Transformation of one-dimensional silicon photonic crystal into Fabry-Pérot resonator

Vladimir A. Tolmachev[a,b], Tatiana S. Perova*[b] and Anna V. Baldycheva[b]

[a]Ioffe Physical Technical Institute, Polytechnicheskaya 26, St.-Petersburg, Russia
[b]Department of Electronic and Electrical Engineering, Trinity College Dublin, Dublin 2, Ireland

ABSTRACT

The model of transformation of one-dimensional, high-contrast silicon photonic crystal (PC) into a Fabry-Pérot resonator is considered. This transformation is achieved either by decreasing the number of periods up to 1.5 or the introduction of optical defect in the ordinary multi-period PC while retaining a high modulation of the resonance peaks up to ~0.95. The simultaneous use of maps of photonic band gaps (PBG) and transmission bands can predict the appearance or disappearance of PBGs in the optical spectra as well as to determine their position and width depending on the order of the band-gap (or stop-band) and the value of the filling fraction. The variation of the refractive index by 0.2 results in significant shift of the resonance peaks of high order up to 10% of the frequency corresponding to the center of the peak.

Keywords: Fabry-Pérot resonator, Liquid Crystal, Silicon Photonic Crystals, One-dimensional Photonic Crystal, Microstuctured Silicon

1. INTRODUCTION

One-Dimensional Photonic Crystal (1D PC) is a periodical structure containing, in general, two components, for example, of high (Si), and low (Air) refractive indices n_{Si} and n_{air}, respectively (Fig.1a). In order to build a resonator with desired resonance transmission modes, the defect in the centre of the PC structure with a greater optical thickness of one component can be formed. Fabry-Pérot Resonators (FPRs) of this type can reach the quality factor, Q up to a few hundreds and are utilised in numerous practical applications such as, for example, optical resonators, filters, light modulators etc.[1,2]. Another method of obtaining the resonance transmission peaks associated with the use of the classical Fabry-Pérot etalon[3], which contains two highly reflective surfaces (mirrors), and the cavity between them. FPRs of this type have low quality factor close to the theoretically estimated value of $Q \cong 100$. If in the PC, shown in Fig. 1a, the

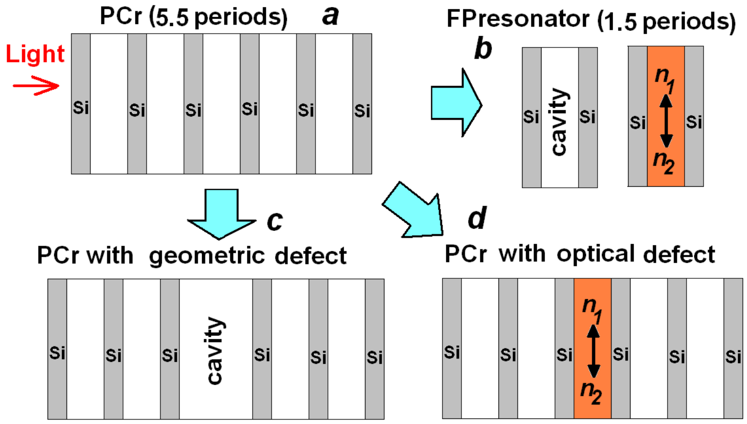

Fig. 1. Schematic of transformation of the 1D PC, based on microstructured Si, into Fabry-Pérot resonator either by introduction of a defect or by lowering the lattice period number to m=1.5.

*perovat@tcd.ie; phone 3 531 896-1432; fax 3 531 677-2442; http://mee.tcd.ie

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number of lattice periods m reduced to the minimum, the resulting 1.5-period PC can be also consider as FPR, since it consists of two Si mirrors and air cavity. Recently we have investigated [4] the FPRs of this type, constructed from Si-Air-Si and Si-Liquid Crystal-Si structures, that demonstrates the effectiveness as a tunable filter with a relative shift of the resonance peak up to $\Delta\lambda/\lambda=3$ %. This result has demonstrated the largest experimental shift, achieved up to date for the microstructured resonators based on Si and Liquid Crystal (LC), and utilized for molding of light propagated parallel to the substrate. In Ref. 4 it was also shown theoretically and experimentally, that for the high optical contrast structure on silicon the reflection from Si wall is reached the value of ~0.95, that provide a high quality resonance peaks in a wide range of spectra, measured from 1.5 to 15 μm. Most importantly that a significant shift, $\Delta\lambda/\lambda$ is achieved by using the resonance peaks of high order. Another advantage of this type of FPRs is the possibility to fine tuning the resonance modes by means of a moderate electric field of 10 - 15 V applied to the LC filler.

In the present work the theoretical investigation of FP-resonators of both types has been undertaken using the reflection and transmission maps approach. Special attention was paid to analysing the behaviour of resonance peaks of high-order depending on variation of the refractive index of the cavity.

2. DESIGN OF 1.5-PERIOD PHOTONIC CRYSTAL (FABRY-PÉROT RESONATOR)

Let us consider first a 5.5-period 1D PC shown in Fig. 1a. Let the optical thickness of the layers to be equal to $n_{air}d_{air}= n_{Si}d_{Si} =\lambda_o/4$, where d_{air} and d_{Si} are the geometrical thicknesses of the air-gaps and Si walls, accordingly, and λ_0 is the operating wavelength. The values of the refractive indices for Si and air are taken as $n_{Si}=3.42$ and $n_{air}=1$, respectively, and the refractive indices for incoming and outgoing media are equal to n_{air}. Using the Transfer matrix Method (TMM)[5], we calculate the reflection spectrum, R, depending on the normalized frequency $NF=a/\lambda$, where $a=d_{air}+d_{Si}$ is the lattice constant of PC. The equally spaced and almost Π-shaped bands of high reflection or the so-called stop-bands (SBs), are seen from spectrum shown in Fig. 2a. The shape of these bands is a characteristic feature of high contrast one-dimensional periodic structures. Between these peaks, the more frequent and less-pronounced (with smaller amplitude) resonance peaks (typically called as Fabry-Pérot resonances) are seen, which caused by interaction of the light beam with the entire layered structure.

If the number of periods is reduced to the minimum value of 1.5, the calculated spectrum will looks quite similar to the one obtained at m=5.5 for the exception of more narrow and rounded edges, a slightly lower value of R_{max} (decreased from 0.999 to 0.97) and a reduced number of oscillations with R_{max}~0.32 between SBs (Fig. 2b). By taking the product of $n_{air}d_{air} = \lambda/2$ at the same value of a, in case of the 5.5-period PC (Fig. 1b) we obtain an increased fraction of air and, as a consequence, a different R spectrum for this PC with characteristic resonance peaks of low (R_{max} ~ 0.2) and moderate

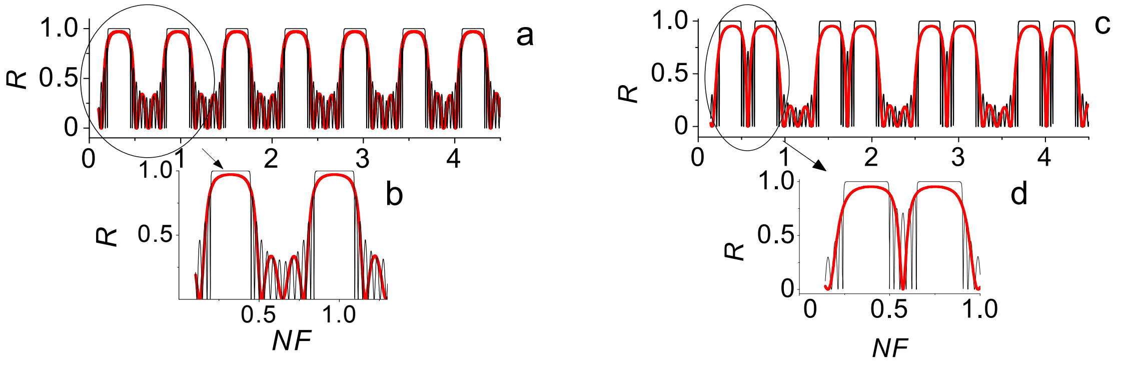

Fig. 2. a) Spectrum R for 5.5-period (thin line) and 1.5-period (thick line) PC made of periodical layers of Si and Air with optical thickness of λ /4. b) The first two SBs and Fabry-Pérot resonances between them. c) Spectrum R for 1.5-period PC (Fabry-Pérot resonator), calculated for optical thickness of air in the cavity $d_{cav} = \lambda$ /2, d) the first two SBs from figure (c).

(R_{max} ~ 0.75) modulations (Fig. 2c, thin line). For 1.5-period PC the obtained spectrum demonstrates a characteristic resonance peak (Fig.2d, thick line) with high modulation $\Delta R \cong 0.95$ (the difference between the intensity in maximum and in minimum $\Delta R=R_{max}-R_{min}$) in the narrow range of *NF* (or the wavelength). Figure 2d demonstrates that the single resonance peak of 1.5-period PC with increasing *m* up to 5.5 is transformed into a set of frequent FP resonances that is a characteristic feature of multi-period structure. In fact, the obtained 1.5-period PC can be considered as some kind of "elemental structural unit". Increasing the number of such units leads to the appearance of PBG, for which R=1 and transmission of photons is forbidden as a consequence of formation of complete photonic crystal.

In order to answer the question what is actually happened with SBs with variation of the filling fraction $f_{Si}= d_{Si}/ a$ (the structural parameter of 1D PC), the set of reflection (or transmission, T) spectra can be calculated and plotted together in order to establish certain relationships for SBs formation. From our point of view, the analysis of large set of spectra is quite complicated and the results demonstration is not always very effective. We prefer to use the gap map approach[6,7,8] that enables us to eliminate a non-important for the analysis information from the large set of the calculated spectra and to demonstrate only the part of information necessary for drawing the important connections. This approach will be demonstrated in the following section.

3. GAP MAP OF STOP-BANDS AND PASS-BANDS OF FABRY-PÉROT RESONATOR "SI-AIR-SI"

In order to draw the map of SBs for 1.5-period PC (FP-resonator), we use the filling fraction of Si, f_{Si} , the part, occupied by Si with respect to the lattice constant a: $f_{Si} = d_{Si}/a$. Then, a set of R spectra (~ 100 in total) are calculated for f_{Si} varied from 0 to 1 with step of 0.01, assuming that the parameter $a = d_{air}+d_{Si}$ is constant. The obtained values of the reflection coefficient, R, are cutting off using the criterion[7,8] R_{cutoff}>0.85, while the rest of the R values are plotted on the graph versus normalized frequency, *NF* as shown in Fig. 3. The lower than usual value of cut off criterion for SBs (R_{cutoff} = 0.85) for selection of data for gap map formation is chosen for clearer presentation.

As shown in Fig. 3a, the obtained SBs are relatively wide, that is due to the high optical contrast and the low value of R_{cutoff}. Then the set of calculated T (or R) spectra is filtered using another cut off criterion, namely T_{cutoff}>0.99. From thus obtained data the map of transmission bands (TBs)[9] (or pass-bands) is plotted on the obtained above map of SBs. This is used for more clear demonstration of the location of transmission bands with respect to the SBs depending on f_{Si}. Figure 3a shows the stop-bands surrounded by the transmission bands. In TB regions the resonance modes with high signal modulation ($\Delta R \approx 0.95$) as well as low signal modulation are included. The second (right side) *y*-axis of the plot, shown in Fig. 3a, is presented the filling fraction of air, f_{air}, as a part occupied by air relatively to a value, i.e. $f_{air} =d_{air}/(d_{air}+d_{Si})$.

Since the resonance peaks in 1D PC are characterised by a sharp change of transmission (from ~0 to ~1) in vicinity of the resonance peak, as a consequence, the TB region must be surrounded by two stop-bands on the gap map. Let us consider the structure with the filling fraction f_{Si} = 0.11, for example. For this value we draw the horizontal line, which intersects the regions of SB as well as the TBs. For convenience we numerate the SBs in order shown in Fig. 3a. It is seen in Fig. 3a, that in the region of *NF*=0.6 there is a TB region, which is surrounded by SBs **1a** and **1b**. Indeed, the resonance peak in spectrum R, depicted in Fig. 3b, is observed at *NF*=0.6. We can also see from Fig. 3b the presence of the resonance peaks with high modulation in the regions *NF* = 1.7 and *NF* = 2.2, which are predicted from the map between SBs **1b**-**1c**, **2c**-**2d** and **2d**-**2e** for the corresponding values of f_{Si}.

Let us now to concentrate more precisely on what is observed between SBs **1a** and **2a**, for example, for f_{Si}=0.4 (Fig. 3c). We can see that the transmission peaks observed between these stop-bands are considerably wide. Similar result is obtained for other values of f_{Si} for transmission bands located between SBs **2a** -**3a**, **1a**-**2b**, **2b**-**3c** and others.

Therefore, for the considered type of FP resonator the formation of resonance peaks with high signal modulation can occur in the region of f_{Si} ~ 0.2 and below. This rule is related to the fact that for obtaining the total constructive interference for transmission a substantial part of low-refractive index component in a structure is necessary.

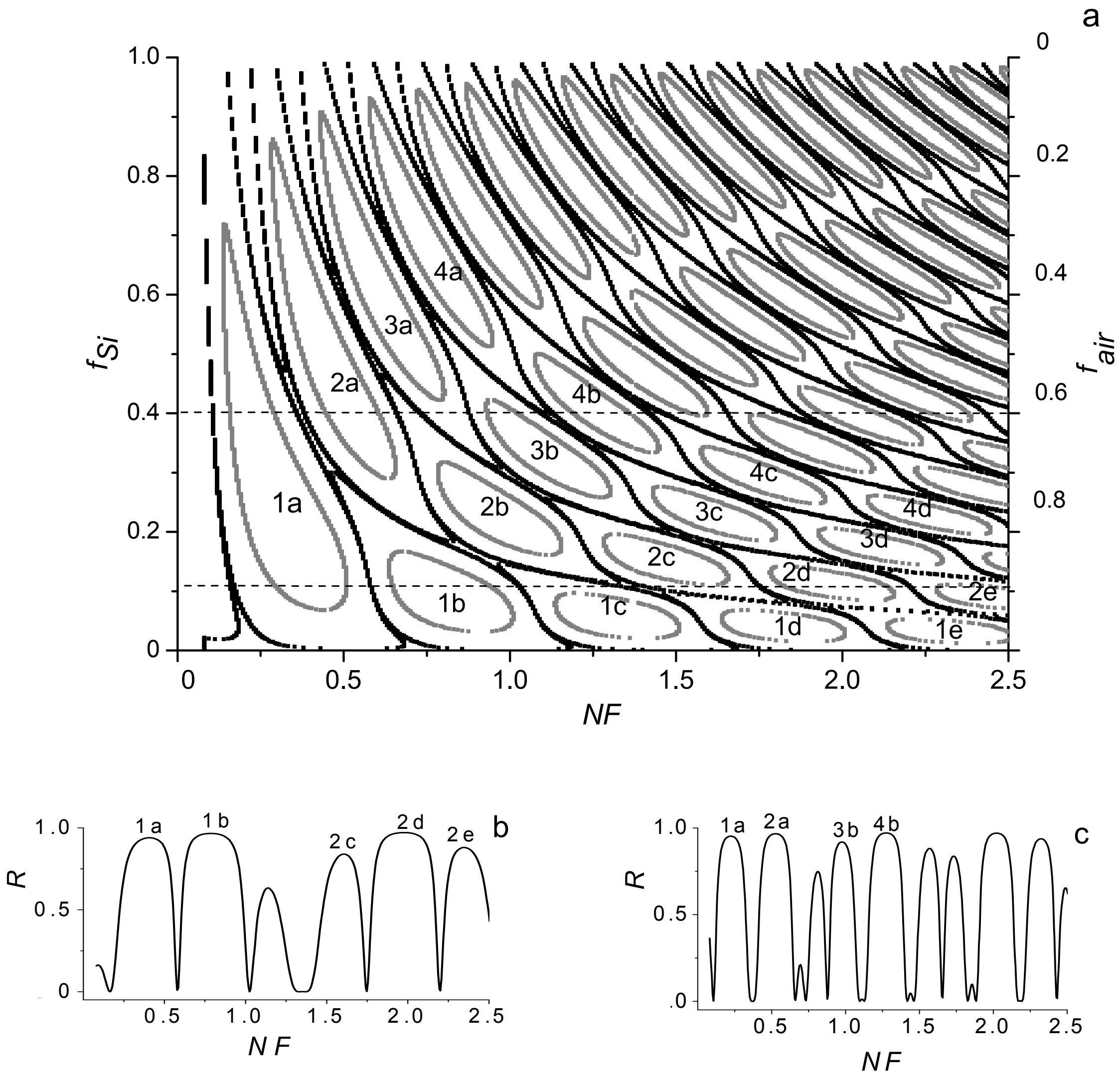

Fig. 3. a) The map of stop-bands (shown by grey contour with symbolic numeration) with $R>0.85$ and regions of transparency (black contour) with $T>0.99$ for 1.5-period PC (FP resonator) based on 3-layer model from Fig.1b. The incoming and outgoing media is air, $n_{Si}=3.42$, the incidence of light is normal. b) The R spectrum for $f_{Si}=0.11$ with few highly-modulated resonance peaks between SBs **1a-1b**, **2c-2d** and **2d-2e**. c) Spectrum R for $f_{Si}=0.4$ demonstrated the absence of narrow resonance peak between SBs **1a-2a** and **3b-4b**.

4. FABRY-PÉROT RESONATOR “SI-LC-SI”

Taking into account the analysis and conclusions made in the previous section, let us now investigate a 1.5-period FP-resonator, in which the compound with refractive index birefringence (that can be changed by the external forces) will replace the air in the cavity (Fig. 1b). For this purpose we can use the LC E7 with extreme values of the ordinary and extra ordinary refractive indices, n_{LC} equal to $n_{LC}=1.49$ and 1.69 in mid-IR[10]. We calculate the set of R and T spectra and draw the maps of SBs and TBs for two values of n_{LC} in the resonator cavity (Fig. 4a).

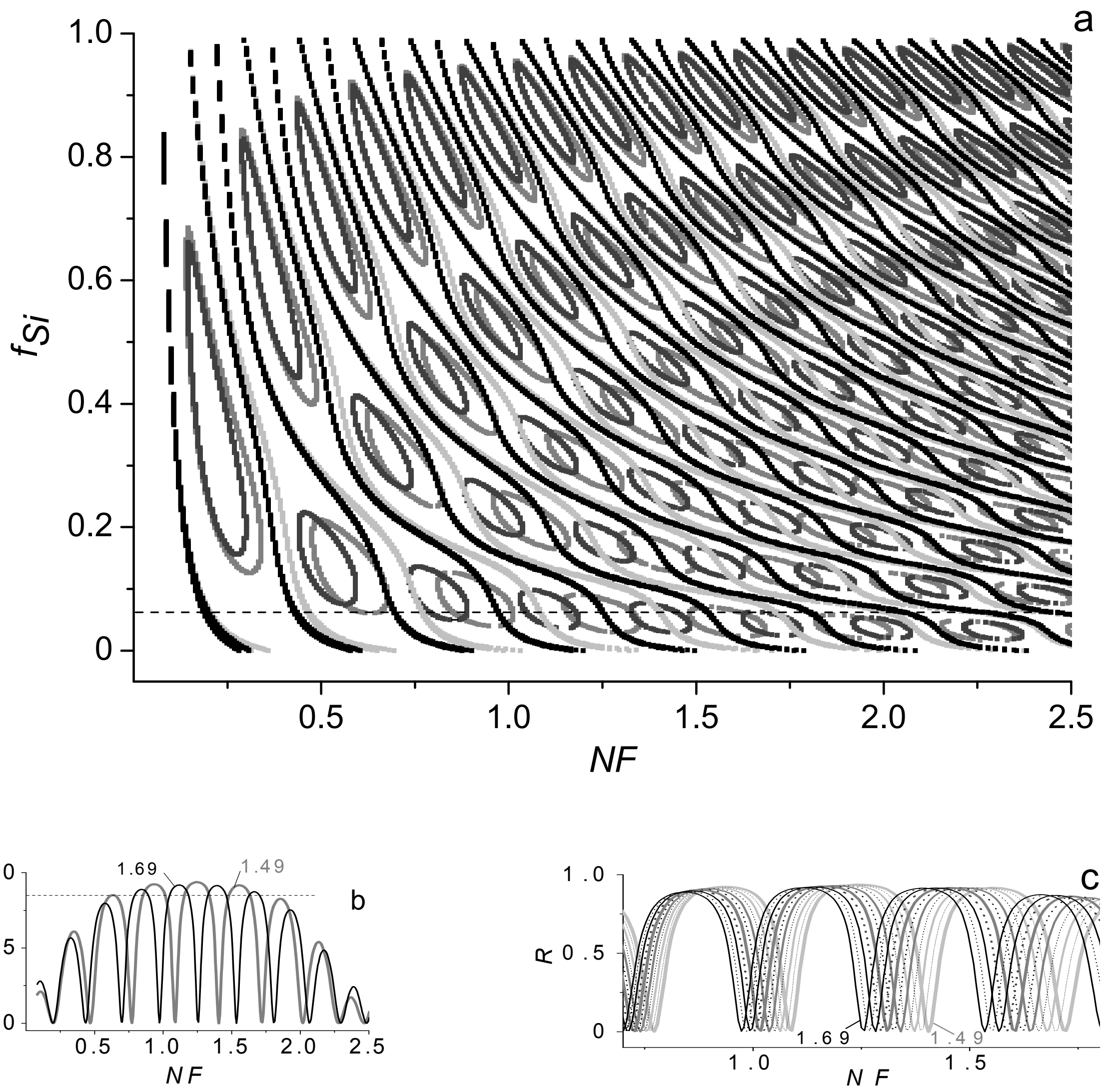

Fig. 4. a) The map of stop-bands (oval-like areas) and transmission bands (shown by lines) for 1.5-period PC (FP-resonator) based on 3-layer model "Si-LC-Si" with two values of n_{LC}=1.49 (grey contours for SBs and light grey lines for TBs) and 1.69 (dark grey contours for SBs and black lines for TBs). R_{cutoff} = 0.85, T_{cutoff} = 0.99. b) Spectrum *R* calculated for f_{Si} =0.06 and demonstrating the cut off line for *R* values, used for drawing of the SBs map. c) Fine tuning of peak positions and stop-bands with variation of n_{LC} from 1.49 to 1.69. The relative peak shift $\Delta NF/NF$=0.14/1.33=10.5%.

Two types of SBs with the defect cavity modes correspondent to *n*=1.49 and 1.69 can be seen in Fig. 4a. These stops are narrower than SBs demonstrated above in Fig. 3a due to the lower optical contrast (in ~ 1.4 times). From Fig. 4a in the region of f_{Si}=0.06 one can observe the intersection of SBs with transmission bands in vicinity of *NF*=1, 1.2 and 1.6, for example. More clearly this can be seen in spectra shown in Fig. 4b. The presented spectra demonstrate that the variation of *n* by 0.2 in the resonator cavity leads to the shift of the resonance peak from 1.40 to 1.26 *NF* resulting in a relative shift value of $\Delta NF/NF$ = 0.14/1.33 = 10.5%. The width of the stop-band in this diapason is ~ 0.2*NF*, as well as the SBs are shifted by approximately the half of their width during the tuning. The LC molecules can be oriented in such way, that their refractive index possesses not only the extreme values (n_o and n_e), but also some intermediate values. Fig. 4c is depicted the spectra for these intermediate values of n_{LC} that demonstrates the possibility of fine tuning of resonance peaks and stop-bands position.

Thus, using TMM, the reflection spectra of FP-resonator (or 1.5-period PC) based on structures "Si-Air" and "Si-LC" can be calculated. The filtering of these spectra using two criteria for maximum and minimum values of the reflection coefficient provides the set of data for drawing the maps of stop-bands and transmission bands. The resonance bands with high modulation of R values ($\Delta R \approx 0.95$) are located between the certain stop-bands regions. By means of this approach, the range of optimal filing fractions of FP-resonator can be established in the range of f values from 0.05 to 0.4 for achieving a significant shift of SBs and the resonance peaks up to 10% using LC birefringence of Δn=0.2 with R modulation up to 0.95.

5. PHOTONIC CRYSTAL WITH OPTICAL DEFECT

To receive a well pronounced resonance peak, it is necessary to increase at least twice the optical thickness of one of the components in the centre of PC structure (Fig. 1a). This can be achieved by increasing the geometrical width of the structure (Fig.1c) or by replacing the air gap (n=1) in the centre of structure with a compound of the refractive index n = 2 (Fig.1d). If it is not possible to use a compound with n = 2, but only with smaller value of the refractive index, for example with n = 1.49, the resonance peak still appears, but not in the centre of SB. Although, the peak width (quality factor) and the amplitude of signal modulation for this resonance peak are nearly the same as the calculated values for an ideal resonator (at n = 2). For the formation of the corresponding defect typically the first stop-band is used. In this work the investigation on the possibility of using the resonance peaks, formed in SBs of high order, is performed with GM approach.

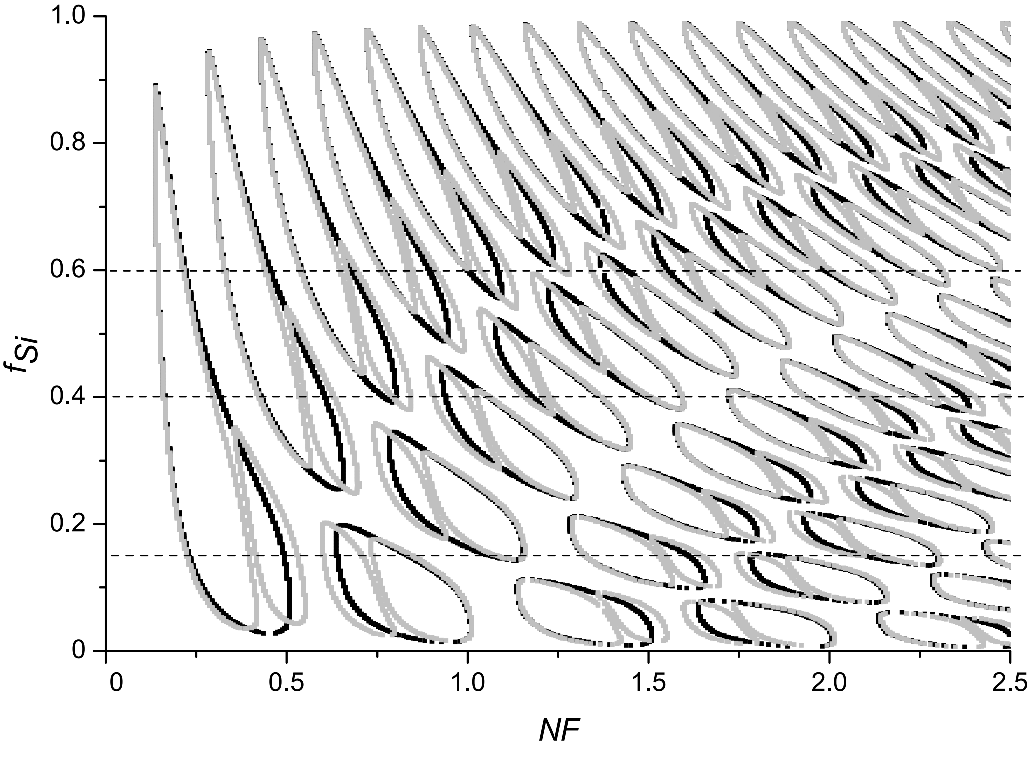

Fig. 5. The map of SBs for 3-period PC "Si-Air" (black contour) and 3-period PC "Si-Air" with LC optical defect (grey contour) with n_{LC}=1.49.

Let us consider the FP-resonator based on 3-period PC with optical defect shown in Fig.1d. We calculate the gap map for PC with three periods and draw the map for PC with central air gap infiltrated by LC, i.e. with refractive index of defect n_{cav}=1.49 (Fig. 5). The gap map demonstrates, as expected [8], that the infiltration of LC into one central groove changes n value from 1 to 1.49, increasing the optical width of this groove and, therefore, results in the formation of the defect mode region by splitting of SBs of any order. For example, the horizontal line on the map, correspondent to f_{Si}=0.15, intersects the SBs and defect bands in the regions of *NF*= 0.41, 0.72 and 1.54. For these *NF* values, very pronounced resonance peaks are observed in spectrum *R* shown in Fig. 6a. Similar appearance of the defect modes is also depicted in Figs. 6b and 6c for f_{Si}=0.4 and 0.6 as an example.

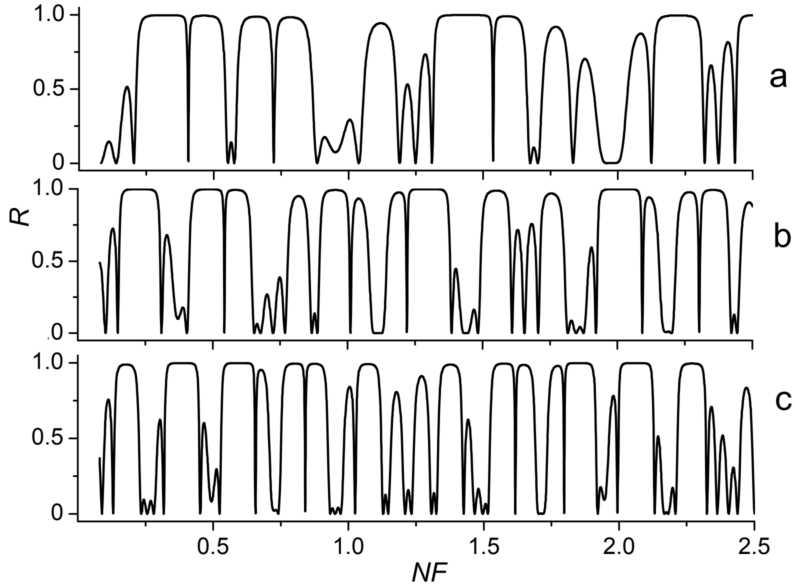

Fig. 6. *R* spectra for (a) f_{Si}=0.15 (b) f_{Si}=0.4 and (c) f_{Si}=0.6 (compare with GM shown in Fig.5).

These examples demonstrate a possibility to cover with resonance peaks the entire range of *NF* values for PC resonator of the above mentioned type with using the first stop-band as well as the SBs of high order. We believe, that during the fabrication of such FP-resonators (or PCs with optical defect), there is a certain advantage in retaining the same thickness of Si walls and air gap. At the same time in case of PC with a resonator where a geometrical defect is used, it is necessary to control additionally the size of the resonator cavity.

The map of the transmission bands which are located between the stop-bands is depicted in Fig. 7. Careful examination of the TBs for this type of resonator shows that it is impossible to generate the resonance peaks with high signal modulation between the stop-bands for any values of f_{Si} (Fig.7).

Next, we investigate how the PBGs map will look like when the refractive index of LC defect is replaced by the value of n_{LC} =1.69 (instead of n_{LC} =1.49 used in the previous case). As can be seen from Fig. 8a, the first stop-band as well as SBs of high order have changed insignificantly by this replacement of n value, as expected, since the optical contrast of PC was not change. The major changes are observed in the regions of the defect modes, namely, the smaller the f_{Si} values, the larger variations are. For example, in SB obtained at f_{Si} ~ 0.1 and *NF*=0.6-1.0 for the modelled transformation at the given value of Δn=0.2, two defect modes with a consequent band shift may appear instead of one mode. This can be explained by increase of the optical width of the cavity. As the purpose of this work is to consider how the maximal shift of the resonance peaks can be achieved, we have to analyse the following points. What values of f_{Si} must be selected to obtain the resonance peaks in the limited range of SB width? To make this solution easier, the f_{Si} values must be in the range where SB is the widest. In Fig. 8a this region of SB corresponds to f_{Si}<0.5. We note that this range of f_{Si} is larger than in case of Si-Air structure (see Fig. 3a) in which case f_{Si}<0.2.

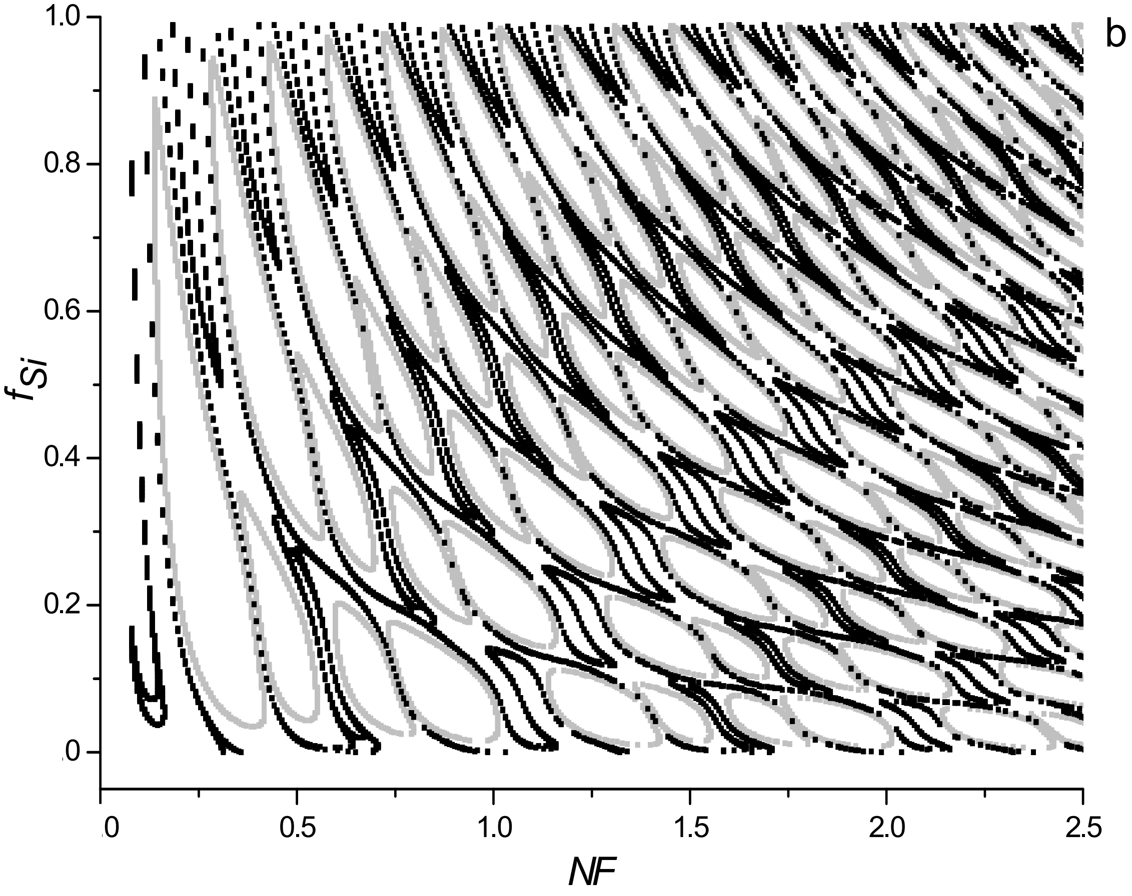

Fig. 7. The map of SBs for 3-period PC "Si-LC" with n_{LC}=1.49 (gray contour) and map of TBs (black lines).

The regions of defect modes must also demonstrate the maximal shift and at the same time the resonance peak with minimal *n* (1.49) must be located near the blue edge of SB, since when the value of *n* increases to 1.69, the resonance peak should not leave a selected stop-band. As can be seen from Fig. 8a, the width of SBs **1a**, **1b**, **1c**, and **1d**, for example for f_{Si}=0.05, is large enough to accommodate both resonance peaks. Based on these conditions, it is more preferable to select PC with f_{Si}=0.05, for which the spectrum *R* is depicted in Fig. 8b. Figure 8b shows that the maximal shift of the resonance peak is provided by SBs **1c**. Figure 8c demonstrates the extended transmission spectra, *T* for SB **1c**. The resonance peak is shifted from 1.27 to 1.4 *NF* correspondent to Δ*NF/NF* = 9.7 %.

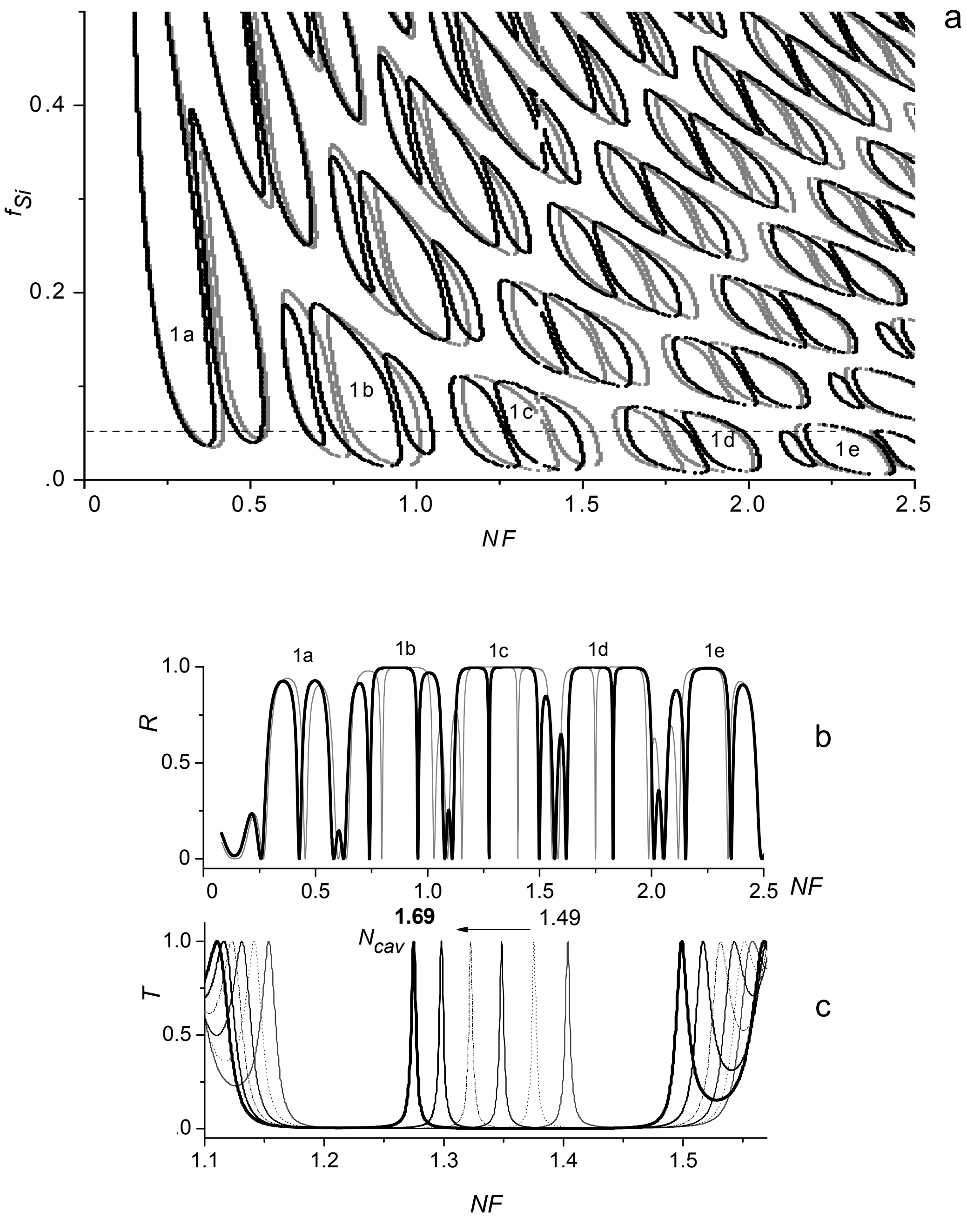

Fig. 8. a) The map of stop-bands (R_{cutoff}=0.85) for 3-period PC "Si-LC" with tunable LC optical defect at n_{LC}=1.49 (gray contours) and 1.69 (black contours). b) Reflection spectra, R for f_{Si}=0.05 with n_{LC} =1.49 (grey line) and 1.69 (black line); (c) the extended transmission spectra, T with relative peak shift of $\Delta NF/NF$ = 9.7 % for SB **1c**.

The obtained dependencies enable us to design the tunable filters for selection of the required transmission band. For this design it is necessary to select the operational range of frequencies (or wavelengths), to determine the lattice constant *a,* a refractive index of a filler and the filing fraction of Si, f_{Si}.

6. CONCLUSION

The utilisation of common maps of PBGs (or stop-bands) and transmission bands allows to predict their appearance or disappearance in the optical spectra and to determine their position and width depending on the order of SB and the value of filling fraction. In this work different methods for design of Fabry-Pérot resonator, obtained from silicon photonic crystal, are demonstrated. In one case this was achieved by the reduction of the number of PC periods to the minimum value of 1.5, while in another case by introduction of the optical defect in PC structure. Since 1.5-period PC (or FP-resonator) has imperfect reflectors (Si single walls) the quality factor of the resonance peak is relatively low. The replacement of the refractive index, n in one of the PC components leads to the variation of optical contrast and, as a consequence, to the shift of both the SB edges and the resonance peaks. It is a main feature of this type of resonators.

The features of FP-resonator based on PC with optical defect are: i) the possibility to avoid the specific procedure for defect fabrication but rather use the possibility of introducing the filler in the centre of PC and ii) the shift of the resonance peak due to variation of n of the defect compound occurs within the band-gap. We note that the quality factor in this case can reach typical values of around several hundred. The important common feature of both types of Fabry-Pérot resonators is the possibility of tuning the refractive index of the filler. Using the example of the nematic liquid crystal E7 with Δn=0.2, a significant shift of the resonance peak up to 10% at high modulation of reflection/transmission coefficient (ΔR (or (ΔT)) $\approx$ 0.95) was demonstrated.

Acknowledgments

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