# Thickness dependence of the effective damping in epitaxial Fe<sub>3</sub>O<sub>4</sub>/MgO thin films

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The precessional magnetization dynamics of high quality epitaxial magnetite (Fe<sub>3</sub>O<sub>4</sub>) thin films growth on MgO are investigated by inductive magnetization dynamic measurements in time and frequency domain. An upper bound for the intrinsic Gilbert damping parameter of  $\alpha_0 = 0.037 \pm 0.001$  is derived, which is significantly lower than previously reported for epitaxial Fe<sub>3</sub>O<sub>4</sub> on GaAs. With increasing film thickness from 5 up to 100 nm a strong increase in the effective damping up to 0.2 is observed which cannot be explained by simple nonuniform spin wave excitations. Possible origins of this effect are discussed. © 2011 American Institute of Physics. [doi:10.1063/1.3531989]

## I. INTRODUCTION

The half-metallic character of magnetite (Fe<sub>3</sub>O<sub>4</sub>) with a predicted full spin polarization at the Fermi level and its high Curie temperature ( $\sim$ 850 K) (Ref. 1) makes it an attractive candidate for a variety of spintronic applications.<sup>2</sup> In addition magnetite is nontoxic thereby enabling a variety of biomedical application.<sup>3</sup> For both fields the magnetization dynamics and especially the dissipation of magnetization precession by Gilbert damping are important: in spintronics the damped precession limits the ultimate speed of device operation, whereas for biomedical applications like, e.g., hyperthermia cancer treatment,<sup>3</sup> the efficient dissipation of energy from external oscillating magnetic fields is also determined by the Gilbert damping parameter. Additionally, Gilbert damping limits spin wave propagation in the emerging field of magnonics.<sup>4</sup> Therefore, magnetization dynamics and damping is presently investigated in a variety of spintronic materials like, e.g., half metallic Heusler compounds.<sup>5</sup> Concerning precessional magnetization dynamics of Fe<sub>3</sub>O<sub>4</sub> frequency domain experiments of thin films have been used to determine the film anisotropy.<sup>6-8</sup> More recently time resolved magneto-optical Kerr effect measurements have been carried out on 6 nm thick Fe<sub>3</sub>O<sub>4</sub> grown on GaAs to investigate the time resolved precession. The resulting Gilbert damping parameter of  $\alpha \approx 0.1$  is more than one order of magnitude larger than typically found for 3d ferromagnetic metal thin films.<sup>10</sup> Note, however, that the large lattice mismatch of  $Fe_3O_4$  and GaAs (around -5%, when the  $Fe_3O_4$  cell is rotated by 45° with respect to the GaAs cell in the (100) plane nakes GaAs less suitable for growing high quality

Here, we report pulsed inductive microwave magnetometry (PIMM) (Ref. 12) and vector network analyzer ferromagnetic resonance (VNA-FMR) (Ref. 13) measurements of the magnetization precession and damping of  $\rm Fe_3O_4$  thin films grown on MgO. The low lattice mismatch of only 0.34% of this system allows to grow high quality fully epitaxial magnetite thin films.  $^{14}$ 

In the low thickness limit a significantly lower effective damping of  $\alpha_{\rm eff}$ =0.037 ±0.001 is found than previously reported for Fe<sub>3</sub>O<sub>4</sub> on GaAs. With increasing film thickness up to 100 nm a strong increase in  $\alpha_{\rm eff}$  up to about 0.2 is observed which cannot be explained by simple nonuniform spin wave excitations. Possible origins of this effect are discussed.

## II. SAMPLE FABRICATION AND EXPERIMENT SETUP

Fully epitaxial Fe<sub>3</sub>O<sub>4</sub> thin films with different thickness (5, 15, 50, 60, 65, and 100 nm) were grown on MgO (001) single crystal substrates using an oxygen plasma assisted molecular beam epitaxy (MBE) (DCA MBE M600, Finland) with a base pressure of  $5 \times 10^{-10}$  Torr. Details of the growth procedure are given elsewhere.<sup>8,15</sup> Reflection high energy electron diffraction (RHEED) was employed to confirm the epitaxial growth and establish the growth mode. The presence of the RHEED intensity oscillations confirms that the films grew in a layer-by-layer mode with a growth rate of 0.3 Å/s. The representative RHEED pattern for the substrate and Fe<sub>3</sub>O<sub>4</sub> were recorded in (100) azimuth [Figs. 1(a) and 1(b)]. The patterns for the MgO substrate [Fig. 1(a)] showed vertical lattice rods and sharp Kikuchi lines representative of well ordered and smooth surface. Figure 1(b) shows the RHEED image after the growth of 5 nm Fe<sub>3</sub>O<sub>4</sub> film show half-order streaks corresponding to the magnetite unit cell positioned between the locations of MgO RHEED lattice

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epitaxial Fe<sub>3</sub>O<sub>4</sub> thin films. Hence, also the Gilbert damping could be extrinsically enhanced compared to high quality epitaxial magnetite thin films.

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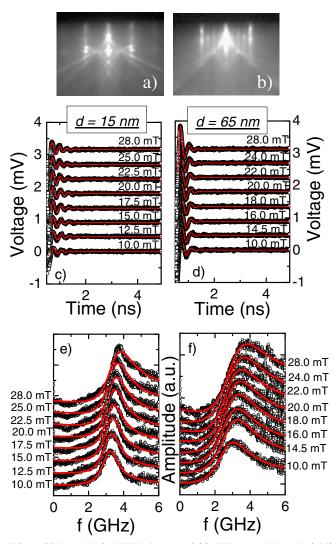


FIG. 1. (Color online) RHEED images of (a) UHV annealed MgO (100) substrate and (b) after 5 nm Fe $_3$ O $_4$  growth on MgO substrate. The images were recorded in (100) azimuth. PIMM data (c), (d) (black dots) and VNA-FMR data (e), (f) of 15 and 65 nm thick Fe $_3$ O $_4$  films. Red lines are fits to the data by an exponentially damped sinusoid (c), (d) and a Lorentzian (e), (f), respectively.

rods. This presence of half-order streaks indicates a pseudomorphic growth of magnetite and reflects the double periodicity of its unit cell as compared to MgO. The magnetic behavior of all Fe $_3$ O $_4$  thin films was examined by means of a physical property measurement system. The in-plane magnetic field was applied along the  $\langle 001 \rangle$  direction and all films showed a Verwey transition around 120 K, thereby confirming that the magnetite was stochiometric.

For PIMM measurements the  $Fe_3O_4$  films were placed onto a 50  $\Omega$  coplanar waveguide connected to a pulse generator and an 18 GHz bandwidth sampling oscilloscope on

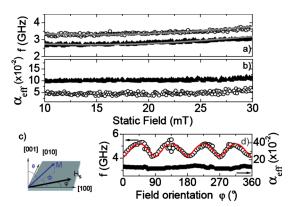


FIG. 2. (Color online) (a) Field dependence of the resonance frequency f of 15 nm (dots) and 65 nm (triangles) thick Fe<sub>3</sub>O<sub>4</sub> films. Field is applied along [001]. Solid line is a fit to the dispersion relation. (b) Field dependence of effective damping parameter  $\alpha_{\rm eff}$ . (d) Sketch of angular coordinates used in model and experiment. (d) Dependence of f and  $\alpha_{\rm eff}$  on in-plane field angle  $\varphi$  at  $H_S$ =50 mT. Blue line is a fit to f.

either side. <sup>16</sup> Current pulse application through the waveguide generates a transverse magnetic field pulse ( $H_p \approx 0.2$  mT, 100 ps pulse duration) exciting precession of the magnetization M. The induced oscillatory voltage due to precession of the M is captured by the oscilloscope. For frequency domain VNA-FMR measurements, the waveguide is connected to the two ports of the VNA. The harmonic voltage from one port of the VNA excites the precession and the induced voltage amplitude and phase is detected by the other port.

## III. RESULTS

Examples of time and frequency domain data for 15 and 65 nm thick films at different static fields are shown in Figs. 1(c)-1(f). PIMM data [(c), (d), open dots] can be well fitted to an exponential damped sinusoid (red lines) indicating small angle precession in the linear regime. <sup>17</sup> VNA-FMR data [(e), (f), open dots] can be well fitted to a Lorentzian function with symmetric and asymmetric component. The FMR frequency f and the effective damping parameter  $\alpha_{\rm eff}$  derived from fitting the two data sets are in good agreement.

Figure 2(a) shows the such derived field dependence of f for the two 15 and 65 nm thick films as function of applied field along the [001] orientation. To derive the film parameters the samples are modeled as (001) thin films with a cubic anisotropy field  $H_c = 2K_c/M_s$  (with  $K_c$  the cubic anisotropy constant and  $M_s$  the saturation magnetization), an uniaxial anisotropy field  $H_u = 2K_u/M_s$  (with  $K_u$  the uniaxial anisotropy constant), an out-of-plane demagnetizing field  $H_d = 4\pi M_s$ , an in-plane external field  $H_s$ , and the easy axis along the  $\langle 100 \rangle$  directions (see inset in Fig. 2). The resulting dispersion relation is (see Eq. 5 in Ref. 8)

 $f = \frac{\gamma}{2\pi} \times \sqrt{\left[H_s \cos(\phi - \varphi) + H_c \cos 4\phi + H_u \cos 2\phi\right] \times \left[H_s \cos(\phi - \varphi) + H_d + \frac{H_c}{4}(3 + \cos 4\phi) - \frac{H_u}{2}(1 - \cos 2\phi)\right]}$ (1)

where  $\gamma = 1.76 \times 10^{11}$  A m<sup>2</sup>/s J is the gyromagnetic ratio and  $\phi$  and  $\varphi$  are the in plane angles of M and  $H_s$ , respectively, measured from the [100] orientation [see sketch in (c)]. Fitting the measured f to this model yields good agreement as shown by the full lines in Fig. 2(a).

TABLE I. Anisotropies and saturation magnetization extracted from fitting the field and angular dependence of f derived from time and frequency domain data. Also  $M_s$  derived from SQUID measurements is shown.

d (nm)	$4\pi M_s$ , SQUID (kOe)	$4\pi M_s$ , inductive (kOe)	$H_c$ (Oe)	H <sub>u</sub> (Oe)
5	$11.4 \pm 2.3$	$7.6 \pm 0.3$	$-200 \pm 5$	$20 \pm 2$
15	$7.7 \pm 0.5$	$7.4 \pm 0.2$	$-245 \pm 15$	$72 \pm 2$
50	$6.4 \pm 0.15$	$5.4 \pm 0.2$	$-172 \pm 7$	$35 \pm 2$
60	$6.1 \pm 0.2$	$5.2 \pm 0.2$	$-110 \pm 10$	$2 \pm 2$
65	$6.3 \pm 0.1$	$5.5 \pm 0.4$	$-240 \pm 6$	$90 \pm 2$
100	$6.1 \pm 0.1$	$5.2 \pm 0.5$	$-80 \pm 15$	$0 \pm 2$

Complementary to the measurements at fixed field orientation VNA-FMR measurements at a higher field amplitude of  $H_S$ =50 mT under variation in field angle  $\varphi$  have been performed. Figure 2(d) shows the such derived angular dependence of f and  $\alpha_{\rm eff}$  for a 60 nm film.  $\alpha_{\rm eff}$  only weakly varies with  $\varphi$ . In contrast f strongly oscillates as expected for a thin film with strong cubic anisotropy. Also here, film parameters can be derived from fitting to the dispersion relation (red line).

The physical parameters of all films deduced from such fits are compiled in Table I. The values are in good agreement with previous studies on  $\text{Fe}_3\text{O}_4/\text{MgO}$  thin films, with an enhancement of  $M_s$  of thinner films. This effect can be ascribed to a noncompensation of spin moments at the surface of the film. Note, however, that the derived  $M_s$  is systematically lower that obtained from SQUID measurements. Here, one possible reason might be the surface oxidization of the films between both measurements leading to the formation of a  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub> film of up to 1.5–2 nm thickness. This oxidized film would lead to a reduced  $M_s$ , especially for thinner samples.

Figure 3 shows the thickness dependence of the effective damping parameter  $\alpha_{\rm eff}$ . Both PIMM (black squares) and VNA-FMR (open dots) data are shown and agree well. With increasing film thickness from 5 to 100 nm  $\alpha_{\rm eff}$  increases by about a factor of five (from about 0.039 up to about 0.2). The observed thickness dependence of  $\alpha_{\rm eff}$  can be well described by a second order polynomial  $\alpha_{\rm eff}$ =0.0365+5.6×10<sup>-4</sup>×d+1.09×10<sup>-5</sup>×d<sup>2</sup> (with d in nm; red line). When attributing the increase in the effective damping with thickness to extrinsic effects related to the film thickness, an upper limit of the intrinsic Gilbert damping parameter can thus be derived by extrapolation to d=0 yielding  $\alpha_0$ ≤0.037±0.001. It is important to note, that the intrinsic Gilbert damping parameter of our high quality epitaxial magnetite thin films is sig-

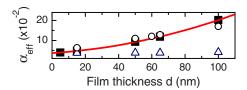


FIG. 3. (Color online) Thickness dependence of the measured effective damping parameter by PIMM (black squares) and VNA-FMR (open dots) by considering saturation magnetization from Table I. Red solid line is a second order polynomial fit. Open triangles show the calculated thickness dependence of  $\alpha_{\rm eff}$  by considering non linear spin wave excitation.

nificantly lower than has been reported for magnetite on GaAs. We ascribe this significant improvement to the reduced lattice mismatch of MgO with respect to  $Fe_3O_4$  [-0.34% (Ref. 14)].

## **IV. DISCUSSION**

One possible origin of such thickness dependence could be the generation of spin waves due TO non homogeneous field excitation by the coplanar wave guide. When the magnetic film is much wider than the width of the stripline non-uniform spin wave modes can be excited resulting in a broadened linewidth (and thus an increased  $\alpha_{\rm eff}$ ). The resulting dependence of  $\alpha_{\rm eff}$  on f and d is then given by,  $^{13,19}$   $\alpha_{\rm eff}$  =  $\alpha_0 + 1/2\alpha_0 \times (\gamma \mu_0 M_s k_{\rm max}/8\pi)^2 \times (d/f)^2$ , with  $\mu_0$  the vacuum permeability, and  $k_{\rm max} = \pi/w$  the upper limit of the transverse spin wave vector  $k_{\parallel}$  that can be excited by a waveguide with inner conductor width w. Following this dependence the measured  $\alpha_{\rm eff}$  should decrease with increasing precession frequency and thus with applied static field. Such dependence is however not observed in Fig. 2(b).

As pointed out by Counil *et al.*<sup>13</sup> nonlinear spin wave excitation results in a more complex time response of M given by

$$M(t) \cong \int_{\omega_0}^{\omega_{\text{MAX}}} e^{-(i\omega+1/\tau)t} d\omega \cong \frac{\sin(\Delta\omega t)}{t} \times e^{-t/\tau} \times \cos([\omega_0 + \Delta\omega]t), \tag{2}$$

where  $\Delta\omega$  is the frequency shift induced by the spin waves and  $\omega_0$  is the resonance angular frequency derived from Kittel's formula. Considering the long wavelength limit  $k_{\parallel}d \ll 1$  the dispersion relation of spin waves propagating orthogonally to the magnetization for a thin film of thickness d is  $\omega(k) = \omega_0 \times \sqrt{1 + (\gamma \mu_0 M_s/2\omega_0)^2 \times (2k_{\parallel}d)}$  and the frequency shift  $\Delta\omega$ , can be written as  $\Delta\omega(k) = 1/2[\omega(k) - \omega_0] \cong (\gamma \mu_0 M_s/2)^2 \times (k_{\parallel}d/2\omega_0)$ .

As described before experimental determination of  $\alpha_{\rm eff}$  is done by fitting the time resolved data by a damped sinusoid. When assuming such fitting for a time resolved precession according to Eq. (2) together with the given thickness dependent frequency shift  $\Delta\omega$  due to spin wave generation, the increase in the measured effective damping  $\alpha_{\rm eff}$  with thickness can be computed. The such derived thickness dependence of the effective damping for our experimental geometry (with  $w=50~\mu{\rm m}$ ) is given by the blue triangles in Fig. 3. Obviously the increase in  $\alpha_{\rm eff}$  by nonlinear spin wave excitation is by far too small to explain the experimental findings and nonlinear spin wave generation can be ruled out as an origin of the observed increase in the effective damping.

Note that the reported thickness dependence of the effective damping parameter significantly differs from those observed in ferromagnetic metals like, e.g., permalloy: while Schneider *et al.* <sup>19</sup> observed an increase in the intrinsic Gilbert damping parameter  $\alpha_0$  with increasing film thickness Council *et al.* <sup>13</sup> and Ingvarsson *et al.* <sup>10</sup> found a decrease in  $\alpha_0$  with increasing thickness for the same material. In the latter case the increase in  $\alpha_0$  with decreasing *d* was ascribed

to a dominating effect of the film surfaces on the damping (e.g., due to surface roughness) which was also correlated with an increased resistivity of the thinner films. In contrast Schneider et al. 19 suggest that their observed increasing thickness dependence of  $\alpha_0$  could be related to a different magnetostriction of their films in comparison to the permalloy thin films used, e.g., by Counil et al. 13 Although perfect permalloy ideally shows no magnetostriction the occurring differences of the actual Ni-Fe composition of the deposited films could result in an effective magnetostriction of either positive or negative sign. A consecutive study of the magnetization dynamics of thin films of different Ni-Fe alloys showed that  $\alpha_0$  can be modified by more than 100% by changing the Ni-to-Fe ratio.<sup>21</sup> It was found that a negative magnetostriction coefficient ( $\lambda_s < 0$ ) results in a larger damping than a positive one  $(\lambda_s > 0)$ . The abovementioned thickness increase in the Gilbert damping could thus be due to a decreasing magnetostriction with film thickness, as observed in magnetostriction studies of NiFe and CoFe thin films.<sup>22</sup>

Magnetostriction measurements of bulk magnetite reveal a magnetostriction coefficient along the  $\langle 100 \rangle$  direction of  $\lambda_{100}^{\text{Fe}_3\text{O}_4}\!\sim\!-19.5\!\times\!10^{-6}$  comparable to those observed for Ni (Ref. 23) and Fe.  $^{24}$  Unfortunately, to the best of our knowledge, magnetostriction measurements of magnetite thin films are not available and the respective thickness dependence is not known. Here, only a decreasing magnetostriction with increasing film thickness could explain our experimental findings.

Another possible origin could be related to the presence of the so called antiphase boundaries (APBs), which are growth defects resulting from mismatch between the periodicity of the film and the substrate as well as from the difference in rotational symmetries between both. Because the magnetic exchange interaction depends on the distance, the angle, the spin and the electronic configuration of neighboring magnetic moments, there can appear, across the boundary, magnetic interactions that are not present in bulk Fe<sub>3</sub>O<sub>4</sub> which will be strongly dependent on the substrate. However, the influence of APBs on  $\alpha_{\rm eff}$  has not yet been investigated. Note that in general the density of APBs in magnetite thin films on MgO is expected to decrease with thickness.<sup>25</sup> Therefore, only an increase in  $\alpha_{\rm eff}$  with decreasing density of APBs would result in the observed thickness dependence which is quite unlikely considering the lattice defect origin of APBs. However, APBs also contribute to the strain in magnetite thin films.<sup>26</sup> Such strain could also influence magnetostriction<sup>27</sup> and hence modify  $\alpha_{\rm eff}$  as a function of film thickness as discussed above.

## V. CONCLUSION

In summary, we have investigated the thickness dependence of the magnetization dynamics in high quality epitaxial Fe<sub>3</sub>O<sub>4</sub>/MgO thin films by inductive measurements. For this, PIMM and VNA-FMR measurements were carried out. Our measurements reveal an upper limit for the intrinsic Gilbert damping parameter of  $\alpha_0 \le 0.0365$  which is significantly lower than previously found for Fe<sub>3</sub>O<sub>4</sub> thin films on GaAs. Such low damping together with GHz precession frequencies

could enable fast future magnetite based spintronic devices. With increasing film thickness up to 100 nm a strong increase in the effective damping up to about 0.2 is observed. The origin of this effect remains subject of speculation. However, its future understanding could enable the growth of magnetite thin films and nanoparticles having an effective damping specifically tailored to the needs of the given application

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