Accepted Manuscript

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PII: S0142-1123(11)00029-6
DOI: 10.1016/j.ijfatigue.2011.01.012
Reference: JIJF 2607

To appear in: *International Journal of Fatigue*

Received Date: 9 September 2010
Revised Date: 15 January 2011
Accepted Date: 26 January 2011

Please cite this article as: Susmel, L., Taylor, D., The Theory of Critical Distances to estimate lifetime of notched components subjected to variable amplitude uniaxial fatigue loading, *International Journal of Fatigue* (2011), doi: 10.1016/j.ijfatigue.2011.01.012

The Theory of Critical Distances to estimate lifetime of notched components subjected to variable amplitude uniaxial fatigue loading

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ABSTRACT

The present paper summarises an attempt of reformulating the so-called Theory of Critical Distances (TCD) to make it suitable for estimating finite life of notched components subjected to variable amplitude (VA) uniaxial fatigue loading. In more detail, similar to the design strategy we have suggested as being followed under constant amplitude (CA) loading, the VA linear-elastic formalisation of the TCD proposed here takes as a starting point the assumption that the critical distance value is a material property whose length decreases with increasing of the number of cycles to failure. Through a systematic validation exercise done using *ad hoc* generated experimental results, it was shown that the TCD, applied in the form of both the Point (PM), Line (LM), and Area Method (AM), is successful in estimating VA lifetime of notched metallic materials by simply calculating an equivalent critical distance having length which depends not only on the features of the assessed load spectrum, but also on the profile of the post-processed stress field. The accuracy and reliability of this alternative formalisation of the TCD was checked by using numerous experimental results generated by testing, under fully-reversed VA axial loading, notched cylindrical samples of a commercial medium-carbon steel containing three different stress raisers. Further, in order to more accurately verify the sensitivity of the proposed definition for the VA critical length to the features of the assessed load history, two spectra were investigated: the first one was characterised by a conventional Rayleigh distribution, whereas the second was used to study the effect of those cycles of low stress amplitude. Such a validation exercise allowed us to prove that the TCD is highly accurate also in estimating fatigue damage in notched components subjected to VA loading: this result is definitely encouraging, fully supporting the idea that the TCD can safely be used to design real notched components damaged by in-service VA load histories.

Keywords: Theory of Critical Distances, notch, variable amplitude loading

NOMENCLATURE

d_g	Gross diameter
d_n	Net diameter
k	Negative inverse slope of the Wöhler curve
m	Negative inverse slope of the Wöhler curve in the high-cycle fatigue regime
n_i	Number of cycles at the i-th stress level
n_{tot}	Sequence length
r, θ	Polar coordinates
r_n	Notch root radius
A, B	Medium-cycle fatigue regime material constants
D	Cumulative damage sum
D_{cr}	Critical value of cumulative damage sum
$D_{cr, exp}$	Experimental value of the critical cumulative damage sum
D_i	Damage associated with the i-th stress level
E	Young's modulus
K_{Ic}	Plane strain fracture toughness
K_t	Stress concentration factor referred to the net area
L	Critical distance for high-cycle fatigue problems
L_L	Critical distance determined in the low-cycle fatigue regime
L_M	Critical distance determined in the medium-cycle fatigue regime
L_S	Critical distance for static problems
L_{VA}	Critical distance value under VA loading
N_f	Number of cycles to failure
$N_{b,e}$	Estimated number of blocks to failure
$N_{f,e}$	Estimated number of cycles to failure
$N_{f,i}$	Number of cycles to failure at the i-th stress level
N_{kp}	Number of cycles to failure defining the high-cycle fatigue knee point
N_A	Reference number of cycles to failure in the high-cycle fatigue regime ($N_A=10^6$ cycles to failure for the statistical reanalyses)
N_L	Number of cycles to failure delimiting the low-cycle fatigue regime
N_S	Reference number of cycles to failure at $\sigma_a=\sigma_{UTS}$ (R=-1)
O, x, y	Frame of reference
P_S	Probability of survival
R	Load ratio ($R=\sigma_{min}/\sigma_{max}$)
T_σ	Scatter ratio of the amplitude of the reference stress for 90% and 10% probabilities of survival.
α	Notch opening angle
σ_0	Inherent material strength
$\sigma_{1,a}$	Amplitude of the maximum principal stress
$\sigma_{eff,a}$	Amplitude of the effective stress calculated according to the TCD

$\sigma_{eff,a-i}$	Amplitude of the effective stress calculated, at the i-th stress level, according to the TCD
$\sigma_{n,a}$	Amplitude of the nominal net stress
$\sigma_{n,a-i}$	Amplitude of the nominal net stress at the i-th stress level
$\sigma_{n,a-max}$	Maximum amplitude of the nominal net stress in the spectrum
σ_A	Amplitude of the reference stress at N_A cycles to failure (plain material)
σ_{An}	Amplitude of the net nominal stress at N_A cycles to failure
σ_{An-max}	Maximum amplitude of the net nominal stress at N_A cycles to failure
σ_y	Yield stress
σ_{UTS}	Ultimate tensile stress
ΔK_{th}	Threshold value of the range of the stress intensity factor

1. Introduction

Even though examination of the state of the art shows that many different hypotheses can be formed to estimate cumulative fatigue damage under variable amplitude (VA) fatigue loading (see, for instance, Ref. [1] and references reported therein), certainly the most frequently adopted parameter in situations of practical interest is the classical linear one due to Palmgren [2] and Miner [3], i.e.:

$$D = \sum \frac{n_i}{N_{f,i}} = D_{cr} \quad (1)$$

where n_i is the applied number of cycles at the i-th stress amplitude level, whereas $N_{f,i}$ is the number of cycles which would result in fatigue breakage if the assessed material were subjected, at the i-th stress amplitude level, solely to CA loading (see Fig. 1).

The classical theory as formalised by Palmgren and Miner postulates that, when materials are damaged by VA load histories, failure occurs when the damage sum equals unity, i.e., D_{cr}=1 in Eq. (1). Even though such an hypothesis is too often used by engineers engaged in designing real components subjected to VA loading, it has to be said that the critical value of the damage sum, D_{cr}, is seen to vary in the range 0.02-5, where the average value of D_{cr} is equal to 0.27 for steel and to 0.37 for aluminium [4]. This implies that, from a statistical point of view, systematically taking D_{cr} equal to unity may result in an unsafe fatigue assessment. Since there exists no sound theory allowing the critical value of the damage sum to be estimated *a priori*, it is evident that the only way to correctly evaluate it is by running appropriate experiments. However, even if the problem is addressed by performing accurate experimental investigations to characterise the material being assessed, it has to be pointed out also that, when moving to real mechanical assemblies, D_{cr} is seen to vary not only as the geometry of the assessed component varies, but also as the profile of the considered load spectrum changes [5]. Further, also the degree of multiaxiality of the applied VA loading path strongly affects the critical value of the cumulative damage sum [4, 6].

Another important aspect which has to be taken into account when performing the fatigue assessment under VA fatigue loading is the fact that the negative inverse slope of fatigue design curves is seen to change moving from the medium- to the high-cycle fatigue field (Figure 1), the inverse slope being much larger in the long-life regime ($N_f>N_{kp}$). The most common hypothesis which is usually adopted to account for the above aspect is the so-called “2k-1 correction” due to Haibach [7], that is the negative inverse slope for $N_f>N_{kp}$ is determined as follows (see also Figure 1):

$$m = 2 \cdot k - 1. \tag{2}$$

A tricky aspect behind the use of the above correction is the correct definition of the position of the knee point (N_{kp} in Figure 1). In particular, the number of cycles to failure at which fatigue curves change their slope is seen to vary in the range 10^5-10^7 cycles to failure [8], such a value depending on both fatigue behaviour of the assessed material, type of applied loading, and load ratio. This implies that, if the value of N_{kp} cannot be determined experimentally, the only reliable way to estimate it is by following the available design recommendations (see, for instance, Ref. [8] and references reported therein). Another alternative strategy may be keeping the negative inverse slope constant (“constant k assumption” in Figure 1): it is evident that if, on one hand, such a *modus operandi* allows a higher margin of safety to be reached, on the other hand, its use in the presence of load spectra characterised by a large number of cycles of low stress amplitude may result in components which are heavier than necessary.

To conclude the present section, it can be said that, in this complex scenario, the ultimate goal of the investigation summarised in the present paper is to reformulate the TCD to make it suitable for estimating fatigue damage in notched components subjected to VA uniaxial fatigue loading by directly post-processing the linear elastic stress fields acting on the material in the vicinity of crack initiation locations (i.e., without the need for defining any nominal quantities).

2. Experimental details

As said at the end of the previous section, the aim of the present investigation is to formalise an alternative TCD based fatigue assessment technique capable of estimating cumulative fatigue damage in notched metallic materials. In order to check the accuracy and reliability of such a novel approach, whose theoretical aspects will be discussed in great detail below, a systematic experimental investigation was carried out by testing, under CA and VA fully-reversed axial loading, cylindrical samples of medium-carbon steel C40 containing three different geometrical features. In more detail, the sharply V-notched samples had gross diameter, d_g, equal to 12mm, net diameter, d_n, to 9.15mm, root radius, r_n, to 0.225mm and, finally, notch opening angle, α, to 35°, resulting in a net stress concentration factor, K_t, of 4.42. The intermediate V-notched specimens had instead d_g=12mm, d_n=9.32mm, r_n=1.2mm, and α=90° (K_t=2.20); finally, the bluntly U-notched cylindrical bars had d_g=12mm, d_n=9.2mm, and r_n=3.0mm (K_t=1.66).

The investigated material was commercial carbon steel C40 having an ultimate tensile stress, σ_{UTS}, equal to 852 MPa, a yield stress, σ_y, of 672 MPa and a Young's modulus of 209000 MPa, the above mechanical properties being determined from four static tests run according to the ASTM standard procedure.

Both CA and VA force controlled testes were performed under a nominal load ratio, $R=\sigma_{min}/\sigma_{max}$, equal to -1 at a frequency of 4 Hz.

All the generated results are summarised in the Wöhler and Gassner diagrams reported in Figure 2. In more detail, the chart of Figure 2a shows the results obtained, under CA loading, by testing both the un-notched specimens (d_g=12mm, d_n=6 mm, K_t = 1.06) and those V-notched samples having root radius equal to 0.225mm (K_t=4.42). According to the fatigue behaviour shown by the parent material (see the two run out points in Figure 2a), the reference number of cycles to failure, N_A, was taken equal to 10^6. The above fatigue curves, estimated for a probability of survival, P_S, equal to 50%, are also summarised in Table 1 in terms of negative inverse slope, k, reference stress amplitude extrapolated to $N_A=10^6$ cycles to failure (σ_A for the plain and σ_{An} for the notched samples, where, in the latter case, the stress is referred to the net cross-sectional area) and scatter ratio of stress amplitude, T_σ, at N_A cycles to failure for 90% and 10% probabilities of survival. As to the calculated values for T_σ, it is worth observing here that the statistical dispersion of the fatigue results generated by testing plain samples is comparable to the one obtained by testing the V-notched cylindrical specimens having notch root radius equal to 0.225 mm, the latter fatigue curve being characterised by a slightly larger value of T_σ.

As to the performed statistical reanalyses, it can be noticed that all the scatter bands shown in Figure 2 are delimited by two straight lines corresponding to a probability of survival, P_S, equal to 90% and 10%, respectively, and they were determined under the hypothesis of a log-normal distribution of the number of cycles to failure for each stress level and assuming a confidence level equal to 95% [9]. Regarding the above fatigue curves, it is worth highlighting here that the VA formalisation of the TCD discussed in the present paper is based on those schematisations usually adopted in Germany [4, 8, 9] to calculate and describe Wöhler curves. However, it can be pointed out that, in theory, there exist no drawbacks in adopting other mathematical laws, together with different statistical procedures, to determine the necessary SN curves, provided that, the approach as described in what follows is applied consistently.

The fatigue tests under VA loading were generated by considering the two load spectra sketched in Figure 3a and a having sequence length, $n_{tot} = \sum n_i$, of 1000 cycles. In particular, the Concave Upwards Spectrum (CUS) was built by using a conventional Rayleigh distribution, whereas the

Concave Downwards Spectrum (CDS) was directly derived from the ones used by Gurney to investigate the damaging effect in steel weldments of those cycles of low stress amplitude [5]. As to the performed VA tests, it is worth noticing that, as shown in Figures 3b and 3c, load cycles were applied in random order. During testing the instantaneous values of the loading were gathered systematically and, for any generated result, the recorded force vs. time signal was post-processed in order to verify *a posteriori* the correspondence between theoretical and generated load spectrum, the cycles being counted by using the classical Rain-Flow method [10]. Lastly, for any geometrical/load spectrum configuration considered, two samples at any of the four investigated stress levels were tested (Figs 2b to 2d).

The obtained VA fatigue results are summarised in Figures 2b to 2d, in the form of Gassner curves, $\sigma_{n,a\text{-}max}$ being the nominal net stress amplitude of the most damaging cycle in the spectrum. For the sake of completeness, the results of the statistical reanalyses are also listed in Table 2 in terms of maximum net stress amplitude in the spectrum, $\sigma_{An\text{-}max}$, extrapolated, for a probability of survival, P_S, equal to 50%, to N_A=10^6 cycles to failure, negative inverse slope k and scatter ratio T_σ.

To conclude, Table 3 lists the results generated by testing plain samples of C40 under the two adopted load spectra (Fig. 3). The same Table also reports the corresponding experimental values of the critical damage sum, $D_{cr,exp}$, calculated by using the 2k-1 correction, the knee point, N_{kp}, being taken at $2{\cdot}10^6$ cycles to failure as suggested in Ref. [8]. It is worth observing that, according to Table 3, $D_{cr,exp}$ under the CUS spectrum was slightly lower than the critical damage sum under the CDS spectrum, the resulting average value being equal to 0.63.

3. The TCD to estimate finite lifetime of notched components under CA fatigue loading

As far as the authors are aware, the well-known fatigue design method formalised by Neuber [11] represents the first attempt of using the TCD to perform the fatigue assessment of notched components. In particular, this approach takes as its starting point the idea that an engineering stress quantity suitable for estimating fatigue damage in notched components can directly be calculated by averaging, in the vicinity of the notch tip, the linear-elastic maximum principal stress over a linear domain whose length depends on the microstructural features of the material being assessed: such a method is known as the Line Method (LM). In a similar way, Peterson [12] has argued that the high-cycle fatigue strength could be estimated also in terms of the so-called Point Method (PM), that is, by determining the necessary reference stress at a given distance from the tip of the stress raiser being assessed.

In recent years, the TCD has gained new popularity thanks to the work done by different researchers who have reformulated it in a variety of ways (see Ref. [13, 14] and references reported therein). In particular, in the notch fatigue discipline, the above improved formalisations of the TCD were devised not only to perform the high-cycle fatigue assessment [15-17], but also to estimate finite life [17-20]. As to the specific problem of predicting fatigue lifetime, the linear-elastic TCD takes as its starting point the assumption that the critical distance value to be used to calculate an effective equivalent stress is a material property whose value increases with decreasing of the number of cycles to failure, i.e. [18, 20]:

$$L_M(N_f) = A \cdot N_f^B \tag{3}$$

In the above power law, A and B are material fatigue constants to be determined by running appropriate experiments. In more detail, A and B are seen to be different for different materials and different load ratios, but their values do not depend on the features of the notch being assessed. The theoretical aspects on which definition (3) is based as well as the procedure to be followed to estimate constants A and B will be discussed below in great detail.

If the critical distance vs. number of cycles to failure relationship, Eq. (3), is assumed to be known for the material being assessed, the TCD can then be formalised in different ways by simply changing the definition of the integration domain used to calculate the amplitude of the effective stress, $\sigma_{eff,a}$. In particular, if such a stress quantity is estimated according to the PM, then $\sigma_{eff,a}$ can be calculated as (see Figures 4a and 4b):

$$\sigma_{eff,a} = \sigma_{1,a}\left(\theta = 0, r = \frac{L_M(N_f)}{2}\right). \tag{4}$$

As suggested by Neuber, an alternative way to determine $\sigma_{eff,a}$ is also by averaging the amplitude of the linear-elastic maximum principal stress, $\sigma_{1,a}$, along a line over a distance equal to $2L_M(N_f)$ [13, 15, 16], i.e. (see Figures 4a and 4c):

$$\sigma_{eff,a} = \frac{1}{2L_M(N_f)} \int_0^{2L_M(N_f)} \sigma_{1,a}(\theta = 0, r) dr . \tag{5}$$

As to the fact that the critical distance to be used with the LM is for times larger than the critical distance value recommended to be adopted to apply the PM, it is worth observing here that such a ratio between the above two quantities strictly applies only to the case of a sharp crack [16]. On the contrary, for any other stress distribution the PM will give a different prediction from the LM. However, it was seen through a systematic validation exercise that the use of both the PM and LM results in accurate estimates when such methods are employed to predict fatigue strength of engineering materials containing any kind of geometrical features (i.e., both short, sharp and blunt notches) and subjected to either axial or bending loading [13, 14].

Lastly, the amplitude of the effective stress can also be calculated by averaging $\sigma_{1,a}$ over a semicircular area centred at the notch tip and having radius equal to $L_M(N_f)$ [16, 21]. Such a form of the TCD is known as the Area Method (AM) and it can be formalised as follows (see Figures 4a and 4d):

$$\sigma_{eff,a} = \frac{4}{\pi L_M(N_f)^2} \int_0^{\pi/2} \int_0^{L_M(N_f)} \sigma_{1,a}(\theta, r) \cdot r \cdot dr \cdot d\theta \tag{6}$$

Finally, independently of the strategy followed to calculate the amplitude of the effective stress, the number of cycles to failure can directly be estimated through the Wöhler curve describing the fatigue behaviour of the parent material the component being assessed is made of, that is [18]:

$$N_{f,e} = N_A \cdot \left(\frac{\sigma_A}{\sigma_{eff,a}}\right)^k \tag{7}$$

With regard to the use of Eqs (4) to (6) to estimate fatigue lifetime of notched components, it is evident that they have to be applied through appropriate recursive procedures, since the number of cycles to failure needed to calculate the critical distance value according to Eq. (3) is, obviously, never known *a priori* [18]. In particular, consider the notched sample subjected to uniaxial fatigue loading that is sketched in Figure 4a, $\sigma_{n,a}$ being the amplitude of the applied nominal stress. Through appropriate tools, either analytical or numerical, the maximum principal stress amplitude vs. distance curve can be determined explicitly (Fig. 4e). Subsequently, by assuming a first tentative value of the number of cycles to failure, Eq. (3) can directly be used to estimate the corresponding critical distance value. If, for instance, the TCD is applied in the form of the PM, the amplitude of the maximum principal stress has to be extrapolated to a distance from the notch apex equal to $L_M(N_f)/2$ - see Eq. (4) and Figure 4e. Subsequently, through the Wöhler equation obtained by testing plain samples of the same material, the value of $\sigma_{1,a}$ extrapolated from the stress-distance curve allows the number of cycles to failure to be re-calculated. It should be evident now that such a procedure has to be reiterated until the number of cycles to failure used to calculate the critical distance has become equal to the number of cycles to failure estimated from the plain material fatigue curve. Finally, it is worth noticing here that similar strategies can be followed to predict finite life of notched components using the LM or the AM instead of the PM, the difference being that calculating the amplitude of the effective equivalent stress becomes much more laborious.

To conclude, it is worth observing that, by performing a systematic validation exercise done considering not only different materials but also different geometrical features [18], the formalisation of the TCD reviewed in the present section was found to be highly accurate when applied to notches under CA loading, resulting in estimates always falling within the parent material scatter band. This outcome was definitively encouraging, since, from a statistical point of view, a predictive method can not be more accurate than the experimental information used to calibrate the method itself.

4. The TCD to estimate finite life under VA fatigue loading

The way the critical distance has been used to address the CA notch fatigue problem, Eq. (3), should make it evident that, in the presence of VA load histories, its value will change as the amplitudes of the different cycles forming the applied load spectrum vary. This clearly implies that, to correctly extend the use of the TCD to those situations involving VA loadings, the critical distance itself has to be redefined coherently: the adopted definition must result in a unique value which fully depends on the specific features of the assessed load spectrum.

Consider then the notched sample sketched in Figure 4a and assume that it is subjected to the VA load history described, in terms of amplitudes of the nominal stress, by the load spectrum reported in Figure 5. Such a spectrum is formed by j different stress levels that are characterised by a nominal stress amplitude equal to $\sigma_{n,a\text{-}i}$ as well as by a number of cycles equal to n_i (i=1, 2, …, j), where $n_{tot} = \sum_{i=1}^{j} n_i$. Any single stress level forming the above load spectrum can now be treated as an independent CA sub-case, so that, according to the procedure briefly reviewed in the previous section, the corresponding number of cycles to failure, $N_{f,i}$, as well as the corresponding critical distance value, $L_M(N_{f,i})$, can directly be determined according to either the PM, LM, or AM. By so doing, as postulated by Palmgren and Miner, Eq. (1), the fatigue damage content associated with any stress level takes on the following value:

$$D_i = \frac{n_i}{N_i} \qquad \text{for i=1, 2, …, j}$$

It is proposed that an equivalent critical distance length suitable for estimating fatigue lifetime of notched components subjected to VA loading can then be calculated as follows:

$$L_{VA} = \frac{\sum_{i=1}^{j} L_M(N_{f,i}) \frac{n_i}{N_{f,i}}}{\sum_{i=1}^{j} \frac{n_i}{N_{f,i}}} = \frac{\sum_{i=1}^{j} L_M(N_{f,i}) \cdot D_i}{\sum_{i=1}^{j} D_i} \tag{8}$$

In more detail, the above reference length is simply defined as a weighted mean, where weight $D_i = \frac{n_i}{N_i}$ is the damage content associated with the i-th stress level and evaluated by taking the critical distance value equal to $L_M(N_{f,i})$. The most important implication of the definition for L_{VA} proposed above is that, given the geometrical feature, its value is clearly expected to vary as the assessed load spectrum changes in terms of both profile and amplitudes of the involved stress cycles.

As to identity (8), it is important to highlight here that the same idea could also be applied by adopting a different definition to calculate the fatigue damage content associated with any counted cycle: for instance, it is straight forward to observe that the use of a non-linear cumulative damage sum [1] would lead to a critical distance value which still depends not only on the assessed geometrical feature, but also on the characteristics of the investigated load spectrum.

After defining the equivalent critical distance value as above, the problem has now to be readdressed in terms of local quantities. In particular, the amplitude of the effective stress can be recalculated directly in terms of either the PM, LM, or AM (Fig. 6), with the advantage over the situation described previously that now the critical distance value is unique and known *a priori*, that is, it is equal to L_{VA}, Eq. (8). Note that, for the sake of simplicity, in Figure 6a the effective stress is clearly determined according to the PM, however the same reasoning applies also when either the LM or AM are used (see Figures 4c and 4d).

After recalculating the load spectrum in terms of amplitudes of the effective stress (Fig. 6b), for any resulting stress level the corresponding number of cycles to failure, $N_{f,i}$, can directly be estimated through the Wöhler equation describing the fatigue behaviour of the parent material, i.e. (Fig. 6c):

$$N_{f,i} = N_A \cdot \left(\frac{\sigma_A}{\sigma_{eff,a-i}} \right)^k \qquad \text{for i=1, 2, …, j} \tag{9}$$

the resulting total damage being equal to (Fig. 6d):

$$D_{tot} = \sum_{i=1}^{j} \frac{n_i}{N_{f,i}} \tag{10}$$

Finally, the number of cycles to failure, $N_{f,e}$, can be estimated from the predicted number of blocks to failure, $N_{b,e}$, that is (Fig. 6e):

$$N_{b,e} = \frac{D_{cr}}{D_{tot}} \Rightarrow N_{f,e} = n_{tot} \cdot N_{b,e} \tag{11}$$

where D_{cr} is the critical value of the damage sum.

To conclude, it has to be highlighted that, as recommended in Ref. [4], fatigue lifetime under VA loading is suggested as being estimated by always changing the negative inverse slope of the plain material fatigue curve according to the 2k-1 correction (Fig. 1).

5. The critical distance vs. number of cycles to failure relationship

The reasoning summarised in the previous two sections was fully based on the assumption that the critical distance vs. number of cycles to failure relationship can directly be formalised through a simple power law, Eq. (3) [18]. Further, in our theory the critical distance is always treated as a material property which is different for different materials and load ratios, but the values of constants A and B in Eq. (3) do not vary as the features of the assessed notch change.

According to the law suggested as being used to define the L_M vs. N_f relationship, two different experimental results should be enough to correctly evaluate constants A and B. For instance, they could directly be determined through the critical distance values determined under static loading [13, 22-24] and in the high-cycle fatigue regime [13, 16], respectively. In more detail, the critical distance value suitable for addressing the static problem can directly be calculated as [13, 22-24]:

$$L_S = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_S}\right)^2 \tag{12}$$

where K_{Ic} is the plain strain fracture toughness, whereas σ_S is the so-called inherent material strength, that is, a material property which is, in general, larger than the material ultimate tensile strength [13].

In a similar way, the critical distance value under fatigue loading takes on the following well-known form [13, 15, 16]:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_0}\right)^2 \tag{13}$$

In the above identity ΔK_{th} is the range of the threshold value of the stress intensity factor and $\Delta\sigma_0$ is the plain fatigue limit range, both material properties determined under the same load ratio as the one characterising the CA load history damaging the component being assessed.

Even though no theoretical inconsistency emerges when attempting to calibrate Eq. (3) through critical distances (12) and (13) [18], such *a modus operandi* is very difficult to be used in situations of practical interest, on one hand, because, by nature, the stress based approach is not adequate for describing metallic materials' behaviour in the low-cycle fatigue regime, and, on the other hand, because, given the material, the position of the knee point in the high-cycle fatigue regime can change as the geometry of the tested samples varies [8, 14].

In order to overcome the above problems, we have argued that constants A and B can simply be determined from the plain fatigue curve and from another fatigue curve generated by testing samples containing a known geometrical feature [18], where, according to our in-field experience, it is always advisable to consider stress raisers as sharp as possible. In more detail, by taking full advantage of the PM, for a given value of the number of cycles to failure, N_f, it is straightforward to determine the distance from the notch tip, $L_M(N_f)/2$, at which the amplitude of the linear-elastic maximum principal stress is equal to the stress amplitude, $\sigma_{1,a}$, that has to be applied to the unnotched material to break it at the same number of cycles to failure (see Figure 7). By so doing, the critical distance value can be determined for all the numbers of cycles from the low- to the high-cycle fatigue regime, allowing the values of constants A and B to be determined unambiguously [14, 18].

The chart of Figure 8a shows the results obtained when the above procedure is applied to determine the critical distance vs. number of cycles to failure relationship for low-carbon steel En3B, that is, for that metallic material we used in Ref. [18] to check the accuracy of the TCD in estimating fatigue lifetime of notched components failing in the medium-cycle CA fatigue regime. In more detail, the notch curve adopted to build the above diagram was generated by testing single V-notched flat samples with root radius equal to 0.12 mm (resulting in a gross stress concentration factor of 16.2). The above chart makes it evident that a unique L_M vs. N_f relationship allowed fatigue lifetime to efficiently be estimated for N_f larger than about $2.5 \cdot 10^3$ cycles to failure. On the contrary, for $N_f < 2.5 \cdot 10^3$ cycles to failure, the calculated values for $L_M(N_f)$ grew very rapidly, immediately becoming larger than the critical length determined under static loading, Eq. (12) [22]. Note that the critical distance values reported in the low-cycle fatigue regime of Figure 8a's chart were calculated by mathematically extending the plain as well as the notch fatigue curve toward to the static failure condition. As to the accuracy of the TCD in predicting the results summarised in Ref. [18], it is worth observing here that, since the investigated fatigue lives ranged in the interval 10^4-$2 \cdot 10^6$ cycles to failure, satisfactory results were obtained by using a unique critical distance vs. number of cycles of failure relationship, i.e.: $L_M = 67.4 \cdot N_f^{-0.342}$ [mm].

The same procedure was applied also to determine the L_M vs. N_f relationship for carbon steel C40 used in the present investigation. As calibration information, the two curves generated under CA fully-reversed axial loading and sketched in Figure 2a were used (see also Table 1). The relevant stress fields, as done for the En3B samples [18], were determined by doing refined FE models. The chart of Figure 8b summarises the obtained results which are, in a way, different from those discussed in the previous case. In particular, the most relevant aspect was that the value of the static characteristic length experimentally determined according to the procedure discussed in Refs [22-24] was seen to be very large compared to the critical distance values calculated by extending the two calibration curves down to the very low-cycle fatigue regime. According to the observed behaviour, two different relationships are then suggested here as being used to model the way the critical distance varies as the number of cycles to failure increases, i.e.:

$$L_L = 19.85 \cdot N_f^{-0.535} \text{ [mm]} \qquad \text{for } N_f<N_L=119 \text{ cycles to failure} \tag{14}$$

$$L_M = 6.05 \cdot N_f^{-0.286} \text{ [mm]} \qquad \text{for } N_f>N_L=119 \text{ cycles to failure} \tag{15}$$

where the constants in Eq. (14) and Eq. (15) were determined by a best fit procedure done by interpolating the calculated solutions in the interval 10-100 cycles to failure, and 10^3-10^6 cycles to failure, respectively.

To conclude, it can be highlighted here that such a strategy based on two different critical distance vs. N_f relationships was followed, on one hand, to correctly estimate the damaging effect of the highest stress level cycles in CDS spectrum (Fig. 3), and, on the other hand, to check whether the damaging effect of those cycles of low stress amplitude could accurately be assessed by directly extending the L_M vs. N_f relationship up to the very long-life regime.

7. Validation by experimental data

The experimental results generated by testing the notched samples of C40 under both the CUS and CUD spectra (see Figs 2 and 3) were used to check the accuracy and reliability of the VA formalisation of the TCD proposed in the present paper.

As to the stress analysis, the relevant linear-elastic stress fields were determined by using commercial software ANSYS©. The cylindrical samples were modelled by using axisummetric bi-dimensional elements and, for any notched geometry, the mapped mesh in the vicinity of the stress raiser's apex was gradually refined until convergence occurred.

The critical distance vs. number of cycles to failure relationship adopted to estimate fatigue lifetime had the profile as shown in the chart of Figure 8b, and described by Eqs (14) and (15). As regards the fatigue behaviour of the plain material, the knee point was taken at $2 \cdot 10^6$ cycles to failure [8] and the negative inverse slope in the high-cycle fatigue regime was corrected according to Haibach's suggestion, Eq. (2).

As to the critical value of the damage sum, D_{cr}, initially the hypothesis was formed that it could be taken equal to its average value experimentally determined from the plain samples, i.e., $D_{cr,exp}$=0.63 (see Table 3). The obtained results are summarised in the experimental, N_f, vs. estimated, $N_{f,e}$, diagrams reported in Figures 9a to 9c (where the reported data are classified according to the value of the notch root radius). Such charts make it evident that the use of both the PM (Fig. 9a) and the AM (Fig. 9c) resulted in estimates characterised by a slight degree of conservatism, but mainly within the target error factor. On the contrary the predictions made using the LM (Fig. 9b) were seen to fall always within the parent material scatter band, but on the non-conservative side. Another important outcome from the above error charts is that, given the specific formalisation of the TCD, the overall accuracy was seen not to depend on the geometrical features of the considered stress raiser, and this holds true even though the L_M vs. N_f relationship was calibrated using the CA results generated by testing the sharpest V-notches. This seems to fully support the idea that the critical distance value can be treated as a material property.

For the sake of completeness, the same analysis as above was performed also by taking, as suggested by the classical theory due to Palmgren and Miner, a critical value for the damage sum equal to unity. The diagrams reported in Figures 9d to 9f show that, as expected, the level of conservatism of the three different formalisations of the TCD decreased, resulting in highly accurate estimates for both the PM (Fig. 9d) and the AM (Fig. 9f). On the contrary, the predictions made using the LM were slightly non-conservative.

According to the charts of Figure 9, from the direct comparison between the accuracy of the estimates and the scatter in the VA results obtained by testing plain samples, nothing definitive can be said about the accuracy and reliability of the idea of using, to estimate cumulative fatigue damage in notched samples, the value for D_{cr} determined by testing the parent material.

Finally, the charts of Figure 10 show the sensitivity of the definition proposed here to be used to calculate the critical distance value under VA loading, L_{VA}, to the profile of the assessed spectrum as well as to the sharpness of the investigated geometrical features. According to the above diagrams, it is possible to conclude saying that the highly accurate estimates summarised in the error charts of Figure 9 were obtained through a novel definition for the critical distance value which is seen to be fully sensitive not only to the actual profile of the assessed linear-elastic stress fields, which in turn depend on the characteristics of the investigated geometrical features, but also to the specific damage content of the considered VA load history.

To conclude, it can be pointed out that the experimental data and predictions considered here extended to a maximum life of approximately one million cycles. More work needs to be done in order to correctly extend the use of the TCD to the giga-cycle fatigue regime [25], by investigating, in particular, the most efficient way to define in that region the critical distance vs. number of cycles to failure relationship.

8. Discussion

According to the reasoning summarised in the previous sections, the novel reformulation of the TCD proposed here to address the VA notch fatigue problem is based on simple and well-known concepts [4, 5, 8] which are already used daily by structural engineers engaged in performing the fatigue assessment of real mechanical components. In other words, the VA TCD discussed in the present paper was devised by taking full advantage of those classical stress based concepts formalised, and experimentally validated, by the German school of thought [4].

However, for the sake of completeness, it has to be said that, according to the state of the art, several other methods, based on different concepts, have been proposed, and somehow validated, to specifically address the notch VA fatigue problem. In particular, a massive work has been done over the last three decades to formalise methodologies based on the use of local elasto-plastic stresses and strains (see, for instance, Refs [26, 27] and references reported therein). As to the in-field use of the strain based approach to design notched components against VA fatigue, one of the most tricky issues which must always be addressed correctly is the appropriate estimation of root stresses and strains. In more detail, if the above quantities cannot directly be measured (by using, for instance, strain gages), they are usually estimated either through numerical methods, or by taking full advantage of well-known rules like, for instance, those proposed by Neuber [28] and Glinka [29, 30], respectively. Even if the results obtained by applying the strain based approach to address the VA notch fatigue problem are definitively encouraging, the main limitation in using them to address problems of practical interest is the difficulty, on one hand, of correctly modelling the material transient behaviour in the presence of stress/strain gradients [26] and, on the other hand, of accurately estimating root stresses and strains through the classical rules commonly adopted in practise, like, for instance, the rules due to Neuber and Glinka [31, 32].

Other than the strain based approach, in recent years other alternative strategies based on the Linear Elastic Fracture Mechanics concepts [26, 27] have been explored in order to formalise reliable design approaches suitable for estimating fatigue damage in notched components subjected to VA fatigue loading. In this scenario, amongst the different formalisations available in the technical literature, the work done by Topper and co-workers certainly has to be mentioned explicitly [33-35].

Another important issue which deserves to be discussed here in great detail is the use of the linear rule due to Palmgren [2] and Miner [3] to estimate fatigue damage under VA fatigue loading. According to the experimental evidence, when such a rule is employed to design real components against VA fatigue, its critical value is seen to vary in the range 0.02-5 [4]: such a high variability suggests that the only way to correctly evaluate D_{cr} is by running appropriate experiments. A possible alternative solution to address such a tricky aspect of the VA problem may be adopting non-linear cumulative damage theories (see, for instance, Ref. [1] and references reported therein). Even though such alternative theories may allow the accuracy in estimating fatigue lifetime under VA loading to be increased, at the same time a correct estimation of the critical values of non-linear damage sums should in any case be based on *ad hoc* experimental investigations, resulting in a further increase of the time and costs associated with the design process: accordingly, as suggested by Sonsino [4], we decided to make use of Palmgren and Miner's linear damage sum to formalise our VA TCD, not only in light of its well-known simplicity, but also because it is already widely used in the industrial reality to address problems of practical interest. However, it is evident that the TCD as proposed here to address the VA notch fatigue problem could easily and directly be applied by also adopting different definitions to evaluate the cumulative damage sum, provided that, such alternative definitions are employed coherently.

Turning back to the way the critical distance is defined in our medium-cycle fatigue formalisation of the TCD [18], it is worth recalling here that L_M is assumed to vary as the number of cycles to failure increases simply because, given the material, the inverse negative slope of the fatigue curve generated by testing un-notched samples is seen to be larger than the inverse negative slope of any fatigue curve generated by testing notched specimens. Further, the TCD is suggested as being applied to estimate finite lifetime by post-processing the relevant stress fields which have to be determined by assuming that engineering materials obey, independently from their level of ductility, a linear-elastic constitutive law. This should make it evident that such a formalisation of the TCD represents nothing but an engineering solution specifically devised to greatly simplify the problem of estimating fatigue strength of notched ductile engineering materials. At the same time, it can be noticed that, through the L_M vs. N_f relationship as proposed in Ref. [18], the effect of cyclic plasticity is in any case taken into account indirectly: according to Eq. (3) the size of the fatigue process zone, which is assumed to somehow depend on the size of the plastic region, increases as the number of cycles to failure decreases, that is, as the magnitude of the applied cyclic force increases.

As to the critical distance value in the medium-cycle fatigue regime, it is worth remembering here also that, as expected, if the elasto-plastic behaviour of the material being assessed is taken into account explicitly, the critical distance is seen to remain constant, that is, its value is no longer dependent on the number of cycles to failure [19].

Another advantage of the elasto-plastic TCD is that, through appropriate fatigue damage models like, for instance, the well-known SWT parameter [36], the mean stress effect under VA fatigue loading can be taken into account directly [19]. On the contrary, the use of the linear-elastic solution proposed in the present paper is somehow limited by the fact that, in the presence of VA fatigue loading characterised by different values of the mean stress, the constants in the L_M vs. N_f relationship change as the applied load ratio varies [18]: it is evident that under such circumstances the above constants must be known from the experiments in order to apply the TCD by coherently taking into account the presence of superimposed static stresses.

To conclude, it is possible to point out that, as proven by the error charts reported in Figure 9, the novel reformulation of the TCD discussed in the present paper is seen to be capable of accurately estimating fatigue damage under VA loading, and this holds true independently of the specific features not only of the considered notch, but also of the assessed load history. This result is definitively encouraging and strongly supports the idea that the TCD is a powerful engineering tool which can safely be used in situations of practical interest also to design notched components experiencing in-service variable amplitude fatigue loadings.

9. Conclusions

1) The novel formalisation of the TCD proposed here was seen to be highly accurate in estimating fatigue lifetime of notched components damaged by VA uniaxial fatigue loading;

2) The alternative definition for the critical distance suggested here as being used to address the VA fatigue problem has proven to be capable of correctly taking into account not only the profile of the linear-elastic stress field arising from the investigated notch, but also the specific features of the assessed load spectrum;

3) More work needs to be done in this area in order to reformulate the TCD to make it suitable for addressing the VA notch fatigue problem by taking into account the actual elasto-plastic behaviour of the material being assessed.

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Table 1: Summary of the experimental results generated under fully-reversed CA loading.

Table 2: Summary of the experimental results generated by testing the notched samples under fully-reversed VA loading.

Table 3: Summary of the experimental results generated by testing the plain samples under fully-reversed VA loading and experimental values of the critical damage sum, D_{cr}.

Figure 1: Wöhler curve corrected to estimate lifetime under VA fatigue loading.

Figure 2: Fatigue results generated under CA (a) and VA (b, c, d) fully-reversed axial fatigue loading.

Figure 3: Adopted Load Spectra.

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Figure 5: Proposed procedure for the determination of the critical distance value, L_{VA}, under VA fatigue loading.

Figure 6: Estimating fatigue lifetime under VA loading by post-processing the local linear-elastic stress field in the vicinity of the notch tip.

Figure 7: In-field procedure to determine the critical distance vs. number of cycles to failure relationship.

Figure 8: Critical distance vs. number of cycles to failure relationships determined, according to the PM, for low-carbon steel En3B [18] (a) and for medium-carbon steel C40 considered in the present investigation (b).

Figure 9: TCD's accuracy in estimating the fatigue lifetime of the notched samples of C40 tested under both the CUS and CDS load spectra.

Figure 10: Sensitivity of critical distance L_{VA} to the features of the applied load spectrum as well as to the sharpness of the tested notch.

Tables

Specimen Type	N. of Data	r_n [mm]	d_g [mm]	d_n [mm]	α [°]	R	k	σ_A, σ_{An}* [MPa]	T_σ	K_t
Plain	10	-	-	6.00	-	-1	9.4	292.8	1.211	1.00
Sharp	10	0.225	12	9.15	35	-1	4.2	97.8	1.361	4.42

*Reference stress amplitude extrapolated at $N_A=10^6$ cycles to failure and referred to the net cross-sectional area.

Table 1: Summary of the experimental results generated under fully-reversed CA loading.

Spectrum	N. of Data	r_n [mm]	d_g [mm]	d_n [mm]	α [°]	R	k	$\sigma_{An\text{-}max}$* [MPa]	T_σ	K_t
CUS	8	0.225	12	9.15	35	-1	3.3	163.1	1.139	4.42
CDS	8						4.5	358.9	1.228	
CUS	8	1.2	12	9.32	90	-1	5.05	265.5	1.243	2.20
CDS	8						5.4	428.0	1.192	
CUS	8	3.0	12	9.20	-	-1	4.9	278.2	1.121	1.66
CDS	8						6.05	482.1	1.084	

*Reference stress amplitude extrapolated at 10^6 cycles to failure and referred to the net cross sectional area.

Table 2: Summary of the experimental results generated by testing the notched samples under fully-reversed VA loading.

Spectrum	$\sigma_{a\text{-}max}$ *[MPa]*	R	N_f *[Cycles]*	$D_{cr,\ exp}$
CUS	728.6	-1	14763	0.59
CUS	728.6	-1	13525	0.54
CUS	594.2	-1	49450	0.29
CUS	512.8	-1	264594	0.38
CUS	512.8	-1	219086	0.32
CDS	891.3	-1	32620	1.31
CDS	735.6	-1	136620	0.91
CDS	618.9	-1	356620	0.47
CDS	891.3	-1	23620	0.95
CDS	618.9	-1	380085	0.50
			Average $D_{cr,\ exp}$ =	0.63

Table 3: Summary of the experimental results generated by testing the plain samples under fully-reversed VA loading and experimental values of the critical damage sum, D_{cr}.

Figures

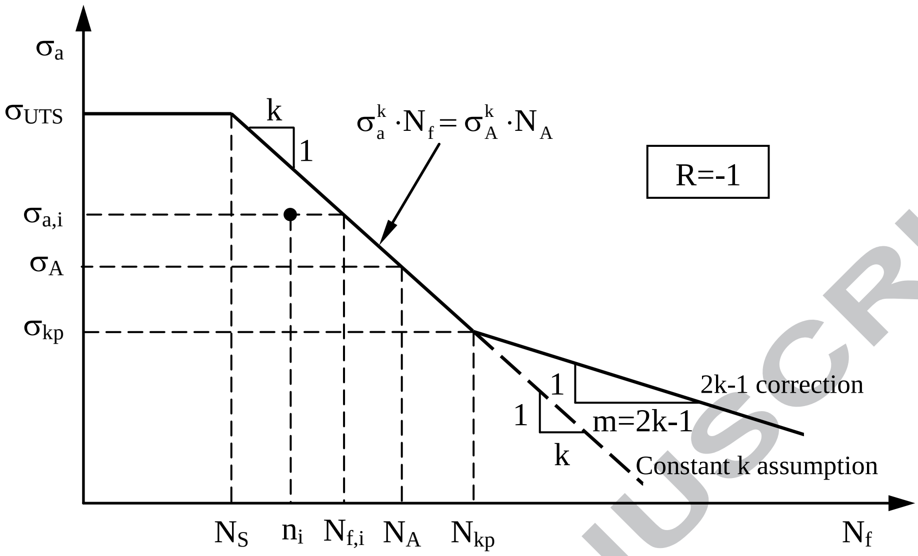


Figure 1: Wöhler curve corrected to estimate lifetime under VA fatigue loading.

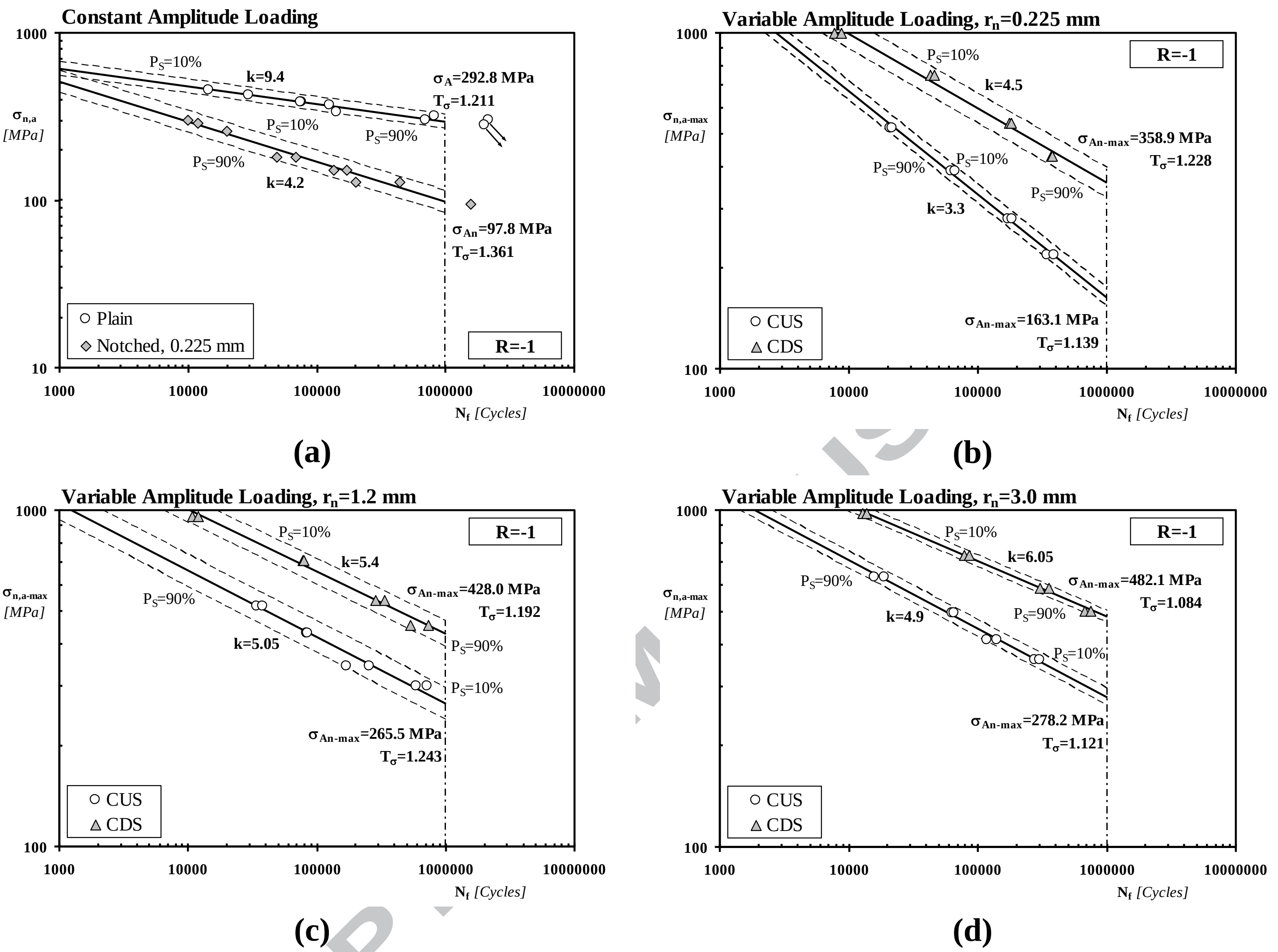
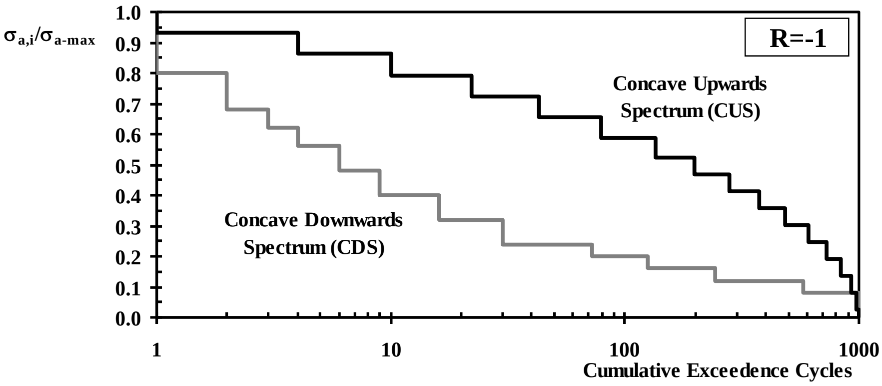
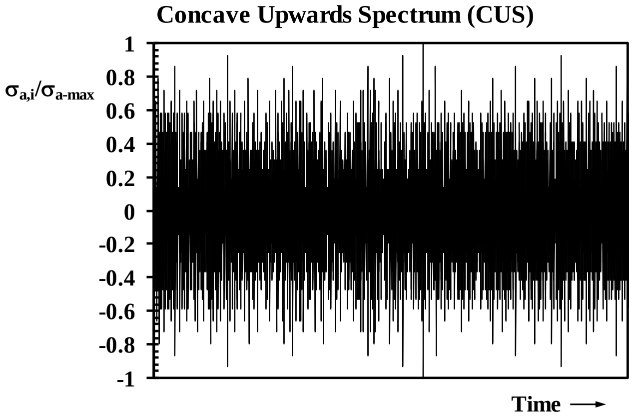


Figure 2: Fatigue results generated under CA (a) and VA (b, c, d) fully-reversed axial fatigue loading (where in the latter three charts all the experimental results were generated by testing notched samples).



(a)



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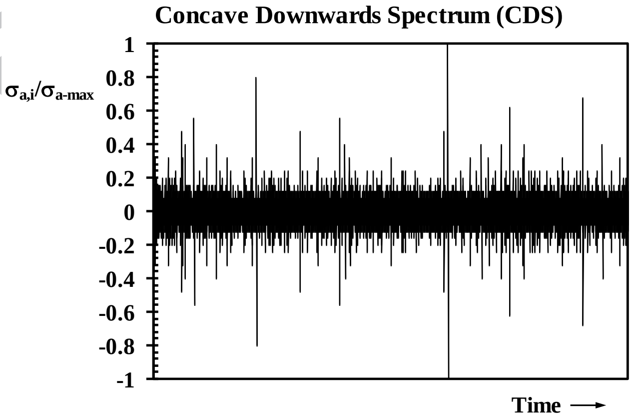


(c)

Figure 3: Adopted Load Spectra.

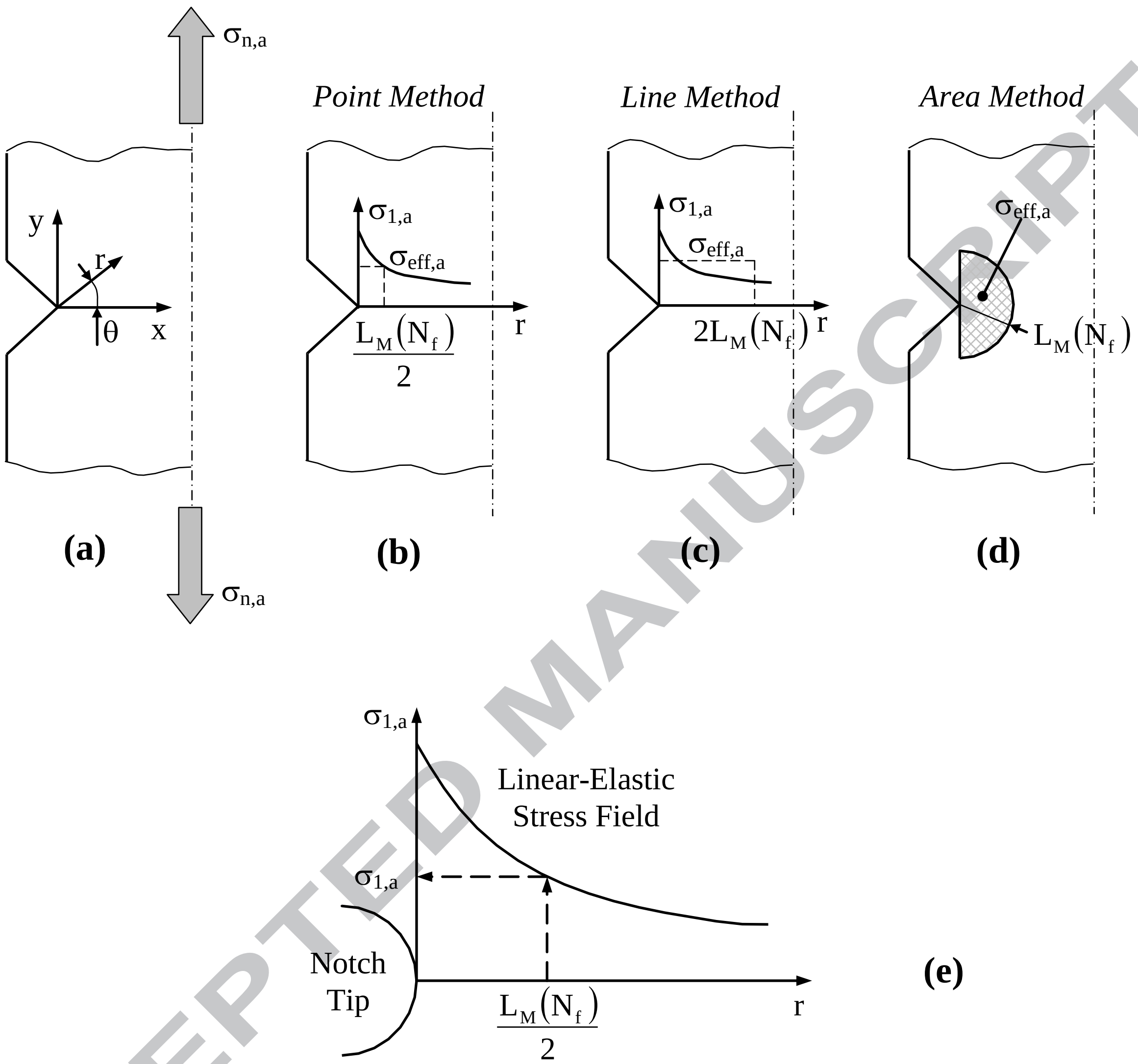
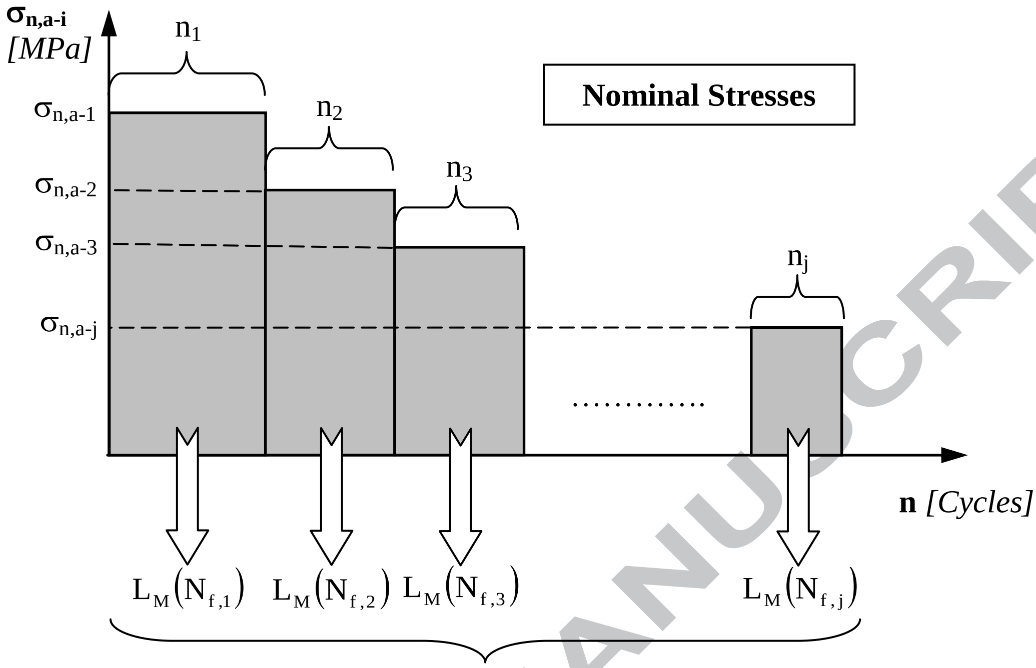


Figure 4: Notched sample subjected to uniaxial nominal loading (a) and definition of the amplitude of the effective stress, $\sigma_{eff,a}$, according to the Point (b), Line (c) and Area Method (d). In-field use of the PM to estimate finite lifetime of notched components (e).



$$L_{VA}=\frac{\sum_{i=1}^{j}L_M\left(N_{f,i}\right)\frac{n_i}{N_{f,i}}}{\sum_{i=1}^{j}\frac{n_i}{N_{f,i}}}=\frac{\sum_{i=1}^{j}L_M\left(N_{f,i}\right)\cdot D_i}{\sum_{i=1}^{j}D_i}$$

Figure 5: Proposed procedure for the determination of the critical distance value, L_{VA}, under VA fatigue loading.

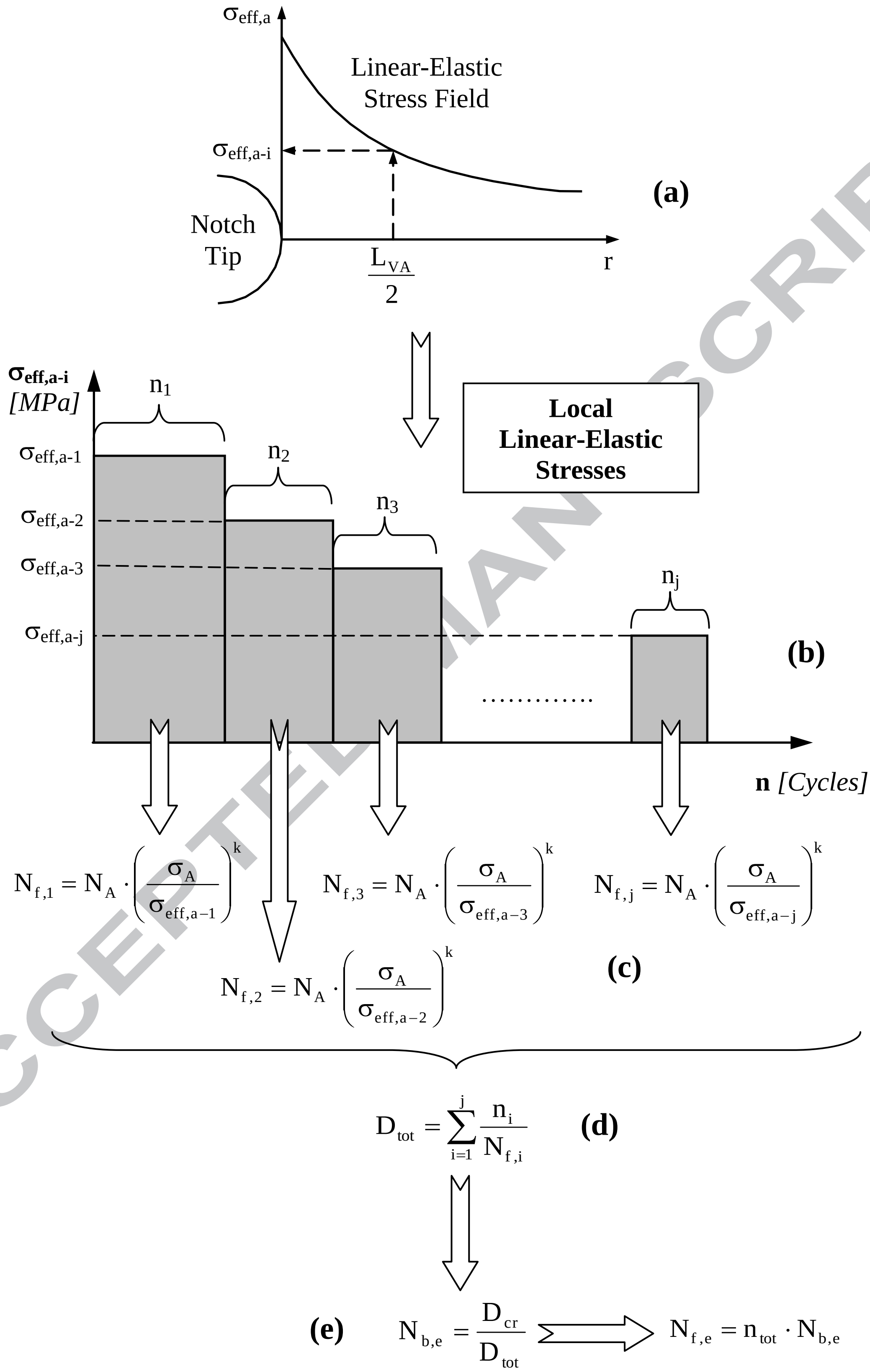


Figure 6: Estimating fatigue lifetime under VA loading by post-processing the local linear-elastic stress field in the vicinity of the notch tip.

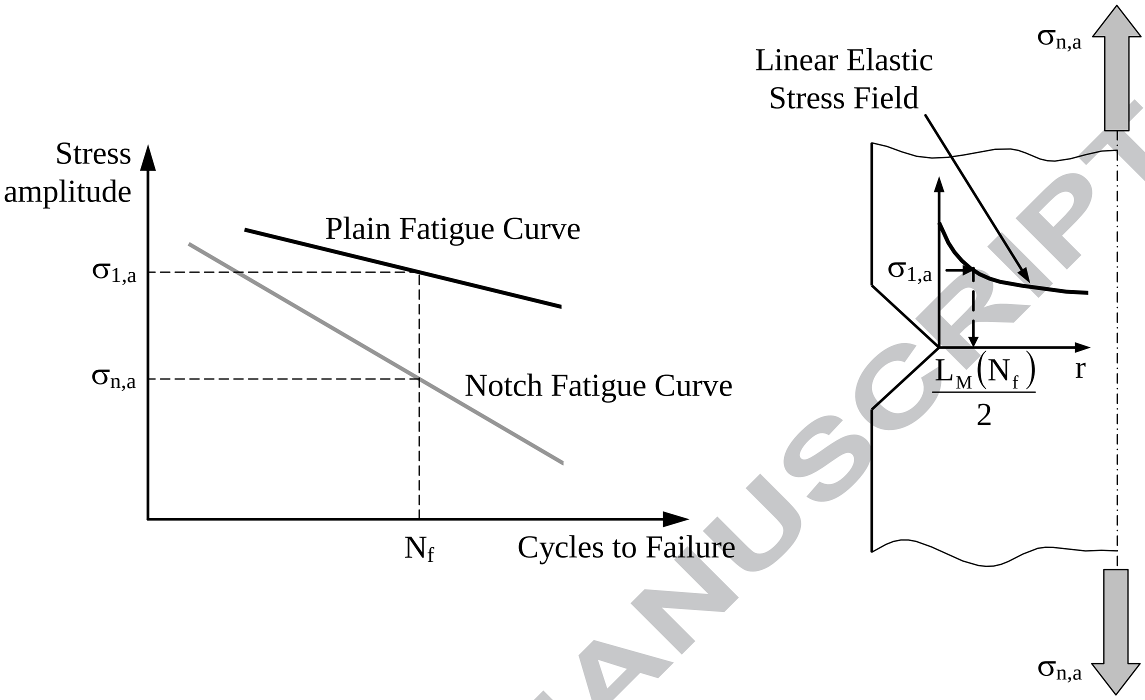
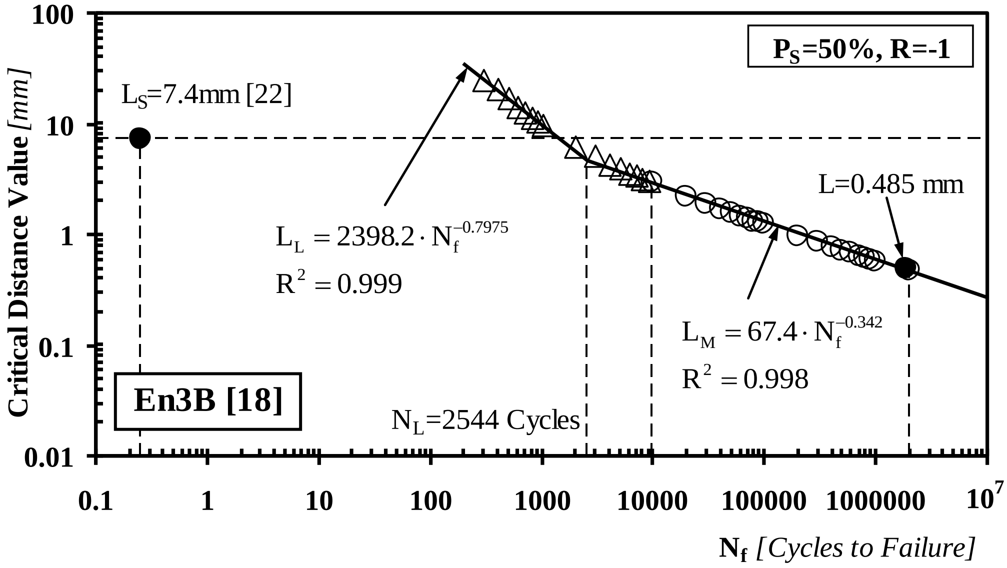
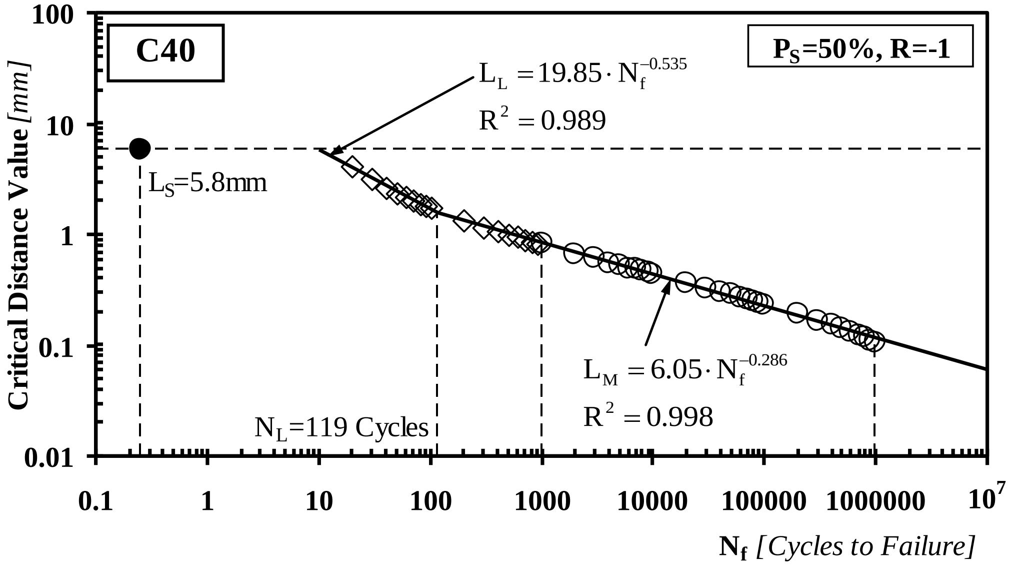


Figure 7: In-field procedure to determine the critical distance vs. number of cycles to failure relationship.

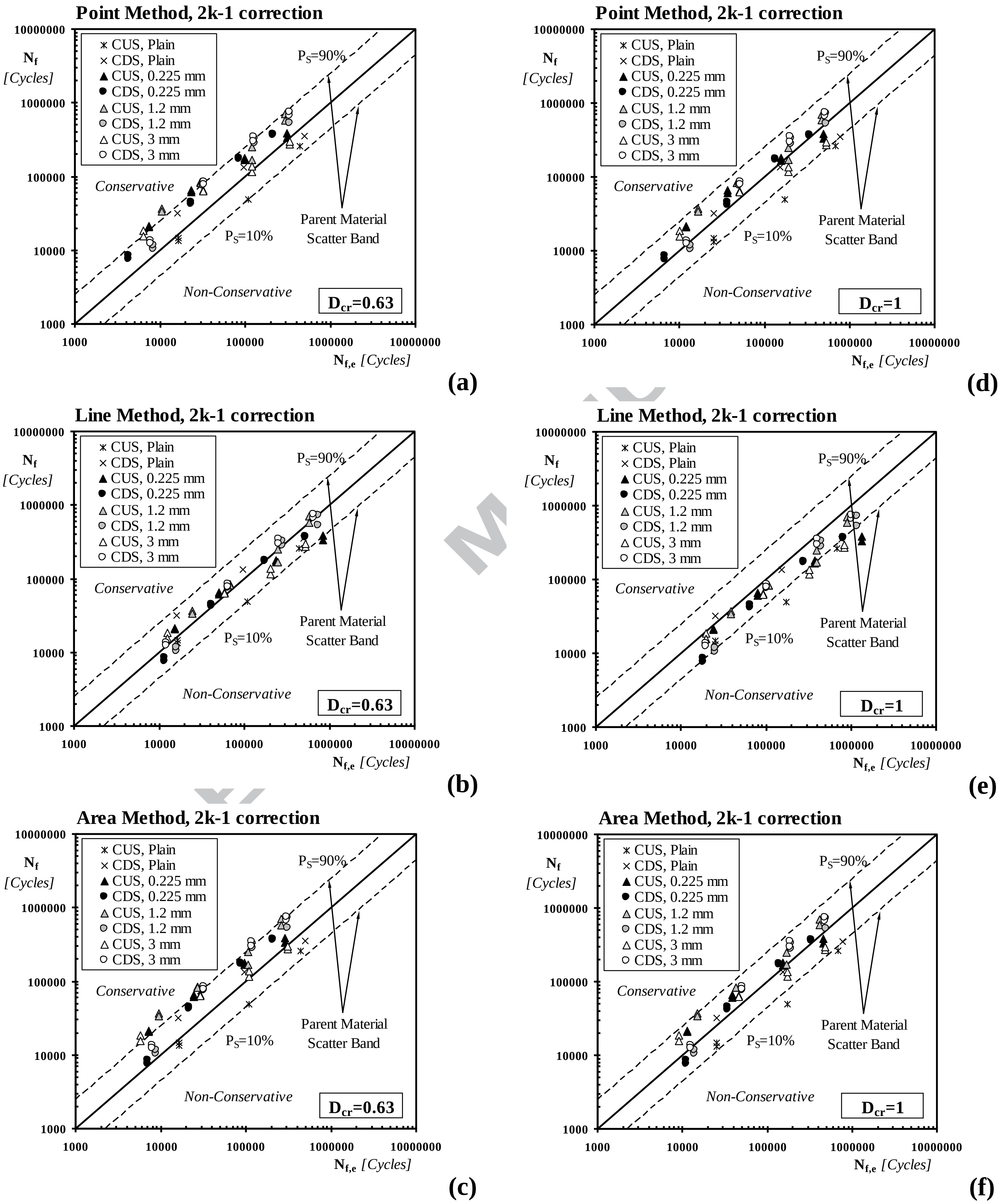


(a)



(b)

Figure 8: Critical distance vs. number of cycles to failure relationships determined, according to the PM, for low-carbon steel En3B [18] (a) and for medium-carbon steel C40 considered in the present investigation (b).



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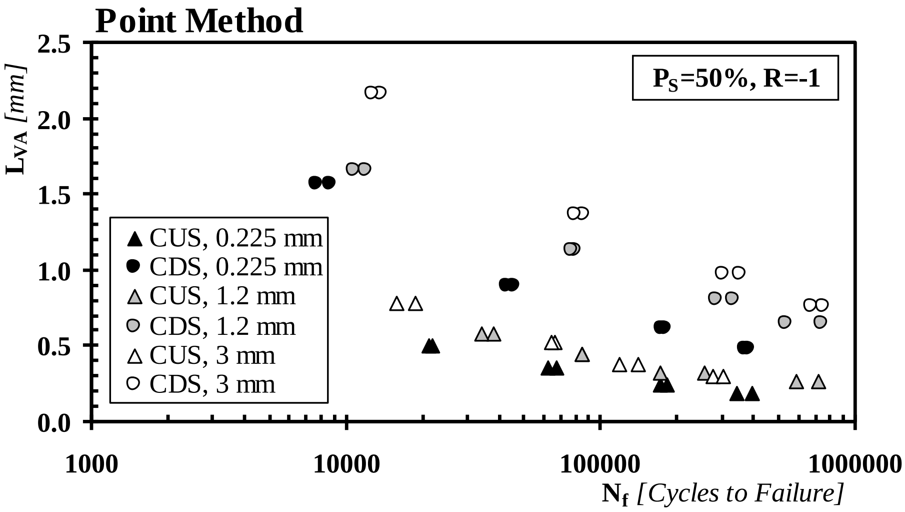
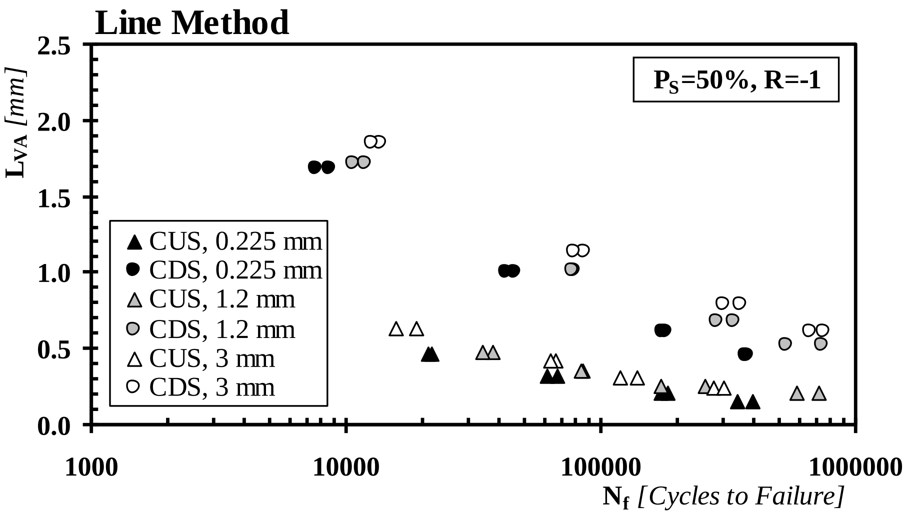
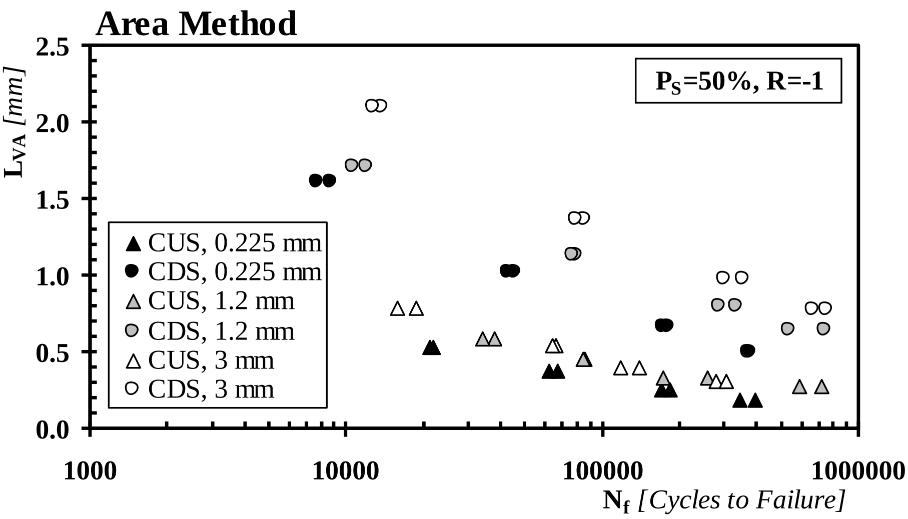


(a)



(b)



(c)

Figure 10: Sensitivity of critical distance L_{VA} to the features of the applied load spectrum as well as to the sharpness of the tested notch.