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MODELLING THE IRISH ECONOMY: A PROGRESS REPORT ON THE CENTRAL BANK'S MACROECONOMETRIC MODEL*

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> "Mathematical Demonstrations built upon the impregnable foundations of Geometry and Arithmetic are the only Truths that can sink into the Mind of Man void of all Uncertainty, and all other Discourses participate more or less of Truth according as their Subjects are more or less capable of Mathematical Demonstration".
>
> *Christopher Wren*

1. INTRODUCTION

Methods of empirical economic analysis have undergone enormous changes in recent decades. The availability of improved statistics and of high-speed computers have facilitated this process, but the increasing ambition and complexity of economic policy itself have perhaps been the major stimuli.

In most of the economically advanced countries and, increasingly, in developing countries also, analysis of macroeconomic policy options is carried out with the aid of large-scale econometric models. Such models are systems of simultaneous equations, hundreds of them in the larger models, designed to encapsulate the interrelationships of the main economic variables. The models, in principle anyway, will predict the impact of changes in policy, say in tax rates, on variables like the levels of output and employment.

The intellectual origins of econometric modelling can be traced with rather more precision than the models themselves sometimes attain. The coincidence of the "Keynesian revolution" in macroeconomic theory and the development of national income accounting in the nineteen thirties led naturally to empirical investigations of the Keynesian system. To this day, almost all large-scale econometric models are built around the Keynesian income-expenditure framework, although not all are thorough-going Keynesian models. But Keynes was unenthusiastic about econometrics and mathematical methods in general, dismissing them as –

> "... Symbolic pseudo-mathematical methods of formalizing a system of economic analysis ... which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols".

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* As with all of the documentation relating to the Central Bank's model, the views expressed in this paper are the responsibility of the authors and do not constitute an official view of the Central Bank of Ireland.

When Keynes wrote these lines, econometric techniques and computational facilities were primitive by modern standards and the first macroeconometric models were a decade in the future. Nowadays, it is precisely the ability to reduce the "complexities and interdependencies of the real world" to manageable proportions that makes the model-building approach so popular.

Up to about 1960 it would have been difficult to locate more than a handful of studies of the Irish economy which used econometric techniques (31). Appropriately, one of the earliest, Professor Quinlan's pioneering model of the Irish economy (28) was read to this Society in 1961. Today, as a perusal of the *Economic and Social Review*, or of this Society's Journal will confirm, empirical studies which eschew "pretentious and unhelpful symbols" are very much the exception. The Central Bank's econometric model, the largest model thus far constructed in this country, borrows wherever possible from earlier empirical studies and it can be regarded to some extent as a summary of the available evidence on the behaviour of the Irish economy in the short run and also provides an organised framework for further empirical research.

The next section discusses the structure of the model. Section 3 deals with data, estimation and solution methods while section 4 is devoted to a consideration of the performance of the model in simulation. Section 5 considers the practical applications of the model and the concluding section considers what further developments and improvements in the model are envisaged.

Macroeconometric models are complex and we can give no more than an overview of the Central Bank's model this evening. The series of technical reports, many of which are listed in the References, give a more detailed account, and are obtainable on request.

2. STRUCTURE OF THE MODEL

All econometric models go through a process of continuous revision. This is no less true of the Central Bank's model which, although comparatively new, has already been revised by way of certain respecifications and re-estimations. The version discussed in this paper is documented in (6) and (7) but in a number of respects is itself undergoing revision. The basic orientation of the model is a short-run Keynesian one with the level of activity being demand determined. In its present version, therefore, the model has no explicit supply side: more formally, the implicit assumption is that supply is infinitely elastic and responds to meet any changes in demand. The model is an annual one and is built around a National Accounts framework. It has six sectors which endogenise the main features of the goods market, foreign trade sector, the labour market, the determination of prices, the government sector and the monetary sector.

In all, there are 107 equations (and hence 107 endogenous variables) of which 53 are stochastic or behavioural relations, while the remainder are identities or definitional relations. There are 89 exogenous variables, 25 lagged endogenous variables and 13 lagged disturbances giving a total of 127 predetermined variables. Many of the equations are either linear or linear in the logarithms of the model variables. The model as a whole is simultaneous, dynamic, nonlinear and, by model standards, of fairly large size. A common type of nonlinearity results from the combination of 'real' and 'nominal' variables in the system. Before proceeding to a detailed consideration of the individual sector specifications it is, perhaps, worthwhile to get an overall view of the workings of the model.

Overall View of the Model

The demand sector of the model is concerned primarily with the determination of consumers' expenditure and gross physical capital formation. These two demand components, together with government expenditure, the greater part of which is determined exogenously, and the net trade balance, determine the volume of output. The value of output is obtained by combining with the volume of output the major National Accounts price deflators. In equations for these latter, external forces play a major role with domestic variables playing a more subordinate one.

The employment and income distribution sector distributes the level of nominal income, determined in the goods and prices sectors, between employee incomes and profits – the level of agricultural incomes being given as exogenous. Wages and salaries are determined by means of equations explaining earnings and employment, with profits being determined residually. This functional distribution of income interacts with the government sector and other exogenous information to give the level of disposable income which feeds back into the demand sector through the consumption function. Thus, the levels of expenditure and income are determined simultaneously.

The government sector endogenises income tax and expenditure tax receipts as well as some items of public expenditure. The disaggregation of consumers' expenditure by means of a Linear Expenditure System enables a detailed disaggregation of expenditure tax receipts to be made. This sector also recognizes the relationship between indirect tax rate changes and consumer prices. Through this relationship, indirect tax increases, by reducing real disposable income, have a deflationary effect on aggregate demand.

Finally, the monetary sector of the model is of the standard type for small open economies with a pegged exchange rate, in accordance with which the money stock is endogenously determined by the private sector. The reason for the endogeneity of the money stock is that monetary flows through the balance of payments ensure that the private sector can always attain its desired holdings of money. This implies that actions by the monetary authorities with the object of altering the level of the money stock will have no impact on the level of real activity in the economy or on the price level. However, the level of domestic credit extended by the banking system is a variable that is exogenous and can be altered by the authorities. Exogenous changes in domestic credit, one counterpart to the endogenously determined money stock, have an effect on the other counterpart, external reserves. The absence of a transmission mechanism between monetary and 'real' variables may strike one as anomalous in a model constructed by a central bank. In addition to theoretical arguments based on an analysis of a small open economy supporting this position, the speed of adjustment in the financial sector appears to be too rapid to permit credit policy to have discernible real effects in an annual model. Attempts to model such effects proved to be unsuccessful with annual data, although it is possible that such effects could well be present over a shorter time span.

Sector Specifications

A full listing of equations and variable names is not provided with this paper, since this would necessitate a paper of inordinate length. Those interested can obtain this material from the Research Department of the Central Bank. In this section, however, we will discuss briefly the stochastic (behavioural) equations of the model.

Demand Sector

The first behavioural equation in the demand side of the real goods market is that for

aggregate consumers' expenditure. This is one of the more important equations of the model from the viewpoint of multiplier analysis, and unfortunately, is one whose specification has not proved to be robust in the face of re-estimation with additional National Accounts data. In the first version of the model, a distinction was made between agricultural and non-agricultural income in order to permit any differences in the propensity to consume to be picked up. Such a distinction appeared to be no longer significant, however, when the relationship was re-estimated using more recent revised data. The consumption function in (19) therefore, has, as arguments, aggregate personal disposable income and a lagged liquid assets variable which is included to pick up a liquidity effect, whereby deviations in real liquid assets from their desired level have repercussions as consumption expenditure is altered in order to restore the desired level of liquid assets. It will be seen that (as is noted later), since the money market is always in equilibrium, only disequilibria in non-money liquid asset markets have effects on consumer expenditure. The equation is discussed in detail in (19).

Aggregate consumers' expenditure is then divided into nine components in the next block of equations. These are Food, Drink, Tobacco, Clothing, Fuel, Petrol, Durable Household Goods, Transport Equipment and a residual category. The division is accomplished through the estimation of a Stone-Geary Linear Expenditure System for the nine categories. The motivation for this disaggregation, which is not essential to the structure of the model, is to permit a corresponding breakdown of indirect tax-yield equations in the endogenisation of government revenue. The specifications and estimation of this commodity demand model are the subject of (24), where certain limitations of the Stone-Geary model for the purpose at hand are noted.

There are two behavioural equations for private non-residential and private residential components of fixed investment. The equation for the former derives from a structural model that relates the desired capital stock to GNP and the actual change in the capital stock to current and lagged changes in the desired stock. Gross investment then becomes a function of current and lagged changes in GNP, plus depreciation, which is here taken to be a fraction of the lagged capital stock. To obtain a relation for private residential investment, it was assumed that the desired stock was related to real personal disposable income with a partial adjustment of actual to desired quantities. The equation for non-agricultural stock changes (inventory accumulation) derives from a structure which has desired inventory levels a linear function of current and lagged GNP with a partial (Koyck) adjustment of actual to desired stocks. A full discussion of fixed investment and stock changes is given in (16) and (23) respectively.

External Trade Sector

In the external trade sector, the demand for manufactured exports is a linear function of world income and relative prices, while the rate of change of manufactured export prices depends on the rate of change of import prices (see (27) for details). Imports of goods are log-linear in the final demand, while imports of services are a log-linear function of merchandise imports; price terms, which were statistically significant in an earlier version of the former equation, have now lost significance and have been dropped. The final demand variable employed in the goods equation is a weighted average of components where the weights reflect direct and indirect import contents as derived from the 1969 input-output tables. The balance of payments on current account is obtained when the remaining exogenous items in the current external account are included.

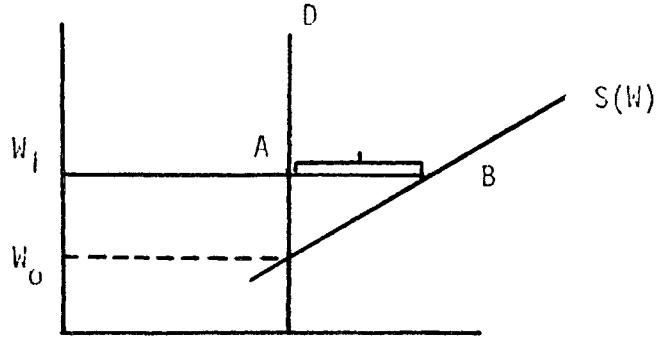
Employment and Income Distribution Sector

The first behavioural equation in the labour market sector endogenises the rate of change of average hourly industrial earnings, which depends on the rate of price inflation. The equation endogenising the demand for labour is cast in terms of hours worked and can be interpreted as an inverted production function. Hours worked per employee are in turn explained by a lagged dependent variable and a (negative) trend. Cyclical variables such as the unemployment rate failed to attain significance in hours-worked equations. Industrial output is explained by final demand, as is output of services. The ratios of average earnings in public administration, and in services, to average earnings in industry are related to the unemployment rate and a time trend. Employment in services (number employed) is related to output and a time-trend. (The time trends in the two employment equations are capital stock proxies). Employment in public administration is exogenous and effectively is a fiscal policy instrument. This completes the demand side of the labour market.

The supply of labour is endogenised through a migration equation which has Irish and British real wages and unemployment rates as arguments, an equation which relates the change in working-age population to net migration and, finally, a labour force participation equation which has the unemployment rate and a trend as arguments.

The working of the labour market can be illustrated conveniently by referring to Figure 2.1 below where real wages and employment are measured on the vertical and horizontal axes, respectively. The demand for labour function, D, is vertical, since, for the model in its present form, real wages do not affect the demand for labour directly. Changes in the real wage do shift the vertical demand curve, however, through effecting changes in real disposable income and output, and consequently the demand for labour. In the model, therefore, increases in the real wage increase the demand for labour, a feature of many short-run models. The supply of labour S(W) is a positive function of the real wage reflecting the effect of changes in the real wage on migration and hence on the supply of labour. For purposes of the diagrammatic representation, the real wage is

Figure 2.1: Labour Market



set exogenously (in fact, it is determined by the rate of inflation) at W_1. It can be seen that this is greater than the market-clearing wage of W_0. At the real wage of W_1, therefore, the supply of labour exceeds the demand for labour, so that unemployment to the extent of AB ensues.

Prices Sector

The prices sector has equations for the rate of inflation of consumer prices, net of indirect taxes and subsidies, and for the rate of change of the deflators of the various GNP components. The rate of change of consumer prices depends on current and lagged import and export prices. Investment goods contain a sizeable portion of nontradeable items such as construction, and a unit labour cost variable appears in the equation for the rate of change of investment goods prices along with import prices. The equation for total export prices has agricultural prices and import prices as arguments. The deflator of non-wage government expenditure depends on unit labour costs in the services sector of the economy. The contribution that changes in indirect taxes and subsidies make to consumer prices is explicated in the government sector of the model. As we saw in the account for the labour market sector, the rate of change of consumer prices, inclusive of indirect taxes and subsidies, is the determinant of wage increases. Price determination in the model is discussed in more detail in (22).

Government Sector

The government sector can be conveniently summarised in two parts. The first part explains variations in government tax receipts under various headings. The first behavioural equation explains flat-rate social insurance contributions in terms of an index of rates and total employment outside agriculture and public adminstration, where the social insurance scheme yields negligible amounts. Personal income tax receipts depend on an adjusted personal income measure and on personal allowances. A block of equations deals with customs and excise duties which distinguish motor vehicles, tobacco, alcohol, light oil, heavy oil and a residual category. Each equation uses one of the categories from the system of demand equations discussed earlier. The correspondence is exact in some cases, but only a rough matching is possible in others. Tax rate indices are also employed.

Motor vehicle duties (the annual ownership tax as distinct from the purchase tax) are related to the stock of cars under licence and a tax rate index. The stock of cars is in turn endogenised through a relationship between the change in the car stock, the lagged stock and consumers' expenditure on transport equipment, itself explained in the linear expenditure block. Value added tax receipts depend on total consumer expenditure and a tax rate index. The tax equations are the subject of two papers, (17) and (18).

The second part of the government sector is concerned with government expenditure. Three items in the latter are endogensied: unemployment benefit, secondary teachers' wages and salaries, and central government wages and salaries. Unemployment benefit depends on numbers unemployed and an index of rates of payment. Secondary Teachers' wages and salaries are related to the number of teachers employed and an average earnings variable. Central government wages and salaries are made to depend on wages and salaries in public administration and defence which are endogenised in the labour market block. The purpose of separately distinguishing the central government component is to facilitate the structuring of the adjusted income variable in the direct personal taxation equation – different rules have been applied over the years to different categories of public servants. The borrowing requirement of the government, finally, is equal to the difference between the expenditure and receipts of the government.

The Monetary Sector

As indicated earlier, the monetary sector of the model is a standard one for a small, open economy, operating a fixed exchange rate. The three stochastic equations explain the

components of the demand for money. In addition, there are identities for the government budget constraint, the balance sheets of the Central Bank and the licensed banks, and official external reserves. The effect of the monetary sector specification is to posit a horizontal LM curve, i.e. an infinitely elastic money supply at the exogenous interest rate, with the implication that the authorities can affect only the counterparts – net domestic credit and external reserves – of the endogenously determined money stock.

This is not to say that the money stock could not be varied in the model – it could, but only through influencing the level of activity and hence the arguments other than the rate of interest in the money-demand equations. The equations for the components of money demand distinguish currency, current accounts and deposit accounts and have scale variables as well as an interest rate variable in the case of current accounts, and are the subject of (8), (9) and (10).

3. DATA SOURCES, ESTIMATION AND SOLUTION METHODS

The economic specification of the model equations, given in Section 2, leaves open such questions as the appropriate functional forms to be used, the number of explanatory variables in any equation, the length of lags, and the numerical values of coefficients and parameters. These questions are usually resolved by means of the statistical technique of regression analysis using time-series of data for the necessary variables. However, the use of regression techniques raises the following issues –

(i) Are consistent time-series available, on at least an annual basis, for the relevant variables and, if so, are these time-series long enough to allow sufficient "degrees-of-freedom" for statistical estimation of parameters?

(ii) Which of the many available statistical techniques of regression analysis is most suitable?

(iii) How best can one set up a system for data gathering and model estimation to facilitate the constant re-estimations of, experimentation with, and improvements to the model equations?

(iv) Having estimated a version of the model, how can it be solved?

(v) What is the best set of tests one should carry out, over the "within-sample" period and the "out-of-sample" period, to establish whether the model represents the behaviour of the real economy sufficiently accurately, in some sense to be specified?

(vi) How can the model be integrated into the existing well-tried policy analysis and forecasting processes?

In this section, we propose to deal with the first five of the above points, leaving the last issue (vi) to a later section. In this way we shall provide the analytical framework necessary to interpret the empirical results to be presented in Section 4.

(i) Data Sources: Since the model is built around the National Accounts, the basic sources of data are the various issues of National Income and Expenditure, together with the Reports of the Revenue Commissioners, the Irish Statistical Bulletin, the Review and Outlook, the Budget Speech, the Central Bank Bulletins, etc. However, before the data are usable for statistical estimation, various types of adjustments are necessary, such as the following:

(a) Series expressed in constant prices, and price indices, must be recalculated on a consistent basis.

(b) The data on Public Authorities in the pre-1974 NIE, etc. must be adjusted from a

financial year basis to a calendar year basis (using a form of linear interpolation), and the major National Accounts tables re-worked.

(c) Where the methodology of the construction of a series changed, and where the CSO-published data did not go back to the beginning of our data period of 1953, adjustments were necessary. Such a case is the treatment of Shannon, and "Other Transportation", in exports and imports of goods and services.

(d) Although our policy is to use always the most recent methodology for data, the major revisions necessitated by the new European System of Accounts (ESA) were so extensive as to make adjustment of data prior to 1976 almost impossible. Here, the CSO came to our assistance, and very kindly provided us with the 1976 NIE and consequent revisions, on the old, pre-ESA basis.

The entire process of gathering and adjusting the data for the model is documented each year, and is made available to other researchers. In this document (13), all the time-series are listed and complete information on sources and methods is given. The work involved is quite onerous, and is carried out on a regular annual cycle as the new NIE is issued.

(ii) Estimation Methods: Virtually all large-scale econometric models involve problems of simultaneity, nonlinearities in variables and/or parameters, lagged endogenous variables, and autocorrelated disturbances. Indeed restricting consideration to simple linear systems, where estimation theory is better developed, usually involves doing violence to economic theory. Aside entirely from "degrees-of-freedom" and computational problems, no consistent estimation procedures have so far been developed which simultaneously deal with all the above problems. Linear simultaneous models, with autocorrelated errors, may be consistently estimated using either Limited or Full Information Methods (14), (15), and recent work in the National Bureau of Economic Research (NBER) has resulted in computer codes for nonlinear FIML estimation (2). However, in each case there is a severe "degrees-of-freedom" problem, so they are only applicable to situations where sample sizes are large (e.g. quarterly data).

As a practical matter, it must be accepted that the consistent estimation of our model must await theoretical, software and data developments. Indeed, to our knowledge, no large-scale model, recognising all these difficulties, has been consistently estimated (refer (15)).

The procedure used to estimate our model was as follows:

(a) Each equation was estimated separately using OLS.

(b) The DW test, or the Durbin t-test in the presence of lagged endogenous variables, was used to test for first-order autocorrelation. Correction for first-order autocorrelation, where necessary, was performed using the Hildreth-Lu grid-search method, which is a maximum-likelihood method.

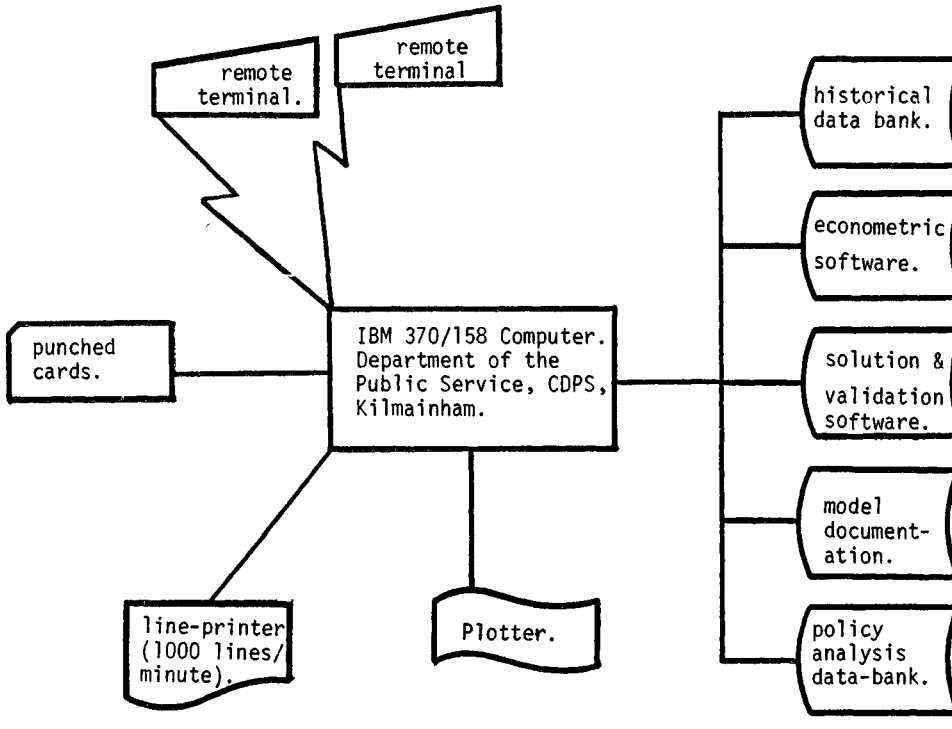
(c) Some use of Indirect Least Squares was made where the simultaneity problem was felt to be acute.

(d) A Linear Expenditure System (LES) of nine commodity demand equations was estimated separately, using a special maximum-likelihood procedure (24).

To summarise, the estimator used is a hybrid, and is obviously not consistent. However, the question of single-equation autocorrelation is dealt with, and an ad-hoc attempt made to deal with the simultaneity problem. As further research proceeds, attempts will be made to use consistent estimators, starting with the small-scale version of the model (3), where the "degrees-of-freedom" requirements are not as stringent.

(iii) Computerisation of Data Bank and Estimation Procedures: If the data gathering and model estimation and documentation were to be done once only, not much thought would have been given to setting up procedures. However, the need for constant up-dating, revision and modification was envisaged, so we have given much thought to these tasks, and to the further problems of making access to the model fairly simple for other economists within the Bank. In Figure 3.1 we illustrate our overall model-

Figure 3.1: Econometric Model Computer Processing System



related computer system. Communication with the computer is by means of inter-active slow-speed terminals (for small volumes), and via punched-cards (for large volumes). At any given time, the most up-to-date version of the data bank of historical time-series is located on a direct-access computer disc file, as are the various econometric estimation packages, the model solution software (see below) and the model documen-tation. Consequently, the data-bank can be interrogated on-line, (i.e. directly linked to the computer), regression runs submitted to the econometric packages, the model solution software altered, and simulation runs performed as part of an integrated system. Output too voluminous to be processed over the slow-speed terminals can be routed to a fast line printer, located at the computer centre in the Department of the Public Services, Kilmainham, and is posted back to us a half day later. The facilities provided to the Bank by the CDPS have greatly assisted in the smooth running of the model project, and we would like to express our appreciation. Below, when we consider the use of the model in policy analysis and forecasting, we will return to the computer system and its use.

(iv) Solving the Model: When we refer to "solving" the model, we mean the process of

selecting certain fixed values for the exogenous variables, and using the model equations to calculate the corresponding values of the endogenous variables. For a linear model, this is a simple task. Using standard notation, a linear model of the form

$$Ay_t + By_{t-1} + Cx_t = u_t$$

can be solved (equivalently, written in its reduced form) as

$$y_t = \Pi_1 y_{t-1} + \Pi_2 x_t + v_t$$

by means of matrix inversion procedures. Generalising, a model with n equations, which is linear in the n endogenous variables, can be solved essentially by inverting an $n \times n$ matrix of coefficients, and all the model multipliers (see below) are found by this one operation, which may be easily computerised.

However, at least two types of nonlinearity enter into our model, the first following from the use of various different functional forms for an endogenous variable (e.g. y, log y, y^2, etc.), and the second arising out of the use of endogenous constant price variables (e.g., y), the corresponding price index (P_y) and the combination $P_y.y$ to represent the nominal value.

Various numerical techniques are available to solve such nonlinear models. The most commonly used method is called the Gauss-Seidel iterative technique, and consists essentially of choosing arbitrary starting estimates of the desired solution, and using the equations of the model one-by-one to calculate a new estimate of the solution, and iterating. Subject to certain equation orderings, and using certain damping factors on the iterations, the iterations eventually converge to fixed values, which can be shown to be a solution of the model. Typically, between twenty and thirty iterations are needed for convergence. It must be borne in mind, of course, that a non-linear model need not have a unique solution, but may have multiple solutions for given inputs. Care must be taken to select the solution which is "economically" sensible in such a case, but multiple solutions have not occurred in any of our simulations.

Three different classes of model solution can be distinguished –

(a) *Single-Equation Solution:* This is calculated simply as a check on the model estimation and programming. It takes each equation in isolation, behavioural and identity, and calculates a solution using historical data for *all* variables on the right-hand side of the equality sign. It is a useful check, since the model identities should hold exactly, and the model solution program can be used to generate input to the model validation software (see below).

(b) *Single-Period Solution:* Here, all the exogenous variables (current and lagged) and all the lagged endogenous variables are considered as given for each year and, at least for within-sample data, take their historical values. The complete system is then solved simultaneously for the current endogenous variables.

(c) *Multiple-Period Solution (with time horizon n years):* Here, all exogenous variables (current and lagged), and the base-year (initial) values of the lagged endogenous variables take on given values. As the system solution sequence moves forward in time, from the base year to the terminal year n, the system solution endogenous variables for year t are used to form lags for year (t + 1).

In the results described in Section 4, simulations of the above types will be examined, and their respective properties discussed.

(v) Model Validation: Having specified, estimated, and solved a model, one is then faced with the problem of establishing whether the model is, in some sense, a "good" one.

Sowey (29) makes the distinction between "verifying" a model (i.e. establishing whether the model is "true") and "validating" a model (i.e. determining whether the model fulfils well the demands made on it, in the sense of whether the model, with all its imperfections, does an adequate job of prediction, both within and beyond the estimation period). Clearly, model verification is impossible, so we restrict ourselves to model validation. Validation is not in any way incidental to model building. It would be irresponsible to use a model for practical applications without first carrying out an exhaustive validation.

There are many possible methods of model validation, and we discuss below some of the possible approaches:

(a) Statistical validation at the estimation stage: A type of validation is carried out automatically at estimation, using the standard regression statistics (t–ratios, adjusted R^2, DW, etc.). However, an equation-by-equation validation is not sufficient since, while individual equations may fit quite well, the equations, solved simultaneously, may have different, undesirable properties.

(b) To effect a within-sample comparison between the model's simulated values for the endogenous variables, and the historical data, a special computer package, VALIDAT (21), was developed. This package produces a graphical representation of the actual and simulated time series (some of which are presented in Section 4), and a set of non-parametric statistics which facilitate the comparison between actual and simulated results (e.g. mean absolute errors, root-mean square error, the comparison between the performance of the model forecasts and the forecasts of a series of "naive" models, such as a "no-change" or a "same-change" model). Also, a test for bias and efficiency of the model forecasts is carried out by regressing the actual data on the model predictions. For a detailed account of the VALIDAT package, we refer to the user-guide (21).

(c) In estimating a model, one should ideally leave some of the most recent observations out of the estimation period, both because it is desirable to be able to test the equation in an out-of-sample period and, more seriously, because the most recent data will be subject to large revisions (for example, the figure for real consumers' expenditure for 1975 was revised upwards by almost 5 per cent between the 1975 NIE and 1976). However, given the lag of over one-and-a-half-years in producing the NIE, and given the large revisions that are usually made even after three years, our freedom of action is limited. However, in future re-estimations of the model, we may exclude the most recent year's data and use these data for an out-of-sample test. This will also facilitate the error-adjustment of the model for forecasting purposes.

(d) A non-statistical validation, using the model multipliers, is highly desirable. Even if the individual equations and the model simulations appear to track well within and out of sample, their reactions to exogenous shocks may be unreasonable. Some of the model's coefficients will be directly interpretable, on a single-equation basis, as marginal propensities to consume, import, etc. However, a system-wide analysis of the effects of such shocks can only be carried out by calculating multipliers. Dynamic multipliers are essentially period-by-period response rates of certain endogenous variables to exogenous shifts in levels or flows of certain exogenous variables. They answer such questions as: "by how much will real GNP increase if real government expenditures are increased by one million 1970 pounds?", and are calculated as follows:

(1) A base starting point in time is chosen (t_o).

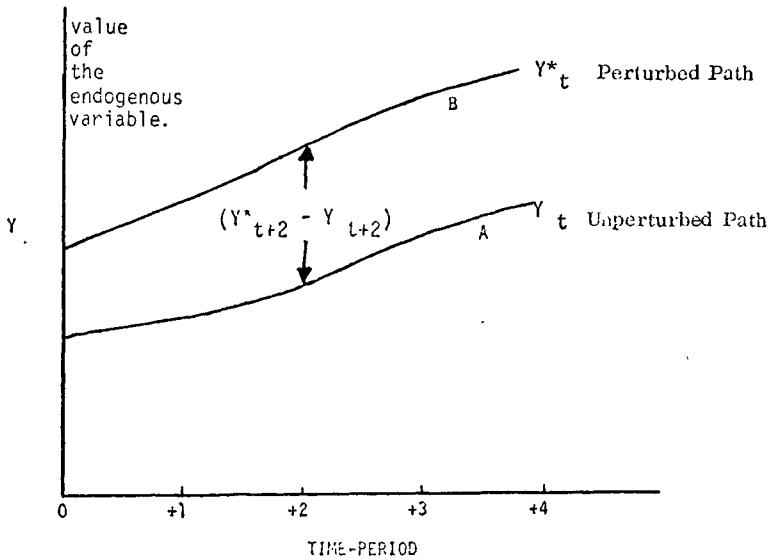
(2) A multiple-period simulation is performed, starting at t_o, and keeping all exogenous variables fixed for $t > t_o$ at their t_o values.

(3) The multiple period simulation is repeated, starting again at t_0, keeping all exogenous variables, bar one, at t_0 values. One exogenous variable is changed (perturbed) from its t_0 value, and is fixed, for $t > t_0$, at this new value.

(4) For each point in time, t, the unperturbed solution for relevant endogenous variables is subtracted from the perturbed solution, to calculate the change induced by the exogenous perturbation.

(5) Using Δx to represent the exogenous perturbation, and Δy to represent the induced change, we calculate $\Delta y/\Delta x$ for multipliers, $(\Delta y/\Delta x)$. (x/y) for elasticities, and $(\Delta y/\Delta x)$. k for, what we call, fiscal effectiveness multipliers, where k is the change in x that is needed to alter the tax revenue associated with x by one million pounds, in real terms. (For example, x could be the tax rate on alcohol, and the corresponding revenue would be the customs and excise duty on alcohol)

Figure 3.2: Time-Paths of an Endogenous Variable in Computing a Dynamic Multiplier



The whole process is represented in Figure 3.2. It should be noted that these multipliers are "cumulated" multipliers, representing the accumulated effect, at time t, of a change made in a previous period, $t_0 < t$. First differencing, yields, what might be called, "delayed" multipliers. It should also be noted that cumulated multipliers for a non-linear, dynamic model need not necessarily have finite equilibrium values, but usually do. The examination of multipliers for a range of instruments and targets, provides a most rigorous and searching test of the macromodel specification. Indeed, it has been responsible for many fundamental changes in the model which would not have come to light otherwise.

(e) A final type of model validation is provided by what we call its "track record". By this we mean the accumulated experience, over a period of years, gained by exposing the model to policy analysis and forecasting exercises. It is too early to talk about the model's track record in this sense, but some of the problems of using the model in this context are discussed later.

4. MODEL PERFORMANCE

In this Section we discuss the results of a comprehensive within-sample validation exercise. The section is essentially a precis of the corresponding section in (7), to which the reader is referred for a more detailed account. The version of the model which was validated in that exercise is now out of date; the new version will be similarly tested in due course, but one would not expect drastic alterations in the model's overall performance.

Summary Statistics

Table 4.1 gives mean absolute errors (MAE) and mean absolute percentage errors (MAPE) for some of the more important variables in the model. These refer to the single-period and multiple-period simulations for the post-1960 period. The mean absolute percentage error, normally an easily interpreted summary statistic, can sometimes be rendered meaningless because of the range of variation in the endogenous variable under study. For example, if the true value of the balance of payments is –£10m., a model which predicts +£10m. has an absolute percentage error of 200%. If the payments balance were +£200m. and the model predicted a figure of +£100m., the absolute percentage error is only 50 per cent, even though the first of the two forecasts would surely be deemed more accurate than the second. In the table, the statistic MAPE is accordingly not shown for variables where the true value can be very small or negative. For this type of variable the mean absolute error is, of course, interpretable in terms of the units of measurement of the variable itself.

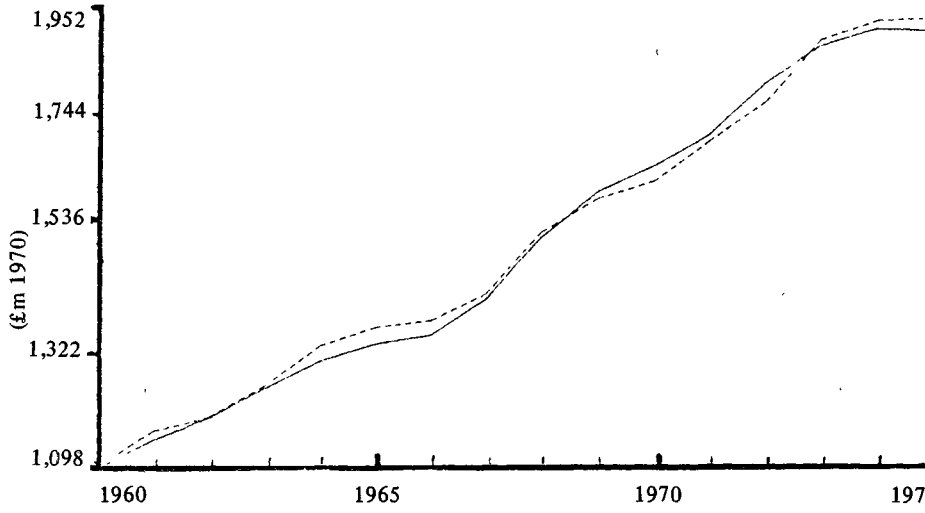
TABLE 4.1: Single and Multiple Period Simulations: Selected Validation Statistics

Variable \ Time-period	1960–75 Single period	1961–75 Multiple period 1-year ahead	1962–75 Multiple period 2-years ahead	1963–75 Multiple period 3-years ahead
Real GNP (£m. 1970)	MAE 17.28	26.34	37.14	42.54
	MAPE 1.14	1.81	2.48	2.79
Unemployment ('000)	MAE 4.25	3.85	3.54	3.63
	MAPE 6.74	6.45	5.93	5.63
Balance of payments (£m. current)	MAE 22.64	27.00	27.86	37.78
Borrowing requirement (£m. current)	MAE 9.57	11.85	13.83	16.54
External reserves (£m. current)	MAE 30.12	42.94	47.20	61.52
	MAPE 10.22	15.63	17.67	20.71
Change in real GNP (£m. 1970)	MAE 17.28	22.89	22.13	24.25
Consumer price inflation (Percentage change)	MAE 1.28	1.27	1.20	1.25

Single-Period Simulations

For the static simulations, the mean absolute percentage errors for the main macroeconomic magnitudes are between one and two per cent. The model's ability to track GNP is illustrated in Figure 4.1.

Figure 4.1: Actual and Simulated Real GNP, 1960–1975



The solid line gives the actual and the broken line the simulated values. It is interesting to note that the mild recession of the mid-sixties is missed, although the simulation does dip at the right time. The same sort of behaviour is evident in 1974-75, where the simulated values turn down, but not quite far enough.

There are certain endogenous variables in macroeconomic models which are critical for checking the model's performance. These are variables whose values in model solution reflect influences from many other equations in the model. An example is unemployment, which, being the difference of labour supply and labour demand, summarises the performance of the labour market section of the model. Other examples are the balance of payments on current account, the government borrowing requirement and the level of reserves, which are in the nature of summary checks on the trade sector, the public authorities' sector and the money demand equations respectively.

The performance of the model in simulating these four variables is illustrated in four more diagrams, beginning with unemployment. Unemployment is tracked well in general, with a mean absolute error of just 4,000. However, some of the turning points are missed. The balance of payments is tracked closely, apart from a large error for 1975. The mean absolute error in the prediction of the payments deficit was just £25m.

The model predicts government borrowing as the difference between revenue, most of whose components are explained in the model, and expenditure, which is largely exogenous. There were some relatively large errors in the early years but the mean absolute error over the sample period was under £10m.

The plot for reserves given in Figure 4.5 has the unusual characteristic, for econometric model simulations, of exaggerating rather than attenuating the changes in the series. The mean absolute error is £30m.

The most important inflation rate in the model is that for consumer prices and the

performance of the static simulation in tracking this variable is shown in Figure 4.6.

Given that the series is a rate-of-change, the model tracks well, with a mean absolute error of only 1¼ percentage points. Turning points are sometimes missed but the take-off in inflation at the end of the period is modelled reasonably accurately.

We conclude the discussion of the static simulation with a look at one final diagram, that for the change in real GNP shown in Figure 4.7.

Figure 4.2: Actual and Simulated Unemployment, 1960–1975

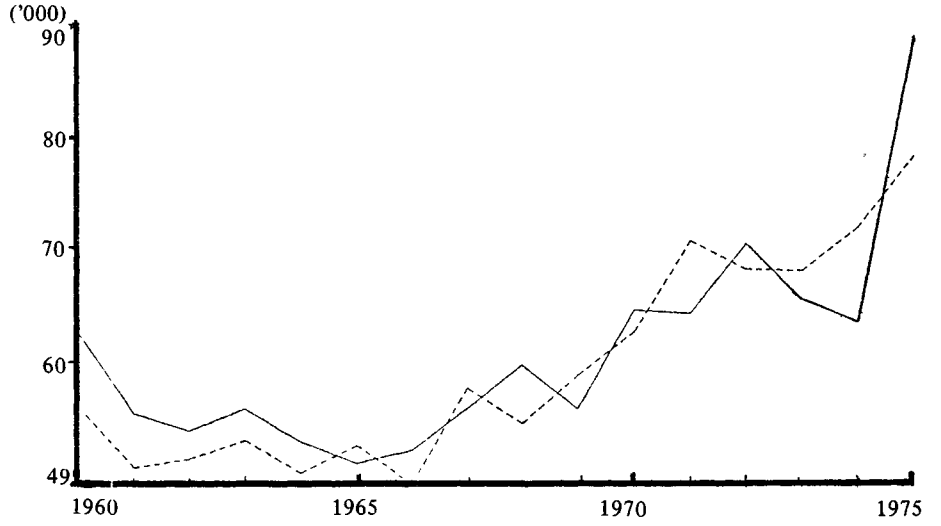


Figure 4.3: Actual and Simulated Current Account Balance of Payments, 1960–1975

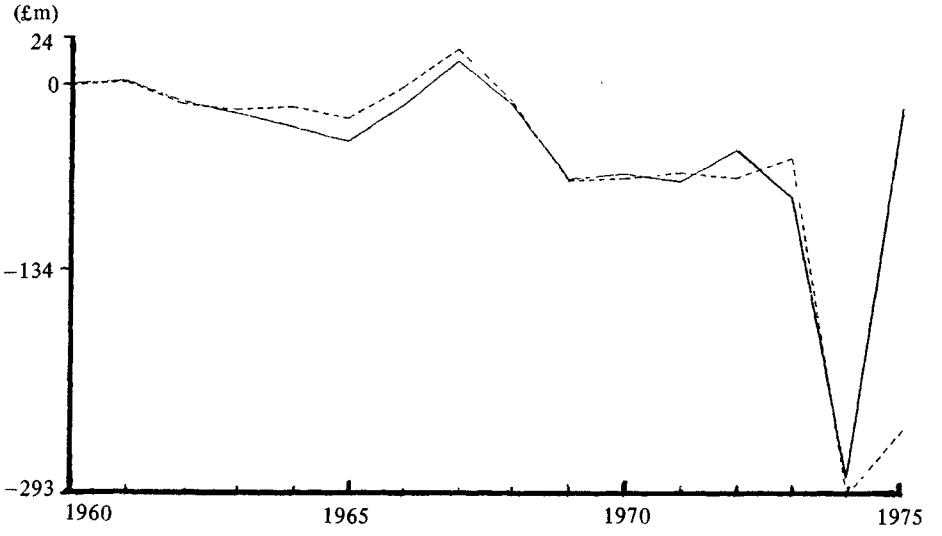


Figure 4.4: Government Borrowing Requirement, Actual and Simulated, 1960–1975

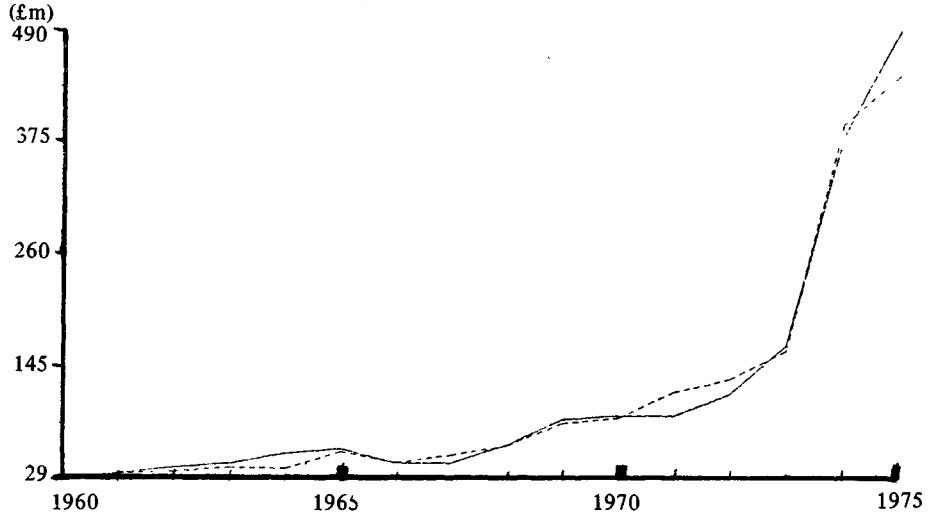


Figure 4.5: Actual and Simulated External Reserves, 1960–1975

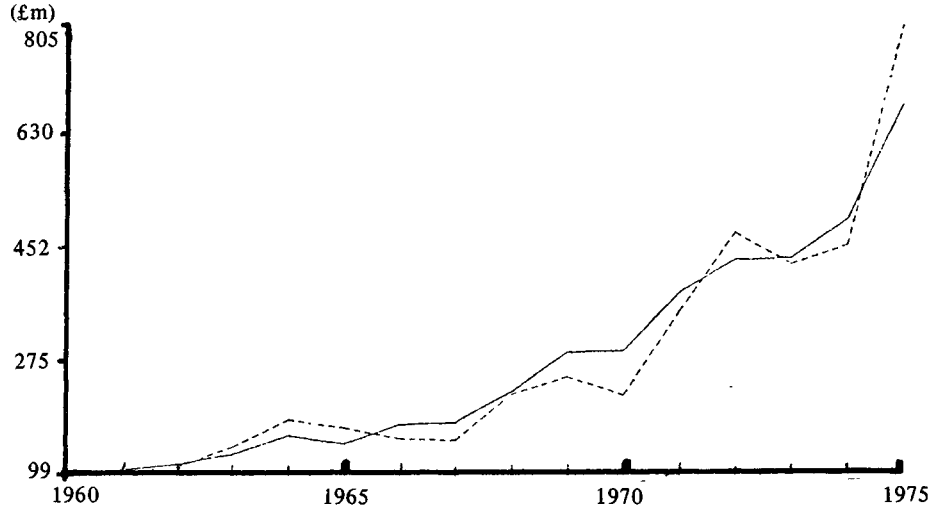


Figure 4.6: Rate of Change of Consumer Prices, Actual and Simulated, 1960–1975

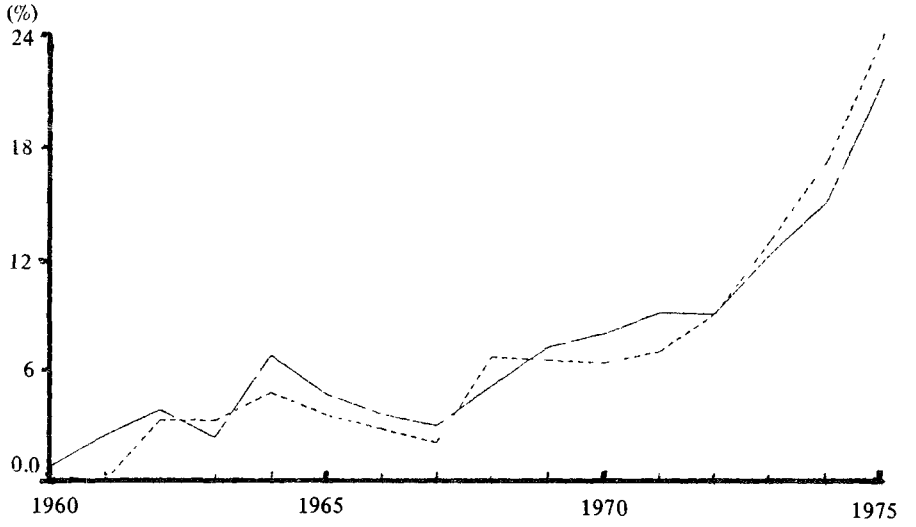
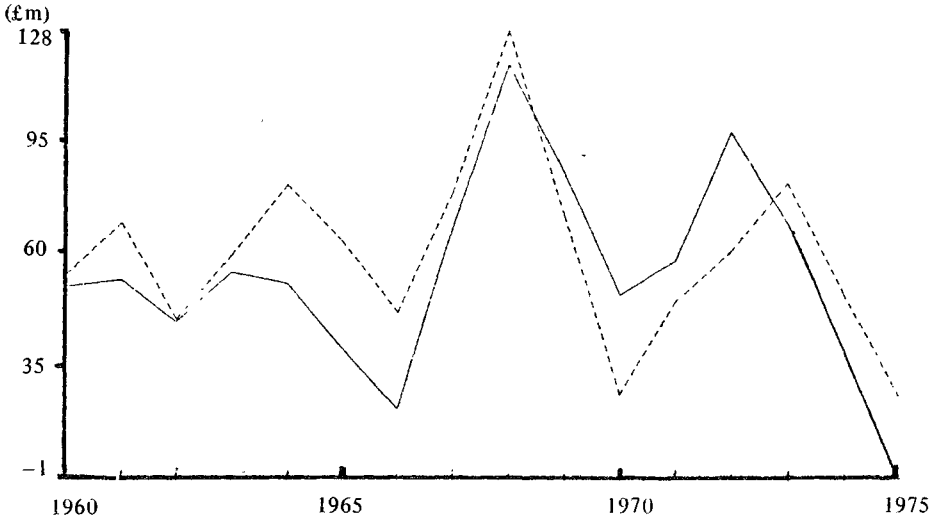


Figure 4.7: Changes in Real GNP – Actual and Simulated, 1960–1975



The tracking of the change in GNP is probably the best overall test of a stabilisation policy model, since changes in the major macroeconomic aggregates are the principal objects of interest. The ability of the model to pick up the turning points in this first-difference series is very good overall, aside from the mis-timing of the 1972 turnaround in ΔGNP, which the model places in 1973. A minor turning point in 1963 is also missed. The 1965–1966 recession, as we mentioned in discussing the level of GNP, is not tracked well: the model turns at the right points but the change in GNP is overpredicted for these years.

In focussing attention on the handful of major economic magnitudes just discussed, much of the detailed information provided by the validation exercise has been ignored. Validation statistics and plots were prepared for all 100+ endogenous variables and for both single-period and three multiple-period simulations, and all of the detailed results can be inspected in the Research Department.

Multi-Period Simulations

Since multiple-period simulations, particularly over a horizon of two or three years, correspond more closely than single period simulations to the way in which an econometric model would be used in practical work, a number of multiple-period simulations were performed with the model. Starting in 1960, one year ahead simulations were performed for each year up to 1975. Two year ahead and three year ahead simulations were performed for 1961–1975 and 1962–1975. Since the base year prediction is of no interest in a multiple-period simulation, this yields time-series of predictions for 1961–1975, 1962–1975 and 1963–1975, corresponding to the one, two and three year horizons.

The three time series of predictions for all the 107 endogenous variables in this version of the model were fed through the model validation package.

As may be seen from Table 4.1, there is a tendency for the prediction errors to grow as the simulation period lengthens, although none of them seem to behave in an explosive manner. Some of the prediction errors must perforce move in step. Thus, the error in the borrowing requirement rises as the GNP error rises, just as one would expect, since tax receipts are directly related to the level of economic activity.

The error in R, external reserves, is large but this is to be expected given the manner in which reserves are determined. Since the stock of credit is given, the prediction error in reserves will equal the sum of the prediction errors in the components of the demand for money. The money stock is a much larger magnitude than reserves, so the error will be magnified when expressed as a percentage of the smaller base. Moreover, there is little likelihood of the errors in the components of money demand offsetting one another.

Multiple-period simulations can display time-paths for the endogenous variables which diverge from the actual values by increasing amounts as the time-span of the simulation lengthens. If the model behaved poorly in this respect, we would expect to observe a clear warning in the pattern of the one, two and three year dynamic simulations. However, there is no indication of any problems in this respect. A much longer (15 year) dynamic simulation was performed on an earlier (but only slightly different) version of the model. Again there was no evidence of major divergence over time in the tracking of the major magnitudes.

If the model had unsatisfactory local properties, this would also be expected to have shown up in the multiplier calculations but this did not happen either. We now turn to a detailed consideration of the multiplier results.

Multipliers

Multipliers with respect to 15 exogenous variables and on eight endogenous variables were calculated for the years 1960, 1965, 1970 and 1975. Impact multipliers and cumulative multipliers with a time horizon ranging from one to four years were computed in each case. The detailed results, which are all given in (7), would be too lengthy to include here, so we concentrate on the government expenditure multipliers and will give only a cursory account of the others.

TABLE 4.2: Government Expenditure Multipliers

		EXOGENOUS VARIABLE G Endogenous Variable							
Base year	Time horizon	Y	BP	R	U	N	PC	BR	TOTEMP
1960	0	1.1688	−0.2313	0.2901	−0.0870	−0.1020	0.0	0.4877	0.1379
	1	1.4841	−0.4407	0.3951	−0.1255	−0.1476	0.0	0.4555	0.2624
	2	1.4617	−0.3760	0.4103	−0.0824	−0.0990	0.0	0.4608	0.2840
	3	1.3475	−0.2161	0.3973	−0.0212	−0.0337	0.0	0.4859	0.2530
	4	1.3008	−0.1653	0.4019	0.0075	−0.0022	0.0	0.5160	0.2293
1965	0	1.1494	−0.2773	0.3989	−0.0747	−0.0874	0.0	0.6674	0.1183
	1	1.4570	−0.5277	0.5353	−0.1100	−0.1298	0.0	0.5915	0.2281
	2	1.4464	−0.4570	0.5686	−0.0712	−0.0988	0.0	0.5972	0.2477
	3	1.3645	−0.2809	0.5646	−0.0197	−0.0325	0.0	0.6293	0.2217
	4	1.3302	−0.2280	0.5768	0.0058	−0.0049	0.0	0.6637	0.2019
1970	0	1.1121	−0.3616	0.6324	−0.0622	−0.0730	0.0	0.8884	0.0985
	1	1.4397	−0.7063	0.8068	−0.0958	−0.1150	0.0	0.7420	0.1964
	2	1.4335	−0.6169	0.8230	−0.0633	−0.0628	0.0	0.7389	0.2162
	3	1.3608	−0.3871	0.8003	−0.0174	−0.0327	0.0	0.7891	0.1936
	4	1.3252	−0.3121	0.8064	0.0068	−0.0067	0.0	0.8399	0.1754
1975	0	1.0855	−0.8318	1.3563	−0.0526	−0.0613	0.0	1.8985	0.0830
	1	1.3832	−1.6500	1.5911	−0.0807	−0.0980	0.0001	1.5557	0.1639
	2	1.3520	−1.4037	1.6299	−0.0510	−0.0711	0.0001	1.5724	0.1759
	3	1.3169	−0.8933	1.6780	−0.0122	−0.0317	0.0001	1.6848	0.1576
	4	1.3136	−0.7794	1.7356	0.0067	−0.0129	0.0001	1.7610	0.1470

Key to Table 4.2:
G = Government expenditure
Y = Real GNP
BP = Balance of payments
R = External reserves
TOTEMP = Employment
U = Unemployment
N = Net migration
CP = Consumer prices
BR = Borrowing requirements

The impact multiplier of G on Y in 1975 is around 1.1 and it rises to approximately 1.4 after the second year is completed. Thereafter it declines somewhat. An experiment with a nine-period multiplier indicates that it rises again after 5 periods and stabilises slightly below the peak level it reaches after the second year. The fluctuating nature of the multiplier derives mainly from the dynamic characteristics incorporated in the investment equations. The dynamics of these equations are such that an endogenous cycle has been built into the model.

Fluctuations in the Balance of Payments (BP) multipliers exhibit similar characteristics

to those on Y. However, those for the level of External Reserves (R) and the Borrowing Requirement (BR) are more even. Because of the nature of the monetary sector, movements in external reserves (with domestic credit fixed) reflect movements in the demand for money. Both R and BR are thus directly affected by the level of consumers' expenditure which exhibits less fluctuation than Y.

The multipliers on those endogenous variables that are measured in real terms decline over time. This is as expected since the marginal propensity to import and the marginal tax rates rise with the level of the system. Those variables measured in nominal terms will have multipliers which increase over time. This is because the impulse, which is held constant in real terms, will rise over time with the rising price level.

Problems arise with the multipliers on U and N because of the manner in which the dynamic simulations were performed. In these simulations the level of the exogenous variables were held at their base year values. This applied also to the time trend variable wherever it appeared. The rationale for this course of action was that the time trend was acting as a proxy for other exogenous variables which should have been held constant. Some endogenous variables appear as rates of change on the left-hand side of equations. If an intercept appears in these equations, then implicitly the level of the variable is dependent upon time. Accordingly, these intercepts should be set to zero after the base period solution. This was not realised before the present multipliers were calculated.

The level of Migration (N) is determined by, amongst other variables, the difference between earnings in Ireland and those in the UK. Earnings in Ireland are, in turn, determined by an equation which has the rate of change on the left-hand side, and an intercept. Since the intercept was not set to zero, these earnings were rising over the dynamic simulations. With the level of earnings in the UK fixed, this had implications for the pattern of net migration. Since Unemployment (U) is closely related to Migration (N), the multipliers on U were also affected.

There is, of course, no difficulty in avoiding this problem in future. Problems associated with assessing the impact fiscal expansion is likely to have on the Irish labour market are discussed in (25).

The motivation for the calculation of multipliers is twofold. Not alone are they of direct interest but the exercise is an excellent mechanism for highlighting weaknesses in the specification of the model. One such weakness is clear from Table 4.2. It concerns the relationship between government expenditure (G) and public employment. Wage and salary payments are a major component of G and an increase in G will stimulate employment directly, unless the entire increase is concentrated in the non-wage component.

This direct effect on public employment has been ignored in the multipliers presented here. Thus the multipliers for total non-agricultural employment with respect to G are very much understated. The impact multiplier for 1970, for example, suggests that a £1m. increase in G will raise employment by 99 jobs. In fact, when account is taken of the direct employment effect on employment in public administration, the total increase in employment could be as much as 500 (at 1970 wage levels, of course). This, in turn, will cause an understatement of the multiplier of G on Y.

We conclude this section with a look at the multipliers associated with some of the other exogenous variables. Public spending on residential investment has slightly lower multipliers than G, because of the higher import content. If government spending took the form of subsidies, the multipliers are low in the first year, since there is no direct effect on GNP, but they rise later as consumer spending comes through. The same is true

of a spending boost in the form of increased personal tax allowances, for example.

The multipliers associated with an increase in overseas demand are lower than those for G, which might appear odd on the face of it. The explanation is straightforward; these multipliers, given the specification of the trade sector in the model, must reflect the higher import content of Irish exports.

The model validation exercise from which these results have been taken revealed several deficiencies in the model specification, which, in our opinion, would never have come to light through the ordinary equation by equation diagnostic checks. Many of these deficiencies have now been remedied and a revised version of the model has been issued (6).

An important preliminary conclusion from the validation exercise, although not a very surprising one, is that the multipliers associated with different expenditure and tax instruments differ substantially both in magnitude and in the timing of their impacts, a fact which is of considerable consequence in the design of stabilisation policy. The implications of the model for the conduct of stabilisation policy are being studied and will be the subject of a separate paper.

5. USING THE ECONOMETRIC MODEL

The direct uses of econometric models lie in the areas of forecasting, policy analysis and control. Given the structure of the Central Bank's model, reflecting the Irish economic policy environment, the main uses we visualise would concern stabilisation policy and short term macroeconomic forecasting, including monetary forecasting. However, before discussing the uses of the model in any detail, it is advisable to isolate areas where it would be inappropriate to use the Bank's model, or areas where caution should be exercised.

A useful distinction can be made between those models that focus primarily on the problems of stabilising an economy around a long-run trend, and those which are concerned with the conditions determining the underlying trend itself. Ideally one would wish for these two aspects of policy-making to be integrated within the one model, but this has proved difficult to achieve wherever it has been attempted. It is clear from the model specification given in Section 2 that the Bank's model would be entirely inappropriate for studying medium or long term developments, since, as it stands, it lacks a supply side. Hence, the model's legitimate area of application is limited to the short run, essentially to demand-management policies. Even in the short run the model by itself would not be a reliable guide to a policy analysis when supply constraints obtain.

The question of the quality and reliability of the model should also be a matter of concern to users. We have seen how the Bank's model has been tested, in an *ex-post* context, using the various validation techniques discussed in Section 3. It is our firm opinion that, given the risks of human error involved, no econometric model should be considered for practical applications unless a well-documented and comprehensive validation exercise is available for inspection. Model users should not be confronted with a "black-box", the accuracy of which has to be taken on trust. Furthermore, there appears to be an upper limit to the degree of accuracy obtainable in representing the complex workings of an economy. This limit, which may vary between countries and over time, is set partly by the state of economic theory, of econometric techniques, and of computer capabilities, but is to a much greater extent determined by the quality and availability of the economic statistics upon which all models are built. Even if perfection were attainable in these

areas, there would remain inherently unpredictable elements in the behaviour of economies due to unforeseeable events and the elements of volatility always present in human behaviour.

For some of the model endogenous variables, part of the behaviour being explained is represented by a "residual error", which ideally should be small and random. However, for the recent past, some of these "residuals" may not appear random, but rather show systematic bias due either to a misspecification in an equation or to some specific event that can be identified and allowed for. Adjustment of residuals must be monitored continually and represents a common-sense way of adapting the model to changing circumstances. Indeed, the stochastic (or random) residuals can be directly incorporated in what are called "stochastic" simulations, and some work in this area is described in (5).

Finally, it will be recalled that out model was estimated using data for the period 1953–1975, except for certain equations where data did not become available until a later date (e.g. VAT, and the output series). This period of twenty-three years saw major changes in the structure and characteristics of the Irish economy, and it might be questioned whether it is reasonable to use such a long period to estimate equations, when it is the characteristics of the post-1970 economy which are of most interest to policy analysts. There is an obvious trade-off between structural stability and "degrees-of-freedom" requirements for estimation: the shorter the data sample period, the less likely the prospect of structural change, but the less reliable will be the coefficient estimates. At this stage we can only say that we are aware of the problem, and where the availability of data permits, have carried out some studies of structural stability (11).

Before proceeding to a consideration of the uses of the model, it should be noted that a small-scale macroeconometric model is also maintained by the Research Department of the Central Bank (3). This contains all the essential features of the larger model, but is easier to manipulate because of its smaller size. This smaller model is being used for testing computer programs before they are adapted for the larger model and for preliminary model work in policy analysis and optimal control methods.

Policy Analysis and Forecasting

There are two direct ways in which the model can be used:

(i) To produce a forecast of the development of the economy.

(ii) To explore the effects of changing some of the assumptions on which a forecast has been based, and thus to discover whether, by altering some of the exogenous variables, (particularly those under the control of the policy making authorities) an improved outcome for the economy can be achieved.

We now go on to discuss briefly the logistics of using the model for exercises in policy analysis and forecasting. It must be emphasised that we are still "feeling our way" in this area, and have been guided by advice from colleagues in the Bank, in government departments, and from researchers in other countries.

The computerised system with which the model is operated was shown in Figure 3.1. In that diagram we distinguish between a data-bank of historical time-series used in the model estimation and a special data-bank used for policy analysis. This latter data-bank would at present contain the best estimates known to us for all the exogenous variables in the model outside the estimation period, i.e. for the years 1976 to 1977, with informed (or inspired!) guesses for 1978 and 1979. A major problem in economic forecasting relates to the starting point of the forecast, since economic statistics are often characterised as being *late* and *unreliable*. Hence we must attempt to project the future

without knowing precisely where we are at present and, worse still, precisely where we have been!

Suppose it is desired to monitor the performance of the economy during 1978 looking forward also to 1979. Since an analysis would involve consideration of at least one previous year, the current year and, perhaps, one future year, a possible format for the task would be as follows:

(i) Using the most up-to-date information on exogenous variables, the model would be used to simulate the period 1977 to 1979, using a multiple-period simulation, after adjusting the model equation intercepts to ensure good tracking performance in the recent past.

(ii) At the same time, another forecast would be made by judgmental means, without the aid of a formal model. Judgmental forecasters usually reflect a good feel for what is going on in the economy and their short term forecasts are often better than formal model forecasts.

(iii) The two forecasts would be compared, and any differences investigated, usually leading to insights and revisions in each.

(iv) A consensus forecast is arrived at, which is a blend of the forecasts prepared by both methods.

(v) One of the advantages of a formal model over informal methods is that having arrived at a consensus forecast one can quite easily investigate the sensitivity of the forecast to the assumptions made about key exogenous variables. For example, optimistic and pessimistic values of exogenous variables can be used, and an estimate obtained of possible upper and lower bounds of the forecasts. In addition, model multipliers on a range of target variables for a selection of policy instrument variables, can easily be calculated, and the implications for fiscal and monetary policy examined. In the UK Treasury, such calculations are used to produce what are called "ready-reckoners" which show at a glance the implications of variations on existing policies (reference (1)).

In the above process, communications between the model-based forecasters and the judgmental forecasters is greatly facilitated if the output from the model is in an easily readable and useable format. To this end, in collaboration with the Department of Finance, we have designed a "report-generator" computer code to produce output from the model in a format analogous to the NIE tables. An example of one such table is shown in Table 5.1. It is hoped that the wide use of such reports will help give a "human" face to the model, and will remove much of the unnecessary mystique that surrounds models and computers.

A variety of analyses can be performed using the present model. These include examining the implications of external disturbances (such as changes in the terms of trade), investigation of the effects of alternative incomes, monetary and fiscal policy mixes. For example, the model might be used to quantify the economic impact of a budget. Although the budgetary instruments are not always incorporated explicitly into the model, it is usually possible to quantify most of the important budgetary changes in terms of the policy instruments used in the model. One would first simulate the model, setting the budget instruments at their pre-budget values. One then makes the necessary changes to the instruments, resimulates with the model, and extracts the effect of the budget by subtracting the latter solution from the former. When the instrument changes are small, one can use the model multipliers to estimate the effect of a budget, even though many instruments are changed simultaneously (26).

TABLE 5.1: Example of a Balance-of-Payments Table

	A		B			C	
	1970 £m	Change £m	1971 £m	Change %	Change £m	1972 £m	Change %
Agri. exports (incl. interv.)	184	44	229	24.0	14	243	6.1
Industrial exports	271	21	291	7.7	83	374	28.5
Services exports	144	5	149	3.6	7	156	4.7
Total exports	599	70	669	11.7	104	773	15.6
Merchandise imports	696	92	788	13.2	80	868	10.2
Services imports	52	−3	49	−6.1	6	55	12.2
Total imports	748	89	837	11.9	86	923	10.3
Trade deficit	−149	−18	−167	12.4	18	−149	****
Net factor income	28	−2	27	−5.9	3	30	11.2
Transfers from EEC	0	0	0	****	0	0	****
Other international transfers	30	1	37	3.3	5	42	12.6
Balance of payments deficit	−85	−19	−104	22.3	26	−78	****
Reserves	290	91	381	31.3	51	432	13.4

Optimal Control

It may be desirable to encourage ways of thinking about different policy objectives in a more integrated way, and this is one of the main advantages of considering economic policy-making within an optimal control framework. Suppose that a policy maker has an econometric model which, he believes, captures the essential relationships between a number of target variables (e.g. the unemployment rate, the growth rate of real GNP etc.) and a number of policy instruments (e.g. tax rates, government expenditure, etc.). He has direct or indirect control over his instruments and desires to manipulate them so as to cause some or all of his target variables to behave in a predetermined fashion, e.g. to track some ideal path or to minimize some function. The question addressed by optimal control is the following: how can the policy maker select the values of his control or instrument variables so that his objectives regarding the target variables will be realised. The mathematical technique which is used to solve this type of problem is called "optimal control". A recent UK House of Commons Report (1) studied the feasibility of applying this technique in the Treasury, and some preliminary work with the Central Bank's small scale model is described in (4). Indeed control theory can be regarded as another type of validation technique, where the model is confronted with a difficult situation in which the behaviour of its components is thoroughly tested by bringing into sharp relief the general interdependencies of the economy and the impact of one equation on the rest of the model, and vice versa. As Daniel Suits has observed (30) "The econometric model is not a substitute for judgement, but rather serves to focus attention on the factors about which judgement must be exercised, and to impose an objective discipline whereby

judgement about these factors is translated into an economic outlook that is consistent both internally and with the past observed behaviour of the economic system".

6. FURTHER DEVELOPMENTS

It should be clear from the earlier discussion that model-building in the Central Bank is seen as a continuing process and that no model will ever be "finished". In this final section, we discuss the work presently under way on the model and consider the directions which it might take in the future. We concentrate on the content of the model itself, ignoring applications: applications will be a major influence on the model's development.

In attempting to improve the performance of an econometric model, a distinction may be drawn between *deepening* the model and *broadening* it. By deepening a model we mean improving it at its present coverage: by broadening it, we mean expanding its scope and thus lengthening the list of policy questions which it addresses.

The model could be deepened in several ways. It has already been noted that the model virtually ignores supply behaviour, and may well overstate multipliers as a result. This weakness could be rectified in two ways: by modelling the supply of goods and factors directly, or by using a capacity utilisation variable in certain of the model's equations. The first approach would be a major undertaking; the second, which is in the nature of a short-cut, can be attempted more quickly. For example, we would expect income boosts to suck in more imports when capacity utilisation is high than when it is low and this type of process can be modelled by re-casting the import equation so that the marginal propensity to import varies with the cycle. Preliminary investigations along these lines have been carried out and the results to date have been encouraging. Whichever approach is ultimately adopted, it seems clear that greater attention to the supply side is desirable.

The performance of the model in tracking the different endogenous variables is uneven and there are some unsatisfactory equations which require attention. Our current strategy is to concentrate attention on the equations which really matter for quantifying multipliers and studies are already in hand on consumption, investment, imports and exports. As has already been mentioned, the consumption study is particularly important in view of the poor performance of the existing equation. The results of these various studies will be to hand in time for inclusion in the next version of the model which should become available in early 1979

Some of the poor tracking performance in the model may be due to the levels of aggregation used. Data availability limits the possibilities in this regard, but there remain opportunities for further disaggregation.

An important aspect of the model is that it is a dynamic model; the level of the system is never independent of its history, as the coefficients attaching to the lagged endogenous variables drive the model through time. The dynamic behaviour of the model depends on these coefficients and one must consider the precision with which they have been estimated. Given that only annual data are available, we are forced to operate at a high degree of time aggregation and it is likely that the dynamic responses are being smudged. Ultimately, the only solution to this problem is sub-annual data. However, the problems created by smudged dynamic responses and by the parameter instability mentioned in the last section are limiting the degree of precision which economic models can attain. The factual basis for econometric models, and indeed for empirical economics generally, is the basic data on which the models are estimated. Good statistics do not guarantee good

models, but no amount of technical virtuosity can make good models with bad data. It may eventually be necessary if improved models are deemed a priority, to make a further investment in economic statistics.

The final aspect of deepening the model that we wish to mention concerns estimation techniques. Given the degree of freedom limitation, it will obviously be difficult to develop a practicable estimation method with superior properties to that currently being used. With existing software there is a limit to what can be achieved in this area but, short of developing a completely new estimation procedure, more attention could be paid to dealing with simultaneity.

If it were deemed desirable to broaden the model, a priority might be in building on an agricultural sector. Given the importance of agriculture in the Irish economy, the lack of an agricultural sector in the existing model is a serious limitation to its usefulness. There are other areas where broadening the model might be thought worthwhile. For example, the modelling of residential investment is fairly *ad hoc* as things stand – a full-blown housing sector would be useful.

Our present general inclination is towards deepening rather than broadening the model. However, reflecting the Bank's primary focus of interest, emphasis will be placed on developing the financial sector. Several studies have been undertaken in the Research Department on monetary topics but these have been based largely on quarterly or sub-quarterly data – see (11) and (12), amongst others. We have been unable thus far to develop a satisfactory model, using annual data, of the credit market, which explains why credit is exogenous in the model as it stands. It is our intention to conduct further studies of financial markets. If, as seems likely, these will continue to exploit the richer vein of quarterly data, work on financial topics may not be reflected fully in the annual model specification. Thus, macroeconomic analysis in the Bank employing the annual model will need to be supplemented by drawing on the more detailed financial studies based on sub-annual data.

The development of the Central Bank's macroeconometric model has already benefitted considerably from outside comment and criticism. We look forward to hearing the views of this evening's audience.

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DISCUSSION

Professor N. J. Gibson:* The Society should I believe be congratulated on inviting the Research Department of the Central Bank to read a paper on its Macroeconometric Model. For the production of this model is, I think, an important landmark in the development of Irish economics. It betokens and epitomises an increasing professionalism which has been evident in Irish economics for some years now and augers well for the quality and calibre of advice that is likely to be offered on fundamentally important economic issues. This is not to say, of course, that economic considerations are all important and should override all others.

In discussing the paper I begin by looking somewhat cursorily and selectively at the theoretical framework of the model, and, in doing so, make use of some of the additional research papers, referred to in the Research Department's presentation, especially numbers 6 and 7. Next, I consider some particular specification issues, concentrating on the monetary sector and its relationship to the rest of the model. Finally, I make some general comments about the whole important enterprise and how it might be further encouraged.

The authors tell us that "the model is a short-run Keynesian one with the level of activity ... demand determined (with) no explicit supply side ... (and) is an annual one built around a National Accounts framework".

It may be helpful to emphasise what is meant by a short-run Keynesian model, though in important respects, the Central Bank model goes far beyond this type of framework. But in general, a short-run Keynesian model lays great stress on the expenditure sector, notably consumption, investment and government expenditure and, in addition in the open economy, on exports and imports. It tends to stress, particularly when there is a fixed exchange rate, the importance of fiscal policy as an instrument of economic management, places little emphasis, as the paper states, on supply considerations or the possibility of factors constraining supply, and frequently gives an unimportant role to monetary influences. It would be unfair to interpret the Central Bank model precisely in this way, though its ultimate origins are to be found within this kind of context.

Typically, the Keynesian type model concentrates on what has been called instantaneous flow equilibrium. That is, it determines the equilibrium values of such magnitudes as real income, investment and savings for given values of the capital stock and money stock (Branson, 1976). But investment and savings are the rates of change of the capital stock and wealth, respectively, and, hence, the equilibrium can only be instantaneous or very short-run. Otherwise, the capital stock, wealth in its various forms, including money balances and securities, will be changing and these changes will in turn generally affect the flows previously referred to and so upset the instantaneous equilibrium.

This problem does I think, in principle, pose difficulties for the model with its reliance on, through no fault of its own – and this is a point I return to – annual data. This period seems very long in relation to the model's theoretical structure.

The specification of the model very carefully includes certain important constraints such as the budget constraint and the balance of payments constraint as well as many balance sheet and other identities. However, it does not contain a wealth constraint for

* I would like to record my appreciation to the Central Bank of Ireland for the opportunity to spend part of my study leave in the Bank and to acknowledge financial support from the Leverhulme Trust.

the private sector and so wealth does not appear explicitly as affecting private sector behaviour. This omission would, I think, upset the susceptibilities of those with leanings towards Yale type model formulations, especially as regards the monetary sector, but also as regards consumption behaviour (Brainard and Tobin, 1968). It is true that liquid assets appear in the consumption function and may well act as a proxy for wealth but this does not fully meet the point. It is, of course, the case that there are no data series for wealth but there are ways of trying to meet this difficulty in terms of the inclusion of wealth as a constraint variable in all the relevant relationships of the private sector. I return to the question of the monetary sector and the liquid assets variable below.

A further feature of the model is its relative neglect of price expectations. But once more, this may in part be explained by the use of annual data. However, price expectations have been found to be of substantial importance in other models, notably the FMP model, in particular, with its strong inter-action between real interest rates and price expectation variables (Modigliani, 1975). The model also neglects expectations as regards the exchange rate. This is (or at least was until recently!) possibly plausible with respect to sterling but is presumably doubtful in relation to other currencies. How to model exchange rate expectations in an ostensibly fixed exchange rate situation is clearly not a straightforward undertaking but may be extremely important where a country or government is a persistent and heavy foreign borrower.

I now return to the monetary sector. As the paper makes clear, this sector of the model is based on what has become an almost standard one for small, open economies with a fixed exchange rate. In its full equilibrium form, this implies that changes in domestic credit affect only the level of reserves and the supply of money is perfectly elastic at the exogenously given interest rate. That is, there are no long-run effects on output, employment, the interest rate and so on. Furthermore, there are no limits on the scale of balance of payments deficits or foreign borrowing, and governments can increase foreign reserves by increasing their spending. Clearly, this is a worrying formulation and perhaps a dangerous one if governments were foolhardy or reckless enough to be tempted to act on it. Once more, to cope with these issues poses difficult modelling problems. It may well be that what is required is a relaxation of the exogenous interest rate assumption and, as a minimum, the specification of demand and supply relationships for domestic and foreign securities.

The monetary sector of the model is highly condensed and apart from the liquid assets variable referred to earlier, does not formally impinge on the rest of the model; and as the paper states "the money market is always in equilibrium". This approach to the money market contrasts strongly with that to be found in what is called the minimal model or, more precisely, RBA76 – albeit a quarterly model – of the Reserve Bank of Australia where they take the view that:

> "In a world with uncertainty and large monetary disturbances, money may rationally be used as a buffer stock to soak up the effects of the outcome of decisions in all other markets in the short run. This is not to imply that economic agents are indifferent about the amounts of money which they hold, but, rather, that the gap between actual and desired money balances signals the need for a change in their decisions on household expenditures, prices, asset and factor demands".
>
> (*Reserve Bank of Australia*, p.87)

This approach to the role of money, of course, owes much to people like Friedman, Brunner, Meltzer and other so-called monetarists, but is by no means exclusive to them

as may be seen from the Brainard and Tobin paper referred to above. Does it have relevance in Irish conditions? At least, it should not, I think, be ruled out on *a priori* grounds. However, if the Central Bank were to follow it, it would mean a fairly radical change in the specification of their model; it may be doubted if this would be justified in an annual model but might well be in a quarterly one.

But whether or not money performs as a buffer asset, there is, I believe, a strong case, and indeed, the paper accepts this, for an elaboration of the model's monetary sector and its integration with other sectors. I suspect the monetary sector needs at least to include not only a domestic credit market for both public and private credit but also, as already indicated, to allow for domestic residents to hold foreign assets and incur foreign liabilities. This would have the effect of introducing a number of interest rates into the model, whereas, at the moment, only one rate appears, the ordinary deposit rate, and is confined to one equation – the demand for current accounts. This again raises important "Yale type" specification questions and, incidentally, in terms of the familiar IS–LM framework, gives the model a horizontal LM schedule, which coincides with the balance of payments schedule, and a vertical IS schedule. However, like the presenters of the paper, it is, I think, doubtful if a substantial elaboration of the monetary sector would be worthwhile unless quarterly data are available.

I mentioned previously the liquid assets variable which is included as part of the monetary sector but appears only in the aggregate consumption function. Liquid assets (LIQ) are defined as currency in the hands of the public (CUP), current accounts (CA) and deposit accounts (DA) in associated and non-associated banks, less total deposits of the non-associated banks (TDNAB), plus other liquid assets (OLIQ), the latter covering deficits in hire purchase companies, building societies, state financial institutions, the post office savings banks, national instalment saving, savings certificates and holdings of prize bonds, i.e.

$$LIQ = CUP + CA + DA - \overline{TDNAB} + OLIQ$$

Thus, liquid assets exclude deposits with the non-associated banks, presumably on the grounds that these are largely company deposits. However, this division has the consequence that a transfer of deposits from the associated banks to the non-associated banks, whilst leaving money balances unchanged, has the effect of reducing liquid assets. But a reduction in the latter will affect consumption expenditure even though the money market is still in equilibrium and other liquid assets are unchanged. This formulation is clearly not entirely satisfactory. Furthermore, since the money market is in continuous equilibrium, disequilibrium as regards liquid assets must apparently appear only in other liquid assets – despite what was just said – and the only way adjustment can take place is via consumption expenditure; specifically, there cannot be shifts between other liquid assets and bank deposits. Also, TDNAB are treated as exogenous. This has the consequence that an increase in GNP in the model increases deposits in the associated banks but not the non-associated ones, which again is scarcely satisfactory. Too much should not be made of these points. Some of them are currently under examination as the model is continually being improved.

Before I finish I would like to refer briefly to the absence of an explicit supply side in the model. This, I think, is a pity for a number of reasons. It presumably implies that the output multipliers which are of great interest are somewhat overstated. Incidentally, they may also be overstated in so far as government expenditure or part of it is a substitute for private expenditure. More fundamentally, I think an omission of supply considerations almost necessarily concentrates attention on demand and demand stabilisation policies. Yet, it is arguable that for a small open economy it is far more important in terms of the

general well being, at least in the longer term, to emphasise the importance of supply considerations and to direct more of our energies towards increasing productive performance. I am not, however, suggesting that the latter should be extensively elaborated in the kind of model under discussion.

Finally, I believe the Central Bank and its Research Department can be extremely proud of their achievement; it is both impressive and, in my view, highly important. However, there seems to me little doubt that the model could be made richer and of far more importance to policy makers if quarterly data were available. I believe it would be a good investment on the part of the State if it were prepared to make available the relatively small resources which would be required to produce and maintain basic quarterly data series. It is a privilege to have been invited to propose the vote of thanks to the Research Department of the Central Bank; I so propose.

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Professor John E. Spencer: I am delighted to second the vote of thanks to Dr Bradley and the Central Bank for this thoughtful and thought provoking description of the Central Bank Econometric Model of the Irish Economy. Central Banks are often noted for conservatism and so it is particularly valuable and pleasing that this Model has been so frankly and openly presented. It will surely stimulate economic research in Ireland in not inconsiderable measure.

I wish to comment on four aspects of the econometrics.

(1) *Size and Structure of the Model*

The Central Bank (CBI) Model is by modern standards of moderate size. While smaller then the large US quarterly models such as the Data Resources Inc. Model, the Brookings Model or the FRB/MIT/Penn Model, it is comparable in size to the London Business School and UK Treasury Models, both quarterly and the Wharton annual and the Hickman-Coen annual US Models. The latter two models are concerned with long term growth and accordingly pay greater attention to the supply side than the CBI do. They aim at policy analysis and forecasting over at least a 5 to 10 year period. In common with many other model-buildiers, the CBI emphasize short term forecasting and stabilisation policy and their model is sufficiently large to include many variables of interest while avoiding major problems of understanding and maintenance associated with many of the very large models.

(2) *Estimation*

The CBI generally uses least squares (OLS) as an estimation technique. The justification for this might be presented negatively by demanding to know what method is better in

some sense for a small sample situation in a nonlinear, interdependent dynamic model based on imprecise data with possible misspecification and nonspherical disturbance term structure.

More positive arguments might include the following considerations (noting that many equations are linear and nondynamic).

(a) OLS will give conditionally unbiased coefficient estimators in a linear nondynamic structural equation, conditioning on the values of the right hand side endogenous, given that the disturbance has zero mean. While it is true that unconditional lack of bias is preferable, this seems to be unavailable by any method. At least, under fairly general assumptions, the existence of the moments up to a relatively high order seems much less problematical for OLS than for the more sophisticated estimation techniques, especially maximum likelihood methods.

(b) Asymptotically, OLS methods are typically inferior but the extent of the inferiority as measured by the inconsistency is surely the relevant issue with respect to point estimates. We know from Wold's Proximity Theorem that the inconsistency varies directly with the variance of the disturbance term and the probability limits of the correlations between the disturbance and the regressors. In practical terms, one can hope for small disturbance variance by careful specification of the equation but the extent of correlation between regressors and disturbance is hard to assess. Its existence can be tested for in linear models following the recent methods of Wu *(Econometrica*, July 1973 and May 1974), but if one is prepared to assume that the covariances are low, OLS can be applied with confidence and the known robustness of the t and F tests will justify hypothesis testing in the usual way. Unfortunately Wu's tests seem to involve more computational effort than the estimation procedures that a successful passing of the tests is designed to obviate. The point to be stressed, however, is that simultaneity need not imply that OLS is an inadequate procedure (especially given imprecise data).

It should not be though however that there is no case for trying more sophisticated estimating methods. For example, Fair has claimed that increased complexity in estimation increases prediction accuracy in his experience. Further, Goldfeld and Quandt's Monte Carlo simulations with nonlinear simultaneous models suggest that OLS is inferior to other methods, the extent of the inferiority increasing with sample size. Since the CBI work with a small sample size of around twenty observations, this evidence, inconclusive as it is, does suggest that a simultaneous method of estimation might be usefully tried. Full information methods should surely not be considered partly on grounds of complexity and partly for fear of inter-equation contamination. Indeed, in the case of maximum likelihood, the estimator would not even be well defined (Sargan, *International Economic Review*, 1975). A form of instrumental variables should be readily feasible, however.

Write the i th equation (following Howrey *et al.*, *International Economic Review*, June 1974) as

$$y_i = F_i(Y_i)\, a_i + x_i\, b_i + u_i$$

where F_i is the linear or nonlinear functions of the right hand side endogenous and x_i are the included predetermined variables. Then an estimator of a_i, b_i is

$$\begin{pmatrix} F_i^{*\prime} F_i & F_i^{*\prime} x_i \\ x_i' F_i & x_i' x_i \end{pmatrix}^{-1} \begin{pmatrix} F_i^{*\prime} y_i \\ x_i' y_i \end{pmatrix}$$

where F_i^* is the predicted F_i derived from a regression of F_i on selected predetermined

variables (or functions thereof). There must be at least as many of these as the order of a_i (a necessary condition for the inverse to exist) and, for consistency they should be chosen as asymptotically uncorrelated with u_i.

(3) *Solution of the Model*

As in common practice with nonlinear econometric models, the CBI Model is solved using Gauss-Seidel iterative methods. The authors suggest that they have not been troubled by finding multiple solutions. One wonders if multiple solutions can arise in their model. If so, the Gauss-Seidel procedure might be producing different roots in the sense of taking, say, the positive square root for some time periods and the negative for others in the course of simulation. If something of the kind were happening in the simulations, how would it be recognized and how would one decide which is the "right" root?

(4) *Model Validation*

Hypotheses on individual coefficients are tested in the paper by using t-ratios. Credence might be added to this procedure by the analysis of Wu as mentioned above. While the authors, of course, recognize that the t-ratios have a problematical interpretation, a glance at their estimated equations (presented elsewhere) suggest that typically the t-ratios of included variables are at least two. This may be unduly conservative for at least two reasons. Model builders elsewhere have noticed startling changes (in some cases of more than a factor of four) in coefficient estimates and t-ratios arising from mere data revision. This kind of experience suggests giving a lot of weight to *a priori* information, based on economic theory or perhaps intuition, in variable choice. This point is reinforced by noting that a demand for smaller t-ratios will result in lower probabilities of Type II Error on null hypotheses of zero coefficients. This in no way denies the necessity of some experimentation, since we have virtually no *a priori* information on, for example, dynamic specification.

Since precise significance levels of t-ratios in a model of the CBI type are unknown, it should be of interest to attempt to adapt a suggestion of Anderson and Rubin (*Annals of Mathematical Statistics*, 1949). Consider a linear structural equation in standard notation $y = X_1 a + Y_1 b + u$, with X_2 the excluded predetermined appearing elsewhere in the model. In order to test the null hypothesis $H_o : b = b_o$ against $H_1 : b \neq b_o$, define $y^* = y - Y_1 b_o$.

Under H_o, $y^* = X_1 a + u$.

Under H_1, $y^* = Y_1 (b - b_o) + X_1 a + u$

$= X_1 c_1 + X_2 c_2 + v$.

The test involves regressing y^* on X_1 and X_2 using OLS and testing c_2 for significance. If the vector c_2 is significantly different from zero, then reject H_o. Insignificance of c_2 leads to acceptance of H_o. Apparently this simple test which involves no complications and is valid for small samples given disturbance term normality (and not too much experimentation if t and F tables are to be used uncritically!), has quite good power (Maddala, *Econometrica*, September 1974) and applies readily to many of the CBI equations. It is particularly useful where the right hand side contains just one endogenous variable. It is simple to adapt for a joint test on b and a sub-vector of a. An approximation could apply to nonlinear equations since y^* is equally well known under H_o in this case. Under H_1, $y^* = f(X_1, X_2) + v$ which might be approximated by $X_1 c_1 + X_2 c_2 + v$ plus further powers of X if desired. The test which I would then suggest would be to test all the coefficients other than c_1 for joint significance.

Practical and important tests of model validation include forecasting ability, in particular *ex-post* predictions in which future exogenous variables are known. Thus, the model might be estimated from a subset of the data, the rest of the data being used for the forecasting test. *Ex-ante* predictions where the future exogenous are forecast, test more than the model, especially where the model builder engages in constant-term adjustment to allow for some important new feature such as a strike, or to make the prediction more reasonable. It would be interesting, as a kind of forecasting test, for the model to be allowed to run into the future holding the exogenous variables constant (apart from logical changes such as time trend movement). Would the resultant path of the main endogenous variables be plausible?

A standard application is to compare forecasts with those of a naive model. This is of interest provided the naive model is not too naive. An ARIMA model is an interesting competitor since such models typically do well over short periods.

Various tracking measures are also often applied by model builders and the CBI report on several of these. These involve using the estimated model and given exogenous variables (a) to generate endogenous variable values over the sample period to compare with the actual values (b) to estimate certain multipliers. In order to assess this procedure write the dynamic model as

$$y_t = F(y_t, \ y_{t-1}) \ a + X_t \, b + u_t, \, Eu_t = 0$$

Supposing the estimates equalled the true parameters, the estimated Y's would be

$$y_t^* = F(y_t^*, y^*_{t-1}) \ a + X_t \, b, y_o^* = y_o.$$

The estimated errors are then

$$\left[F(y_t, y_{t-1}) \ - F(y_t^*, y_{t-1}^*)\right] a + u_t$$

which do not have mean zero (in contrast with the linear case). Since $Ey_t^* \neq Ey_t$ the tracking ability is hard to assess. The problem is that y* traces out a time series of what history would have revealed if the disturbances were all zero, given y_o. The y's, on the other hand, trace out the particular disturbed path that history did reveal, each disturbance affecting future y-values through the model's dynamics. Why should they be similar? This point, which is connected with analysis of Howrey and Kelejian (reported in Naylor's Computer Simulation Experiments with Models of Economic System, Wiley 1971) suggests that a few replications of stochastic simulation would provide more interesting sequences of y-values with which one could contrast reality and compute multipliers and which could be used to gain further understanding of the mechanics of the model and its dynamic implications. The CBI have made a start on such work with promising results (refer Bradley and Sexton, 1978).

Dr R. C. Geary: I compliment the team on the paper. It is good to know that their experiments are to continue.

A few specific points. It is stated that amongst the 129 predetermined variables 89 are exos. Is not this number excessive (though I notice from the paper Professor Spencer has just circulated that other models have as large a proportion of exos?). Even allowing for policy variables, isn't it a lot to expect that one is supposed to know so many values in the year to be forecasted? It is good to learn that agricultural income is to change sides. Few of the so-called exos have this character: I am really sure only of time *t*.

A major use of the model should be cautionary forecasting, showing the undesirable outcome of disorderly claims.

I am interested in the notion of lagged disturbances amongst the predetermined. Could

we be told what this is meant to do?

While to the eye the fit of estimated to actual of real GNP looks good in Figure 4.1, this is not the case, by the tau test (which would surely be confirmed by DW). There are only 2 (or at most 4) changes of sign amongst the residuals, with 16 observations, indicative of probable residual auto-regression. The model does not explain real GNP. On the other hand the showing of Figures 4.3, 4.4 and 4.7 is impressive. Why is 4.7 so good and 4.1 so much less so?

I may be wrong, but I wonder does the multiplier treatment (too small a *parvo* in too large a *multum*) make the *ceteris paribus* error, obvious in single equation multivariate OLS regression? It is true that the multi-equation system should eliminate this source of error but does the multiplier bring it back? Bluntly, matrices of behavioristic coefficients are meaningful but individual coefficients rarely so.

My next comment must be on the data, their accuracy and up-to-dateness. I heartily endorse the team's statement "no amount of technical virtuosity can make good models out of bad data". Having spent most of my active life in CSO I am aware of the difficulties. The solution is not simply to increase the resources of CSO. It is a matter of making all sectors of the public realise the vital importance of furnishing prompt and reasonably accurate returns. It is fortunate for us statisticians that absolute accuracy is not necessary for wise policy determination. If it were we would lose our jobs. Perhaps a professional advertising campaign on RTE would help in securing better returns.

All figures given in the paper relate to the in-sample period. Could the team not state what their forecasted ends would have been for 1976, to compare with the officially published actuals (some of which, it is unfortunately true, may be changed later).

In such uncertain fields as forecasting and planning every method should be used, small stochastic systems and large, as well as hunching, to be respected because it implies good sense and knowledge, and is ignored at the experimenter's peril whatever his methods. I still have a liking for the non-stochastic type of approach I advocated in this Society in 1965, based largely on input-output. Would the team care to give this method a run? Admittedly it is designed for planning and longer-term forecasting, while the team's model is short-term.

On validation, a useful practice is that of comparing estimates from the model with naive estimates (single factor extrapolations and the like) I have found the practice chastening when naive and sophisticated estimates are compared with ultimate actuals.

I would prefer percentage changes to absolutes in all variables used. Absolutes can deceive because of the inevitable high relationship between them.

At one time there was interest in developing a system of financial statistics, basic data and model, the lot, not merely to supplement the non-financial entities but to act as a mirror image of them. The point is that financial statistics have the advantage of being up-to-date, and accurate, if definitionally suspect sometimes. Could such a system be isolated using the team's data and methods?

Mr John Fitzgerald: The model, as described in this paper, represents an important step forward. With very limited resources the Central Bank have succeeded in producing a sophisticated tool which will prove useful in many aspects of economic analysis and policy formulation.

The first question any user or potential user must ask about such a model is how reliable it is. In this case the model has been subjected to extremely rigorous testing. While there are some areas where it has not performed satisfactorily these have been

clearly identified. As a result any user knows what to expect and can, in using the model, allow for the greater probability of errors in certain areas. The amount of testing carried out on the model. also gives any potential user a good feel for how the model works.

The model is particularly well designed for the purpose of the fiscal policy simulation with a time horizon of one to two years. While some sectors of the model may have proved unreliable over the test period, when it is used to examine the effects of relatively small changes in individual fiscal policy instruments it should provide a reasonable guide to their impact.

For areas other than fiscal policy simulation there may be some additional problems in using the model. For example, in examining the effects of a change in the rate of the Irish Green Pound the absence of a fully developed agricultural sector in the model poses problems. However, I believe that the model, if handled appropriately, will prove an extremely useful tool for economic analysis.

Mr Justin Wallace (AnCO): The paper has been extremely well dealt with from the economic standpoint. I would like to make a comment from the standpoint of someone involved on the supply side, that of trained labour!

Once any move is made beyond the use of this (or any other model) for forecasting, one is in the policy area. It seems that three kinds of policy orientation can be distinguished in the present context, two of which are mentioned in the paper:

- Monetary and Fiscal Policy – e.g. stabilisation.
- Development Policy – e.g. optimal growth.
- Social Policy – e.g. Maximise GNP/capita or perhaps *equalise* GNP/capita.

It is unlikely that the present model could extend to cover the third area, but there are some ways in which it could be used to assist research on the supply side, particularly where policy is concerned.

(1) *Enveloping*: Setting parameters within which particular policies would have to be planned, e.g. reserves will stay above present amount as long as proportion of GNP devoted to health expenditure remains at some fixed level.

(2) *Structural Analysis:* By measuring the effects of changing coefficients arbitrarily. This has already been suggested variously as the introduction of temporal variation, or stochastic shocks. The role of technology is of particular interest, and perhaps this factor is more important now when the after effects of OPEC energy cost increases are being felt in the western world. The deliberate introduction of discontinuities designed to reflect the effects of these external changes would be of interest.

On a minor point I would question the reality of the assumption (p. 4) that there is no significant distinction between the propensity to consume *vis-a-vis* agricultural and non agricultural income. Observers of the rural scene, particularly those who sell to rural consumers, would have no hesitation in affirming that sales of consumer goods have greatly increased in rural areas as against urban areas in recent years. Furthermore, it is well known that the Irish banking system is largely based on deposits from agriculture, so that the investment funds flow from west to east. Surely these phenomena need to be reflected in some way in the model.

Professor Dermot McAleese: I would like to join previous speakers in congratulating the Central Bank research team on the admirable paper they have presented this evening.

In the course of developing their model, the team have not only produced an impressive amount of empirical research on the Irish economy, as reflected in the references cited in this paper, but their work has also done much to encourage complementary research activity on the part of academic and government economists. We must be especially grateful for the open and frank manner in which the team has discussed the achievements and limitations of this impressive exercise in model-building.

My first question relates to the role of price variables in the model. Price terms have been dropped from the import equation and the interest rate does not appear in the investment function. Furthermore, the real wage in this model has no adverse effect on demand for labour. On the contrary, a rise in real wages working through aggregate demand has the effect of shifting the vertical demand for labour curve to the right, thereby implying that real wage increases lead to an expansion in employment. It is worth emphasising that this analysis applies strictly to the short-run. In the medium- to long-run, a rise in real wages (*ceteris paribus*) must eventually bring about a decline in employment. I take it that the authors would agree that raising the real wage is not a sensible way of stimulating aggregate demand? Also, may I ask how price variables (interest rate, trade prices and the real wage) have fared in models of other small countries?

Second, regarding the government employment multiplier, we are told (p. 20) that a £1m. increase in G will raise non-public sector employment by 99 jobs and public administration employment by 401 jobs, amounting to a total increase of 500 jobs. This ratio of four jobs in public administration to every one job in the non-public sector seems rather large.

Third, the limited availability of quarterly series has been adverted to in the paper. Would it be possible to utilise such quarterly series as are available by estimating structural coefficients from quarterly data, adjusting them in some appropriate manner to an annualised basis and then incorporating them in the annual model?

Reply to the Commentators (with some afterthought): We would like to begin by thanking all those who spoke and proceed to deal with their comments in turn.

Professor Gibson draws attention to the omission of wealth variables from the spending and asset demand equations, and also to the fact that the model specification admits stock disequilibrium over perhaps too long a period. The absence of data on wealth holdings and on quarterly magnitudes respectively are responsible. As to the impact of price expectations on real interest rates one would expect that in Ireland, with both nominal interest rates and the rate of price inflation determined almost entirely by external forces, the scope for domestic price expectations to affect real interest rates is limited. Under a different exchange rate regime, this might no longer be the case. In any event, with annual data a stationary expectations assumption is not as implausible as it might be at a lower level of time aggregation.

Professor Gibson was worried, understandably, at an implication of the monetary specification. An increase in Government spending, financed by foreign borrowing, will increase both income and reserves, a truly seductive prospect. However things are not as straightforward as they seem. First of all, reserves will only rise if domestic credit is held constant. The mechanism is obvious – income is stimulated, the demand for money goes up, so reserves must rise. However we see this as very much a short-term proposition – clearly, sustained Government foreign borrowing must, if carried to excess, lead to a re-rating of a country's credit worthiness and a higher interest rate. Eventually the well runs dry, as some developing countries discovered in the wake of the oil crisis. The assumption

of interest-rate exogeneity, and the consequences which flow from it, are reasonable, we believe, in the short-run, and for moderate changes in the values of policy instruments. We would not intend to apply the model in other circumstances.

We agree with Professor Gibson that our inability to model the market for domestic credit detracts from the monetary side of the model, but we find it difficult to see how the Irish private sector, particularly in the annual framework, can be denied its desired stocks of money balances.

The role of the liquidity variable in the consumption equation attracts some criticism from Professor Gibson and we would not disagree with what he has to say. The real balance variable employed was seen as a wealth proxy, but it has unsatisfactory implications. In any event, a new approach to modelling consumption is being developed.

We would not disagree either with Professor Gibson's contention that, in a small open economy, policy should pay more attention to the supply side of the economy. We do not believe, however, that short-run models should develop elaborate production sectors – longer-term policy might be better based on long-term models.

Professor Spencer's comments are confined to econometric matters. We did not set out to build a model of a particular size (say, 100 equations) but rather specified a model of a particular type for given data availability, which turned out to need 100 or so equations. We suspect that this is how most model-builders proceed. Our motives in using OLS were pragmatic – of course, knowledge of the small-sample behaviour of estimators in "complicated" situations is so limited that there can be no presumption that OLS is inferior on, say, a minimum mean square error criterion. But in some equations the regression involves right side variables of which the dependent variable is a component (imports on final demand, say); in these cases a procedure along the lines suggested by Professor Spencer is likely to be most fruitful – in other cases, where there is no reason to expect high correlations between the regressors and the disturbances, there would be less point in going beyond OLS.

We have not encountered problems of multiple roots in solving the model so far – where such a problem arises, it will often be clear which is the correct root on grounds of economic sense. If the alternative roots are close together, the choice is difficult, but making the wrong choice now matters less, except in calculating multipliers, where great care must be exercised.

Professsor Spencer suggests that an alternative to the ordinary t-test might be appropriate for hypothesis testing. This is an area to which we have yet to devote any attention, but the approach he suggests appears promising and we would hope to follow it up. Finally, our experiments in stochastic simulations have led us to conclude that the inequality of Howrey and Kelejian is of little practical importance for models of the type studied.

Dr Geary is concerned about the large number of exogenous variables and the practical difficulties involved in forecasting them. It is a big job, but it is rendered more tractable because (a) some of the exogenous variables have a very small impact on the endogenous variables of interest, so great care in forecasting them is not necessary, and (b) some of the exogenous variables are forecast separately by various agencies. For example, the annual budget speech contains enough information to set many of the Government policy instruments appearing amongst the list of exogenous variables.

The lagged disturbance terms appear amongst the predetermined variables only as a consequence of the autocorrelation in the system. As to the relative showing of Figure 4.1 (Real GNP) and Figure 4.7 (Change in Real GNP), the information content of the

two diagrams is identical, so one should not read too much into any apparent superiority of one over the other.

A multiplier is a partial derivative, very definitely a *ceteris paribus* construct. But it is (or approximates to) the partial derivative of the reduced form of the system, so the model's simultaneity is not ignored when multipliers are being calculated.

Using percentage first differences, as some continental model-builders have done, assumes a particular autocorrelation structure as well as removing flexibility in functional form specification. Our preference is for making autocorrelation adjustments explicitly where this appears to be necessary.

Professor McAleese is concerned about the implications of the model in regard to wage inflation. The impact of wage inflation in the short run is always expansionary in this type of demand-oriented model. Of course, in the long run, it is difficult to see how high rates of wage inflation can fail to reduce the demand for labour. We would not advocate wage manipulation, if it were feasible, as a means of regulating aggregate demand.

Professor McAleese's second point is a consequence of some ambiguous drafting on our part. What we meant to say, on page 20, was that, where a public expenditure increase arose as a result of public employment creation, the impact on employment will be much larger than where the expenditure boost is in the non-wage portion of public spending.

Mr Wallace mentions a number of areas of policy application of econometric models. We see the present model as useful only in the first of the policy areas he mentions – stabilisation. For development or social policy, a different approach would be necessary. We have begun some work on the areas he mentions (optimal control and stochastic simulations) but it is still at an early stage. Mr Wallace's final remark is concerned with urban/rural consumption patterns. All we wished to state (on page 4) was that the regression equation no longer records any significant difference between urban and rural marginal propensities to consume, a state of affairs that is not inconsistent with the phenomena he reports.

Mr Fitzgerald regrets the inability to simulate Green Pound changes, given the lack of an agricultural sector. However, it appears that agricultural supply response is weak, and it might be possible, abstracting from changes in agricultural output, to analyse the effects of the changes in agricultural income with the present model.

Finally, several speakers have mentioned the lack of quarterly macroeconomic data. It is our conviction that quarterly data would improve the accuracy and reliability of econometric models – the availability of data only at the annual level poses particularly severe problems for the estimation of dynamic responses, for example, and in modelling the financial sector. The Committee on Statistical Requirements and Priorities, which reported in November 1974, attached a high priority to the provision of quarterly national accounts statistics. While this report was accepted by the Government, the resources needed to implement its recommendations appear not to have been forthcoming. We believe that the limits to what can be achieved with existing data, in understanding the workings of the Irish macroeconomy, are being approached. If it is felt that major improvements in the quality of economic analysis and forecasting are necessary, we may first have to spend more money on economic statistics.