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Web-Appendix to

Demand for Money: A Study in Testing Time Series for Long Memory and Nonlinearity

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This Web-Appendix contains supplementary material to the above-titled paper. Supplementary tables, figures and references can be found in Appendices A, B and C, respectively.

J.E.L. Classification: C22, C52, E41

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Appendix A

Tables

Table A.1: Unit root tests, Denmark ($T = 55$)

Variable	Test Statistic probability†	Constant	Trend	Lag Length using sic
m_t	1.123 (0.930)	Not Significant	Not Significant	2
y_t	0.776 (0.878)	Not Significant	Not Significant	0
p_t	0.437 (0.664)‡	Significant	Significant	0
i_t	−0.616 (0.446)	Not Significant	Not Significant	0
b_t	−0.982 (0.288)	Not Significant	Not Significant	1

†MacKinnon (1996) one-sided p-values.
‡Normal probability.

Table A.2: Unit root tests, Finland ($T = 106$)

Variable	Test Statistic probability†	Constant	Trend	Lag Length using sic
m_t	1.720 (0.979)	Not Significant	Not Significant	4
y_t	−1.951 (0.054)‡	Significant	Not Significant	4
p_t	−4.200 (0.006)	n/a	n/a	1
i_t	−4.874 (0.001)	n/a	n/a	0

†MacKinnon (1996) one-sided p-values.
‡Normal probability.
n/a denotes not applicable.

Table A.3: Number of cointegrating relations by model, Danish data

Test Type	no inpts no trends		rest'd inpts no trends		unrest'd inpts no trends		unrest'd inpts rest'd trends		unrest'd inpts unrest'd trends	
0.05 and 0.10 significance levels, excluding seasonal dummies										
Trace	3	4	3	3	3	4	3	3	1	2
Max-Eig.	1	2	3	3	3	4	3	3	0	1
0.05 and 0.10 significance levels, including seasonal dummies										
Trace	4	4	3	3	3	3	2	3	1	2
Max-Eig.	1	2	2	3	2	3	2	3	1	1

Table A.4: Number of cointegrating relations by model, Finnish data 1

Test Type	no inpts no trends		rest'd inpts no trends		unrest'd inpts no trends		unrest'd inpts rest'd trends		unrest'd inpts unrest'd trends	
0.05 and 0.10 significance levels, excluding seasonal dummies										
Trace	3	4	2	4	2	2	2	2	2	2
Max-Eig.	2	4	2	2	2	2	2	2	2	2
0.05 and 0.10 significance levels, including seasonal dummies										
Trace	3	4	3	4	2	2	2	2	2	2
Max-Eig.	3	4	2	4	2	2	2	2	2	2

Table A.5: Number of cointegrating relations by model, Finnish data 2

Test Type	no inpts no trends		rest'd inpts no trends		unrest'd inpts no trends		unrest'd inpts rest'd trends		unrest'd inpts unrest'd trends	
0.05 and 0.10 significance levels, excluding seasonal dummies										
Trace	1	1	1	1	1	1	1	1	1	1
Max-Eig.	1	1	1	1	1	1	1	1	1	1
0.05 and 0.10 significance levels, including seasonal dummies										
Trace	1	1	1	1	1	1	1	1	1	1
Max-Eig.	1	1	1	1	1	1	1	1	1	1

Table A.6: Number of cointegrating relations by model, Finnish data 3

Test Type	no inpts no trends		rest'd inpts no trends		unrest'd inpts no trends		unrest'd inpts rest'd trends		unrest'd inpts unrest'd trends	
0.05 and 0.10 significance levels, excluding seasonal dummies										
Trace	0	1	0	1	0	0	0	0	0	0
Max-Eig.	0	1	0	0	0	0	0	0	0	0
0.05 and 0.10 significance levels, including seasonal dummies										
Trace	1	1	1	2	0	0	0	0	0	0
Max-Eig.	1	1	1	1	0	0	0	0	0	0

Table A.7: Johansen results for Danish data plus modified critical value

Unrestricted Cointegration Rank Test (Trace)

Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value
$r = 0$	$r \geq 1$	111.641	82.230	77.550	112.803
$r \leq 1$	$r \geq 2$	55.809	58.930	55.010	-
$r \leq 2$	$r \geq 3$	29.843	39.330	36.280	-
$r \leq 3$	$r \geq 4$	7.640	23.830	21.230	-
$r \leq 4$	$r = 5$	0.094	11.540	9.750	-

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	55.831	37.070	34.160
$r \leq 1$	$r = 2$	25.966	31.000	28.320
$r \leq 2$	$r = 3$	22.203	24.350	22.260
$r \leq 3$	$r = 4$	7.546	18.330	16.280
$r \leq 4$	$r = 5$	0.094	11.540	9.750

Note: The correction factor is 1.372.

Table A.8: Johansen results for Finnish data 1 plus modified critical values

Unrestricted Cointegration Rank Test (Trace)

Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value
$r = 0$	$r \geq 1$	85.122	58.930	55.010	63.998
$r \leq 1$	$r \geq 2$	42.621	39.330	36.280	42.712
$r \leq 2$	$r \geq 3$	12.207	23.830	21.230	-
$r \leq 3$	$r \geq 4$	2.247	11.540	9.750	-

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	42.501	31.000	28.320
$r \leq 1$	$r = 2$	30.414	24.350	22.260
$r \leq 2$	$r = 3$	9.961	18.330	16.280
$r \leq 3$	$r = 4$	2.247	11.540	9.750

Note: The correction factor is 1.086.

Table A.9: Johansen results for Finnish data 2 plus modified critical value

Unrestricted Cointegration Rank Test (Trace)

Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value	Modified 0.05 Critical Value
$r = 0$	$r \geq 1$	43.798	39.330	36.280	40.313
$r \leq 1$	$r \geq 2$	10.795	23.830	21.230	-
$r \leq 2$	$r \geq 3$	2.052	11.540	9.750	-

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	33.004	24.350	22.260
$r \leq 1$	$r = 2$	8.742	18.330	16.280
$r \leq 2$	$r = 3$	2.052	11.540	9.750

Note: The correction factor is 1.025.

Table A.10: Johansen results for Finnish data 3 plus modified critical values

Unrestricted Cointegration Rank Test (Trace)				
Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r \geq 1$	8.827	23.830	21.230
$r \leq 1$	$r \geq 2$	0.300	11.540	9.750
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)				
Hypotheses		Trace Statistic	0.05 Critical Value	0.10 Critical Value
$r = 0$	$r = 1$	8.527	18.330	16.280
$r \leq 1$	$r = 2$	0.300	11.540	9.750

Table A.11: Fractional integration analysis, Danish data

	MLE	NLS	GPH	GSP	FDF†	FADF†
m_t	1.159 (0.123)	1.176 (0.133)	1.168 (0.171)	0.993 (0.096)	-0.850	-1.490
y_t	0.360 (0.281)	0.577 (0.276)	1.23 (0.171)	1.08 (0.096)	-0.076	2.267
p_t	0.741 (0.301)	0.674 (0.231)	0.994 (0.171)	0.870 (0.096)	0.077	-1.569
i_t	0.574 (0.275)	0.521 (0.260)	1.171 (0.171)	1.084 (0.962)	-0.223	-1.125
b_t	0.738 (0.278)	0.727 (0.218)	1.377 (0.171)	1.275 (0.096)	2.339	-2.191

†Based on the MLE estimator of d.

Table A.12: Fractional integration analysis, Finnish data

	MLE	NLS	GPH	GSP	FDF†	FADF†
m_t	0.778 (0.090)	0.762 (0.090)	0.830 (0.112)	0.590 (0.069)	-2.94	-2.046
y_t	0.559 (0.084)	0.570 (0.086)	0.745 (0.113)	0.523 (0.693)	1.250	8.502
p_t	0.236 (0.099)	0.210 (0.096)	0.410 (0.114)	0.394 (0.113)	-6.45	-2.63
i_t	0.621 (0.108)	0.622 (0.103)	0.759 (0.112)	0.796 (0.069)	-2.73	-3.54

†Based on the MLE estimator of d.

Table A.13: Simple Chow breakpoint tests

	Denmark	Finland
F-statistic	10.127 (0.000)	15.205 (0.000)
Log-likelihood ratio	69.400 (0.000)	70.273 (0.000)

Note: Two breakpoints each model.

Appendix B

Figures

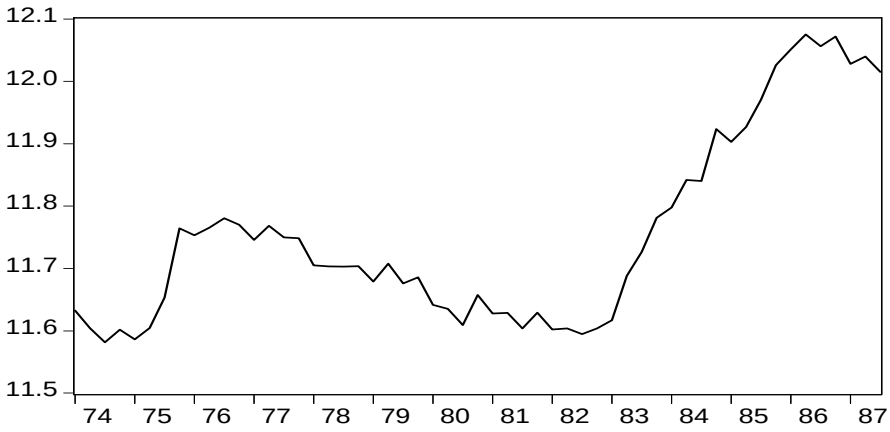

Figure B.1: Money demand over time, Denmark

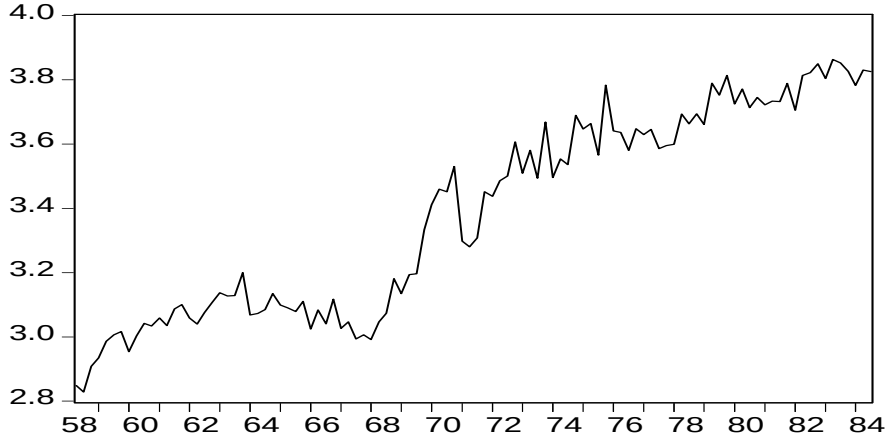

Figure B.2: Money demand over time, Finland

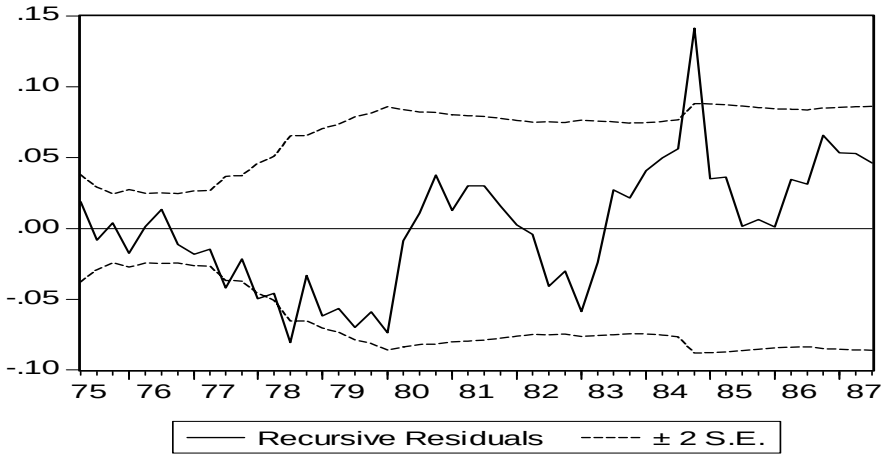

Figure B.3: Recursive residuals plot, Denmark

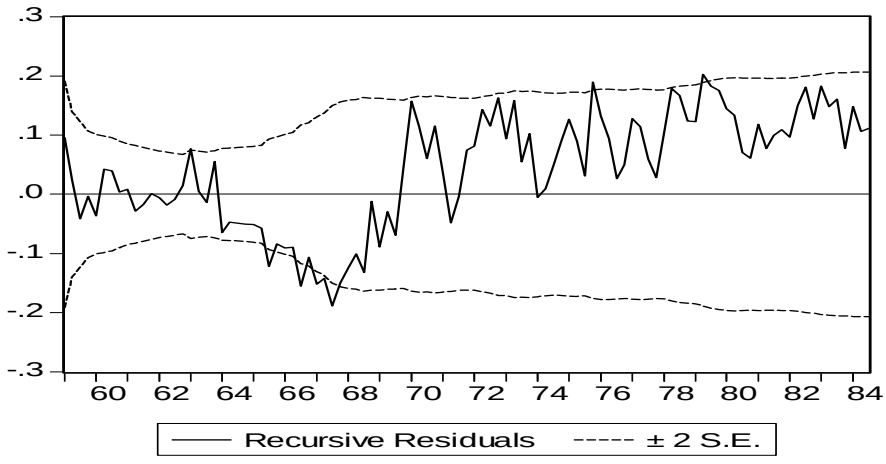

Figure B.4: Recursive residuals plot, Finland

Appendix C

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