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Climate Policy Under Fat-Tailed Risk: An Application of Dice

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Abstract: Uncertainty plays a significant role in evaluating climate policy, and fat-tailed uncertainty may dominate policy advice. Should we make our utmost effort to prevent the arbitrarily large impacts of climate change under deep uncertainty? In order to answer to this question we propose an new way of investigating the impact of (fat-tailed) uncertainty on optimal climate policy: the curvature of carbon tax against the uncertainty. We find that the optimal carbon tax increases as the uncertainty about climate sensitivity increases, but it does not accelerate as implied by Weitzman's Dismal Theorem. We find the same result in a wide variety of sensitivity analyses. These results emphasize the importance of balancing of the costs and the benefits of climate policy, also under deep uncertainty.

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Key words: Climate change; decision making under uncertainty; fat-tailed risk; integrated assessment

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1. Introduction

Everything about climate change is uncertain. Although this has been acknowledged for a long time, a recent paper by Weitzman (2009a) emphasized the importance of conceptualizing climate policy as risk management. Weitzman (2009a) formalizes an earlier suspicion by Tol (2003): there is good reason to believe that the uncertainty about the impacts of climate change is fat-tailed.[1] That is, the variance or even the mean of the distribution of the objective value (welfare) may not exist. This violates the axioms of decision making under uncertainty.

Unfortunately, Weitzman (2009a) and Tol (2003) diagnose the problem but do not offer a solution. Anthoff and Tol (2010) propose alternative decision making criteria. In this paper, we follow a different track: we keep the standard decision making criterion of utility maximization but investigate how the uncertainty around key parameters of the climate change affects the optimal decision of economic agents. This is a worthwhile course of action because Weitzman (2009a) and Tol (2003) only consider scenarios without climate policy. It may be that greenhouse gas emission reduction thins the tail.

Considering the impacts of climate change but ignoring the impacts of greenhouse gases (GHG) emission reduction, Weitzman's characterization of climate policy is incomplete. As shown by Hennlock (2009), this materially affects the results. Intuitively, the reasoning is something as follows. Weitzman (2009a) argues that the certainty-equivalent of the marginal damage cost of carbon dioxide emissions is arbitrarily (or infinitely) large. Taken at face value, this implies that an arbitrarily large carbon tax should be imposed – or that emissions should be driven to zero immediately. That is a silly implication. It is currently impossible to grow sufficient amounts of food to feed the world population and transport crops from the fields to the population centres without fossil fuels. It may well be possible to do without fossil fuels in 50 years time, but imposing a carbon-neutral economy in 50 days time would lead to widespread starvation. In the short term (but not in the long term), fossil fuels are a necessary input. Put differently, the costs of ultra-rapid abatement are arbitrarily large as well, with the difference that they are known with more certainty than the impact of climate change. The theoretical argument of Weitzman could thus be reversed in case of very high costs of ultra-rapid abatement since the expectation of the damages of a rapid emission reduction policy may be infinite. This raises the question of how policy makers should weight two extreme (or infinite) costs in their decision making. In many aspects, this is more a philosophical than an economic question since infinity is not economically quantifiable.

[1] There is no consensus on the exact definition of the term 'fat (or heavy) tail' (Nordhaus, 2009). However, most climate change economists currently use the term proposed by Weitzman (e.g., Newbold and Diagneault, 2009; Dietz, 2010; Pindyck, 2010): "a PDF has a fat tail when its moment generating function is infinite - that is, the tail probability approaches 0 more slowly than exponentially" (Weitzman, 2009: 2). We also follow this definition in this paper.

Therefore, the economically optimal climate policy under uncertainty, whether fat-tailed or not, requires a balancing of the impacts of climate change and the costs of emission reduction. In this paper, we analyze Weitzman's Dismal Theorem in an economic decision framework. We use a well established numerical Integrated Assessment Model (IAM), namely the DICE model. While less general, our approach is more flexible, more realistic, and more accessible. As uncertainty is bounded by definition in a numerical framework, we introduce a method for analyzing fat-tails using thin-tailed distributions.

As far as the uncertainty about climate change is concerned, simplified analytical models (e.g., Ulph and Ulph, 1997; Ulph and Maddison, 1997; Gollier et al., 2000; Fisher and Narain, 2003; Baker, 2005; Ingham et al., 2007) or numerical models such as IAMs (e.g., Manne and Richels, 1992, Peck and Teisberg, 1993, 1995; Kolstad, 1996; Nordhaus and Popp, 1997; Ulph and Maddison, 1997; Gjerde et al., 1999; Kelly and Kolstad, 1999; Pizer, 1999; Tol, 1999; Webster, 2002; Baranzini et al., 2003; Keller et al., 2004; Yohe et al., 2004; Albertth and Hope, 2007; Leach, 2007; McInerney and Keller, 2007; Bosetti et al., 2009) based on cost benefit analysis (CBA) have been mainly used since the early 1990s. Fat-tailed uncertainty, however, is not considered in those models.

Meanwhile, CBA based on expected utility (EU) theory have been criticized for failing to account for structural (or deep) uncertainty (Tol, 2003; Weitzman, 2009a). As a response to this criticism, on the one hand, alternative approaches such as the ambiguity aversion framework (e.g., Lang and Treich, 2008) and the minimax-regret criterion (e.g., Anthoff and Tol, 2010) have been tried. On the other hand, within the framework of EU, there have been applications of IAMs introducing the fat-tailed uncertainty about key parameters (e.g., Roughgarden and Schneider, 1999; Mastrandrea and Schneider, 2004; Newbold and Daigneault, 2009; Ackerman et al., 2010; Dietz, 2010; Ikefuji et al., 2010). However, these studies differ from ours because they do not directly investigate how the optimal climate policy responds to uncertainty. Instead they generate samples of draws of key parameters assuming a fat-tailed distribution or stochasticity and then perform a deterministic simulation of their model for each value of the parameters. Such an experiment is more a wide sensitivity analysis than a real definition of the optimal choice under uncertainty. They give the optimal choice for each possible value of the parameters but do not say what would actually be the optimal decision given this uncertainty. That is, they compute the expectation of maximum welfare rather than maximize expected welfare. Moreover, they do not investigate the impact of alternative levels of uncertainty, and therefore do not gain insight into the effect of increasing uncertainty on climate policy.

In contrast, we focus on the optimal choice given uncertainty by assuming that the agent chooses the optimal abatement speed that maximizes the expected utility. We also investigate how the optimal choice is affected if the uncertainty on the climate sensitivity increases. Hennlock (2009) does this analytically. Instead we use a simulation model by assuming a fat-tailed distribution of the risk. We run different simulations of DICE for increasing value of the variance. In other words, we increase the tail of the distribution and look at the evolution of the optimal choice in order to answer the following questions. How does the optimal carbon-tax behave when the uncertainty about climate change varies?

Does it have an upper bound or not? Do we get the Weitzman effect where the optimal tax increases to an arbitrarily high number if the variance grows? Is the link between the optimal tax and uncertainty sensitive to the specification of the damage function, the abatement-costs function or of the utility function? For simplicity we ignore learning effects - the fact that at least a part of uncertainty may vanish over time -which would highly complicate the resolution of the maximization program.

The paper proceeds as follows. Section 0 presents the model and the scenarios. Section 0 discusses the results for the case in which the climate sensitivity is the only uncertain parameter. Section 0 shows seven sensitivity analyses: [1] an alternative parameterization of the uncertainty about the climate sensitivity; [2] an alternative method of increasing uncertainty; [3] the shape of the damage function; [4] the uncertainty about both the climate sensitivity and the impact of climate change; [5] the uncertainty about both the climate sensitivity and the cost of GHG emission reduction; [6] an alternative utility function accounting for nonmarket (ecological) goods, and [7] an alternative coefficient of constant relative risk aversion. Section 0 concludes.

2. The model

2.1. Representation of uncertainty

As detailed in Appendix A, we amend the DICE model using the notion of 'state of the world' in order to introduce uncertainty into the model (Manne and Richels, 1992). Although almost all parameters in an economic model on climate change are more or less uncertain, we introduce uncertainty only about climate sensitivity for mainly three reasons. First, this parameter plays a significant role in the results of IAMs. Second, its uncertainty has been relatively well investigated (Solomon et al., 2007) where very little is known about the uncertainty surrounding most other parameters. Third, the limits of the GAMS language do not allow for including simultaneously all uncertainties in IAMs.

We denote a set of possible values of climate sensitivity by SW and an individual state by s. We assume that SW is finite and the lower and the upper bound of climate sensitivity is 0°C and 25°C, respectively.[2] The number of states of the world is set at 1,000, and thus climate sensitivity in each state of the world increases by 0.025°C in our model.[3] This paper also recalibrates the speed of adjustment in the atmospheric temperature equation of the DICE model so that the modeled temperature-increases from pre-industrial to present times is in line with the observed warming. This procedure also circumvents the infeasibility, which has been reported in applications of DICE when the value of climate sensitivity is lower than

[2] We find that the main implications of this paper are hardly affected by increasing the upper bound of climate sensitivity, even up to 500°C/2x[CO_2]. The computational limit prevents from increasing the upper bound further without violating the 1% threshold of maximum loss of information we allow in this paper (see the explanation in Section 0).

[3] Our model is solved with a nonlinear programming algorithm named CONOPT (version 3.14S) in the GAMS modeling system. The model involves 391,451 endogenous variables for a single-uncertainty run and it is not possible to enlarge the number of states over 1,000 using the Core 2 Duo (3.0 GHz, 2.99 GHz) Intel Processor with 3.46 GB of RAM.

about 0.5°C (e.g., Ackerman et al., 2010). See Appendix B for the method of calibration in detail.

The objective function of our model including the uncertainty about climate sensitivity is as follows.

$$\max_{c(t,s)} W = \sum_{s=1}^{S} \sum_{t=1}^{T} p(s) U\big(c(t,s), L(t)\big) R(t) \qquad (1)$$

where W denotes expected net present social welfare, S denotes the total number of states of the world s, T denotes the total number of time periods t, $p(s)$ is the probability of each state of the world, $U\big(c(t,s), L(t)\big)$ is the utility function, $c(t,s)$ is the flow of consumption per capita at time t and state s, $L(t)$ is the level of labor force at time t, and $R(t)$ is the discount factor.

Attaching subscript s into the relevant variables, we associate of each state s with the utility of per capita consumption. Then, solving Equation (1) we analyze the impacts of the uncertainty about climate sensitivity on the discounted sum of expected utility of per capita consumption or social welfare.[4]

2.2. Probability distribution

We need to specify the probability of each state of the world in order to solve the objective function. To this end, we use the probability density function (PDF) of climate sensitivity derived from the framework of feedback analysis.[5] In this framework, the probability distribution of climate sensitivity can be drawn from the probability distribution of feedback factors. The term 'feedback factors' refers to the impacts of physical factors such as water vapor, cloud, albedo etc. to radiative forcing in a way of amplifying the response of climate system (Hansen et al., 1984). The total feedback factors has the following relationship with the equilibrium climate sensitivity (Roe and Baker, 2007).

$$CS = CS_0/(1-f)$$

(2)

where f denotes the total feedback factors, CS denotes the equilibrium climate sensitivity, and CS_0 =1.2°C denotes the climate sensitivity in a reference system (without any feedback factors like a blackbody planet).

Assuming that feedback factors has a normal distribution with mean $\bar{f}$ and standard deviation σ_f as usually assumed in general circulation models (GCMs) and that climate sensitivity is related to feedback factors according to Equation (2)

, the probability density of climate sensitivity is given as follows (Roe and Baker, 2007).

[4] This is assured by the 'substitution rule' in probability theory. If climate sensitivity $CS_i \in SW$ takes on a value of probability p_i utility U, by a series of mapping from CS_i to temperature increases, to damage costs, to consumption, etc., also has the same probability p_i.

[5] For more detailed discussion on the framework of feedback analysis, see Roe (2009) and the references therein.

$$f(CS) = \left(\frac{1}{\sigma_f\sqrt{2\pi}}\right)\frac{CS_0}{CS^2}exp\left\{-\frac{1}{2}\left[\frac{\left(1-\bar{f}-\frac{CS_0}{CS}\right)}{\sigma_f}\right]^2\right\} \quad (3)$$

Figure 1 depicts the probability distributions of climate sensitivity with different values of σ_f. ($\bar{f}$=0.65 and CS_0=1.2°C remain unchanged). Climate sensitivity is asymmetrically distributed (left-skewed) and σ_f strongly affects the distribution, especially the tails: the higher σ_f, the fatter the tails.[6] Climate sensitivity below a certain value, say 1.5°C, has a negligible density. This is because, by Equation (2)

, climate sensitivity cannot take on values less than CS_0 unless we let negative feedback (f<0), which is neither interesting in terms of global warming nor physically relevant (Roe, 2009).[7]

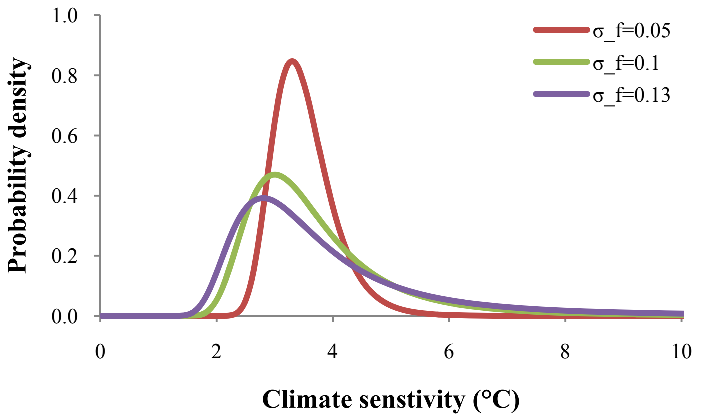

Figure 1. Climate sensitivity distributions from the framework of feedback analysis. σ_f denotes the standard deviation of feedback factors ($\boldsymbol{CS_0}$=1.2°C, $\boldsymbol{\bar{f}}$=0.65). Note that the graph is truncated and thus does not show the densities of climate sensitivity > 10°C.

The probability of each state of the world $p(s)$ in the objective function is derived from the following formula: $p(s) = f(CS) \times \Delta CS$, where $f(CS)$ is the probability density calculated from Equation (2)

and ΔCS is the interval of each climate sensitivity. This provides a discrete approximation of the continuous density function since the probability of a random variable, for small intervals, is nearly equal to the probability density times the length of the interval. This approximation, unless the interval is extremely small, causes an unfortunate loss of information. Thus this paper considers the results which have no more loss of information than 1%: the '1% criterion' for later reference. More precisely, the cumulative probability across the whole range of climate sensitivity should not be lower than 0.990. This 1% threshold is arbitrary but small enough to maintain a good approximation of the underlying

[6] The mean of feedback factors $\bar{f}$ also affects the climate sensitivity distribution, but this paper deals only with σ_f because $\bar{f}$ plays the similar role in increasing variance to σ_f.

[7] Note that the total feedback factors is assumed to be in the range of 0<f<1 in typical GCMs. If f<0, the system response is damped (or not amplified) by inclusion of feedback factors. If f≥1, an equilibrium state cannot be established (Roe, 2009).

uncertainty. Finally, in order to make the cumulative probability 100% in simulations, we use $p(s)/\sum p(s)$ as the probability of each state of the world.

2.3. Increasing uncertainty

We increase the uncertainty about climate sensitivity by stepwise increments of the standard deviation of feedback factors σ_f in Equation (3). This increases the variance of climate sensitivity which is a measure of the spread of the possible values of climate sensitivity. This method is frequently used in statistical applications, but it has a disadvantage in that it may not preserve the mean if the probability distribution of an uncertain variable is asymmetric. In our analysis, for instance, even if we assume that the total feedback factors f is normally distributed and fix the mean of feedback factors as a constant, the nonlinear relationship between feedback factors and climate sensitivity, specified by Equation (2)

, leads to the highly skewed distribution of climate sensitivity. In this case the mean alters while the variance increases.

The so-called mean-preserving spread (MPS) (Rothschild and Stiglitz, 1970) may be an alternative since it fattens tails without changing the mean. The MPS, however, has problems adapting to our model. In coherence with the climate model retained in Roe and Baker (2007) where the uncertainty is not directly about the climate sensitivity but about the value of the feedback factors, it seems logical to preserve the mean of feedback factors instead of the mean of climate sensitivity.

Moreover, the MPS approach raises intractable technical difficulties. First of all, in case of fat-tailed distributions, which we focus on, it is not possible to apply the method because fat-tailed distributions, by definition, do not have moment generating functions. In other words, there is no mean to preserve in fat-tailed distributions. Secondly, even in bounded distributions where we can calculate the mean and the standard deviation from the simulated PDF, it is not easy to apply the MPS procedure since there is no established way of doing it. One may think of an iterative way of taking densities from the centre and transferring them into the tails (Mas-Corell et al., 1995: ch.6), but this method produces several discontinuous jumps on probability distributions, which is scientifically irrelevant. Furthermore, the fact that we are not able to let the upper bound of climate sensitivity approach infinity in numerical models restricts the usage of the MPS. The process of variance-increasing allots an additional density to both tails of the distribution (see Figure 1). The probability additionally attached to the left tail dominates the one attached to the right tail in a bounded distribution. This is because the effect of ‘shift of density to the left tail’ would not be compensated for the effect of the ‘shift of density to the extreme right tail’: in a bounded model, the latter effect is rarely big enough. Unfortunately, the MPS procedure amplifies this imbalance by shifting more density to the left tail during the process of variance-increasing (See Figure 1).

For these reasons, and although the method of increasing variance raises some issues, we use it as the basic method of increasing uncertainty. In the application below, the mean increases with the variance so that we may overstate the effect of increasing uncertainty – in the context of our research, our basic method is therefore conservative. As a sensitivity analysis, however, we apply the MPS method in order to investigate how our main results are affected by the method of increasing uncertainty.

2.4. Detecting arbitrarily high carbon taxes

Increasing uncertainty affects the optimal level of emission reduction. Indeed, Weitzman (2009a) argues that climate policy should be arbitrarily ambitious if the tails of the uncertainty about welfare are fat. We use the carbon tax as our measure of the intensity of climate policy.[8]

In a numerical model with a finite number of states of the world, tails are necessarily thin in the sense that all empirical moments exist and are finite. Therefore, we devise an alternative way of analyzing the impacts of uncertainty on policy instead. We increase the variance of climate sensitivity in the truncated fat-tailed distribution, and then plot the resulting optimal carbon tax against the variance. If the optimal carbon tax increases and its curvature is convex (for instance, like that of an exponential function), we can deduce that the carbon tax become arbitrarily large when the uncertainty about climate change goes to infinity. This can be translated into the argument that we should put our utmost efforts to reduce carbon emissions at the present time. By contrast, if the carbon tax function of uncertainty is increasing and concave, that is, it has a diminishing rate of growth in carbon tax, we can postulate that there may be an upper bound on carbon tax even if uncertainty increases unboundedly.

2. Results

We gradually increase the standard deviation of feedback factors $\boldsymbol{\sigma_f}$ and then calculate the variance of climate sensitivity from the simulated PDF. The calculated mean and variance of climate sensitivity are listed in Table 1. The cumulative probability across the whole range of climate sensitivity is not equal to 1, especially when σ_f becomes greater than 0.11, because we use discrete programming and a truncated distribution. With the 1% criterion, σ_f cannot be higher than 0.13 in this simulation. The mean of climate sensitivity increases from 3.43 to 4.06 while the variance of climate sensitivity increases from 0.01 to 5.51.

Table 1 Simulated mean and variance of climate sensitivity

σ_f	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13
m_{cs}	3.43	3.44	3.45	3.48	3.50	3.54	3.59	3.65	3.72	3.80	3.89	3.98	4.06
σ_{cs}^2	0.01	0.04	0.09	0.17	0.29	0.46	0.71	1.11	1.69	2.47	3.41	4.45	5.51
$P(cs \leq cs_{up})$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.994	0.990

[8] The carbon tax is a diagnostic variable in the DICE model. It is indirectly calculated as the Pigou tax by the following equation: $carbon\ tax = -(\partial W/\partial E_t)/(\partial W/\partial K_t)$, where E_t and K_t denote carbon emissions and the capital stock at time t, respectively.

Note: σ_f denotes the standard deviation of feedback factors, m_{cs} and σ_{cs}^2 denote the mean and the variance of climate sensitivity, respectively, $P(cs \le cs_{up})$ is the cumulative probability across the range of climate sensitivity (0°C, 25°C]. Throughout all simulations the mean of feedback factors remains fixed at 0.65.

Figure 2 illustrates the relationship between the optimal carbon tax and the variance of climate sensitivity. We only present the initial carbon tax (in 2005) because it represents the optimal choice of current generations. In addition, the selection of a specific time-period does not affect the nature of our results. Selecting another time period gives a higher tax level but a similar evolution of the tax as the one presented in Figure 2. The first thing we can observe in Figure 2 is that the optimal carbon tax increases when the uncertainty about climate sensitivity increases. This can be interpreted as follows: the risk-averse social planner would be willing to make more efforts to reverse the adverse impacts of climate change if the uncertainty increases. Secondly and more importantly, we can notice that the carbon-tax function is concave in uncertainty. This feature holds even when the variance of climate sensitivity goes up to 589.[9] Figure 2 does not show the behavior we would expect from the dismal theorem of Weitzman (2009a). Rather it implies that the optimal carbon tax may be upper bounded when uncertainty increases.

In order to examine the robustness of our findings, we also run our model with alternative assumptions in the next section: different probability distribution, different method of increasing uncertainty, different parameterization of damage function and abatement-costs function, different form of utility function, and different level of risk aversion.

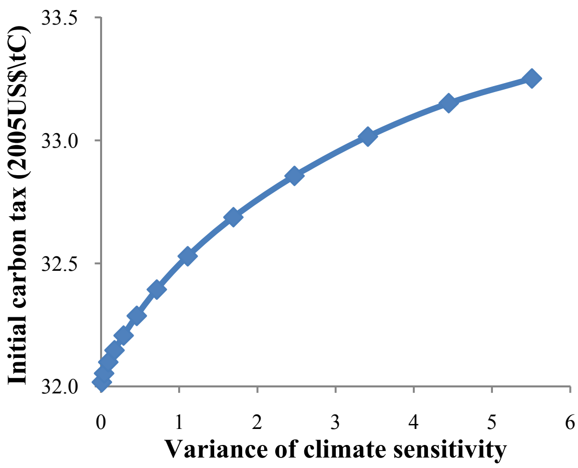

Figure 2 Carbon tax against uncertainty about climate sensitivity. All results within the 1% criterion are shown in this graph. Note that the initial carbon tax in deterministic run ($28.6/tC, when $\boldsymbol{\sigma_f}$ =0), is not depicted in the graph and thus the first dot in the left-bottom of the figure represents the case when $\boldsymbol{\sigma_f}$ =0.01.

4. Sensitivity analysis

[9] When the upper bound of climate sensitivity is set to 500, the variance of climate sensitivity can be increased up to 589 within our 1% criterion.

4.1. Alternative distribution

We assume that climate sensitivity has the lognormal distribution given by Equation (4) with parameters $\mu = 1.071$ and $\sigma = 0.527$ as in Ackerman et al. (2010).[10]

$$f(CS) = \frac{1}{\sigma\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left[\frac{(\ln(CS) - \mu)}{\sigma}\right]^2\right\} \tag{4}$$

We increase σ from 0.05 to 0.90 and then calculate the variance of climate sensitivity from the simulated PDF. The reason that we stop increasing σ at 0.90 is the same as in the previous section: the 1% criterion. Other assumptions including the number of states of the world, the range of climate sensitivity, the equations of the model remain the same as in the previous section.

The PDFs of climate sensitivity following Equation (4) are depicted in Figure 3. The tails become fatter as the parameter σ gets higher. One of the main differences between the lognormal distribution and the Roe-Baker distribution is that the lognormal distribution allows low climate sensitivity (say, below 1.5°C) to have non-negligible densities. There is also a loss of information when σ becomes higher, but in this case the loss is mainly at the lower end of climate sensitivity. This implies that there is a limit to the variance of climate sensitivity.

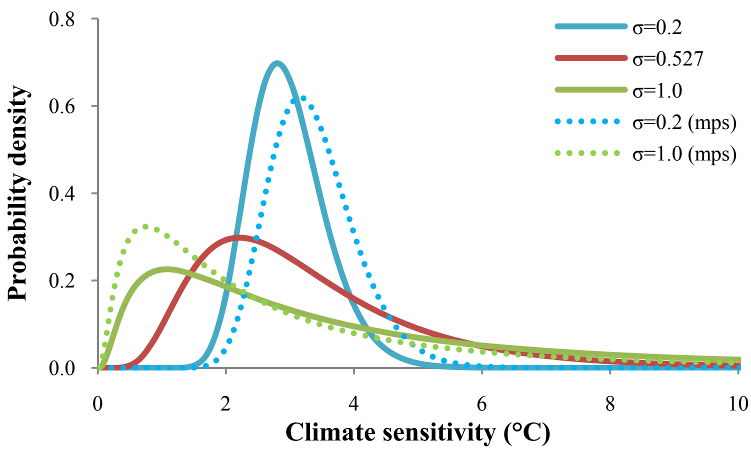

Figure 3 Lognormal distribution of climate sensitivity. σ denotes the standard deviation of the logarithm of climate sensitivity, mps denotes a probability density function drawn from the method of mean-preserving spread. The mean of the logarithm of climate sensitivity is set to 1.071°C for all distributions. Note that the graph is truncated and thus does not show the densities of climate sensitivity higher than 10°C.

Figure 4 gives the relationship between the optimal carbon tax and the variance of climate sensitivity. For low variances, the optimal carbon tax decrease in the variance. This is explained by the fact that we use a truncated probability distribution. As we can see from Figure 3, increasing the variance fattens the tails both right and left. If the variance of the

[10] They calibrated the parameter values to the probability-estimates in Weitzman (2009a) and Solomon et al. (2007).

climate sensitivity is low (high) – below (above) 0.58 in our parameterization – the fattening of the left (right) tail dominates and the carbon tax falls (rises) as the variances increases. For substantial uncertainty about the climate sensitivity, the optimal carbon tax increases but its rate of growth diminishes as the variance of climate sensitivity becomes higher.

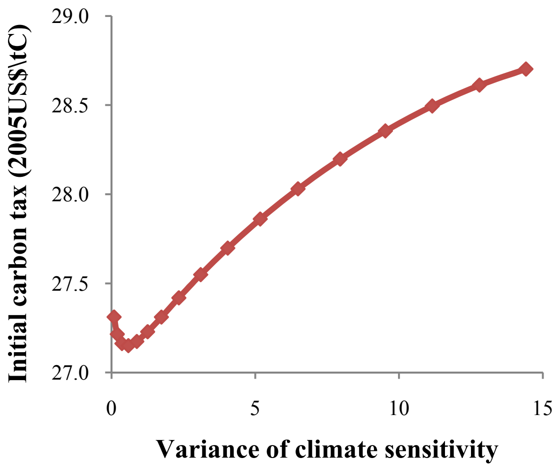

Figure 4 Carbon tax against uncertainty about climate sensitivity: calculated from the lognormal distribution.

4.2. Mean- and mode-preserving spread

We also run our model with a MPS procedure. Since the mean, mode and variance of the lognormal distribution are calculated as Equation (5), we can preserve the mean (mode) while we increase the variance by adjusting the parameter values μ and σ.

$$E[CS] = \exp(\mu + \sigma^2/2) \tag{5}$$
$$M[CS] = \exp(\mu - \sigma^2) \tag{6}$$
$$var[CS] = \exp(2\mu + \sigma^2) \times (\exp(\sigma^2) - 1) \tag{7}$$

Using the same specification as in Figure 4, the optimal carbon tax decreases as the variance increases with the MPS procedure. See Figure 5. This is because the mode of the climate sensitivity falls as the variance increases. If we run the model with the mode-preserving spread, the optimal carbon tax increases and its rate of growth decreases as the variance of climate sensitivity increases.

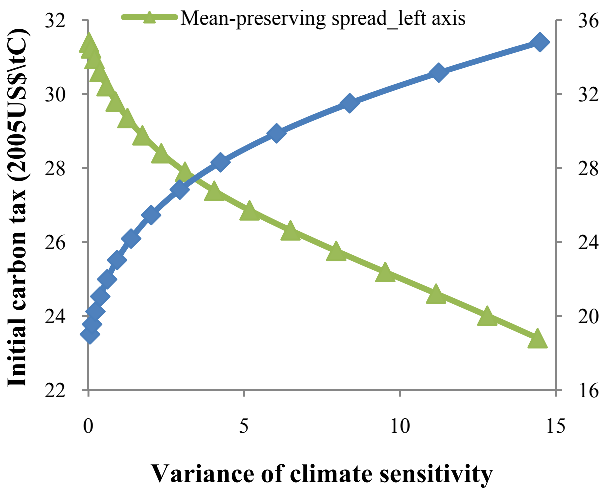

Figure 5 Carbon tax against uncertainty about climate sensitivity: according to the methods of mean-preserving spread and mode-preserving spread.

4.3. The shape of the damage function

The shape of the damage function is obviously important to the results. If large warming would have more severe impacts on damages, the effect of a fatter right tail of the climate sensitivity would be larger. Thus we replace the damage function in DICE with the functional form and the parameter values from Weitzman (2010):

$$\Omega(t,s) = 1/[1 + \pi_1 T_{AT}(t,s) + \pi_2 T_{AT}(t,s)^2 + \pi_3 T_{AT}(t,s)^{\pi_4}] \qquad (8)$$

where $\Omega(t,s)$ denotes the impacts of temperature-increases on damage costs (1-$\Omega(t,s)$ is the damage costs as a fraction of gross world output), $T_{AT}(t,s)$ is the global mean surface air temperature increases from 1900, π_1=0, π_2=0.0028388, π_3= 0.0000050703, π_4=6.754 are parameters. The original damage function in the DICE model obtains for π_3=0 and π_4=0.

Introducing the polynomial term hardly affects the damage ratio at low temperature increases, but the ratio greatly increases from the warming of 3°C or more as in Figure 6. Thus the damage function becomes more reactive to high temperature increases than the original DICE model.

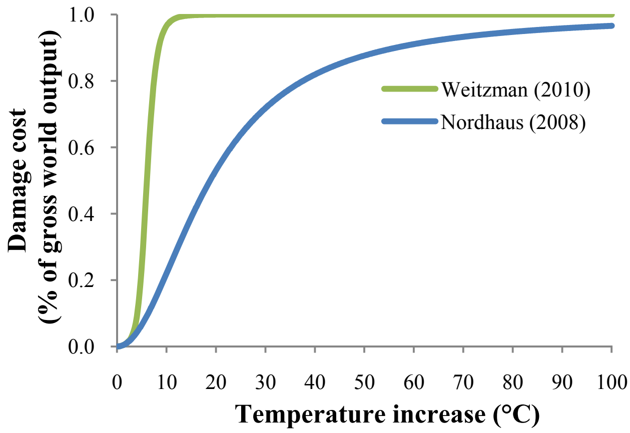

Figure 6 Damage costs against atmospheric temperature-increases.

Figure 7 shows the results of the MPS procedure. Whereas the optimal carbon tax decreases in the variance of the climate sensitivity with the original, quadratic damage function, the carbon tax increases in uncertainty for the more non-linear function of Weitzman (2010). Strikingly, although the carbon tax is concave in the uncertainty of climate sensitivity, it is barely so. When the upper bound of climate sensitivity is 25ºC, the concavity is hardly visible (green line in Figure 7) but it becomes visually detectable when the upper bound is increased to 100ºC (blue line in Figure 7).

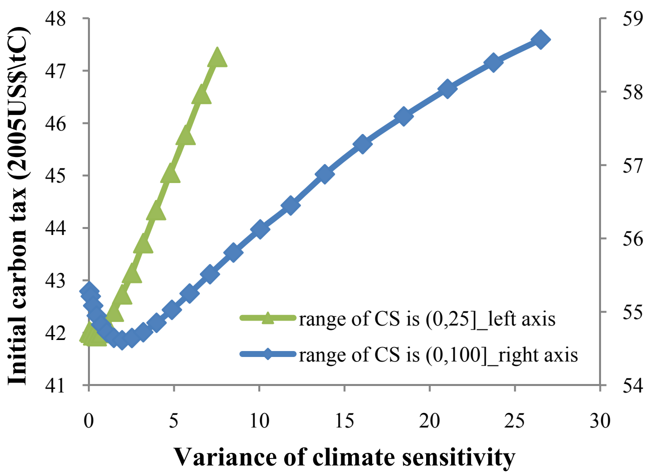

Figure 7 Carbon tax against uncertainty about climate sensitivity: calculated from the method of MPS using Weitzman (2010)'s damage function. The limit of the variance of climate sensitivity (horizontal axis) is different because we apply the 1% criterion.

4.4. Uncertainty about the impact of climate change

The damage function in the DICE model is as Equation (8) without the polynomial term $\pi_3 \times T_{AT}(t,s)^{\pi_4}$. Nordhaus (2008) calibrates π_1=0, π_2=0.0028388 to his own estimates for climate damages: that is, the ratio of damages to global output for a warming of 2.5°C equals to 1.77%. Beside Nordhaus (2008), a number of estimates of the damage costs have been published and the parameter values of the damage function will be different according to their estimates. We introduce the uncertainty about the damage costs through a PDF of the coefficient of quadratic term π_2.[11] In order to calibrate π_2, we use the published estimates for climate damages induced by a warming of 2.5°C (Tol, 2009).[12] We assume that each estimate is a random draw from the normal distribution; the mean μ and the standard deviation σ of the relative damage are calculated as 0.40% and 1.37% of GDP, respectively. Some of the calibrated π_2 are listed in Table 2. Since the DICE does not allow net benefits from temperature increases, we restrict π_2 to take on a nonnegative value. That is, we fix the probability of every negative damage ratio to be equal to zero.[13]

Table 2 Alternative parameterization of damage function

	Global mean losses for warming of 2.5°C (% gross world output)		Coefficient of quadratic term of the damage function π_2
Original DICE		1.77	0.002839
Calibration	μ	0.40	0.000640
	μ-1σ	1.77	0.002830
	μ-2σ	3.15	0.005040
	μ-3σ	4.52	0.007230
	μ-5σ	7.27	0.011600

Note: μ and σ denote the mean and the standard deviation of damage ratio respectively.

Then we investigate how the joint uncertainty about the damage function and the climate sensitivity affects the optimal carbon tax. Considering memory constraints, we reduce the number of states of the world on climate sensitivity to 100; thus climate sensitivity increases by 0.25°C from 0.25°C to 25°C. The number of states on damage ratio is 25, of which value increases by 0.4% from -4.2% to 5.0%.[14] Thus the total number of states is 2,500.

Figure 8 shows the behavior of carbon tax under multiple uncertainties calculated from the recalibrated Nordhaus (2008)'s damage function and Roe and Baker (2007)'s PDF. For the same variance of climate sensitivity, the optimal carbon tax increases as the variance of damage costs increases. This is a concave function, and thus the growth rate of carbon tax diminishes as the variance of damage ratio increases. Multiple uncertainties play a role in a

[11] It is possible to introduce the uncertainty about damage function by making a PDF of the curvature-coefficient (default value is 2 in the DICE model) instead of π_2 (e.g., Ackerman et al. (2010)). However, it does not matter which one we choose here because we calibrate the damage costs to the published data. That is, even if we make a PDF of the curvature-coefficient and run the model, we obtain the same results because of the calibration procedure.

[12] In specific, we used 10 estimates concerning a warming of 2.5°C.

[13] The lognormal or the gamma distribution may be applied instead of this truncated normal distribution. Those distributions, however, are not better than the truncated normal distribution for our purpose. If we want to use the lognormal distribution, for instance, we should be able to make the interval of each state of the world on damage ratio as small as possible, which is computationally impossible in this simulation, because the standard deviation of damage ratio is far bigger than the mean. Otherwise, we cannot meet the 1% criterion. Another point is that we cannot rule out the possibility of negative damage ratio because the raw data, from which we derived the mean and the standard deviation, do not exclude the possibility of net benefits from low temperature-increases. Thus it is not theoretically relevant to apply probability distributions which only deal with positive variables.

[14] This range of damages covers about μ±3.5 σ of its distribution.

direction of increasing carbon tax more than in the case of single uncertainty. The shape of the function, however, does not change: it is increasing and concave.

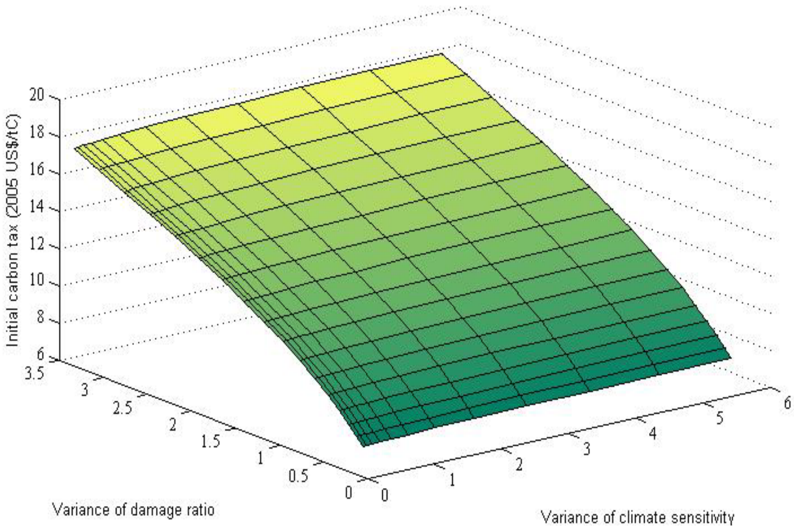

Figure 8 Carbon tax under multiple uncertainties about climate sensitivity and damage function. Note that since the expected value of damage ratio in this simulation (0.4%) is lower than the deterministic value in the DICE model (1.77%), the carbon tax in this graph is lower than the one in Figure 2.

4.5. Uncertainty about the abatement costs

We introduce the uncertainty about abatement costs through a PDF for the cost of the backstop technology. The abatement cost function in the revised model is as follows.

$$\Lambda(t,s) = \pi(t)\theta_1(t,s)\mu(t,s)^{\theta 2} \quad (9)$$

where s denotes the state of the world on abatement costs, $\Lambda(t,s)$ is the ratio of abatement-costs to gross world output, $\pi(t)$ is the fraction of emissions in emission control regime, $\mu(t,s)$ is the emission-control rate, $\theta_1(t,s) = [\theta_0(s)r_{eo}(t)/\theta_2] \times [\theta_3 - 1 + \exp(-\theta 4t - 1)/\theta 3$ is the adjusted cost of backstop technology, $\theta 0s$ is the cost of backstop technology, $r_{eo}(t)$ is the CO2-equivalent-emissions output ratio, θ_2=2.8, θ_3=2, θ_4=0.05 are parameters.

Similar to the method we use for introducing uncertainty in case of damage function, we derive a PDF of the abatement cost from published results. Tavoni and Tol (2010) conduct a meta-analysis of abatement costs from the Energy Modeling Forum 22 (Clarke et al., 2009). The mean and the standard deviation of the abatement costs are μ=0.15% and σ=0.08% of GDP, respectively. We assume that the abatement costs are normally distributed and calibrate the cost of backstop technology as in Table 3. We estimate a quadratic relationship between the abatement costs and the cost of the backstop technology.[15] We assign a zero probability to negative abatement costs. The total number of states of the world is the same

[15] The exact relationship is as follows. $\theta_0 = 65.8x^2 + 33.5x - 0.519$ ($R^2 = 0.999$), where θ_0 and x stand for the cost of backstop technology and the ratio of abatement costs, respectively.

as the case of sensitivity analysis for damage function: that is, 100 for climate sensitivity, 25 for the ratio of abatement costs, which increases by 0.025% from -0.15% to 0.45%.[16]

Table 3 Alternative parameterization of the abatement cost function

		The ratio of abatement costs (% gross world output)	The cost of backstop technology (2005 thousand US$/tC)
DICE original		0.056	1.17
Calibration	μ	0.15	6.50
	μ+1σ	0.23	11.0
	μ+2σ	0.31	16.0
	μ+3σ	0.39	22.0
	μ+5σ	0.55	38.0

Note: μ and σ denote the mean and the standard error of abatement-costs, respectively. Abatement-costs and gross world output are calculated as the net present values over a century with 5% discounting.

Figure 9 depicts the optimal carbon tax under the joint uncertainty about climate sensitivity and the abatement costs. We only add the uncertainty about abatement costs to the results of Section 0. Regardless of the variance of the abatement-costs ratio, the carbon-tax function is increasing and concave. Other things being equal, the uncertainty about the abatement costs reduces the optimal carbon tax but, unlike the case of the damage costs of climate change, its curvature is almost linear. Since the impact of the uncertainty about climate sensitivity dominates, the shape of the carbon tax function is increasing and concave.

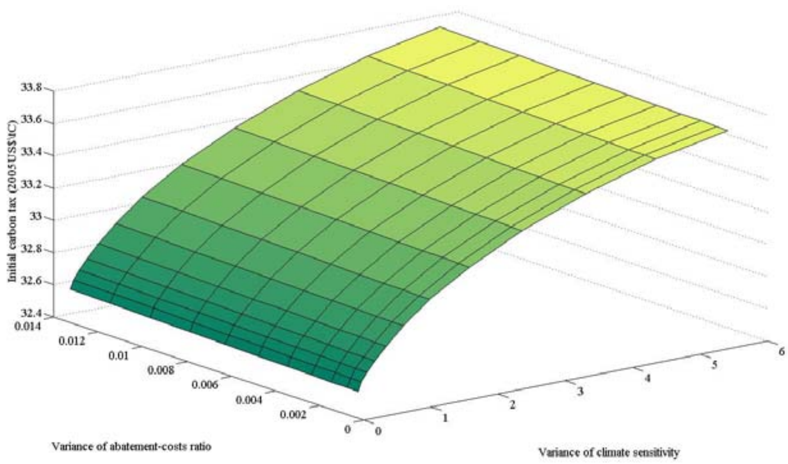

Figure 9 Carbon tax under multiple uncertainties about climate sensitivity and abatement-costs function.

4.6. Alternative utility function

As argued by Weitzman (2009b), the results of economic analysis about climate change may depend on the specification of the interaction between temperature and consumption in the utility function. Assuming that damage only affects consumption goods or assuming that

[16] This range of abatement-costs ratio covers μ±3.75 σ of its distribution.

they also affect 'ecological goods' incorporated in the utility function may lead to radically different conclusion. To test how the results are affected we introduce the alternative constant elasticity of substitution (CES) utility function proposed by Sterner and Persson (2008) as follows. We apply this function to the DICE model using the same parameter values as they used, and then investigate how the carbon tax behaves as the pure rate of time preference varies.[17]

$$U(t,s) = \left[(1-\gamma)C(t,s)^{1-1/\sigma} + \gamma E(t,s)^{1-1/\sigma}\right]^{(1-\alpha)\sigma/(\sigma-1)} / (1-\alpha) \quad (10)$$

$$E(t,s) = E_0/[1 + aT(t,s)^2] \quad (11)$$

where $E(t,s)$ denotes ecological goods, α=2 is the elasticity of marginal utility of consumption, σ=0.5 is the elasticity of substitution, γ=0.1 is the share of nonmarket goods in the utility function, E_0=γC_0 is the level of consumption of environmental amenities in year 2005, a=0.0028388 is a calibration parameter. See Anthoff and Tol (2011) for a critique of the parameterization.

Figure 10 shows how the specification of utility function affects the evolution of atmospheric temperature. As Sterner and Persson (2008) argued, the inclusion of nonmarket goods explicitly in the utility function drastically increases the optimal abatement effort. The impact of relative price of nonmarket goods plays a more significant role than the pure rate of time preference ρ itself.

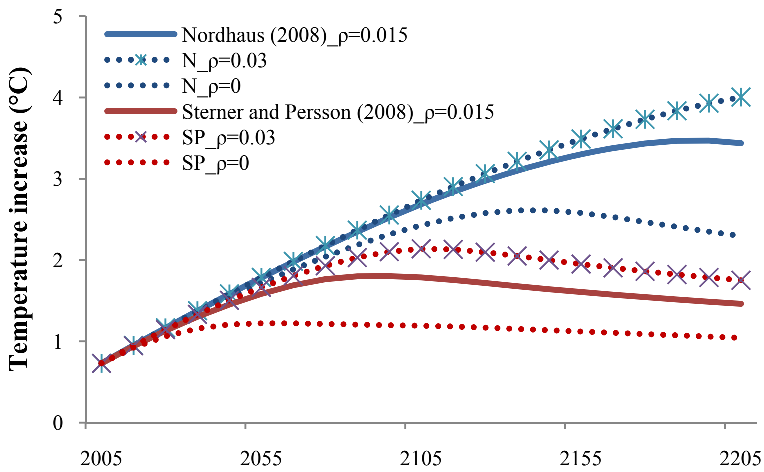

Figure 10: Time profile of global mean air temperature increases using different utility function and the pure rate of time preference. Nordhaus (2008) refers to the results of the original DICE model, Sterner and Persson (2008) refers to the results of the DICE model replacing the utility function to Equation (10), (11), ρ denotes the pure rate of time preference.

Introducing nonmarket goods into the utility function, however, does not disprove our main results: the carbon-tax function of uncertainty is increasing and concave (see Figure 11).

[17] Using the same parameter values as Sterner and Persson (2008), Equation (10) becomes similar to the 'additive' form in Weitzman (2009b).

Lowering the pure rate of time preference makes the magnitude of changes bigger, but it does not alter the curvature of the function.

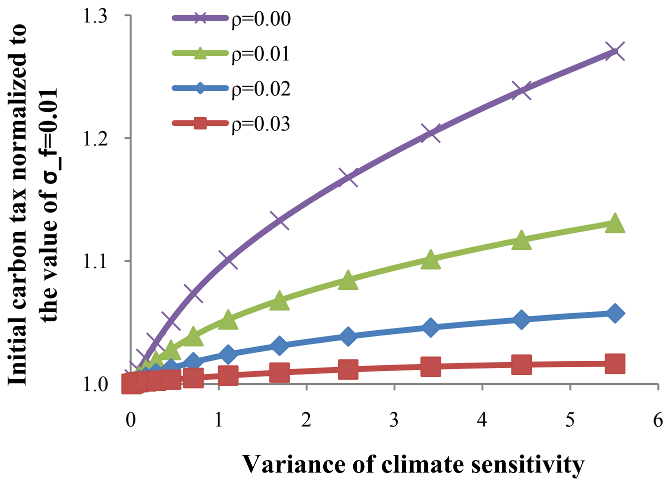

Figure 11: Carbon tax against increasing uncertainty about climate sensitivity using Sterner and Persson (2008) utility function. σ_f and ρ denote the standard deviation of feedback factors and the pure rate of time preference, respectively.

4.7. Alternative risk aversion

The assumed rate of risk aversion α is obviously important to the optimal carbon tax, and perhaps also to the shape of the carbon tax function. We therefore run the model with various values for α. See Figure 2. The behavior of carbon tax according to the value of α is similar to that according to the pure rate of time preference in Figure 11: that is, basically it is increasing and concave, and as α becomes higher, the magnitude of changes of carbon tax becomes smaller. This is because α plays the similar role as the pure rate of time preferences ρ in increasing the discount rate r in a growing economy (recall the Ramsey formula: $r = \rho + \alpha \times g$, where g is the growth rate of consumption per capita). However, if α is greater than 3, the optimal carbon tax decreases as uncertainty increases. For a high rate of risk aversion and a large uncertainty about the climate sensitivity, there is a high probability of a dismal future – and, more importantly, it is hard to avoid such a future through greenhouse gas emission reduction. Therefore, the optimal action is to maximize consumption in the short run – enjoy the good times while they last.

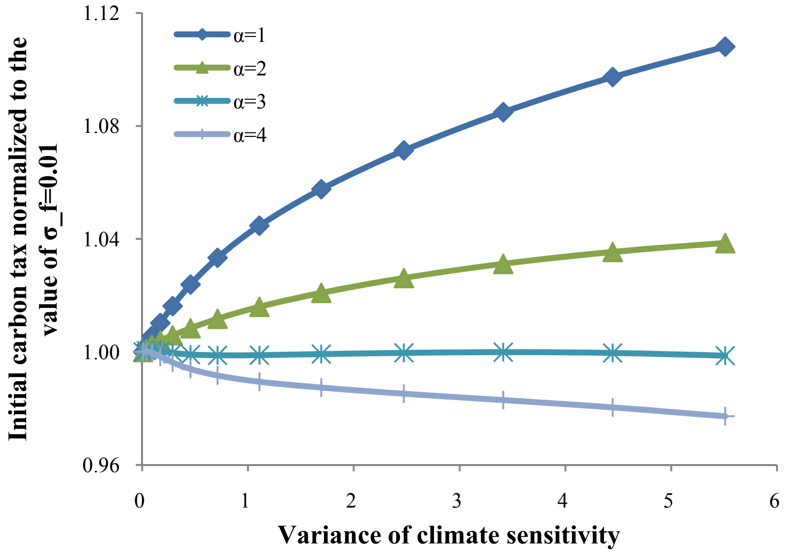

Figure 12: Carbon tax against increasing uncertainty about climate sensitivity using alternative risk aversion. α denotes the coefficient of constant relative risk aversion and σ_f denotes the standard deviation of feedback factors.

5. Conclusion

In this paper, we investigate the optimal choice under fat-tailed uncertainty using the revised DICE model. Since the fat-tailed distribution cannot be fully incorporated into numerical models, we propose an alternative way of analyzing the impacts of fat-tails: the curvature of carbon-tax function according to uncertainty. We find that the optimal carbon tax increases as the uncertainty about climate sensitivity increases and that its curvature is concave. Our main result is generally robust to the alternative assumptions on the model-specification, except that it is sensitive to an unconventionally high value of the coefficient of constant relative risk aversion, that is, $\alpha \geq 3$.[18]

This paper is in line with Anthoff and Tol (2010), which use alternative decision criteria such as minimax regret, tail risk, and Monte Carlo stationarity together with welfare maximization, in that both of them focus on climate policy under fat-tailed risk. Anthoff and Tol (2010) and this paper use different methods and different models, but the results commonly confirm the mathematical analysis of Hennlock (2009): that is, although it is true that fat-tailed uncertainty requires more stringent action for abatement in general, an ultra-rapid abatement like arbitrarily large carbon-tax or instant phase-out of fossil fuels is not justified even when the impacts of climate change are fat-tailed.

[18] Note that conventional values for the coefficient of constant relative risk aversion are $\alpha \approx 2 \pm 1$ (Weitzman, 2007).

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Appendix A: the revised DICE model

In this Appendix we describe our revisions to the original DICE model. Interested readers about the original specification of the DICE model are referred to Nordhaus (2008). We revise the DICE model using the notion of 'state of the world' to introduce uncertainty about climate change. In our model, it is assumed that each state has its own deterministic value of climate sensitivity. Then the probability calculated from the assumed distribution of climate sensitivity is allotted to the corresponding state of the world.

We attach s to relevant variables of the model to denote that the values of the variables are changeable according to the possible states of the world. In this way we can map the possible values of climate sensitivity into the relevant variables such as temperature increases, consumption, utility, and so on. The important revised equations of the model are:

$$\max_{c(t,s)} \sum_{s=1}^{S} \sum_{t=1}^{T} p(s) U\big(c(t,s), L(t)\big)\, R(t) \tag{A.1}$$

$$U\big(c(t,s), L(t)\big) = L(t)[c(t,s)^{1-\alpha}/(1-\alpha)] \tag{A.2}$$

$$Q(t,s) = \Omega(t,s)[1 - \Lambda(t,s)]A(t)K(t)^{\gamma}L(t)^{1-\gamma} \tag{A.3}$$

$$\Omega(t,s) = 1/[1 + \pi_1 T_{AT}(t,s) + \pi_2 T_{AT}(t,s)^2] \tag{A.4}$$

$$\Lambda(t,s) = \pi(t)\theta_1(t)\mu(t,s)^{\theta 2} \tag{A.5}$$

where S and T denote the total number of states s and the time periods t, respectively, $p(s)$ is the probability of each state of the world, $c(t,s)$ is the flow of consumption per capita at time t and state s, $L(t)$ is the level of population at time t, $U\big(c(t,s), L(t)\big)$ is the utility function, $R(t) = (1+\rho)^{-t}$ is the discount factor, ρ=0.015 is the pure rate of time preference, $Q(t,s)$ is the global output net of abatement-costs and damages, $\Omega(t,s)$ is the damage function, $A(t)$ is the total factor productivity, $K(t)$ is the capital stock, $\Lambda(t,s)$ is the abatement-cost function, $\pi(t)$ is the fraction of emissions in emission control regime, θ_1(t) is the adjusted cost of backstop technology, $\mu(t,s)$ is the emission-control rate, $T_{AT}(t)$ is the global mean air temperature increases from 1900, α=2 is the elasticity of marginal utility of consumption, γ=0.3 is the elasticity of output with respect to capita, π_1=0, π_2=0.0028388 are the parameters of damage function, θ_2=2.8 is the parameter of abatement-cost function.

We use the same parameter values as the original DICE model unless otherwise indicated. In order to deal with extreme climate change we remove the upper bound of atmosphere and ocean temperature increases. We also remove the fixed value of an initial emission-control rate because it artificially affects the initial carbon tax.

For multiple-uncertainty runs in the section of sensitivity analysis we attach other subscripts d and a to the relevant variables in order to denote the possible states on damage function and abatement-cost function respectively. Other features of multiple-uncertainty runs, except the method of producing the probability distribution, which is described in the core text, are the same as the one mentioned above.

Appendix B: calibration of the atmospheric temperature equation

The DICE model represents the climate system by a multi-layered system consisting of the atmosphere, the upper oceans, and the lower oceans. It adopts a box-diffusion model (Schneider and Thompson, 1981) as the following equations:

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\} \quad \text{(B.1)}$$
$$T_{LO}(t) = T_{LO}(t-1) + \xi_4\{T_{AT}(t-1) - T_{LO}(t-1)\} \quad \text{(B.2)}$$

where $T_{AT}(t)$ is the global mean surface air temperature increases from 1900, $T_{LO}(t)$ is the lower oceans temperature increases from 1900, $F(t)$ is the total radiative forcing relatively from 1900, ξ_1, ξ_2, ξ_3, ξ_4 are parameters.

This representation is a simple way of incorporating climate system into IAMs but has some problems. Apart from the critics that it fails to capture the real mechanism of the climate system (Marten, 2011), one of the practical problems encountered during simulations is that the model does not produce a feasible solution when the value of climate sensitivity is lower than around 0.5°C. We find that this problem is induced by the fact the original specification creates a fast cyclical adjustment when we change only the parameter on climate sensitivity. To see this, notice that Equation B.1 can be rearranged with simple algebra into Equation B.3, which is an error-correction model (Philips, 1957; Salmon, 1982) with an adjustment speed α_1 and target $\alpha_2 F(t) - \alpha_3[T_{AT}(t-1) - T_{LO}(t-1)]$:

$$T_{AT}(t) = T_{AT}(t-1) - \alpha_1\{T_{AT}(t-1) - \alpha_2 F(t) + \alpha_3[T_{AT}(t-1) - T_{LO}(t-1)]\} \quad \text{(B.3)}$$

where α_1=$\xi_1\xi_2$, α_2=1/ξ_2, α_3=ξ_3/ξ_2.

Since α_2 is the climate sensitivity (CS) divided by a constant (the estimated forcing of equilibrium CO_2 doubling), decreasing the climate sensitivity artificially increases the adjustment speed α_1=ξ_1/α_2. With DICE default (CS=3°C), the adjustment speed α_1 is 0.22. For CS lower than 0.8, the adjustment speed becomes higher than one. This leads to a cyclical adjustment to the equilibrium temperature which does not make much sense scientifically speaking. For CS = 0.5°C, α_1=1.7 which, in addition to implying important and irrelevant jumps up and down in the temperature every period, leads to an infeasible solution.

To avoid this problem, we recalibrate the parameter values in Equation B.1 so as to ensure a coherent adjustment process. We calculate atmospheric temperature $T_{AT}(t)$ according to Equation B.3 using various values of adjustment-speed α_1 and climate sensitivity CS. Then we fit $T_{AT}(t)$ against the historical observation data.[19] Through the experiment we find that the adjustment-speed is linearly related to the inverse of climate sensitivity and the slope of

[19] The historical data used for this calibration are as follows: atmospheric temperature (Hadley center, CRUTEM3), ocean temperature (NOAA, global anomalies and index data), radiative forcing (Hansen et al., 2007).

the function changes around CS=1.5°C and CS=3°C.[20] Thus we obtain three different functional forms according to the range of CS as follows.

$$\alpha_1 = 0.559/(CS - 1.148)\ (if\ CS \geq 3°\text{C})$$
$$\alpha_1 = 0.993/(CS + 0.430) + 0.012\ (if\ 1.5°\text{C} \leq CS < 3°C) \qquad \text{(B.4)}$$
$$\alpha_1 = -0.943/(CS - 3.218) - 0.022\ (otherwise)$$

[20] The detailed data for this calibration can be obtained upon request. Air temperature may decrease over time if CS <1.2°C (Baker and Roe, 2009), but in this experiment the sign of the slope changes at CS=1.5°C. This may be caused by the sign of radiative forcing in the slope equation or observational-errors of the data used (Slope = $\beta_1[T_{AT}(t) - T_{AT}(t-1)]/F(t-1)$, where β_1>0 is a constant).

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2011		
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