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# Quantifying the Non-Stationarity in Irish Real Exchange Rates

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*Abstract:* Empirical work, both in Ireland and elsewhere, has found little evidence for the proposition that log-real exchange rates are stationary, implying that the purchasing power parity (PPP) relation cannot hold, not even in a long-run equilibrium. Using techniques proposed by Cochrane (1988), we aim to quantify the magnitude of that non-stationarity in Irish/German and Irish/UK data during the EMS period. In this way we can assess how empirically important deviations from PPP are. At least in the case of Irish/German data, the non-stationarity in the log real exchange rate appears to be small.

## I INTRODUCTION

The seminal paper of Nelson and Plosser (1982) began the recent focus on unit roots in the applied macroeconomic literature, arguing that US macroeconomic time series, including GNP, are non-stationary. The view of the business cycle as temporary deviations from a trend was thus challenged. The question of the "degree of non-stationarity" of the series was, however, left unaddressed. Cochrane (1988) subsequently developed techniques which measure the size of the non-stationary component in a time series, based on a Beveridge-Nelson decomposition (Beveridge and Nelson, 1981) of a general I(1) series into a random walk with drift[1] (permanent) component and a

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1. Henceforth, all random walks are allowed to have drift, except where the contrary is specified, but we describe them simply as random walks.

stationary (transitory) component. He used this method to argue that the non-stationarity in US GNP is not quantitatively important, in the sense that the random walk component in the decomposition is small.

A considerable literature has developed, both in Ireland and elsewhere, on the relationship between exchange rates and foreign and domestic prices. Although there is a consensus that PPP cannot hold in the short run, ideas of co-integration can be used to detect a long-run equilibrium price-taking relationship. Such a relationship would imply that granted that foreign and domestic prices and nominal exchange rates are I(1), real exchange rates are I(0), so that the prices and nominal exchange rates co-integrate with co-integrating vector (1,–1,–1). The background to this literature is surveyed in a parallel paper (Wright, 1993) and we do not repeat it here. It can however be said that the empirical results in this literature are broadly unfavourable to the idea of real exchange rates being stationary (and so *a fortiori* to PPP co-integration).

Recently, several approaches have been used to attempt to rehabilitate long-run PPP. A number of studies have augmented the PPP relation with interest rates, including Johansen (1992) and Wright (1993), using Irish data. Perron and Vogelsang (1992) find some real exchange rates to be stationary with a level shift although the same series appear to be non-stationary if the structural break is ignored. They thus argue that the PPP relation holds in relative rather than absolute form and that the relative PPP relation is subject to occasional structural breaks. Perron and Vogelsang (1992) do not however impose the break date a priori but rather estimate it from the data by a series of t tests which leaves them very open to a charge of data mining.

We propose a third and conceptually simpler approach. Granted that Irish real exchanges rates are indeed non-stationary, it is our goal, in this paper, to assess the empirical significance of that non-stationarity using the method proposed by Cochrane (1988) without appeal to augmentation of the PPP relation by interest rates, structural breaks etc. This approach was taken with non-Irish real exchange rates in Ardeni and Lubian (1991). Casual analysis of the data indicates that rejection of PPP, even as a long-run relation, cannot be taken as a complete account of the time series properties of foreign and domestic prices and exchange rates: although the exchange rate and foreign price shocks may never feed through to domestic prices fully, it is clearly unreasonable to argue that there is no long-run transmissions mechanism at all. Figs. 1 and 2 show our data on Irish/German and Irish/UK relative prices[2] and exchange rates (in levels). It is clear that if econometrics cannot say anything about this transmissions mechanism in relation to the

2. The relative price indices have been scaled to equal the exchange rates in the first period of the sample.

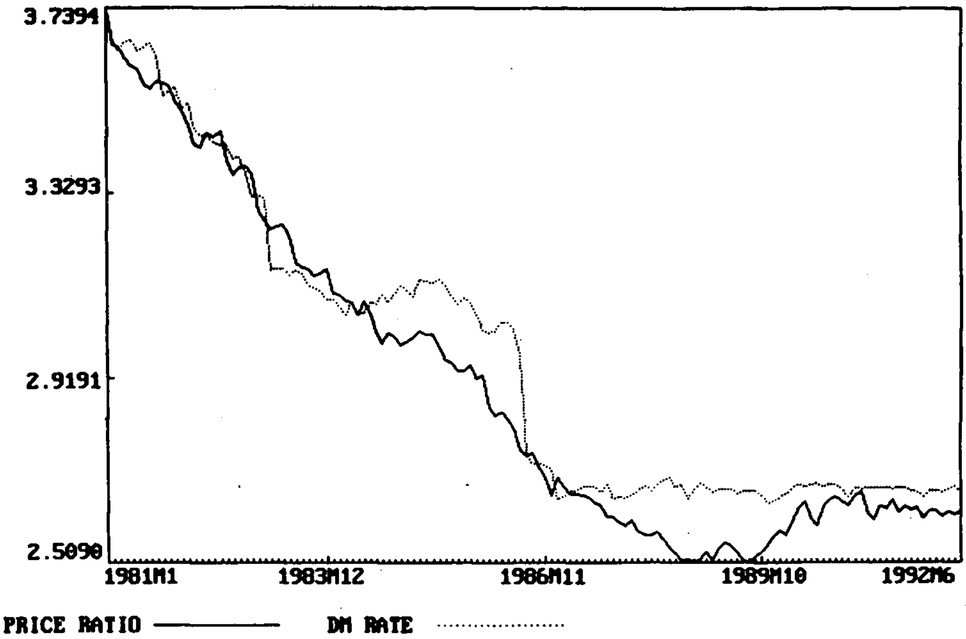


Figure 1: *Irish / German Relative Prices and Exchange Rates*

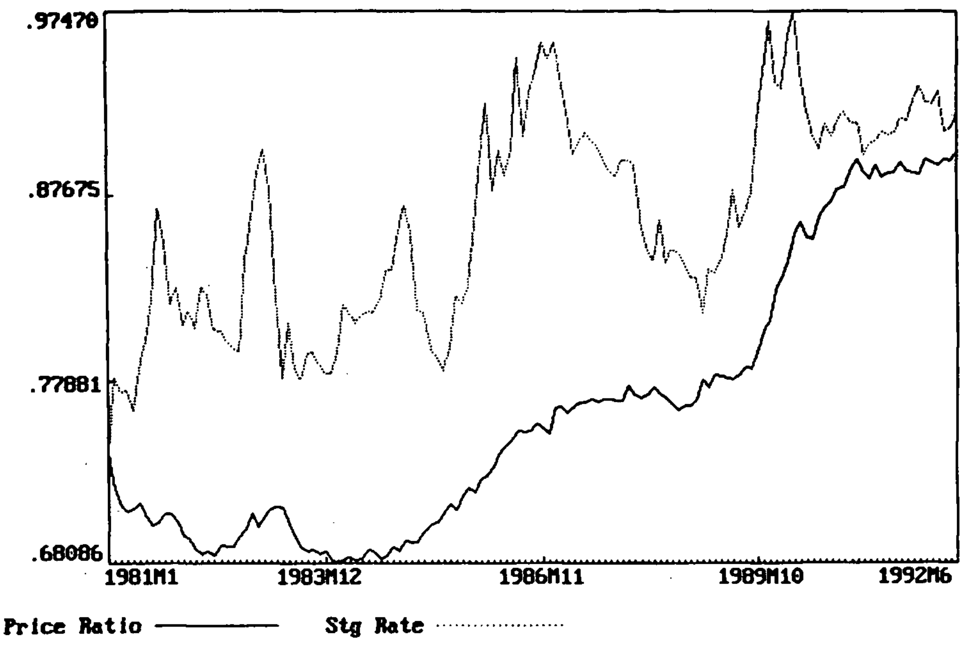


Figure 2: *Irish / UK Relative Prices and Exchange Rates*

Irish/German data other than that we fail to reject the unit root hypothesis in the real exchange rate, then it does not provide a useful time-series characterisation of the relationship shown in Fig. 1. Quantifying the non-stationarity in real exchange rates enables us to give econometric force to this argument: if we expect that long-run deviations from PPP exist, but are not empirically important, then we would expect to find that the real exchange rates, though I(1), have only small non-stationary components. The structure of the remainder of this paper is as follows. In Section II, we discuss the econometric theory underlying the method of Cochrane (1988). In Section III, we apply this theory to the PPP relation using Irish/German and Irish/UK data. Section IV contains a brief conclusion.

## II QUANTIFYING NON-STATIONARITY IN TIME SERIES

Consider a real scalar I(1) time series, $x_t$, $t=1,\ldots T$. According to the Beveridge-Nelson decomposition, $x_t$ can always be written as the sum of a random walk $z_t$ and a stationary component with identical innovations in the two components. The size of the random walk component can be measured by k, the ratio of the variance of the innovations in $z_t$ to the variance of $\Delta x_t$. If $x_t$ were a random walk, k would be unity and if $x_t$ were in fact trend-stationary, k would be zero. Typically, many decompositions of $x_t$ into random walk and stationary components exist, including decompositions in which the two components have different innovations, but any decomposition of $x_t$ into a random walk and stationary component has the same value of k. Were this not the case, we would know immediately that k is an unidentified parameter. Note that $x_t$ can also be decomposed into I(1) permanent and I(0) transitory components where the permanent component is not a random walk. A fundamental identification issue raised by Quah (1992) then arises, however. If $z_t$ is I(1), but is not restricted to be a random walk, then different decompositions of $x_t$ into $z_t$ and a stationary component have different values of k, so that k would then be an unidentifiable parameter. Our interpretation of k as measuring the size of the permanent component in $x_t$ depends on the assumption that $z_t$ is a random walk.

Define the power spectrum of an arbitrary stationary series, $y_t$, as:

$$d(\lambda) \equiv \phi(0) + 2\textstyle\sum_{\tau=1}^{\tau=\infty} \phi(\tau)\cos(\lambda\tau),\ \ 0 \le \lambda \le \pi \qquad (2.1)$$

where $\phi(\tau)$ is the autocovariance function of $y_t$, $E((y_t-E(y_t))(y_{t-\tau}-E(y_t)))$. It is always possible to represent $y_t$ as a sum of an uncountable infinity of sinusoidal components, weighted by uncorrelated random variables: $d(\lambda)$ can be thought of as the variance of the random variable associated with the cycle of frequency $\lambda$ in $y_t$. Let $\gamma(\tau)$ and $f(\lambda)$ denote the autocovariance function and power spectrum of $\Delta x_t$, respectively. Letting $\psi(\tau)$ and $s(\lambda)$ denote the autocovariance function and power spectrum of $\Delta z_t$ respectively, notice that, because $z_t$ is a random walk (and so has iid innovations), $s(0)=\psi(0)$. Because first differencing a stationary process yields a process with zero power spectrum at the origin, it follows that $f(0)=\psi(0)$. It is well known that:

$$\gamma(0) = \pi^{-1}\int_0^{\pi} f(\lambda)d\lambda \tag{2.2}$$

So, if $r(\tau)$ is the autocorrelation function of $\Delta x_t$, $\gamma(\tau)/\gamma(0)$, the standardised power spectrum, defined as:

$$g(\lambda) \equiv r(\lambda) + 2\sum_{\tau=1}^{\tau=\infty} r(\tau)\cos(\lambda\tau), \quad 0 \le \lambda \le \pi \tag{2.3}$$

integrates to $\pi$. Because k was defined as $\psi(0)/\gamma((0)$, it is trivially equal to $g(0)$. We have thus established that k, our measure of non-stationarity, is identified from the second moments of the differenced observed series as the standardised power spectrum of $\Delta x_t$ at the origin.

Campbell and Mankiw (1987) measured the non-stationarity of time series by the multiplier effect of a transitory shock on long-run forecasts. Unfortunately, they found it necessary to impose parametric assumptions on $x_t$, in order to estimate this quantity, unlike our measure, k. However, k has an interpretation as a measure of non-stationarity which does not involve appeal to the Beveridge-Nelson decomposition and is closely related to the approach taken by Campbell and Mankiw (1987) as we can see (e.g. from Durlauf, 1993) that:

$$k = \frac{\delta E(x_\infty | x_t)}{\delta x_t} \tag{2.4}$$

Estimation of the standardised power spectrum is notoriously difficult, even with very large finite samples. All usual estimates are of the form:

$$\hat{g}(\lambda) = c(0) + 2\sum_{\tau=1}^{\tau=m} \omega_\tau c(\tau)\cos(\lambda\tau), \quad 0 \le \lambda \le \pi \tag{2.5}$$

where $c(\tau)$ is the sample autocorrelation function and $\omega_1, \ldots \omega_m$ is a sequence of weights, known as the lag window, which the researcher can choose. The three common choices are:

(i) The Bartlett window:

$$\omega_\tau \equiv 1 - \frac{\tau}{m}, \; 0 \le \tau \le m$$

(ii) The Tukey window:

$$\omega_\tau = \frac{1}{2}\left(1 + \cos\left(\frac{\pi\tau}{m}\right)\right), \quad 0 \le \tau \le m$$

(iii) The Parzen window:

$$\omega_\tau \equiv 1 - 6\left(\frac{\tau}{m}\right)^2 + 6\left(\frac{\tau}{m}\right)^3, \quad 0 \le \tau \le \frac{m}{2}$$

$$2\left(1 - \frac{\tau}{m}\right)^3, \quad \frac{m}{2} \le \tau \le m$$

The choice of m is critical to the procedure. Consistency requires that m be a suitable increasing function of the sample size. There is a bias/efficiency trade-off which will be familiar to students of non-parametric estimation procedures. The higher is m, the lower will be the finite sample bias but the less efficient the estimator will be. In Chatfield (1989), an "optimal" choice of m as $2T^{1/2}$ is proposed, based on a compromise criterion between bias and efficiency.

We can use $\hat{g}(0)$ as a point estimate of k and confidence intervals can be constructed using the well-known fact (see e.g. Brockwell and Davis, 1992) that, under suitable regularity conditions,

$$\sqrt{T}\ (\hat{g}(\lambda) - g(\lambda)) \Rightarrow N\left(0, \left[1 + 2\Sigma_{\tau=1}^{\tau=m}\omega_\tau^2\right]g(\lambda)^2\right), \ 0 < \lambda < \pi \qquad (2.6)$$

$$\sqrt{T}\ (\hat{g}(\lambda) - g(\lambda)) \Rightarrow N\left(0, \left[1 + 2\Sigma_{\tau=1}^{\tau=m}\omega_\tau^2\right]g(\lambda)^2\right), \lambda = 0, \pi \qquad (2.7)$$

In the literature, unit roots are almost invariably nested within an AR(1) model. This gives rise to an unattractive discontinuity in statistical properties when the autoregressive parameter is reduced arbitrarily slightly below one. We can think of Cochrane's method as providing an alternative nesting of unit roots.

## III EMPIRICAL WORK

Our data consists of five series, as follows:

(i) $p_d$: Irish wholesale price index.
(ii) $p_u$: UK manufacturing output price index.

(iii) $p_g$: German wholesale price index.
(iv) $e_u$: UK spot exchange rate in Irish currency terms.
(v) $e_g$: German spot exchange rate in Irish currency terms.

Our observations are monthly, from January 1981 to June 1992 inclusive. Series are taken from various issues of OECD Main Economic Indicators. Our data covers virtually the entire EMS period. We however deliberately omit all consideration of the recent instability within the EMS since this will not help to clarify the properties of the system in a long-run steady-state equilibrium. We use wholesale price indices (essentially traded sector prices) rather than consumer price indices because Ireland has only quarterly consumer price data whereas all other series are monthly and because these are the indices used in Thom (1989) in common with most of the literature in this area. Data analysis was conducted using the MICROFIT package on a DEC 433 PC.

Wright (1993), using Dickey-Fuller and Augmented Dickey-Fuller tests, concludes that all price and exchange rate series used in this paper do indeed have unit roots. Applying Dickey-Fuller and Augmented Dickey-Fuller tests to the Irish/German and Irish/UK real exchange rates, as reported in Table 1, we dramatically fail to reject the non-stationarity null in either case. This accords with earlier empirical work, as explained in the introduction.[3]

Table 1: *Augmented Dickey-Fuller Statistics for Real Exchange Rates (Critical Values are –1.95)*

| | *Irish / German Real Exchange Rate* | *Irish / UK Real Exchange Rate* |
|---|---|---|
| 0 lags | 0.79 | –0.74 |
| 1 lag | 0.83 | –0.33 |
| 2 lags | 0.14 | –0.33 |
| 3 lags | 0.76 | –0.27 |
| 4 lags | 0.51 | –0.28 |
| 5 lags | 0.4 | –0.1 |
| 6 lags | –0.04 | –0.08 |

We now use the measure $\hat{g}(0)$, developed in Section II, to quantify the non-stationarity of Irish real exchange rates. Neither of the real exchange rates has significant drift. In Table 2, we report the values of $\hat{g}(0)$ for both real exchange rates using three alternative spectral estimators and the "optimal" value of m (24, with our data). In Table 3, using the Bartlett window, we compute values of $\hat{g}(0)$ for several choices of m.[4] Note carefully, from (2.7), that

3. The alert reader will notice that we use different critical values from Thom (1989). We test for a unit root under the maintained hypothesis that there is no drift (or trend).

4. Remember that although increasing m causes efficiency loss, it reduces finite sample bias.

Table 2: *Values of $\hat{g}(0)$*
*(Estimated Asymptotic Standard Errors in Parentheses)*

| | *Bartlett Window* | *Tukey Window* | *Parzen Window* |
|---|---|---|---|
| Irish/German Real Exchange Rate | 0.41 (0.20) | 0.42 (0.21) | 0.46 (0.20) |
| Irish/UK Real Exchange Rate | 0.62 (0.30) | 0.60 (0.31) | 0.69 (0.30) |

Table 3: *Values of $\hat{g}(0)$ for the Real Exchange Rates with the Bartlett Window*
*(Estimated Asymptotic Standard Errors in Parentheses)*

| | *Irish / German Real Exchange Rate* | | *Irish / UK Real Exchange Rate* | |
|---|---|---|---|---|
| m=5 | 0.6 | (0.13) | 0.95 | (0.21) |
| m=10 | 0.54 | (0.17) | 0.82 | (0.26) |
| m=15 | 0.52 | (0.2) | 0.75 | (0.29) |
| m=20 | 0.47 | (0.21) | 0.68 | (0.3) |
| m=25 | 0.39 | (0.19) | 0.63 | (0.31) |
| m=30 | 0.29 | (0.15) | 0.62 | (0.34) |
| m=40 | 0.14 | (0.088) | 0.6 | (0.37) |

our asymptotic standard errors themselves depend on g(0): in Table 2 and 3 we have used $\hat{g}(0)$ to construct standard errors (which are quite large, as in all applications of this procedure). As m rises, $\hat{g}(0)$ falls, but there is an asymptotic efficiency loss (although the asymptotic standard errors may actually fall because they scale with g(0)). The inefficiency of spectral density estimates has led many authors (e.g. Cochrane, 1988) to conclude that the main value of this sort of procedure is in obtaining point estimates, not in conducting inference. Although the 95 per cent confidence region in Cochrane (1988) included both k=0 and k=1, we have had better luck, as, for instance, with the Irish/German data the 95 per cent confidence region includes k=0 but excludes k=1, for sufficiently large m. For any stationary (mean reverting) series, the true value of k is zero and, while $\hat{g}(0)$ is upward biased in finite samples, it converges to zero as m increases. Intuitively, in the time domain, the mean reversion results from negative higher order autocorrelations and the higher is m, the more such autocorrelations we consider, from (2.5). For the Irish/German data, we find strong evidence for substantial mean reversion, in fact our results are quite consistent with the Irish/German real exchange rate being stationary. The evidence is much weaker for the Irish/UK data.[5]

The idea that the price-taking relationship is much stronger for Germany than the UK is entirely consistent with the graphical evidence from Figs. 1 and 2. Callan and Fitz Gerald (1990) suggested that the responses of agents to a foreign price/exchange rate shock would depend on whether they perceived that shock to be permanent or not. This may explain the closeness of the link to Germany that we find because, even though Irish trade links with the UK are stronger, sterling is much more volatile than the Mark and exchange rate movements are much more likely to be reversed quickly.

PPP co-integration can be defined as an unrestricted co-integrating relationship among prices and nominal exchange rates, rather than as the stationarity of the real exchange rate, as we have defined it hitherto. We have, in effect, imposed a (1,–1,–1) restriction on the putative long-run relationship. However, there is a serious econometric obstacle to be overcome in removing this: namely that the asymptotic behaviour of $\hat{g}(0)$ would no longer be given by (2.7) since we would be estimating the power spectrum of a generated series. Ardeni and Lubian (1991) investigate this case using some very simple Monte-Carlo experiments for the purposes of calibration. We prefer not to investigate this issue further until the theoretical econometric problem is resolved.

## IV CONCLUSION

Non-stationary time-series analysis has become enormously popular in applied macroeconometrics, but this has arisen mainly in the context of the US business cycle debate where data spanning over a century are available. Even with data covering long periods, a mean-reverting process can be nearly observationally equivalent to one that is not mean-reverting, provided that any mean-reversion is slow enough. Large data sets of high frequency data are of limited value unless they cover a long period. The rejection of PPP, even as a long-run co-integrating relationship found by Thom (1989), Callan and Fitz Gerald (1989) and ourselves using procedures based on nesting a unit root within an AR(1) process can be interpreted simply as a consequence of the low power of Dickey-Fuller and related tests and the short span of Ireland's experience in the EMS. We have summarised the method proposed

5. A separate but equally important point is that both real exchange rates have no significant drift, whereas all the constituent price and nominal exchange rate series except $p_g$ have significant drift. Long-run PPP would require "deterministic co-integration" (Hansen, 1992) among the prices and nominal exchange rate series as the co-integration would have to be a singularity removing both the unit roots and the drift components. At least the real exchange rates do not "inherit" the drift from the constituent price and nominal exchange rate series.

by Cochrane (1988) for quantifying the non-stationarity in a time series and found evidence that any non-stationarity in the Irish/German real exchange rate is small. Looking at Table 3, it is not clear to us that the Irish/German real exchange rate is not stationary, after all. For the Irish/UK data our results are more ambiguous. By the standards of US business cycle data, Ireland's experience in the EMS has been short and longer data may provide even stronger evidence for mean-reversion in real exchange rates. Nevertheless, the reader will readily see from Tables 1-3 that Cochrane's method provides dramatically different conclusions to those yielded by conventional ADF tests, at least for German data.

In a related paper, Wright (1993) argues that augmenting the PPP relation with interest differentials enables us to find a co-integrating relation for both Irish/German and Irish/UK data. As the interest differential was found to be I(0), this implies that the long-run equilibrium relation is simple PPP but that adding the interest differential into the system enables us to remove some I(0) effects that are causing us incorrectly to reject co-integration. The findings in these two papers are clearly consistent.

Our finding of strong mean reversion in Irish/German real exchange rates is of more policy significance than the absence of such evidence for the Irish/UK data. Unilateral actions by Irish authorities will, according to our findings, have little or no effect on long-run competitiveness. The fact that volatility in sterling does not seem to be passed through to Irish prices anything like as fast or as completely is irrelevant to the options open to Irish policy-makers. It would be possible, in principle, to extend the analysis in this paper to cover multilateral price-taking relationships involving Irish/UK/German data. We do not do this because we would need to estimate the parameters of the putative long-run relationship which leads to substantial econometric difficulties already outlined. We conjecture that such an exercise would stress the role of Germany, rather than the UK, in Irish price determination.

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