International Portfolio Formation, Skewness & the Role of Gold

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Abstract

This paper examines the optimal allocation of assets in well diversified equity based portfolio where the investor is concerned not only with mean and variance but also with the skewness of the returns. Beginning with an analysis of the rationale for concerning with skewness, the paper then discusses previous attempts to model multi-objective portfolio problems. The second part of the paper outlines the attractive nature of the gold asset in equity portfolios. The paper then integrates the two elements, showing the changes in portfolio composition that arise when not only skewness but gold are concerned.

Keywords: Portfolio Allocation, Skewness, Gold

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1 - Introduction

For many decades the prevailing paradigm for portfolio selection has been the Mean-Variance approach. However, there are more moments of a distribution, and thus potentially more tradeoffs than between the first (mean) and second (dispersion). Recent work (see Chunhachinda, Dandapani, Hamid and Prakash (1997), Prakash, Chang and Pactwa (2003), Sun and Yan (2003)) has shown the importance, in portfolio selection, of explicitly considering higher moments. An added impetus comes from the fact that it has long been recognised that the returns to equity assets are not normally distributed, and perhaps are at most log-normal (see, among others Fama (1963), Arditti and Levy (1975), Simkowitz and Beedles (1978)). One characteristic of log-normal returns is the significant asymmetric nature of the distribution. This model has also been shown to be a good descriptor of gold returns (Lucey and Tully (2006)). Gold has also been seen by a number of researchers and commentators as a useful diversification tool for equity based investors due to its low or negative correlation with equities (this is discussed more fully below).

This paper addresses these issues – portfolio selection under skewness and the usefulness of gold in portfolio selection - in a unified framework. Section 2 outlines the previous literature on the role of higher moments in portfolio selection; Section 3 outlines the potential role gold might play in equity dominant portfolios, while Section 4 introduces the programming approach that is used. Finally, Sections 5 to 7 describe the data, present the results and discuss the results.

2 - The Role of Higher Moments in Portfolios

Significant evidence exists that higher moments have utility in the pricing and selection of financial assets. Several researchers (Arditti and Levy (1975), Levy and Sarnat (1972), Samuelson 1970, Jean (1971, (1973)) have noted that ignoring moments of the distribution higher than the variance is appropriate only under very restrictive circumstances. These are in essence:

- The investor places no utility on the higher moments, or

- The asset returns are normally distributed, or
- The investor has a quadratic utility function.

All of these assumptions are challengeable. For example, Hanoch and Levy (1970) show that the use of quadratic utility functions has a number of drawbacks. Chief among these is that the use of quadratic utility functions carries an implicit assumption that the investor has *increasing* absolute risk aversion, rather than the more commonly assumed decreasing. Levy and Sarnat (1972) further show that the assumption of quadratic utility is useful only when the assets generate relatively low absolute returns.

The assumption of normality of distributions is also open to question. From Fama (1965) and Mandlebrot (1964, (1966) through Singleton and Wingender (1986) the evidence is strong that assets returns are not normally distributed. Others (Arditti and Levy (1975), Kraus and Litzenberger (1976), Nummelin (1997), Jondeau and Rockinger (2003)) have shown that skewness is an important factor in pricing of equities.

In a similar fashion to the total variance of a portfolio being a product of not just individual variance terms but also co-variances, the role of co-skewness has also been revived. Chunhachinda, Dandapani et al. (1997) discusses the early history of portfolio solutions under the assumption that skewness matters, noting that early attempts, up to Lai (1991) suffered from a number of defects. In particular the simplifying assumptions needed to make these models tractable were of such magnitude to render them unrealistic. Lai's model, using what has become known as Polynomial Goal Programming (hereafter PGP) is discussed in more detail in section 4. In brief however selection of a portfolio where skewness matters is a trade-off. Investors are assumed (see Levy and Sarnat (1972) and Kraus and Litzenberger (1976) *et seq*) to have a preference for positive over negative skewness. Thus simultaneously the portfolio is to maximize return and skewness for a given level of risk.

3 - The Role of Gold in Portfolios

The inclusion of gold holdings has been advocated to lead to a more balanced portfolio by reducing its volatility (see for example Ciner (2001), World Gold Council). According to Sherman (1983) under conditions of uncertainty many investors turn to gold because it is perceived to be a "currency without borders" - a highly liquid and secure asset that can be accessed at any time. A change in investor sentiment during periods of economic distress may lead investors to consider a role for gold in their portfolios as gold tends to hold its value over time. However, it is not just under conditions of equity fragility that gold has been advocated as a part of portfolios.

Shishko (1977)was an early paper that demonstrated the case for gold, noting its desirable properties (low risk especially) for portfolio formation.

Sherman (1982) found that an equity based portfolio with 5% and 10% proportion in gold can lower volatility and improve returns. Jaffe (1989) finds that gold has virtually no measurable correlation relationship with stocks and bonds. Similar to the findings of Sherman (1982) he finds that a small percentage of gold is beneficial, increasing returns while reducing risk.Chua (1990) shows that over the period 1971-1988 a 25% weighting in gold bullion resulted in lowering portfolio risk while increasing its return. Blose (1996) noted the benefits of gold based mutual funds. Davidson, Faff and Hillier (2003) comment that gold has renewed its diversification status in the aftermath of the Asian crisis as it is once again being used as a hedging device by investors. According to Draper, Faff and Hillier (2002) portfolios that contain gold, silver or platinum perform significantly better than standard equity portfolios.

According to Shishko (1977), Johnson and Soenen (1997), and Davidson, Faff et al. (2003) gold is an efficient hedge against inflation, political unrest and currency risk, which all affect the price equilibrium of this metal. Gold becomes an important element of a diverse portfolio mix during times of political disharmony, weakness of the US dollar, equity markets declines, corporate fraud and low interest rates. Gold has traditionally down through the ages played out this role of a hedge against risk in the Middle East and India. Sherman (1983) highlights gold's ability against the eroding effects of inflation as the price of gold rises in times of increasing inflation, it enhances overall rate of return and provides diversification and superior flexibility in portfolio management.

Kaufman and Winters (1989) reiterate gold's role as a hedging device and comment that since gold holds it value over time, the price of the metal should rise with inflation. Chua (1990) states that gold has long been recognised as a hedge against inflation and maintains that gold's most important contribution is its ability to maintain value during a financial crisis.

The summary statistics of gold and the largest equity markets are shown in . While the mean returns for gold are lower than other equities, gold displays the lowest standard deviation of all markets examined. In addition gold exhibits the highest positive skewness of all markets. While most equity markets demonstrate negative results, ranging from small exhibits to highly negatively skewed data, gold consistently achieves highly positively skewed data. We also examined the correlation statistics of gold with these major equity markets[5]. The data reveals that gold does not have a positive relationship with many of the major equity markets and is in general negatively correlated with most of the markets. In particular gold is negatively correlated to some of the biggest equity markets such as the New York Stock Exchange, NASDAQ and London markets. This highlights the important role gold may play in the nature of portfolio diversification, as a safety net against adverse trends in equity and bond markets, thereby making sense to include gold in a well diversified portfolio.

4 - Polynomial Goal Programming

The technical derivation of this model can be found in Lai (1991), and more succinctly in Chunhachinda, Dandapani et al. (1997). Both these papers suggest that as the issue under investigation is the relative set of weights allocated to various assets when higher moments are relevant, the problem can be simplified if optimizations are rescaled on the unit variance space. In essence, what one is attempting to do is select a set of weights to satisfy more than one constraint. Traditional Mean-Variance analysis selects the weights, percentages of wealth, that are to be allocated amongst assets, such that (typically) the variance of the expected returns is as low as possible for a desired expected mean return. When one adds another constraint, such as skewness, one then also desires that the expected skewness be as large as possible for the lowest expected risk consistent with the desired expected return.

Polynomial goal programming operates by breaking the overall problem into soluble elements and then iteratively finding solutions that preserve as closely as possible the individual solutions.

In the case of portfolio selection we have three selection stages. The first stage is analogous to the standard mean variance analysis. We select a

[5] Data available on request

set of weights, for a given variance, that maximises returns. The second stage is the selection of a different set of weights, where now the objective is to maximise skewness. Crucially however, in the second stage, the weights are chosen such that while they maximise skewness they also (regardless of the return) keep the variance the same as that selected in the first stage. The final stage is to form a set of weights that minimises the difference between the returns of the two portfolios so selected. For convenience, unit variance is usually chosen.

The overall portfolio problem then is given by

$$w_{mvs} \quad \textit{solving} \quad \min_{w} \left\{k_{mv}\left(w_{mv}\right) - k_{mvs}\left(w_{mvs}\right)\right\}^{\alpha} + \left\{s_{ms}\left(w_{ms}\right) - s_{mvs}\left(w_{mvs}\right)\right\}^{\beta}$$

$$s.t \quad w_i \geq 0; \quad \mathrm{var}(w_i) = 1$$

where $\mathrm{var}(w_i)$ is the portfolio variance for each and any given set of weights, $k_{mv}(w_{mv})$ is the return on the mean-(unit)variance optimal portfolio with weights w_{mv}, $k_{mvs}(w_{mvs})$ is the return on the mean-(unit)variance-skewness portfolio, $s_{ms}(w_{ms})$ is the skewness[i] of the skewness-(unit)variance optimal portfolio and $s_{mvs}(w_{mvs})$ the skewness of the mean-variance-skewness portfolio. In essence then the three step process is encapsulated in the equation above.

Additional constraints, such as short-selling restrictions, full-investment, target weights etc are easily accommodated as extra restrictions. In this paper we do not explicitly require full investment. The interpretation of any set of weights which therefore sum to less than 1 is that the residual is invested in the risk-free asset. Both Chunhachinda, Dandapani et al. (1997) and Prakash, Chang et al. (2003) also discuss various degrees of investor trade off between the importance of skewness and return. The α and β weights above indicate the relative preferences that the investor places on the elements of the goal. Trivially, the PGP problem collapses to a standard mean-variance optimisation problem if we place zero weight on the skewness element. The standard weight for both elements is 2, as the squaring of the deviations ensures that the deviation to be minimised is strictly greater than zero. An important issue is that there is no guide in theoretic or empirical literature as to the relative size of these weights. Equality of the weights implies that the investor cares equally about the skewness and mean elements of the portfolio (the variance being held at a unit value). Clearly, this may not be the case in reality. We do not here vary the weights. The objective of the paper is to show the applicability of the polynomial goal programming approach, and to highlight the role of gold when this method is used. In any case, trivially, as the weight on the skewness element of the programme is reduced the weights for the assets begin to converge back to the weights of the mean –variance solution.

Both Chunhachinda, Dandapani et al. (1997) and Prakash, Chang et al. (2003)discuss the issues involved with operationalizing this PGP model. These two papers, along with Sun and Yan (2003) appear to be the only published works that present empirical evidence on the actual stocks.

Sun and Yan (2003) examine individual equities while the other papers, in common with this paper, examine indices. Chunhachinda, Dandapani et al. (1997) examine MSCI indices from 1988 to 1993 while Prakash, Chang et al. (2003) examines these indices from 1993 to end 2000. All three papers show that the composition of the mean-variance-skewness optimal portfolio is markedly different to that of the mean-variance portfolio. For instance, Chunhachinda, Dandapani et al. (1997) show that Sweden, Italy and Japan are selected only when the weight on skewness is above 2; they also show that while a considerable weight is placed on the UK in a mean-variance framework it is not selected in a mean-variance-skewness portfolio. Prakash, Chang et al. (2003) shows a similar result for the USA. Unfortunately Sun and Yan (2003) do not present the names of their equities, but again show significant differences as between the various portfolios.

5 - Data

A number of different indices and time periods are examined. The focus of the paper is on large indices and gold. Based on data from the International Federation of Stock Exchanges, the markets chosen accounted on average for in excess of 87% of world market value over the period. Data are analysed from August 1988 to September 2003.

To examine the potential effects of different investment horizons and thus counter the intervaling effect, which as Levy and Sarnat (1972) and Brown and Warner (1985) have shown can have significant effects on investment choice, data are examined on a weekly, monthly and quarterly basis. In addition to the 'national' indices (TOPIX, DAX, etc) the analyses are also carried out on the indices constructed by the Financial Times (the so called FTSE indices) which share common characteristics of construction and thus are perhaps more appropriate for international comparisons. All data are analysed as annualised percentage returns, to allow comparability, and are examined from the perspective of the US investor, being expressed in US$ terms. provides details of the indices, while presents evidence on their non-normality.One use that we can make of the information in is to evaluate likely candidates for the portfolio. The coefficient of variation provides a summary that related the risk and return elements. From the table we can see (treating the world and FTSE indices as separate universes) that Italy has the highest risk per unit of return, measured by the coefficient of variation, CV, (as was found by Chunhachinda, Dandapani et al. (1997)) while, of the non-negative returns, the NYSE Composite/FTSE-US indices have the best risk-return profile. This again is essentially the same result as was found in Chunhachinda, Dandapani et al. (1997). We would therefore expect to see low weights for Italy and higher weights for the US indices in the mean-variance portfolio.

Positive, desirable, skewness, is rarely found, and then consistently only in gold returns. FTSE-Hong Kong (monthly) TOPIX (monthly & weekly), COMIT (quarterly & monthly), and FTSE-Japan (monthly & weekly) returns also show positive skewness. In all cases except COMIT and FTSE-Hong Kong this is at the expense of negative mean returns however. We should therefore expect, inter alia, to find gold having potential to be included as we move from mean-variance to mean-variance-skewness optimization

As can be seen from the degree of non-normality in the data increases greatly as we move from lower to higher frequencies. While on its own the degree of departure from normality is not high and would perhaps not justify the use of skewness

Table 1 **Summary Statistics of indices 1988-2003**

QUARTERLY COMPOUNDED DATA					
	Mean	Std.Deviation	Skewness	Kurtosis	CV
NYSE Composite	0.08	0.29	-0.63	0.99	3.56
NASDAQ Composite	0.10	0.59	-0.55	1.49	6.26
TOPIX	-0.05	0.56	-0.57	0.47	-11.33
FTSE-A	0.05	0.34	-0.28	-0.25	7.50
DAX30	0.07	0.50	-0.92	2.37	6.99
TSX	0.04	0.39	-0.99	2.33	9.61
SMI	0.08	0.39	-0.58	1.34	4.69
HANG SENG	0.08	0.61	-0.16	0.62	7.15
COMIT	0.04	0.48	0.03	1.10	12.22
GOLDAM	-0.01	0.23	0.41	0.56	-15.90
FTSE-US	0.09	0.32	-0.57	0.96	3.75
FTSE-JA	-0.05	0.56	-0.68	0.96	-12.28
FTSE-UK	0.05	0.34	-0.27	-0.33	7.15
FTSE-GE	0.06	0.48	-1.05	2.61	8.59
FTSE-CA	0.05	0.39	-0.86	2.37	8.51
FTSE-SW	0.09	0.38	-0.60	1.10	4.35
FTSE-HK	0.06	0.64	-0.13	0.88	11.04
FTSE-IT	0.04	0.49	-0.01	1.03	13.56
FTSE-AUS	0.04	0.35	-0.22	-0.50	9.04

MONTHLY COMPOUNDED DATA					
	Mean	Std.Deviation	Skewness	Kurtosis	CV
NYSE Composite	0.08	0.45	-0.28	0.67	5.47
NASDAQ composite	0.10	0.88	-0.54	1.39	8.76
TOPIX	-0.04	0.88	0.19	0.94	-22.06
FTSE-A	0.05	0.56	-0.11	0.17	12.25
DAX30	0.08	0.76	-0.57	1.57	10.09
TSX	0.04	0.62	-0.90	3.03	14.35
SMI	0.09	0.64	-0.18	1.48	7.36
HANG SENG	0.09	0.97	-0.13	1.34	10.50
COMIT	0.04	0.84	0.03	0.32	22.16
GOLDAM	-0.01	0.43	0.79	3.84	-43.88
FTSE-US	0.09	0.49	-0.36	0.36	5.67
FTSE-JA	-0.04	0.88	0.17	1.04	-24.72
FTSE-UK	0.05	0.56	-0.07	0.11	11.94
FTSE-GE	0.06	0.74	-0.67	1.78	12.37
FTSE-CA	0.05	0.61	-0.72	2.48	12.55
FTSE-SW	0.09	0.62	-0.21	1.25	6.88
FTSE-HK	0.07	1.00	0.08	1.43	14.66
FTSE-IT	0.03	0.87	-	0.23	25.36
FTSE-AUS	0.04	0.64	-0.20	-0.18	16.71

WEEKLY COMPOUNDED DATA					
	Mean	Std.Deviation	Skewness	Kurtosis	CV
NYSE Composite	0.08	1.08	-0.19	3.66	13.11
NASDAQ Composite	0.10	1.81	-0.68	2.97	17.75
TOPIX	-0.04	1.70	0.24	1.55	-44.27
FTSE-A	0.05	1.20	-0.37	1.64	25.53
DAX30	0.08	1.74	-0.50	2.73	22.89
TSX	0.04	1.22	-0.74	2.87	27.92
SMI	0.09	1.43	-0.43	2.10	16.32
HANG SENG	0.09	2.09	-0.94	7.98	22.52
COMIT	0.04	1.72	-0.24	1.01	42.47
GOLDAM	-0.01	0.93	0.60	8.81	-102.20
FTSE-US	0.09	1.19	-0.19	2.92	13.81
FTSE-JAPAN	-0.03	1.71	0.25	1.56	-50.44
FTSE-UK	0.05	1.24	-0.31	1.67	25.73
FTSE GERMANY	0.06	1.67	-0.50	3.01	27.74
FTSE-CANADA	0.05	1.23	-0.64	2.63	25.05
FTSE-SWISS	0.09	1.40	-0.42	2.32	15.38
FTSE-HONGKONG	0.07	2.11	-0.88	7.64	30.71
FTSE-ITALY	0.04	1.83	-0.16	1.05	50.00
FTSE-AUSTRALIA	0.04	1.33	-0.57	3.38	34.10

NYSE Composite- New York Stock Exchange Composite Index (USA) ; NASDAQ Composite – National Association of Security Dealers Automated Quotations Composite Index (USA) ; TOPIX – Tokyo Stock Exchange Composite Index (Japan) ; FTSE-A – Financial Times London Stock Exchange All Shares Composite Index (UK) ; DAX30 – Deutsche Bourse 30 Largest Shares Composite Index (Germany) ; TSX – Toronto Stock Exchange Composite Index (Canada) ; SMI – Swiss Market Composite Index (Switzerland) ; HANG SENG – Hong Kong Stock Exchange Composite Index (Hong Kong) ; COMIT – Milan Stock Exchange Composite Index (Italy) ; GOLDAM – London Bullion Market AM Indicative Fix for Spot Gold ; FTSE-US – Financial Times Stock Exchange Index – United States All Markets ; FTSE-JA– Financial Times Stock Exchange Index – Japan All Markets ; FTSE-UK– Financial Times Stock Exchange Index – United Kingdom All Markets ; FTSE-GE– Financial Times Stock Exchange Index – Germany All Markets ; FTSE-CA– Financial Times Stock Exchange Index – Canada All Markets ; FTSE-SW– Financial Times Stock Exchange Index – Switzerland All Markets ; FTSE-HK– Financial Times Stock Exchange Index – Hong Kong All Markets ; FTSE-IT– Financial Times Stock Exchange Index – Italy All Markets ; FTSE-AUS– Financial Times Stock Exchange Index – Australia All Markets . There are 60 Quarterly, 182 monthly and 702 weekly observations

Table 2 **Normality Tests of Indices 1988-2003**

	QUARTERLY DATA		MONTHLY DATA		WEEKLY DATA	
	KS	JB	KS	JB	KS	JB
NYSE Composite	.19	0.03	0.32	0.05	0.04	0.00
NASDAQ Composite	.20	0.01	0.12	0.00	0.00	0.00
TOPIX	.80	0.15	0.67	0.02	0.11	0.00
FTSE All Shares	.97	0.63	0.84	0.75	0.01	0.00
DAX	.20	0.00	0.11	0.00	0.04	0.00
TSX Toronto	.51	0.00	0.02	0.00	0.01	0.00
SMI Zurich	.70	0.00	0.42	0.00	0.06	0.00
Hang Sen	.89	0.55	0.29	0.00	0.00	0.00
COMIT Milan	.83	0.21	0.83	0.67	0.04	0.00
Gold AM Fixing	.91	0.28	0.32	0.00	0.00	0.00
FTSE-US	.17	0.06	0.36	0.08	0.01	0.00
FTSE-JA	.72	0.03	0.58	0.01	0.15	0.00
FTSE-UK	.98	0.61	0.95	0.89	0.05	0.00
FTSE-GE	.26	0.00	0.12	0.00	0.01	0.00
FTSE-CA	.78	0.00	0.12	0.00	0.00	0.00
FTSE-SW	.84	0.03	0.48	0.00	0.12	0.00
FTSE-HK	.81	0.33	0.67	0.00	0.01	0.00
FTSE-IT	0.76	0.26	0.73	0.82	0.06	0.00
FTSE-AUS	0.91	0.57	0.98	0.49	0.28	0.00
FTSE-SP	00.45	0.60	0.81	0.00	0.13	0.00

KS is the Kolomogorov-smirnov test, JB the Jarque-Bera test, values are marginal probabilities, Ho of Normality.

6 - Results

We first of all show in details of the optimal portfolios formed over the entire period 1988-2003. We do not initially present results for differing levels of tradeoffs between mean and skewness. As noted, this is motivated by the knowledge that such an exercise would, while interesting, merely reflect ad-hoc selections of weights. In the absence of theoretical justification for particular weights we prefer to defer this. A number of elements are clear.

In common with previous researchers we find that the idea of MV portfolios dominating any other portfolio at a given level of variance is preserved. All MV portfolios provide superior mean returns when compared to MVS portfolios. This holds across the various horizons and across the two different sets of indices. However, we also find that skewness measures of the MVS portfolios are considerably larger than those of the MV portfolios. This appears to confirm the idea that investors may trade off return for skewness.

We also see that there are significant differences between the portfolio weights as between the MV and MVS portfolios. In particular, a number of countries markets are selected only under one or the other. Looking at the FTSE indices first, we note that there is a much greater number of indices selected as we move investment horizon from quarterly through to weekly. The US, UK and Japanese indices are not selected under MV or MVS conditions at quarterly, while the US only is selected under MV conditions at weekly or monthly horizons. However, when we take account of skewness both Japan and the UK (with the UK very much smaller weighted) are chosen. No country chosen under MV portfolios is not also chosen under MVS portfolios, but Germany is the only market that is chosen across all investment horizons and both MV/MVS optimizers. Japan, the UK, Italy and Australia are only ever chosen under the MVS optimizer.

Gold plays a small but consistent part when we consider the FTSE indices as the alternative. It is selected, with a small weight, under all horizons and across both optimizers. Perhaps surprisingly its weight declines under the MVS optimizer. However, this is perhaps not surprising as in only one case, Germany at quarterly horizons, do we see a market increasing its weight when we move from the MV to the MVS portfolio.

When we examine the 'national' indices we see a similar pattern emerging. Again the number of indices selected under the MVS optimizer is generally larger than under the MV. In no case do we see an asset increasing its allocated weight under the MVS optimizer. Surprisingly the domination of the German market is not maintained. In the case of national indices the NYSE composite index and the SMI index from the Zurich exchange are the only such indices selected across all horizons and optimizers. The NYSE index is very lowly weighted however except at the very short horizon, while the SMI index is more heavily weighted, although these weights do decline as between the MV and MVS optimizers. The NASDAQ composite index is chosen in the MV portfolio with a high weight in all horizons, but not in the MVS portfolio, where the weight is lower and it is not selected at the quarterly horizon.

The importance of gold again emerges. In this universe of assets it is not as prevalent, but when it is selected it is with a high weight. Thus in the MVS quarterly horizon a weight of 25% is suggested, the joint highest with the COMIT index from Milan; in the MVS monthly a weight is suggested higher than that given to the DAX, the TSX index from Toronto, or the Hang Sen index from Hong Kong

There is no obvious relationship between the rankings of the indices in terms of their coefficient of variation and the weights suggested in the MV portfolio. Gold is consistently selected against 'better' performing indices. At the quarterly horizon the MV portfolio is mainly composed of middle ranking indices in terms of their risk-return payoff, while more and more indices are added as we shorten the frequency. Skewness, in the MVS portfolio, is paid off immediately at the longer horizon. For shorter investment horizons all indices are included as part. At the shorter horizons the ranking of weights for the MVS portfolio is close to the rankings of the skewness coefficients. The relationship between the rankings for MVS portfolio weights and the skewness coefficients is generally higher than that for the MV portfolio weights and the coefficient of variation rank

Although not strictly comparable, due to the different universes that the portfolios are selected from, it is instructive to compare the results here with those for Chunhachinda, Dandapani et al. (1997) and Prakash, Chang et al. (2003). Both these papers use the MSCI indices, which are, similar to the FTSE indices used here, designed to allow easy comparison across markets and assets. However, Prakash allows short sales and as such is less easily again compared.

On the monthly horizon Chunhachinda, Dandapani et al. (1997) find a high (33%) weight for Hong Kong, which is not dissimilar to that found here. However, while in Chunhachinda, Dandapani et al. (1997)this holds across the MV and MVS portfolios this is not the case here. Another point of dissimilarity is that Chunhachinda, Dandapani et al. (1997)allocate lower weights (13-28%) to the US across the two portfolios than is the case here. Prakash, Chang et al. (2003)allocates a very large (116%) weight to the USA and to Switzerland (67%), which weights are significantly in excess of the weights here. However these weights are only for the MV optimizer – when selecting in the MVS optimizer these countries are not selected.

As noted, there is no theoretical guide as to the choice of tradeoff weights. Shown in are the results of changing weights on the two parameters of interest. We note a number of interesting elements. Low skewness and low mean weights we select less gold and less NYSE, and more FTSE and COMIT. Higher mean weights result in selection of more SMI and HangSeng. These relationships are not linear however, indicating that the algorithm trades-off weights. Increasing the relative weight on the mean return results in increased skewness if we havea high preference for skewness also and less skewness if we have a relatively low. The mean return is very sensitive to skewness if we have a modest taste (Skew=1) but not if we have a high taste (Skew=2). The results indicate that selection of an optimal portfolio when we have a preference for skewness leads to significant changes in asset allocation if we have a modest preference but not as dramatic changes when we have a higher preference for skewness.

Table 3 **Optimal Portfolios 1988-2003: Weight Allocated to Each Asset**

	QUARTERLY DATA		MONTHLY DATA		WEEKLY DATA	
	Mean-Variance	Mean-Variance-Skewness	Mean-Variance	Mean-Variance-Skewness	Mean-Variance	Mean-Variance-Skewness
GOLDAM	0.04	0.04	0.06	0.03	0.05	0.02
FTSE-US	-	-	0.28	0.28	0.20	0.12
FTSE-JAPAN	-	-	-	0.21	-	0.28
FTSE-UK	-	-	-	0.09	-	0.08
FTSE-GERMANY	0.25	0.28	0.12	0.03	0.15	0.06
FTSE-CANADA	0.25	0.14	-	0.05	0.20	0.11
FTSE-SWISS	-	-	0.28	0.11	0.20	0.09
FTSE-HONGKONG	-	-	0.28	0.05	0.20	0.05
FTSE-ITALY	0.21	0.28	-	0.14	-	0.08
FTSE -AUSTRALIA	0.25	0.28	-	0.02	-	0.05
Mean Return %	1.14	1.09	8.04	4.52	4.42	2.08
Skewness	1.2134	134.41	-182.05	18.14	-76.19	0.00
Total Invested	100%	100%	100%	100%	100%	94%
GOLDAM	-	0.25	-	0.07	0.14	0.12
NYSE COM	0.00	0.01	0.00	0.00	0.04	0.32
NASDAQ COMM	0.25	-	0.28	0.10	0.28	0.07
TOPIX	-	0.03	-	0.20	-	0.31
FTSE-A	-	-	-	0.09	-	-
DAX30	0.25	-	0.17	0.05	-	0.02
TSX	-	-	-	0.06	-	0.01
SMI	0.25	0.01	0.28	0.10	0.28	0.10
HANGSENG	0.25	-	0.28	0.07	0.28	0.04
COMIT	-	0.25	-	0.12	-	-
Mean Return %	8.63	1.10	9.01	3.17	7.89	0.03
Skewness	- 7.04	33.65	-1871.63	0.00	-2.33	0.17
Total Invested	100%	29%	100%	80%	100%	1.00

Table 4 **Mean-Skewness tradeoffs**

	Skew=1						Skew=2					
	Mean=.5	Mean=1	Mean=1.5	Mean=2	Mean=2.5	Mean=3	Mean=.5	Mean=1	Mean=1.5	Mean=2	Mean=2.5	Mean=3
GOLDAM	0.11	0.11	0.12	0.12	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.12
NYSE COM	0.21	0.28	0.28	0.28	0.35	0.42	0.34	0.33	0.33	0.32	0.32	0.32
NASDAQ	-	-	0.05	0.07	0.06	-	0.06	0.07	0.07	0.07	0.07	0.07
TOPIX	0.28	0.28	0.28	0.28	0.28	0.21	0.28	0.30	0.31	0.31	0.31	0.31
FTSE-A	0.01	0.03	0.05	0.03	-	0.03	-	-	-	-	-	-
DAX30	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02
TSX	0.05	0.06	0.07	0.06	0.03	0.05	0.02	0.01	0.01	0.01	0.01	0.01
SMI	0.04	0.05	0.07	0.08	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
HANGSENG	-	0.01	0.02	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
COMIT	0.02	0.03	0.03	0.02	-	-	-	-	-	-	-	-
Mean Return	0.01	0.02	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Skew	0.17	0.17	0.14	0.12	0.10	0.02	0.12	0.15	0.17	0.17	0.17	0.17
Percent Invested	0.73	0.85	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

7 - Conclusion

This paper has demonstrated a number of findings. First, in common with others, we show that the incorporation of skewness as an objective in portfolio selection causes the optimal portfolio to change significantly from one formed only under conditions of mean-variance analysis. Second, we show that the selection of the set of assets is important. When we examine national indices as opposed to a set of indices (FTSE World Indices) that are designed explicitly to allow comparisons we find marked differences as between the optimal portfolios. This holds regardless of whether we are interested in mean-variance or mean-variance-skewness optimal portfolios. Third, the shorter the investment horizon the more assets are included in the optimal portfolios. Finally, we show that under most circumstances gold bullion has an important role to play in the creation of an optimal portfolio. We are not aware of any paper to date that has examined the role of gold in a mean-variance-skewness portfolio. The practical implications of the results are, we consider, threefold. First, the PGP approach is tractable with real life data, and thus allows us to consider in a programming framework the selection of weights where skewness is selected as a desirable property of the portfolio. This is important as there may well be agents that are willing to trade off reduced returns for the increased likelihood that these returns will not be negative. Pension funds and endowments are immediate candidates that come to mind. The second implication is that there is a continued role for gold. Whether under standard mean-variance analysis or under more complex PGP methods, we find a consistent role for gold in a portfolio. This is especially evident when the equity elements of the portfolio are directly comparable one with the other such as the FTSE indices. A third important practical consideration is that of the juxtaposition of the first two – gold has a significant attraction to investors that wish to select for positive skewness, even if they wish to keep the vast majority of their wealth in equities.

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