanced equipment.

The use of generating stations to meet a given demand for electricity, referred to as load-despatching, is influenced by the ESB's obligation to consume a certain amount of

native peat each year. However, subject to this constraint, and certain other physical constraints, such as those imposed by the breakdown of equipment or its shut-down for maintenance purposes, the ESB attempts to utilise its capacity so as to produce electricity as economically as possible. In practice this means that base load is covered by the modern oil-fired stations (and some conventional hydro stations) and the intermediate load range by the older oil and peat-fired stations, while peak load is covered by the North Wall oil-fired plant in Dublin (and by the hydro stations with storage facilities). One obvious result of this load-despatching policy is that different plants are subject to different degrees of capacity utilisation. This is taken into consideration in the formulation of the model in Section 4.

The Electricity Production Process[5]
The fundamental function of the generating system, of course, is to convert energy from the various sources into electricity. In the thermal stations, the same basic process is used to carry out this function. Briefly, the fuel is burned to produce hot gases which are used, firstly, in an economiser to pre-heat feed-water entering the boilers, and secondly, in the boilers themselves to convert the water to steam. The temperature of the steam leaving the boilers is increased in a superheater and the steam is then used to operate turbines which turn the generators to produce the electricity. The exhaust steam, after leaving the turbines, passes through a condenser and is converted back to water to be pumped, via the feed-water heater and economiser, back to the boilers.

For the purposes of this study it is convenient to distinguish four inputs to this production process, each being, in principle, a flow variable. Firstly, a capital input (C) which accounts for the service flow provided by the physical stock of equipment: the boilers, turbines, generators, pumps, etc. Secondly, a fuel or raw material input (F) which, as has already been mentioned, may be a flow of peat, coal or oil. Thirdly, an energy input (E) which accounts for the flow of power required to operate some equipment, such as fuel-handling machinery and pumps, and to provide lighting; power stations actually use a portion of their own output for these purposes. Fourthly, a labour input (L) which accounts for the flow of services provided by the staff that operate and maintain the plant. There is of course a single homogeneous flow of output (Q) from the production process.

The process is highly capital intensive and is subject to rigid technological requirements which preclude the ex post possibility of substitution of inputs for a given level of output. It is also thought to be characterised by substantial economies of scale and the available evidence lends firm support for this view. It has, of course, as has already been mentioned, been influenced by the rapid technical progress experienced during recent years. The actual degree of capacity utilisation varies amongst plants and is determined essentially by load-despatching policy.

3. ELECTRICITY PRODUCTION AND THE PRODUCTION FUNCTION

Previous Studies
The production of thermal electricity at the plant level has been analysed in terms of the production function concept by several authors. The work of Komiya (1962), Barzel (1964), Dhrymes and Kurz (1964), and Galatin (1968) is of particular relevance to the present study and will be referred to periodically throughout the paper. Their respective sets of results, though based on data for the United States, constitute a valuable basis of comparison for the results presented in Section 4 and they will be referred to again in that Section. Immediately following is a brief account of the basic approaches employed by these authors.

Komiya, using cross-section data, estimated production functions for several groups of plants categorised by their technological vintage and fuel-type. He found that a substitution model based on the Cobb-Douglas function was an unsatisfactory means of analysis. A more successful model, which he called a 'limitational model', was based on a system of input demand equations relating capital, fuel and labour, respectively, to a measure of

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plant capacity, as follows:

$$X_i = \alpha_i \, \overline{Q}^{\beta_i} \, N^{\mu_i}, \ i = C, F \text{ and } L, \tag{1}$$

where X_i denotes the quantity of the ith input per generating-unit (set) required at the capacity level of operation, $\overline{Q}$ the average size of the generating-unit in megawatts, and N the number of generating-units in the same plant. The α_i, β_i and μ_i are constants; the economic interpretation of such parameters is discussed, in relation to the actual model employed in the present study, in Section 4. The impact of technical change was assessed by Komiya on the basis of the differences in the estimated input functions for his various vintage groups.

The studies of Barzel, Dhrymes and Kurz, and Galatin were also based on sets of input functions similar to (1), but they each incorporated modifications of specification and employed various procedural innovations. In particular, Barzel introduced the degree of capacity utilisation, measured by the observed load factor,[6] into his specification, and, using cross-section and time-series data, attempted to quantify the effect of technical change by the use of dummy variables for the various vintages of plant. Dhrymes and Kurz, unlike all the other authors, derived their equations explicitly, on the assumption of exogenous demand and prices, and cost-minimising behaviour subject to the constraint imposed by the underlying production function, which they postulated to be of the CES-type. Thus their equations contained relative prices, as well as output, as explanatory variables. However, in the light of their initial estimates, which were based on cross-section data on plants grouped by vintage and size, they replaced their original labour input function by an equation akin to (1); by relating the labour requirement to output only, this better reflected the situation in electricity production, namely, that labour is not a substitute for capital or fuel.[7] Galatin, critical of various aspects of the previous studies, especially their neglect of the problem of 'machine-mix', formulated a model which took explicit account of machine-mix and degree of capacity utilisation. Indeed, for his fuel equation, he took the machine – i.e., the turbine-generator set and its associated system of boilers and ancillary equipment – as his unit of observation, rather than the plant. However, like his capital and labour equations, the fuel equation had to be estimated from data on plants, and this posed certain aggregation problems (see Galatin 1968, Sec. 5.2). Largely because of these problems, Galatin used a linear functional form for his equations rather than a form which was linear in the logarithms of the variables as previous writers had done. His estimation was undertaken using cross-section and time-series data on plants categorised according to vintage, size and fuel-type.

It is noteworthy that a similar functional form to (1) has been used in several studies of production in other industries characterised by limited factor substitutability. Of these studies, the one by Lau and Tamura (1972) for the Japanese petrochemical industry is especially significant in that it includes the first proof that a system of input functions such as (1) may be derived from cost minimisation subject to a production function constraint. Previously, such formulations had been distinguished from those, such as the equations of Dhrymes and Kurz, that were derived on the explicit assumption of optimising behaviour (see, for example, Galatin 1968, Ch. 4). Moreover, Lau and Tamura present an explicit derivation of the class of production functions underlying the derived input functions, namely, the non-homothetic Leontief production function (hereafter NHL production function). A brief explanation of this function and its properties is given in the following subsection.

The NHL Production Function

The NHL production function for an output-taking, cost-minimising undertaking, which gives rise to input equations such as those in (1), may be written as

$$Q = \Phi(X) = \min_i \left[f_i^{-1}(X_i) \right], \ i = 1,2,3, \ldots ,k, \tag{2}$$

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where Q denotes the quantity of output, and X_i the quantity of the ith input. $f_i^{-1}(.)$ is a generalised inverse of the function $f_i(.)$ which has the properties that $f_i(Q)$ is a positive real-valued function, defined and finite for all finite $Q > 0$ with $f_i(O) = 0$, and is a non-decreasing lower semicontinuous function in Q which becomes unbounded as Q becomes unbounded.[8] Of course, the $f_i(Q)$ are just the right hand sides of the derived input demand functions for the k factors employed in the production process; these may be written as $X_i = f_i(Q)$, i = 1,2,3, . . . ,k. Embodied and disembodied technical change may be incorporated into the NHL production function simply by writing the $f_i(Q)$ as functions of plant vintage (v) and time (t) as well as of the quantity of output. The system of input functions then becomes $X_i = f_i(Q,v,t)$, i = 1,2,3, . . . k. Lau and Tamura (1972, p. 1174) show that on the assumption of Hicksian neutrality for both forms of technical change this may be written as

$$X_i = f_i(Q,v,t) = f_i^*(Q) \, V(v) \, T(t). \tag{3}$$

Underlying these formal definitions is a basically simple idea. Equation (2), in essence, merely states that the NHL production function is the kind of function which, in terms of the concepts of elementary economics, has L-shaped isoquants — hence the name Leontief. Thus it has the property of zero elasticities of substitution between all pairs of inputs, and the derived input demand functions are independent of prices. In fact, the NHL function is the most general production function with zero elasticities of substitution, and is a member of the CES class of functions. As its name implies, another property of the function is that it is not necessarily homothetic; that is, its expansion path is not necessarily a straight line through the origin. Hence the optimal proportions of factors may vary across plants if plant output levels differ. Another property of the NHL production function is that the degree of returns to scale may be different for each factor input. Some further explanation of this last feature may be useful.

The 'technical' meaning of returns to scale relates to the impact on output of equi-proportionate changes in the quantities of all inputs. In the context of the NHL production function, however, an alternative interpretation of returns to scale is adopted. This, perforce, focuses on the effect on output of scale changes in the form of proportional variations in the quantities of all inputs corresponding to movements along a plant's expansion path. These proportional variations will not be equal, and the technical definition of returns to scale will therefore not apply, unless the expansion path is a straight line through the origin, which, as has been stated, is not necessarily the case for a NHL production function. Incidentally, this alternative interpretation of the concept corresponds with the approach, sometimes used in economics, of relating returns to scale to the behaviour of average costs.

4. ECONOMETRIC ANALYSIS OF ESB PRODUCTION

Model Specification

The characteristics of the electricity production process in thermal plants, which were described in Section 2, themselves suggest that a fixed factor proportions model is a plausible model for econometric analysis. The empirical findings of Komiya, Barzel, Dhrymes and Kurz, and Galatin, in particular, provide considerable support for this view; the theoretical work of Lau and Tamura reinforces it further. For the purposes of this study, therefore, a NHL production structure was hypothesised for thermal electricity generation at the plant level in the Irish generation system. It does not follow of course, as Nerlove (1963, p. 173) has pointed out, that the NHL structure would necessarily be the most appropriate to assume for a higher level of aggregation, such as the firm level.

The input demand functions which derive from the NHL production function, and which provide the means of estimating it, may take various forms. For this study, following Komiya, Barzel, and Dhrymes and Kurz, functions which are linear in the natural logarithms of the variables were chosen. Apart from the computational convenience of

using log-linear or constant elasticity equations, there are sound theoretical reasons for preferring this functional form as Haldi and Whitcomb (1967, pp. 375-376) have pointed out.

Unfortunately, there is not the same consensus as to the precise specification of the variables that should enter the input functions. The formulation adopted here was based on that used by Lau and Tamura, which allows explicitly for the presence of embodied technical change. This form was chosen because, although pooled cross-section and time-series data were available, the number of thermal plants in Ireland is very much smaller than the number in the United States, and consequently both the method of using several vintage groups of plants, and the method of using vintage dummy variables, were considered unfeasible with Irish data for degrees of freedom reasons. Unlike the Lau and Tamura model, however, the model used in the present study also allows for the presence of disembodied technical change. Thus, in accordance with equation (3), the basic system of input equations postulated were as follows:

$$\log_e X_{ipt} = \log_e \alpha_i + \beta_i \log_e Q_{pt} + \gamma_i v_p + \delta_i t + \epsilon_{ipt},$$
$$i = C,F,E,L, \ p = 1,2,\ldots,N, \ t = 1,2,\ldots,T, \tag{4}$$

where X is plant input per time-period, Q is plant output, v is plant vintage, t is the time-period and ϵ is an additive stochastic disturbance; i refers to the four inputs previously defined, p to the particular plant; the α_i, β_i, γ_i and δ_i are unknown constants.

The unqualified use of output as an independent variable is not, however, entirely satisfactory in the context of electricity generation, since plants of widely differing sizes may produce a similar output, but without requiring the same amounts of inputs. It is therefore desirable to attempt to distinguish between the effects of size and the degree of capacity utilisation, especially, it seems, in the case of the labour, fuel and energy input relations. Size and utilisation factors can be introduced directly by means of the relationship between output and the capacity and load factor of a plant. For estimation purposes, therefore, the preferred system of input functions was of the form:

$$\log_e X_{ipt} = \alpha_i^* + \beta_i \log_e Q_p^* + \lambda_i \log_e L_{pt}^* + \gamma_i v_p + \delta_i t + \epsilon_{ipt}, \tag{5}$$

where Q^* denotes plant capacity (size), and L^* denotes the overall plant load factor; α_i^* denotes $\log_e \alpha_i$.

Clearly, $\beta_i > 1$ implies the existence of diseconomies of size with respect to input i; $\beta_i = 1$ implies constant returns to size; $\beta_i < 1$ implies increasing returns to size. Similarly, $\lambda_i > 1$, $\lambda_i = 1$, $\lambda_i < 1$ imply the existence, respectively, of diminishing, constant and increasing returns to degree of capacity utilisation with respect to input i. If $\beta_i \neq \lambda_i$, the decomposition of output into its capacity and load components is, of course, entirely justified. Also γ_i, $\delta_i > 0$ implies, respectively, embodied and disembodied technical retrogression; γ_i, $\delta_i = 0$ implies zero technical change of both kinds; γ_i, $\delta_i < 0$ implies technical progress of both kinds. Furthermore, it can be verified (see Lau and Tamura 1972, pp. 1174-1175) that given the type of specification in (5), a necessary and sufficient condition for homotheticity is that for all i, $\beta_i = \lambda_i = c_1$, where c_1 is a constant; a necessary and sufficient condition for overall constant returns to size is $\beta_i = 1$, for all i, and a necessary and sufficient condition for overall constant returns to degree of capacity utilisation is $\lambda_i = 1$, for all i;[9] a necessary and sufficient condition for Hicks neutral technical change of both kinds is that for all i, $\gamma_i = c_2$ and $\delta_i = c_3$, where c_2 and c_3 are constants; and a necessary and sufficient condition for zero technical change is $\gamma_i = \delta_i = 0$ for all i.

A specification of the properties of the stochastic disturbance term was chosen to yield a pooled cross-section and time-series model which allowed, for each input function, the possibility of heteroscedasticity and correlation amongst plants, and time-wise auto-correlation for individual plants, as follows:

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$$E(\epsilon_{ipt}^2) = \sigma_{pp}^{(i)}, \quad E(\epsilon_{ipt}\epsilon_{ip*t}) = \sigma_{pp*}^{(i)} \quad (p \neq p^*),$$

and $\quad \epsilon_{ipt} = \rho_{ip}\epsilon_{ip(t-1)} + u_{ipt}, \text{ where } u_{ipt} \sim N(0, \Phi_{pp}^{(i)}),$

and $\quad E(\epsilon_{ip(t-1)}u_{ipt}) = 0, \quad E(u_{ipt}u_{ip*t}) = \Phi_{pp*}^{(i)} \quad (p \neq p^*),$ $\qquad$ (6)

$$E(u_{ipt}u_{ips}) = 0 \quad (t \neq s),$$

$$i = C, F, E, L, \quad p, p^* = 1, 2, \ldots, N, \quad t, s = 1, 2, \ldots, T.$$

The initial value of ϵ_{ipt}, ϵ_{ipO}, was assumed to have the properties $\epsilon_{ipO} \sim N(0, \Phi_{pp}^{(i)}/ 1-\rho_{ip}^2)$ and $E(\epsilon_{ipO}\epsilon_{ip*O}) = \Phi_{pp*}^{(i)}/1-\rho_{ip}\rho_{ip*}$ for $p \neq p^*$. The disturbance variance-covariance matrix, Ω_i, for the ith input demand function of the model, may therefore be written, in terms of the typical (pp^*th) element, as

$$\Omega_i = \left[\sigma_{pp*}^{(i)} R_{pp*}^{(i)}\right], \text{ where } R_{pp*}^{(i)} = \begin{bmatrix} 1 & \rho_{ip*} & \rho_{ip*}^2 \cdots \rho_{ip*}^{T-1} \\ \rho_{ip} & 1 & \rho_{ip*} \cdots \rho_{ip*}^{T-2} \\ \vdots & & \ddots \\ \rho_{ip}^{T-1} & \cdots & \ddots \ 1 \end{bmatrix} \qquad (7)$$

A stoachastic specification similar to this is described in Kmenta (1971, pp. 512-513). Such a model is not only much more comprehensive, but, it would seem, much more realistic for the purposes of analysing electricity production data than the kind of stochastic specifications used in previous studies. These previous specifications are alluded to in the following subsection.

Of the explanatory variables in the system of equations (5), which for the purposes of this study is viewed as relating to an ex post underlying NHL production function, vintage and time are of course exogenous. Plant size and load factor are also assumed to be exogenous. Once a generating station is built, output from the plant, and hence its load factor, are largely determined by the overall demand for electricity and the load-despatching policy of the generation authority. While for the firm as a whole the demand for output from a given plant is clearly not exogenous, the view was taken, as in previous studies, that at the level of the plant, output and load factor are determined by an exogenous demand. Therefore the plant was assumed to behave essentially as an output-taking concern which attempts to minimise its cost of production. This does not seem to be too far out of line with stated ESB policy. It would, of course, be more difficult to sustain an argument that in the ex-ante choice of plant design, planned output (capacity) is exogenous.

It should perhaps be noted that the hypothesised input functions abstract from the problem of the machine-mix of plants. As has already been mentioned, Galatin has been critical of such formulations. However, due to the small number of Irish generating stations, Galatin's approach to the problem, namely, of grouping plants by the number and size of their turbine-generator sets, was not feasible for this study, and no alternative approach was found.

Model Estimation

In previous electricity production studies, ordinary least squares (OLS) has invariably been used to estimate the coefficients of the input functions. Although little attention seems to have been given to the stochastic specification of the functions, it may be presumed that appropriate assumptions about the disturbance terms were made. Unfortunately, no test statistics relating to residual analysis were published by which to assess the validity of these assumptions, at least for the various production studies mentioned earlier. However, Galatin did note that in the case of his fuel function, the classical assumptions are

violated by the presence of heteroscedasticity introduced by his aggregation methodology.

It is perhaps more important to note that the studies which used OLS assumed, implicitly at least, that no correlation existed between the disturbances of different input functions. If such correlation exists — and it seems reasonable to assume it does for a given plant, if not between plants — then OLS estimation of the separate input functions would be a statistically efficient technique to use only if all the input functions contained the same explanatory variables and there were no restrictions on their parameters. Otherwise the method of estimating seemingly unrelated regressions due to Zellner (1962) would be required for efficient estimation. In fact, neither Komiya, Barzel, nor Galatin used the same set of explanatory variables for all their input functions, although they all used OLS.[10]

In this study, this uniformity of input equations is assumed, so that estimation of equations individually may be considered. Despite this, OLS would not yield efficient coefficient estimates due to the nature of the stoachstic properties specified in (6). OLS would, however, give estimates which were unbiased and consistent, and for practical expediency could be used. Theoretically, a preferable estimator would appear to be the modified Aitken estimator

$$\hat{\mu}_i = (Z'\hat{\Omega}_i^{-1}Z)^{-1}(Z'\hat{\Omega}_i^{-1}X_i), \tag{8}$$

whose asymptotic variance-covariance matrix is

$$V(\hat{\mu}_i) = (Z'\hat{\Omega}_i^{-1}Z)^{-1}, \quad i = C,F,E,L, \tag{9}$$

where Z is the NT $\times$ 5 matrix of pooled cross-section and time series observations on the explanatory variables, including the dummy variable unity to account for the intercept, X_i is the vector of NT observations on the ith input, and $\hat{\Omega}_i^{-1}$ is the NT $\times$ NT estimated disturbance variance-covariance matrix, and $\hat{\mu}_i = \begin{bmatrix} \hat{\alpha}_i^* \hat{\beta}_i & \hat{\lambda}_i & \hat{\gamma}_i & \hat{\delta}_i \end{bmatrix}'$ the vector of parameter estimates, for the ith input function.

The following procedure based on OLS may be used to provide a matrix $\hat{\Omega}_i$ whose elements are consistent estimates of the elements of the unknown matrix Ω_i. Firstly, OLS is applied to all of the pooled data and the resulting residuals, e_{ipt}, used to estimate ρ_{ip} by the formula $\hat{\rho}_{ip} = \sum_{t=2}^{T} e_{ipt}e_{ip(t-1)}/\sum_{t=2}^{T} e_{ip(t-1)}^2$, p = 1,2, . . . ,N, i = C,F,E,L. The $\hat{\rho}_{ip}$ are consistent estimates of the ρ_{ip}. Secondly, the $\hat{\rho}_{ip}$ are used to transform the data to 'generalised differences': $x_{ipt} = X_{ipt} - \hat{\rho}_{ip}X_{ip(t-1)}$, $q_p^* = Q_p^* - \hat{\rho}_{ip}Q_p^*$, etc. Thirdly, OLS is applied to the transformed data and the residuals from this regression, $\tilde{e}_{ipt}$, used to estimate the variances and covariances of the ϵ_{ipt}, that is, the $\sigma_{pp}^{(i)}$ and $\sigma_{pp*}^{(i)}$, by $s_{pp*}^{(i)} = \hat{\varphi}_{pp*}^{(i)}/1 - \hat{\rho}_{ip}\hat{\rho}_{ip*}$, where $\hat{\varphi}_{pp*}^{(i)} = \sum_{t=2}^{T} \tilde{e}_{ipt}\tilde{e}_{ip*t}/T-5-1$, p,p* = 1,2, . . . ,N, i = C,F,E,L.

This approach to the estimation of (5) can be simplified by applying the modified Aitken formulae to the transformed data, that is, by using the estimator

$$\hat{\mu}_i = (z'\hat{\Psi}_i^{-1} z)^{-1}(z'\hat{\Psi}_i^{-1} x_i), \tag{10}$$

with the associated asymptotic variance-covariance matrix

$$V(\hat{\mu}_i) = (z'\hat{\Psi}^{-1} z)^{-1}, \tag{11}$$

where z denotes the transformed explanatory data matrix and $\hat{\Psi}$ the estimated variance-covariance matrix. The estimate $\hat{\Psi}_i$ is of order N(T-1) $\times$ N(T-1) and, in terms of the typical element, may be written $\hat{\Psi}_i = \begin{bmatrix} \hat{\varphi}_{pp*}^{(i)} I_{T-1} \end{bmatrix}$, where I_{T-1} is the identity matrix

of order T–1, and the $\hat{q}_{pp*}^{(i)}$ may be obtained as previously indicated. The orders of z and x_i are similarly reduced by N due to the transformation of variables. In general the value of $\hat{\mu}_i$ would not be expected to be identical to that of $\hat{\mu}_i$, but the asymptotic properties of the two estimators are the same.

The approach is, of course, well-known, and has been discussed by Zellner (1962) and Telser (1964) who have shown that both variants of the estimator, (8) and (10), have the same asymptotic properties as Aitken's estimator, that is, they are consistent, asymptotically efficient and asymptotically normal.[11] The gain in efficiency over OLS depends on the values of the off-diagonal elements in Ω_i (in the present case, the extent to which the $\sigma_{pp*}^{(i)}$ and ρ_{ip} differ from zero), the correlation of the explanatory variables for different cross-sectional units (plants), and on whether shift variables are included in the equation (see Zellner 1962, or Balestra and Nerlove, 1966, p. 597).

Data

A considerable amount of reliable technical and production data on the Irish generating stations is available from the annual reports of the ESB. Specifically, the reports give, for each plant, information on its capacity in megawatts, its annual gross and net output, and hence its own consumption of electricity, in millions of kilowatt-hours (units), its annual percentage plant load factor, its fuel type, its numbers of turbine-generating sets and boilers, as well as a small amount of works cost information. The reports do not contain information on the fuel consumption, manning and capital cost of plants. However, the ESB kindly made this additional information available to the author. The plant fuel consumption figures provided were in thousands of tons per year, the labour figures were the average number of employees in each plant per year, and the capital figures were the total capital cost of each plant sub-divided into the cost of buildings (wharfs, cooling-towers, etc.), equipment (turbines, boilers, etc.) and outdoor equipment (transmission station, fuel-handling, etc.).

For the purposes of analysis, the fuel input (X_F) of a plant was measured in thousands of tons of oil (or, in the case of peat and coal, oil-equivalents) per year; the energy input (X_E) was taken to be the consumption of electricity by the plant measured, as published, in millions of units per year; the labour input (X_L) was the number of man-years as provided by the ESB. For the capital input (X_C) two alternative measures were used, namely, the overall cost of an installation deflated by the price index for capital goods, and the cost of indoor generating and ancillary equipment deflated by the price index for transportable capital goods for use in industry, both indices having base 1953 = 100.[12] For the explanatory variables, the following measures were used for each plant: for output (Q) the published figure for annual gross output in millions of units; for capacity (Q*) the published megawatt capacity figure, which is based on the name-plate ratings of the sets in a plant; for degree of capacity utilisation (L*) the published annual percentage plant load factor; for vintage (v) the year of installation of a plant, and for time (t) the dummy variable t = 1, 2, 3, . . . , T, with 1953 = 1.

It should be noted that of these various measures, those for labour, capital and capacity may have serious shortcomings. For labour, 'average number of employees' is known to be a potentially poor measure because, amongst other things, it takes no account of differences in the length of the working day or week over time, nor of different types (operating, maintenance) and qualities of labour. It was used, as in previous studies, in the absence of a readily available alternative.

Similarly, the problems associated with measuring the concept of the quantity of capital employed in a plant per time-period are well-known. The deflated value of the cost of capital would appear to be a suitable measure if the price index used as deflator is the appropriate one; but an "appropriate" price index cannot be derived unless the quantity units for capital are already well-defined, which is the original problem. Measures of capital used in other studies of electricity generation have included the number of sets and the installed capacity in a plant, but the former takes no account of differences in sizes, and both ignore "quality" differences which may be important. Dhrymes and Kurz

(1964) used megawatt capacity times the sum of hours a plant was "hot", whether connected or not connected to load. Galatin (1968, p. 91) has criticised this measure and suggested that the aggregate of capacity multiplied by the degree of utilisation in each hour would be a better measure in the context of the Dhrymes and Kurz analysis. This kind of measure could not be used in the present study as the necessary information on capacity utilisation per hour was not available to the author. Galatin himself, and Barzel before him, actually used the undeflated value of capital because of the unavailability of what they considered to be a suitable deflator. In adopting the deflated capital cost as the capital measure, the present study follows the approach of Komiya (1962).

The capacity measure, more surprisingly perhaps, may also be somewhat unsatisfactory, because name-plate ratings refer to the maximum output that can be achieved without over-heating. According to engineering studies, as Nerlove (1963, p. 181) has pointed out, generating units of the same size, general design and actual capability may show as much as 20 per cent difference in rating. To the extent that this factor is significant, the plant load factor, used to measure the degree of capacity utilisation of plants, also becomes unreliable, of course. Since capacity and degree of capacity utilisation are explanatory variables, possible errors in their measurement are of particular concern in that they have important implications for estimation. The likely impact of such errors on the results of the study is mentioned in the discussion in the following subsection.

The data on the remaining variables — X_F, X_E, Q, v and t — are considered to be quite reliable and not prone to measurement error.

With the exception of the four small Western stations at Miltown Malbay, Screeb, Cahirsiveen and Gweedore, the observations on the variables were available for each plant from its first full year of operation up to and including the year ending March, 1976. In the case of the Western stations, only combined production and labour figures were available. Therefore, having been commissioned within a year of each other, they were regarded as a single operation for the purposes of analysis.

Analysis and Results

In accordance with the estimation methodology outlined earlier, the model as specified by (5) and (6) was initially estimated by OLS regression using the pooled cross-section and time-series data on the thermal plants in the generating system. Because the characteristics of input demand functions may be expected to vary somewhat with the type of fuel used — and the results of previous studies appear to confirm this — the data were stratified according to fuel-type and separate functions estimated for peat and oil plants. The single small coal-fired station at Arigna was excluded from the analysis. Moreover, even within the peat and oil categories, certain plants were excluded, and not all of the available observations on plants that remained in the sample were utilised. This was essentially because use of the modified Aitken estimator — variant (8) or (10) — requires that the time-series on each plant contain the same number of observations. In fact, as mentioned in Section 2, there is considerable variation in the age of generating stations.

A further consideration was that in the interests of meaningful estimation, the vintage — i.e. year of installation — of each plant should adequately represent the state of the technology of production throughout the period covered by the data. In fact, many plants have had additional generating sets installed since their original commissioning; and to the extent that the technical specification of these newer sets differs from that of the original equipment, the notion of the vintage of a plant becomes difficult to define, and its representation by the year of initial installation difficult to interpret.

In view of these two considerations, the length (T) of the time-series for each fuel category was determined by the age in years of the newest plant, or the number of years in operation with equipment of the original vintage of the plant which operated for the shortest time with that equipment, whichever was the smaller, and provided a certain minimum number of observations was available. Thus, for the peat category, the value of T was determined by the Lanesborough 'A' sod peat station, the period of operation before the installation of the Lanesborough 'B' milled peat equipment being 7 years. The

Lanesborough 'B' plant was not included in the sample because from the date of its commissioning, separate labour figures for it — and the Lanesborough 'A' plant — were not available. The Ferbane milled peat station was not included because the 5 years of operation before additional equipment was installed was considered too short a period. For the oil category, T was determined by the Tarbert station on which there were 7 annual observations available. The new oil-fired Poolbeg station was excluded because there were too few observations on it. In addition, the now obsolete, old Pigeon House stand-by station was excluded from the analysis.

The results of the initial OLS regressions are given in Table 1. For simplicity of presentation, only the coefficient estimates and the coefficient of determination (R^2) for each equation are presented.[13] Where a coefficient is not significantly different from zero on the basis of the standard two-sided t-test at the 5 per cent probability level, this is indicated by means of an asterisk. The capital input C1 refers to capital as measured by the cost of indoor equipment only, while C2 refers to capital alternatively measured by the total cost of the installation, both measures being at 1953 prices as explained earlier.

Table 1: *Coefficient Estimates for Peat and Oil Input Functions — First Phase of Estimation*

FUEL-TYPE	INPUT	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\delta}$	R^2
Peat	Labour (L)	0.569	−0.050*	−0.060	0.050	0.86
	Fuel (F)	1.051	0.907	−0.017	−0.002*	0.99
	Energy (E)	1.236	0.755	−0.014	0.006*	0.98
	Capital (C1)	1.112	−0.024*	−0.029	0.005*	0.97
	Capital (C2)	1.051	0.117	−0.015*	−0.001*	0.96
Oil	Labour (L)	0.991	0.253	−0.045	−0.014*	0.85
	Fuel (F)	0.926	1.149	−0.057	0.031	0.98
	Energy (E)	0.991	0.786	−0.045	0.034*	0.97
	Capital (C1)	1.304	−0.006*	−0.009*	−0.012*	0.95
	Capital (C2)	1.331	0.257	−0.030	−0.015*	0.97

*Not significant at 5 per cent level.

Although they are statistically inefficient, the results in Table 1 are nevertheless consistent estimates of the parameters of the input functions and are not without interest. However, as they relate only to the first phase of the estimation procedure — to a phase which is required essentially for the purposes of providing residuals with which to compute estimates of the various autocorrelation coefficients ($\hat{\rho}_{ip}$) in each fuel category and thereby to transform the original data to generalised difference form — they are not discussed at any length at this stage. It is sufficient to note that the equations appear to fit the data well and that while there was a suggestion of significant multicollinearity in the explanatory data sets for both the peat and oil regressions,[14] the sizes and signs of the estimates are not out of line with what might be expected on the basis of *a priori* reasoning and the results of previous studies. In the three cases where $\hat{\lambda}$ has a negative sign, the estimates are not significantly different from zero statistically.

The values of the $\hat{\rho}_{ip}$ were calculated using the residuals from the initial regressions and they conformed closely with the expectations embodied in the stochastic specification of the model, suggesting in the large majority of cases a high degree of first-order positive autocorrelation. Using these values, the original data were appropriately differenced and used to perform a second set of regressions. Unfortunately the results of this second phase of estimation were entirely unsatisfactory due to severe multicollinearity in the transformed explanatory data sets; they are not therefore reported here. In the least affected equations, the normalised determinant of $z'z$ was 0.0004 in the case of peat and 0.0002 in the case of the oil category. The corresponding values of Haitovsky's χ^2 statistic were both 0.01; at the 5 per cent probability level and for 6 degrees of freedom the

critical value of chi-square is 12.59. The effect of this degree of multicollinearity was to produce many implausible coefficient estimates with large standard errors, and so this first attempt at estimation was abandoned.

In view of this, the first phase results take on a new significance and importance, despite their inefficiency. Rather than begin to draw conclusions from the figures in Table 1 straightaway, however, a second attempt at efficient estimation was carried out. After all, several methods are available by which the problem of multicollinearity may be circumvented. Of these, two appeared to be feasible in the context of the present study.

First, the size of the sample may be increased, both by including plants that were originally omitted and, in the case of the peat category, by increasing the length of the time-series on each plant. Of course, in doing this, the problem of additional equipment and plant vintage is introduced and any results have to be interpreted and treated with care. Second, a variable or variables may be eliminated from the specification of the input equations. The risk of misspecification which attaches to this course of action is well-known, but the procedure would appear to be acceptable for those equations in which a coefficient is insignificantly different from zero on the basis of the initial regresssion, or, recalling the earlier discussion of specification, where $\hat{\beta}$ and $\hat{\lambda}$ are not statistically different from each other, in which case equation (5) may be replaced by the original equation (4). This latter approach would seem to be far more acceptable in the case of the equations for fuel and energy than for those of the other inputs.

It was decided to attempt the re-estimation of equations using a combination of these two approaches. Specifically, extended samples were used to estimate equation (4) for all inputs in the two fuel categories; and in view of the results in Table 1, a modified form of this equation, with capacity replacing output, was also estimated in the case of the labour and capital inputs. Again, initial OLS regressions were carried out, $\hat{\rho}_{ip}$ values computed and the data transformed to generalised differences. A second set of OLS regressions was then performed on the transformed data. This time the second regressions were in general judged as econometrically satisfactory and so their residuals were used to compute the $\hat{\Phi}_{pp*}^{(i)}$ values and thus to form the estimated covariance matrix $\hat{\Psi}$. Together with the transformed data, this value of $\hat{\Psi}$ was used in (10) and (11) to yield the final GLS estimates. Certain other statistics, including R^2 values, were also computed as part of the GLS regression analyses.[15]

While the degree of autocorrelation, as indicated by the $\hat{\rho}_{ip}$ values, was again high in almost all cases at the first stage of the estimation procedure, the degree of heteroscedasticity indicated by the $\hat{\Phi}_{pp*}^{(i)}$ values at the second stage seemed only small. This was reflected in the small differences between the second stage estimates and the third (and final) stage estimates of the coefficients in most of the equations. Because of this, only the results for the first and final phases of the estimation procedure are given in Tables 2 and 3.

Finally, and mainly for purposes of comparison, the Komiya model and the Lau and Tamura model were fitted to the data. The former model is defined by equation (1); the latter may be represented by equation (4) with the term in time (t) deleted; both models were estimated by OLS regression using cross-section data only, as in the original studies. The full lists of coefficient estimates from these regressions are not reported here. However, the estimates of the scale (size) coefficients, β_i, are given — together with corresponding values selected from the previous tables — in Table 4. The table is divided into two sections to distinguish the models and estimates based on the use of gross output as an independent variable from those based on the use of capacity. In the case of the pooled models, the final GLS estimates are given in brackets alongside the first phase OLS estimates.

As in the case of Table 1, the equations whose coefficients are given in Tables 2, 3 and 4 were on the whole judged to be statistically satisfactory. The exceptions were the GLS labour and capital (C2) equations for oil-fired stations in model (4) (Table 2), the labour equations for peat and oil plants in the modified formulation of model (4) (Table 3), and the capital (C1) equation for plants in the oil category in the Komiya model (Table 4).

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Table 2: *Coefficient Estimates for Peat and Oil Input Functions – Full Procedure*

FUEL-TYPE	INPUT	$\hat{\beta}$ (a)	$\hat{\gamma}$	$\hat{\delta}$	R^2
Peat	L	0.460	−0.022	0.018	0.82
		0.831	−0.039	0.032	0.98
	F	0.806	−0.001*	0.016	0.94
		0.856	−0.009*	−0.022	0.95
	E	0.947	0.012*	0.002*	0.96
		0.544	0.037	−0.018	0.98
	C1	1.116	−0.005*	−0.006*	0.93
		0.856	−0.011*	−0.052	0.77
	C2	1.089	−0.011*	−0.007*	0.94
		1.035	−0.032*	0.075	0.85
Oil	L	0.622	−0.040*	−0.002*	0.94
		0.185	−1.063	0.038	0.93
	F	1.050	−0.040	−0.008	0.99
		0.894	0.346	−0.001*	0.99
	E	0.944	0.035	0.004*	0.99
		0.656	0.180	0.003*	0.96
	C1	0.993	0.008*	−0.022	0.96
		1.076	0.028*	0.026	0.90
	C2	0.849	−0.098*	−0.024	0.97
		0.314	0.152	0.015*	0.56

Note: *Not significant at 5 per cent level
(a) coefficient of gross output.

Table 3: *Coefficient Estimates for Peat and Oil Input Functions – Full Procedure*

FUEL-TYPE	INPUT	$\hat{\beta}$(a)	$\hat{\gamma}$	$\hat{\delta}$	R^2
Peat	L	0.755	−0.045	0.016	0.95
		1.135	−0.084	0.022	0.99
	C1	1.537	−0.055	−0.008*	0.94
		1.817	−0.033	0.010	0.99
	C2	1.487	−0.059	−0.009*	0.93
		1.387	−0.054	0.164	0.99
Oil	L	0.659	0.022*	0.004*	0.96
		1.054	−1.754	−0.014	0.98
	C1	1.072	0.117	−0.013	0.99
		1.182	0.261	0.017	0.97
	C2	0.918	−0.004*	−0.017	0.99
		1.616	−0.519	0.002	0.97

Note: *Not significant at 5 per cent level.
(a) coefficient of capacity.

In the first of these cases, multicollinearity appeared to be a problem; in the second, the fit of the equation was poor. In the next three cases, the second and third phases of estimation were carried out in the presence of substantial multicollinearity which, no

doubt, is the main reason for the unexpectedly high values of the final estimates of β. In the case of the Komiya formulation, the fit of the equation was poor and the estimated value of β insignificantly different from zero at the 5 per cent level as indicated by the asterisk in Table 4.

Table 4: *Comparison of Estimates of the Scale (Size) Coefficient β from Pooled and Cross-Section Models*

INPUT	MODEL	PEAT		OIL
	(a) Independent Variable — OUTPUT			
L	Pooled Model – (4) [Table 2]	0.46	(0.83)	0.62 (0.19)
	Lau and Tamura – LT	0.76		0.57
F	(4)	0.81	(0.86)	1.05 (0.89)
	LT	1.18		1.06
E	(4)	0.95	(0.54)	0.99 (0.66)
	LT	1.16		0.91
C1	(4)	1.12	(0.86)	.0.99 (1.08)
	LT	0.85		0.72
C2	(4)	1.09	(1.04)	0.85 (0.31)
	LT	0.86		0.72
	(b) Independent Variable — CAPACITY			
L	Pooled Model – (5) [Table 1]	0.57		0.99
	Modified Pooled Model – M(5) [Table 3]	0.76	(1.14)	0.66 (1.05)
	Komiya – K	0.46		0.38
C1	(5)	1.11		1.30
	M(5)	1.54	(1.82)	1.07 (1.18)
	K	0.97		0.42*
C2	(5)	1.05		1.33
	M(5)	1.49	(1.39)	0.92 (1.62)
	K	1.00		0.73

*Not significant at 5 per cent level.

Hypothesis Tests and Discussion of Results

There is scope for considerable detailed testing and discussion of the results that have been presented. However, the following comments concentrate on what are considered to be some of the more substantive matters. First, certain issues which involve the testing of hypotheses about the nature of the entire production structure in thermal electricity generation are discussed, namely, the homotheticity of the structure and the nature of overall returns to scale and technical change. Second, there is a brief discussion of individual input functions and of the economic interpretation and implications of their respective parameter estimates. Of course, consideration of both of these sets of topics presupposes that the NHL production function provides some explanation for the Irish data.

The overall explanatory power of each of the models examined in the study was assessed by means of the omnibus F-test of the null hypothesis, H_O, that all of its parameters are zero. Without exception, H_O was rejected at the 5 per cent significance level. In the case of fitted models (5) and (4), which produced the results in Tables 1 and 2 respectively — and which are the prime concern of this paper — the rejection was quite decisive.

Having established that the NHL model does provide an explanation for the ESB data,

testing of the hypothesis of homotheticity followed using both the OLS results for model (5) and the GLS results for variant (4). The null hypothesis for this test corresponded to the homotheticity condition given in Section 4, namely, that the coefficient of output, β, has the same value for all input functions in a given fuel category. For both sets of results, the hypothesis was rejected. In the case of model (5), homotheticity in peat-fired generation was rejected rather more decisively than for oil-fired generation. This is reflected in the relatively low value of $\hat{\beta}_L$ for peat which, on the basis of the standard two-sided t-test of the difference between regression coefficients, is significantly different from the $\hat{\beta}$ values for all of the other inputs at the 5 per cent level. In the oil category, however, $\hat{\beta}_L$ is not statistically different from $\hat{\beta}_F$, $\hat{\beta}_E$ and $\hat{\beta}_C$ at the 5 per cent level. There are similar differences in the $\hat{\lambda}$ values for different inputs, though these were not tested formally.

The rejection of homotheticity is of considerable importance. For a fortiori, homogeneity of the production structure, and hence overall constant returns to scale — and, indeed, overall increasing and overall decreasing returns to scale of any given degree — must also be rejected automatically. This is not to say, however, that the use of individual inputs may not be subject to constant returns to scale. This matter is considered below.

As can be seen from the prevalence of negative values for $\hat{\gamma}$ and $\hat{\delta}$ in Tables 1, 2 and 3, there is a suggestion that embodied and disembodied technical progress has influenced the use of the factors of production in both peat- and oil-fired generation. However, many of the estimates are not significantly different from zero, and those that are, are generally small. The few larger values of $\hat{\gamma}$ and $\hat{\delta}$ in the GLS results in Tables 2 and 3 generally relate to those equations that have been mentioned as being unsatisfactory. On the basis of the results of models (5) and (4), the null hypothesis of overall zero technical change, and that of neutral technical change, could not in fact be rejected at the 5 per cent level. As with the result on overall constant returns to scale, this finding does not necessarily imply that technical change has not had an impact on the use of individual inputs during the sample period. However, it is a somewhat unexpected result, and will be commented on again in Section 5.

The explanatory power of the large majority of results for the individual input functions were also highly significant on the basis of F-tests at the 5 per cent level. For example, in the case of the first OLS regressions for peat in model (5), the computed F values ranged from 26 to 1082, while the 5 per cent critical value, F_{37}^4, is 2.63. In the case of the final GLS equations for peat in model (4), they ranged from 76 to 2547. Similar high values were recorded for the equations in the oil category. Only two equations fitted the data poorly; these were referred to at the end of the last subsection.

In considering the individual input results, it is useful to bear in mind that to the extent that the data on independent variables are subject to errors of measurement, parameter estimates may be expected to be biased downwards. Attention was drawn to the possibility of such errors in the measures of capacity and load factor in the subsection on data. It may also be useful to distinguish between the $\hat{\beta}$ values for output and those obtained using capacity as an independent variable. In what follows, greater attention is concentrated on the former, and the ex post interpretation of the production function. Finally, in view of the range of models used, no particular set of point estimates is singled out for special consideration. Rather, the emphasis is placed on the general orders of magnitude and mutual consistency of the different estimates of individual parameters. In principle, of course, the estimates obtained from the full GLS procedure are favoured, but use of the extended data set to obtain them gives rise to certain difficulties of interpretation as mentioned in the discussion of the empirical analysis in the previous subsection.

Labour

In model (4), the scale coefficient $\hat{\beta}$, i.e. the partial elasticity of the quantity of labour used with respect to gross output, ranges from 0.46 to 0.83 in the case of peat, and 0.19 to 0.62 in the case of oil. The values for this coefficient obtained using the Lau and Tamura model lie within these ranges. Thus, as all of these values are considerably less

than unity, even ignoring the suspiciously low value of $\beta_L = 0.185$ for oil, a consistent indication of substantial economies of scale in the use of labour emerges, with the economies being somewhat greater in the case of generation using oil.

The results for the size coefficient, obtained from the models involving capacity, also reflect economies of scale. In the case of peat, the range of values of $\hat{\beta}$ is 0.46 to 0.76 (disregarding the suspect GLS estimate) which is very similar to the range for the output coefficient. In the oil category, for which the range of $\hat{\beta}$ values is 0.38 to 0.99, the picture is less clear. If the Komiya estimate is disregarded, however, the indications are that economies in the use of labour with respect to plant size are less pronounced than those for the use of labour with respect to output for a given plant size.

The values of the size coefficient for labour obtained by Komiya and Barzel, using data for the USA, were 0.50 to 0.60, and 0.63, respectively. While these figures are similar to those obtained in this study, it should be noted that they relate to coal and oil plants and, in the case of oil, to an earlier sample period.

The finding of large economies in the (ex post) use of labour is quite in accord with prior knowledge and expectations. For after a plant is installed, labour is almost a fixed factor of production. This fact is vividly reflected in the estimated value of λ, the partial elasticity of the quantity of labour used with respect to the degree of capacity utilisation, obtained from model (5). The value of $\hat{\lambda}$ in the labour function for peat plants is zero, statistically; for oil-fired plants, it is 0.25. These figures compare quite closely with the corresponding figure of 0.17 reported by Barzel; they mean that plants of equal size, operating at different levels of capacity, do not vary substantially in the amount of labour used.

The values of the coefficient of embodied technical change, $\hat{\gamma}$, are in almost all cases negative, but small, as noted earlier. The implication of this is that embodied technical progress, or the reduction of the labour requirement in plants of a given size or scale of operation, but more recent vintage, has been quite modest on a year by year basis. Over a longer time period, however, the impact of embodied technical change is more noticeable. For example, the GLS estimate of -0.039 for peat plants in model (4) suggests that the amount of labour required by a plant installed in, say, 1960 was about 48 per cent less than a plant installed in 1950, all other things being equal.[16] Both Komiya and Barzel reported similar-sized vintage effects. Of course, when combined with an expansion of scale, the decline in the labour requirement becomes substantial.

The estimated values of δ, where they are significant, are generally much smaller than those for γ, suggesting only slight disembodied technical change. Indeed, in all of the labour functions in the oil category except the unsatisfactory GLS result in Table 2, the value of $\hat{\delta}$ is not significantly different from zero. This may be interpreted as indicating no discernible change in labour quality or efficiency over the sample period.

Fuel

The partial elasticity of the quantity of fuel used with respect to output, $\hat{\beta}$, ranges from 0.81 to 0.86 in the case of peat stations, and from 0.89 to 1.05 in the case of oil plants, using model (4). The GLS estimates in each of these categories are significantly less than unity on the basis of the standard t-test at the 5 per cent level, unlike the estimates from the Lau and Tamura model which are not significantly different from unity at the 5 per cent level. Despite the closeness of the different numerical values, there is therefore some doubt about whether fuel economies accompany higher levels of output. If they do, as suggested by the preferred GLS results, they are only slight.

The results for the size coefficient in model (5) — 1.05 and 0.93 for peat and oil plants, respectively — are not significantly different from unity, implying constant returns to size in the case of both types of station. On the other hand, the load factor coefficient, $\hat{\lambda} = 0.91$, is significantly less than unity in the case of peat plants, suggesting, like the GLS estimate of $\hat{\beta}$ in model (4), that there are slight fuel economies to be realised by increasing output and the level of utilisation of the capacity of peat plants. The $\hat{\lambda}$ value for oil-fired stations is not significantly different from unity and therefore

does not lend similar support to the associated GLS estimate of the output coefficient. The $\hat{\lambda}$ value is also not significantly different from the size coefficient $\hat{\beta}$ in model (5), which suggests that there is little to be gained by decomposing output into its capacity and load components in the fuel function for oil stations.

The weight of the evidence would suggest acceptance of the hypothesis of constant returns to plant size. This result confirms expectations, no doubt. However, it differs somewhat from the findings of both Komiya and Barzel. Komiya estimated the size coefficient for fuel to be between 0.80 and 0.85; Barzel estimated it to be 0.89. In both cases the values were significantly less than unity, although Barzel stated that the importance of the economies implied by his result should not be overestimated (see his example: Barzel 1964, p. 137.)

The technical change coefficients, $\hat{\gamma}$ and $\hat{\delta}$, in the fuel input functions are on the whole smaller than those recorded for the labour input functions. In the case of embodied technical change, i.e. the change in the fuel requirement resulting from different vintages or qualities of equipment, the significant estimates of $\hat{\gamma}$ are mainly negative. In the case of disembodied technical change, which may be interpreted as the change in fuel quality, there are small negative and positive values of $\hat{\delta}$. Negative values indicate increases, and positive values indicate decreases in fuel efficiency, respectively. It is not particularly surprising to have observed both. In his American study, Barzel (1964, p. 139) also observed falls and rises of fuel efficiency over time, of up to 10 per cent.

However, it is not the signs of the technical change coefficients that are of greatest interest, so much as the general contrast between the sizes of the coefficients in the two fuel categories. For purposes of comparison, the final phase estimate of the fuel function for oil stations in Table 2 may be disregarded because, probably due to multicollinearity, it contains an unacceptably large positive value for $\hat{\gamma}$. The first phase estimate of this equation is considered more reliable. Nevertheless, it may be seen that the $\hat{\gamma}$ value for oil plants is still, relatively, very much larger than that for peat plants. The same may be said about the $\hat{\gamma}$ values given in Table 1. The indications are that embodied technical progress has been more significant for oil plants than for peat plants.

For example, consider the significant $\hat{\gamma}$ values for peat and oil plants in Table 1. The value of -0.017 for peat means that the difference in the fuel requirement for plants whose vintage differs by a decade, is only 18 per cent, whereas the value of -0.057 means the corresponding difference for oil plants is about 75 per cent, all other things being equal. Komiya's conclusion that "the improvement in thermal efficiency can be explained primarily by the increase in the scale of production rather than by the shift in the function" may hold in the Irish case for peat, but it does not appear to apply in the case of oil-fired generation.

Energy

The size coefficient for the energy function estimated using model (5) is indicative of slight diseconomies of scale in the case of peat, the $\hat{\beta}$ value of 1.236 being significantly greater than unity. In the case of oil plants, constant returns are indicated as the $\hat{\beta}$ value of 0.991 is not significantly different from unity. In contrast, there are very clear indications of economies in the use of energy when both types of plant are operated more intensively, the partial elasticity with respect to degree of capacity utilisation being of the order of 0.7 in both cases.

This last feature is reflected in the values of the output coefficient estimated using model (4), although at the first stage of estimation the estimates are only slightly less than unity. At the final stage, however, they are markedly less and suggest economies comparable with those for labour. On the other hand, the estimates obtained using the Lau and Tamura model are much closer to unity, with constant returns in the case of peat plants and slight economies in the case of oil plants being suggested by hypothesis tests at the 5 per cent level.

The values of the vintage and time coefficients for energy in both fuel categories are very similar to the corresponding coefficients in the fuel input functions, at least in the

case of model (5). One may thus be led to very similar conclusions. However, in the case of energy, it is not as easy to dismiss the GLS result for oil plants as it was in the case of the fuel function. There is therefore a conflict between the relatively large positive value for $\hat{\gamma}$ given by model (4) and the smaller negative value of model (5). A similar situation exists for peat plants, although the value of $\hat{\gamma}$ from model (4), though positive, is not nearly so large as that for oil plants. The meaning of positive and negative values of these coefficients has already been stated. The same kind of conflict does not arise from the estimates of δ. Only one of these is significant at the 5 per cent level, namely, the GLS estimate in the peat category, and this, being negative and small, suggests that there has been a slight increase in efficiency in the use of energy over time.

Energy functions were not estimated in any of the studies described in Section 3. As far as the author is aware, no other basis of comparison for the present estimated energy functions exists.

Capital

The estimates of the coefficient of scale, β, in those models which incorporate output as an independent variable are similar for the C1 measure of capital based on the cost of generating equipment only, and the C2 measure based on the total capital cost of plants. In the case of peat stations, the coefficient values range from 0.85 to 1.12, and in the case of oil plants from 0.72 to 1.08, with neither the 1.12 figure nor the 1.08 figure being significantly greater than unity. The coefficients are particularly close for the alternative capital measures in the case of the Lau and Tamura formulation of the function. While, on the basis of the results for model (4), constant returns to scale cannot be ruled out for either type of station, the overriding impression is one of slight economies of scale with respect to the use of generating equipment and the total capital requirement. The total capital economies appear to be somewhat greater in oil-fired generation than in peat-fired production.

The same broad consistency between estimates based on the use of the two measures of capital may be observed in the case of the models which employ capacity as an independent variable. In these cases, however, the indications are of diseconomies in real capital requirements. Values of $\hat{\beta}$ range from 0.97 to 1.54 for peat stations and 1.07 to 1.30 for oil stations when C1 is the dependent variable; and from 1.0 to 1.49 for peat stations and 0.73 to 1.33 for oil stations when C2 is the dependent variable, with the figure of 0.73 from the Komiya formulation being the only estimate which is significantly less than unity at the 5 per cent level. There is some suggestion that the diseconomies are less for oil-fired plants than for peat-fired plants. ·

Such estimates of the partial elasticity of the real capital cost with respect to size (capacity) of plant are in conflict with expectations. They contrast sharply with the kind of value that would be expected on the basis of certain conventions used in engineering fields to estimate the capital cost of plants of different sizes. In particular, they differ considerably from 0.6, the so-called "six-tenths factor" proposed by Chilton (1960) and once widely used by engineers. They are also greater – and in most cases significantly greater – than the values of the corresponding parameter estimates obtained by Barzel and Komiya. Barzel's estimate was 0.82, while Komiya's was the range 0.80 to 0.85. These values are much closer to those of the coefficient of gross output obtained using the model (4) variant of the capital function in this study.

By contrast the estimates of $\hat{\lambda}$, the partial elasticity of the amount of capital services used with respect to degree of capacity utilisation, are entirely in accord with expectations. When C1 is used as the capital measure in model (5), $\hat{\lambda}$ is not significantly different from zero in both peat- and oil-fired production. The interpretation of this result is that essentially the same flow of services is required of generating equipment whether a low or a high level of output is produced. On the other hand, when C2 is used as the measure of capital, $\hat{\lambda}$ is 0.117 in the case of the capital function for peat plants, and 0.257 in the case of that for the oil category. Although small, both of these values are significant at the 5 per cent level and are similar to the value of 0.116 reported by Barzel. Thus, with

respect to total capital services, very large economies may be derived from higher levels of capacity utilisation of peat and oil plants, as was found to be the case with respect to labour.

Finally, the coefficients $\hat{\gamma}$ and $\hat{\delta}$, relating to technical change, are quite small, as in the other equations. The majority of statistically significant values of $\hat{\gamma}$ are negative, indicating embodied technical progress. The majority of significant values of $\hat{\delta}$, however, are positive. While this may seem a rather surprising result, it is noteworthy that Komiya also reported similar small positive shifts in his capital function. Their meaning is that disembodied technical change has been negative, that is, there has been a move over time in the direction of plants of the same size requiring more capital. The implication of the existence of these opposite signs is that in certain of the capital functions, embodied and disembodied technical change have had a cancelling rather than a mutually reinforcing impact on the use of capital.

5. CONCLUDING REMARKS

This study was motivated by an interest in applying a relatively new concept of production theory to a concrete situation. The concept — the non-homothetic Leontief production function — would appear to be very well suited to application to any industry whose production process is characterised by limited factor substitutability. The case of electricity production at plant level in Ireland was chosen because econometric analysis of production in that industry had not previously been undertaken; because good-quality data are readily available for plants in the industry; and because several studies of electricity production have been done for other countries and it seemed useful to compare results for Ireland with their findings. It was not the intention to relate the exercise to an examination of any particular policy issue that might confront the ESB. However, as was stated in Section 1, there are several kinds of economic issues on which quantitative information about the production process may have a bearing. Some of these issues, and the extent of the practical relevance of the results reported in this paper, are briefly discussed below. But first, the main findings of the analysis are summarised.

The NHL production function for electricity generation was estimated in terms of a system of derived factor input equations using an econometric methodology based on pooled cross-section and time-series data. The equations appear to fit the data well. On the basis of the estimates of the scale parameters, the hypothesis of homotheticity, and hence overall constant returns to scale was rejected. In fact, the results clearly suggest the existence of increasing returns to scale in electricity production at plant level, with substantial economies with respect to the labour and energy inputs, lesser economies with respect to the capital input, and slight economies with respect to the fuel input. All of these results are in accordance with previous findings on thermal electricity production in other countries. However, the results on the capital input function incorporating capacity as an explanatory variable are at variance with the results of previous studies, and with certain assumptions used by some engineers in estimating the capital cost of plants of different sizes. Similarly, there is an unexpected result for peat-fired stations in the energy function incorporating capacity, which suggests the possibility of slight diseconomies with respect to plant size. Both of these findings perhaps warrant further investigation.

The estimates of the parameter associated with capacity utilisation are quite in accord with expectations, suggesting sizeable increasing returns to labour, energy and capital as the degree of capacity utilisation of a plant increases, and approximately constant returns with respect to the use of fuel. In contrast, the impact of embodied and disembodied technical change, as measured by the estimates of the coefficients of plant vintage and time, was found to be unexpectedly small. Indeed, while in a few cases the reduction in the quantity of an input required to produce a given level of output is not negligible over a sufficiently long time period, the hypothesis of overall zero technical change could not be rejected.

Comparing results for the two types of plant examined, the scale economies associated

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with labour were found to be larger for oil plants than for peat plants. But in general, the differences between the estimated parameter values of the production structures of the two types of plant were not found to be large.

The results seem to show clearly that the scale effect is probably a far more important factor in improving the production efficiency of plants than pure technical change. This is not to say, of course, that technical change has played only a small part in improving efficiency; that is patently not the case. Rather, it suggests acceptance of the view, expressed by Komiya (1962, p. 166), that "The fact that it has become possible to build larger and larger generating units realising the benefit of increasing returns is to be considered as the major achievement of technological progress in this industry." No doubt such a view would be widely accepted amongst those concerned with the operation of the electricity supply industry in Ireland. Therefore, in providing econometric confirmation of this view, the results may not be without interest for them. It is hoped, however, that the individual numerical estimates of the actual extent of returns to scale and technical progress — though the first of their kind for Ireland and therefore still somewhat tentative — may be of rather more interest than that. Such knowledge may be useful for its own sake, but more important, it may have relevance for various factors which the plant operator may wish to take into account in formulating policy. Therefore, in conclusion, some of the possible applications of the results, and the kinds of policy issue on which they might have a bearing, are briefly considered.

Using the estimates of the derived input demand equations, it is possible, of course, to give numerical substance to the underlying production function as specified in equation (2). However, there seems little to be gained from this as information about the nature of the production structure is available directly from the estimated input functions. Therefore it would appear to be potentially much more profitable to examine further the direct use of these equations; but it is also possible to make important indirect use of them through the derivation of cost functions. It is not the aim here to undertake a systematic exploration of these possibilities, but simply to suggest and comment on some of them.

Thus, for example, the input equations may be used directly to provide an indication of the optimal relative factor intensities of plants. Using the notation of Section 3, these may be written as $X_i/X_j = f_i(.)/f_j(.)$, $i \neq j$, for a given output level from a given type of plant. In principle, knowledge of optimal factor proportions is clearly a matter of importance for the efficient operation of plants. Similarly, in addition to what has already been inferred about returns to scale and technical change, individual parameter estimates may be used to obtain various other measures, such as of the increasing capital intensiveness of plants. Hence, although a substitution model was not used, the ratio of the scale parameters for capital and labour, for example, would appear to indicate a significant long-run trend of substitution of capital for labour. Such considerations as the importance of returns to scale and the likely factor bias of technical change may be of some relevance for planning activities. In particular, they would seem to have some bearing on such questions as whether fewer but larger power stations should be constructed even at the risk of increasing vulnerability, and the estimation of future demand for labour.

From the individual input functions, it is a routine task to derive cost equations, given factor prices. Therefore the results of this study provide a means of deriving estimates of the total operating cost and capital cost of plants of a given fuel-type, which may operate at varying degrees of capacity. Such cost information may be of value in formulating capital investment policy. It should be stressed, however, that econometric cost information would only be one small ingredient in the investment decision-making process. The actual process is, of course, highly complex. It is complicated, for example, by the fact that because the life-span of capital in the industry is so long, optimisation over a substantial period of time is required. Faced by uncertain demands and a life of capital of say 20 years, an acceptable optimisation procedure would involve that choice of plant which, for a set of exogenous demands, minimises the present value of expected costs. Moreover, the requirements of reserve capacity to meet the problem of "outage", i.e. breakdown

and maintenance, the relationship between total capacity and peak load, and tariff policy and revenue considerations, not to mention the recent surge of interest in nuclear generation, would also complicate the process.

Therefore, without wishing to exaggerate their value, the results of econometric production studies may shed some useful light on various kinds of policy issue. To the extent that the statistical results of this study are accepted, it is hoped that they might be of some help in analysing some of the matters that have been referred to, or, at least, that they might be considered a worthwhile basis for further research by those involved with the Irish electricity industry.

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FOOTNOTES

1. In preparing Section 4, I was faced with a dilemma. I was conscious, on the one hand, of the technical, though mostly routine, nature of much of the material it contains, and, on the other hand, of the fairly general nature of the audience. Therefore, at the risk of being somewhat tedious to those familiar with the type of econometric approach used, I have dwelt at rather greater length on the methodology and results than I would have done had the paper been prepared for a more specialised audience. I hope I have struck a reasonable balance.

2. For further details see Booth (1966, Part IIA, especially pp. 9-14), and ESB Investigation Committee (1972, Appendix 1, pp. 2-5).

3. Unless stated otherwise, the statistics given in this Section are based on figures contained in the ESB Annual Report for the year ended 31st March, 1976.

4. The names of the stations, their locations and technical specifications are given in ESB Annual Report (1976, Appendix 6, pp. 40-41).

5. For greater detail see, for example, Deshpande (1966).
6. The load factor = total annual KWh./(installed KW. x total annual hours) x 100 = percentage of time a plant is utilised during a year, where KWh. stands for kilowatt-hour, the standard unit of electricity output, and KW. stands for kilowatt, a unit of capacity.
7. A survey and critique of the studies of Komiya, Barzel, and Dhrymes and Kurz, and others, is contained in Galatin (1968, Ch. 4). A summary of Komiya's methodology and results is also available in Desai (1976, pp. 160-167).
8. These properties are discussed and illustrated by Lau and Tamura (1972, pp. 1170-1171).
9. A necessary and sufficient condition for overall constant returns to scale (i.e. to plant output) follows as $\beta_i = \lambda_i = 1$, for all i, which is equivalent to $\beta_i = 1$, for all i, in model (4).
10. The system of input functions developed by Dhrymes and Kurz gave rise to very serious estimation problems, quite apart from the nature of the disturbances, and these were approached by an "informal" estimation procedure based on the use of two-stage least squares (see Dhrymes and Kurz 1964, p. 294, fn. 5). The Zellner method was used by Lau and Tamura (1972, pp. 1179-1180).
11. See also Kmenta (1971, pp. 513-514).
12. The source of the price indices was *Irish Statistical Bulletin*, Central Statistics Office, Dublin (various issues).
13. The practice of presenting a minimum of statistical detail is followed throughout this Section. Fuller details are available from the author.
14. Multicollinearity was tested for by using the χ^2-test of Haitovsky (1969) at the 5 per cent significance level.
15. The GLS computer program, and the programs for all intermediate calculations, were written by the author. The OLS program used was the modified IBM package of Neary (1972). The machine used was the IBM 360 model 44 at Trinity College, Dublin.
16. The natural antilogarithm of (10 x 0.039) = 1.477.

DISCUSSION

A.G. Kelly: I am happy to propose the vote of thanks to Mr. Harrison for his very interesting paper, and to express my appreciation of the quality of the work behind this research.

The engineer manager measures the success or failure in economies of scale and technical change in power stations under three main headings — efficiency, cost and reliability.

— As proof that the ESB is successful in efficiency of production of electricity, refer to the World's Top Ten Power Stations on efficiency (Kelly 1977) where Poolbeg Station is fifth in the world and best in Europe; this is despite the small size of this station.

— The cost of completed installations is compared with others around the world. As an example, we have not found a similar station that compared with the 500 MW Tarbert B which was completed at £92/KW in 1975/1976. The final cost of the unit of electricity to the consumer is the ultimate measure of "efficiency". On this, the ESB compares favourably with other European utilities and has been cheapest on domestic charges over the past decade. This is without the benefits of interconnection with nearby utilities.

— Reliability of production is compared with other power station units of plant in the world. An analysis (Kelly 1979) shows that few units of plant attain a load factor of over 80 per cent. The ESB has regularly exceeded this figure, e.g., at Poolbeg and Lanesborough B stations.

The overall system load factor is a measure of good planning, design, rates of charge, operation and maintenance. The ESB figure is 56 per cent; the USA is 49 per cent — but was 57 per cent in 1957 and has gradually deteriorated. With interconnection with GB or N. Ireland (the present connection has been severed for some years) our figure could be even better. The load factor of a unit of plant is more important than the efficiency of the unit, e.g., for a coal-fired unit operating at 5,000 hrs/year, 2-weeks extra operation per year is equivalent to 1½ per cent on efficiency; to improve 1½ per cent on efficiency in design is not possible but to design for further reliability certainly is practicable.

Now for some detailed comments on the actual paper:

Plant Ratings: The USA and UK ratings used are varied from year to year thus yielding false results. The study (Kelly 1979) has corrected for these. All ESB figures are genuine.

Labour: Certainly, the men/MW of plant has decreased rapidly over the years; this will continue with further automation. Indeed, the total staff in the ESB is less in 1979 than 1971 even though the output is far higher.

The number of persons employed is, on its own, just a measure of the number of persons employed.

Capital: No rule of thumb can be used to say that doubling the unit size increases the cost by a certain amount.

There is always an upper limit to size, e.g., the cost of a 5,000 MW single unit is not known because such a unit cannot be made. The 250 MW units now operating in Ireland were not possible years ago. Thus, the capital cost rises rapidly as we approach the upper limit of technological development.

Economies of Scale: Economy of scale disappears (Kelly 1979) below 1,000 MW; the economic size of unit is presently in the range 250 to 350 MW.

The study by Mr. Harrison on economy of production by size of power stations using peat gives a false result. This is because, for example, Ferbane 90 MW station has smaller units of plant than Shannonbridge 40 MW station which has a larger unit (40 MW).

Oddly enough, future improvements in efficiency will be on smaller units used in combined cycle plants. The cost of construction of such smaller units is also lower – so economy of scale is upset.

Load Factor: is assumed by the author to depend only on despatching policy. Unfortunately, similar sized units, e.g., 40 MW units in peat stations have not equal reliability; some are excellent and would match the world's best but others are not so excellent. This depends mainly on the design by different manufacturers.

It is axiomatic that production costs reduce with increasing load factor. It is not necessary to do a statistical study to discover that "with zero load factor, the cost per unit produced is infinite."

Fuel: The efficiency of use of fuel had attained a maximum of 40 per cent (on net calorific value) and the higher efficiencies were attained in the range of sizes 100 MW-200 MW in Ireland/UK (Kelly 1979). This upper limit to efficiency has existed for many years.

However, a new technological development has introduced an improvement. This is the development of the combined cycle plant such as the new ESB power plant at Marina, Co. Cork. The new limit for efficiency has now quickly risen to over 45 per cent.

The efficiency of production of a 40 MW peat unit is shown by the fact that overall cost of a unit of electricity is only 75 per cent of that from the smaller 20 MW units; yet the author does not arrive at this conclusion from the statistical study because of the use of total plant rather than units of plant as the basic item examined.

Other investigators (Niebo 1979) attempted to deduce results from statistics of the USA unit results. The results were mostly statistically inconclusive.

Comment: The data used for Ireland could profitably be compared with a larger pool such as the USA, for which all data are available.

This study by the author is a very interesting statistical exercise. The fact that the engineering data and constraints mentioned here are not allowed for, and that thus the conclusions tentatively put forward are often incorrect, does not detract from this.

If the author wished to proceed to correct the study to conform with engineering realities, the ESB would be more than pleased to co-operate.

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Colm McCarthy: Mr. President, Ladies and Gentlemen: It is a pleasure to have the opportunity to second the vote of thanks to Mr. Harrison on this evening's paper. There have been only a few production function studies on Irish data and this is the first attempt to model the production of electricity. I will confine my remarks to just two areas — the econometric issues raised by Mr. Harrison's paper, and some of the economic issues raised by the power generation industry generally.

The log-log formulation of the input demand equations has the implication that Mr. Harrison's no-substitution assumption suppresses variables from a Cobb-Douglas input (factor) demand specification. With $Q = f(K,L)$ in the familiar notation, the Cobb-Douglas factor demand equations are, for K say, of the form

$$\log K = a_0 + a_1 \log Q + a_2 \log L \qquad (1)$$

while Mr. Harrison's choice of functional form (which is arbitrary) would yield

$$\log K = b_0 + b_1 \log Q \qquad (2)$$

Thus the force of the Leontief specification is that the demand for each input is independent of the demands for the other inputs, i.e. that a_2 in (1) is zero. It would hardly be interesting to test for the presence of substitution possibilities in the context of (1), since the Cobb-Douglas elasticity of substitution is unity. Such a high figure would be amazing for the type of productive process described by Mr. Harrison. However, the possibility that the elasticity of substitution is non-zero might be worth testing for in a framework, such as CES or its extensions, where it is permitted to take on small values.

The assumption that the units of observation are cost-minimising is not as innocuous as it might appear. If they are not, and this is a problem common to all production studies, there is a kind of identification problem. In the present context it would involve inferring, from the input demand functions, that the L-shaped isoquants were located further from the origin than they would be for a plant that truly minimised costs. As with any identification problem, extraneous information will resolve the difficulty. If we knew that there was a consistent 10 per cent excess usage of a particular factor, a simple re-scaling would permit identification of the true relationship describing the technology of production. But no such simple alternative assumption is available. This problem has consequences for the conclusions one may draw about such matters as homotheticity, since any size-related pattern in excess usage of particular factors would interfere with the parameter estimates on which the tests are based.

Mr. Harrison's remark that independent variable errors may bias coefficient estimates towards zero is well taken, but the prevalence of multicollinearity in his data is noted several times. Coefficient estimates tend to explode in the presence of multicollinearity and it is difficult to see what the joint impact of multicollinearity and data-errors might be.

The literature on pooling time-series and cross-sections has tended to concentrate on developing GLS methods to deal with heteroscedasticity — Mr. Harrison's finding is that autocorrelation may be more important. To the extent that a simplification of the GLS procedure is desirable and may be necessary for computational feasability, it would be useful if one could develop some generalisations about the circumstances in which one or other "disease" is likely to be dominant. If the time series are untrended and the cross-section units of equal size, for example, it might be legitimate to ignore heteroscedasticity

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altogether.

Turning to the economics of power generation generally, and the policy issues which it raises, there are several areas in which research seems called for. Aside from the work which Susan Scott has undertaken recently, and which is highly aggregative, I am not aware of any econometric evidence on electricity demand in Ireland. Both for forecasting and as an aid to pricing policy, it would be desirable to know more in this area.

The construction of the Turlough Hill project could be seen as an attempt to deal with the peak load problem from the supply side. It would be interesting to contrast the economics of a supply-side solution of this problem with those of a demand-side solution, based on peak-load pricing.

A third area of inquiry which may be worthy of attention concerns the choice of primary energy sources and of plant sizes in electric power supply. Choice of one energy source and one huge plant might minimise costs under certainty: given the likelihood of variation in the relative prices of fuels and the risks of plant shutdowns for technical or industrial relations reasons, there is a kind of portfolio diversification incentive.

The final item in this arbitrary list concerns the pricing of inputs. There has long been a place on the research agenda for this item given the policy adopted in regard to turf. The decision to use Kinsale gas for electricity generation, and the policy adopted in relation to the taxation of oil products generally, have made the issue more important in recent times.

I am aware that the ESB may well have undertaken economic studies in these and other areas for internal consumption, but there has been very little in the public domain.

John W.O'Hagan: I would like to raise three issues resulting both from Mr. Harrison's paper and from the replies by Mr. Kelly and Mr. McCarthy. The first issue concerns this problem of the different meanings assigned by economists and engineers to the word efficiency. I would like to hear some comment from the engineers present on the rather restricted definition of efficiency used in some of Mr. Kelly's charts and I would like to ask whether other aspects of efficiency have been looked at in the papers that Mr. Kelly has mentioned. Second, I wonder if Mr. McCarthy's suggestion of using other CES production functions, apart from the NHL and Cobb-Douglas functions, could be extended to include production functions not in the CES class? The last issue I would like to raise concerns the use of plant load factor as an explanatory variable in the paper, particularly in the input demand equations for labour and capital. I find it difficult to envisage how, *a priori*, there could be any relationship between load factor and the demand for labour and capital and, as such, this variable should not, perhaps, have been included.

May I take this opportunity of warmly associating myself with Mr. Kelly's vote of thanks to Mr. Harrison. A rather difficult paper, researched with the meticulous care and rigorous standards that we now associate with Mr. Harrison's work, was presented in a most interesting manner.

Patrick Honohan: The difficulty in disentangling the respective effects of scale and technological progress on factor demand is particularly important in this study which, after all, is focused on exactly these effects. That technological progress seems to have manifested itself in the availability of larger plant sizes certainly presents an econometric problem. I wonder whether experimentation with different functional forms, or other representations of technological progress than those used, might perhaps have allowed for the separate identification of the two effects.

Reply by M.J. Harrison: I am most grateful to the Society for giving me the opportunity of presenting my paper this evening, and to those who spoke on the paper for their various comments. All but one of the speakers made some points which are not directly related to the subject of the paper. Mr. Kelly's remarks on how engineers assess the efficiency, costs and reliability of electricity production by international comparisons were interesting and informative. I agree fully with Mr. McCarthy's view that there would

seem to be considerable scope for further research into other areas of the economics of power generation in Ireland, not least into the demand for electricity; and I sympathise with Mr. O'Hagan's concern about the engineers' use of the concept of efficiency. In my reply, however, I propose to confine my attention to those points which were directed towards, or have a bearing on, specific aspects of the paper.

With regard to Mr. Kelly's detailed comments, I am interested and pleased to hear that the ESB figures on plant ratings are considered accurate. For this means that my comments on the possible shortcomings of the measures of capacity and degree of capacity utilisation used in the paper are probably unduly pessimistic, and my concern for the associated bias in parameter estimates largely unwarranted. I am also pleased to hear what Mr. Kelly had to say about capital cost. In the paper I express some surprise at the values I obtained for the coefficient of size in the capital equations, which in almost all cases are greater than unity, indicating diseconomies of scale, and greater than the corresponding figures obtained in the USA. However, given that the American figures were obtained in the 1960s, and given what Mr. Kelly has said, the finding now seems less surprising. Presumably the stations with the 250 MW sets (a 270 MW set in the case of Poolbeg) are using sets sufficiently close to the technological limit for unit size, mentioned by Mr. Kelly, for the diseconomies in capital cost to have manifested themselves in my results.

I am very much aware of the problems caused by different generating stations having different numbers of differently-sized units. Indeed, much of the force of the critique of production studies put forward by Galatin (1968), to which I alluded at the start of Section 3, was directed at this problem, referred to by Galatin as the "machine-mix" problem. Given the data available to me, however, I was not, as stated in the paper, in a position to adopt a feasible alternative approach which would make due allowance for machine-mix. Despite this, I am not convinced that my findings on the relative economy of the smaller peat stations is totally fortuitous, as Mr. Kelly's remark on the matter appears to suggest.

I would dispute Mr. Kelly's suggestion that I assume in the paper that load factor depends only on despatching policy. I do say that degree of capacity utilisation is determined *essentially* by load-despatching policy, but, aware of both forced and planned "outage", I also draw attention in Section 2 to the possibility of breakdown of equipment and its shut-down for maintenance purposes. Lest Mr. Kelly's unreferenced quotation be ascribed to me, I also have to quibble with his other point concerning load factor. First, it may well be "axiomatic" that unit costs decline as load factor increases, just as it is "axiomatic" that in the case of most goods the quantity purchased per unit time decreases as price increases, *ceteris paribus*. This, surely, does not make attempts to quantify the rates at which these changes take place unnecessary. Second, the statistical study I report in the paper was not directly concerned with production costs at all, and certainly not with what is stated in Mr. Kelly's quotation. Incidentally, it is not surprising, therefore, that I do not arrive at the conclusion concerning overall costs of a unit of electricity which Mr. Kelly refers to under the heading *Fuel*. To say that I do not arrive at this conclusion for the reason stated by Mr. Kelly would seem to me to be a *non sequitur*, although, as I have already mentioned, I acknowledge the difficulties and implications of the machine-mix problem for the achievement of my actual aims as set out in Section 1.

It would be interesting to derive the cost structure implied by my results, of course. As mentioned in Section 5 of the paper, this would not be difficult. Were it to be done, my suspicion is that the resulting ratio of generating costs for a 40 MW peat station to those of a 20 MW peat station would not be too far out of line with that which Mr. Kelly quotes.

While the paper does contain some comparisons of my results for Ireland with similar, though rather older, results for the USA, I agree with Mr. Kelly that it would be useful to make comparisons using more recent data on the larger number of American, and perhaps British, power stations. I do not have any immediate plans to refine my study and/or to carry out such comparisons, but should I return to do more econometric work on electricity production, I should be eager to avail of Mr. Kelly's kind offer of co-

operation from the ESB on matters of engineering detail and data provision.

Mr. McCarthy began by commenting on my choice of functional form for the input equations and suggesting that it might be useful to employ some other form to allow for the possibility of non-zero elasticity of substitution between factors. I have no doubt that this would be appropriate for a study undertaken at the level of the firm, or at plant level if the concern was with the *ex ante* production function. However, as stated in Section 4 of the paper, I was primarily concerned with the *ex post* production function. Given the nature of the electricity production process, and the not insubstantial body of empirical results obtained for other countries using various functional forms, including the CES function, I am not sure that substitution models warrant serious consideration as models of the *ex post* production function, and I remain satisfied with zero substitution as a maintained hypothesis for the purposes of my study.

Incidentally, while I agree that there is inevitably an element of arbitrariness in the choice of any functional form for an economic relationship, my choice of log-log was not totally arbitrary. First, the choice was based on the theoretical rationale of Haldi and Whitcombe (1967) to which I referred in Section 4 of the paper. This rationale is particularly compelling in the case of the capital equation, I feel. For, without going into the detail of the argument, the relationship between the amount of material required to build equipment, and the capacity of the equipment, is invariably close to a geometrical one. Second, in many previous studies of electricity production, log-log formulations have performed well by comparison with alternative formulations.

In view of these various points concerning functional form, I would want to have very good reasons before trying other forms in what, otherwise, would be a totally arbitrary manner. Does Mr. O'Hagan, for example, have strong feelings about why my chosen functional form might not be the most appropriate, and has he any specific suggestions as to what kind of functions, not in the CES class, might constitute a superior alternative? The problem of untangling the effects of scale and technological change on factor utilisation may well be a good enough reason, as Dr. Honohan suggests. I am not aware of other functional forms having been used in the literature specifically to try to effect this separation, nor, I confess, of any potential types of equation that would allow this to be achieved. The matter would, undoubtedly, seem to deserve further consideration.

Mr. McCarthy raises a number of other points which would apply to most econometric studies of production. The point about the possibility of X-inefficiency and the associated problems of identifying the production surface and hence drawing conclusions about its precise nature from my parameter estimates is well-taken. Unfortunately, as he says, there is no easy way of circumventing this problem. I certainly did not have the kind of extraneous information he mentions to resolve the matter. What I did have, however, were statements from the ESB on the basis of which I felt able to use the assumption of cost-minimisation in the operation of plants as a not too unrealistic approximation.

Mr. McCarthy raised an interesting question concerning the likely consequences of the simultaneous occurrence of errors in variables and multicollinearity. As far as this study is concerned, however, it seems from what Mr. Kelly has said about the accuracy of the ESB plant capacity figures that errors in variables may be much less of a problem than I originally thought it might have been, as I indicated earlier. Furthermore, I took care to check on the incidence of multicollinearity at all stages of estimation; indeed, it will be recalled that I abandoned my first attempt at estimation entirely due to what I considered an unacceptably high prevalence of the problem. Contrary to what Mr. McCarthy suggests, multicollinearity did not seem to be a problem in obtaining my final estimates. The few equations in which it was detected in large measure were recorded in Section 4 of the paper.

I agree with Mr. McCarthy's remark on the potential usefulness of some kind of guidelines on the likely relative significance of autocorrelation and heteroscedasticity in circumstances in which pooled cross-section and time-series data may be available, if such guidelines could be devised. It might be the case that heteroscedasticity tends to be slight where the cross-section units are of similar size, but the interesting thing about my finding

on the matter is that heteroscedasticity was slight, contrary to expectation, in a situation in which the cross-section units were of widely varying sizes. On the other hand the cross-section sample was quite small. I feel that the prospect of being able to ascertain the kind of guidelines Mr. McCarthy envisages is, in fact, remote.

Mr. O'Hagan's point concerning the use of load factor as a variable in the input equations might, perhaps, be met by debating the validity of his *a priori* view of the relationships. For example, it is not obvious to me, particularly in view of what Mr. Kelly has said about reliability of plant and the difficulty of achieving high load factors, that it would not require a larger amount of labour to deal with the increased problems of breakdown and servicing that may reasonably be assumed to be associated with the maintenance of a higher load factor. However, my use of load factor in the labour and capital equations was not based on *a priori* considerations so much as statistical expediency; and while it is, I believe, an important variable in the fuel and energy functions – and my results would seem to provide confirmation of this – load factor *could*, I agree, have been excluded from the labour and capital functions. But, given the stochastic specification of the model, such exclusion would have meant that efficient estimation of individual equations using ordinary least squares would not have been possible. Rather, as mentioned in the paper, efficient estimation would have required Zellner's method for seemingly unrelated regressions. Not having a readily available computer program for Zellner's method, I chose not to exclude load factor essentially in the interests of simple, yet efficient, estimation based on the use of ordinary least squares regression. Incidentally, it transpired that the estimated coefficients of load factor in the labour and capital equations were all small, as expected, and most were not significantly different from zero at the 5 per cent probability level.

It is well-known that inclusion of superfluous variables in an equation does not give rise to any bias in the least squares estimates of the parameters. However, it does entail some cost, of course, namely that the fit of the equation would almost certainly appear better than it would be without the additional variables.

In conclusion, I would do no more than reiterate my thanks to all those who contributed to the discussion of my paper this evening.