The Economic and Social Review, Vol. 25, No. 3, April, 1994, pp. 261-278

A Co-integration Based Analysis of Irish Purchasing Power Parity Relationships Using the Johansen Procedure

JONATHAN H. WRIGHT*
Harvard University

Abstract: Empirical work has found little evidence for purchasing power parity, specified as a long-run co-integrating relationship between Ireland and trading partners. Using Irish/UK and Irish/German data, we find evidence for such co-integration if and only if the system is augmented by short interest rates. We use the Johansen procedure for estimation and inference on the augmented co-integrating system.

I INTRODUCTION

One of the leading applications of co-integration analysis in applied econometrics has been to long-run price and exchange rate interactions. The most basic theory of the price and exchange rate nexus, purchasing power parity (PPP), specifies that if p_d and p_f are log prices (d and f subscripts denote domestic and foreign values throughout) and e is the log exchange rate[1] (domestic currency price in foreign currency terms), then $p_f - p_d = e$ so that all real exchange rates are zero. Econometrically, the theory can be implemented as a long-run relationship by specifying that deviations in real exchange rates from zero form a stationary process so that p_f, p_d and e are co-integrated with co-integrating vector (1, –1, –1), assuming that these series all have unit roots. By adding an intercept to the co-integrating regression,

1. Henceforth all prices and exchange rates are deemed to be in logs.

*Comments by John Bradley, Noel Farley, Patrick Honohan and two anonymous referees on an earlier draft of this paper are gratefully acknowledged. The usual disclaimer applies.

we can allow for the possibility that real exchange rates have a long-run equilibrium value but one that is not necessarily equal to zero.

In this way we can allow PPP to hold in the long run while recognising that it does not hold in the short run. The standard argument sustaining PPP, specified in this way, is that spatial arbitrage can prevent real exchange rates drifting away from equilibrium indefinitely (though a monetary justification for PPP also exists (Dornbusch, 1987)). We might expect adjustment in response to foreign price and exchange rate shocks to be faster in the traded sector than in the non-traded sector but, even though arbitrage cannot apply to the non-traded sector, it too may eventually respond to the shock (if, for instance, economy-wide real wages are set in the traded sector). If agents perceive an exchange rate shock to be temporary, it may not be transmitted into the domestic price level, but this should affect only the short-run properties of the system. Any number of objections can be raised to the idea of PPP holding as an instantaneous relationship, but we might expect these factors not to affect PPP when specified as a long-run co-integrating relationship. It is perhaps surprising then that empirical work testing for PPP co-integration has found little evidence for such a relationship.[2] For instance, Corbae and Oularis (1988) do not reject their non-stationarity null in the real exchange rates between the US and each of five trading partners, nor do they reject their non-co-integration null when examining PPP co-integration without the (1, –1, –1) restriction. Although prior to breaking the link with sterling, PPP applied between Ireland and the UK even in the short run (Bradley, 1977), evidence using data in the EMS period has been broadly unfavourable to the idea of PPP co-integration holding between Ireland and any trading partners. Using Augmented Dickey-Fuller tests, Thom (1989) fails to reject the unit root hypothesis in the real exchange rate using both Irish/US and Irish/German data. This null is rejected for Irish/UK data, but even in this case, co-integration is not found if the (1, –1, –1) restriction is not imposed on the co-integrating vector. Callan and Fitz Gerald (1989) reject PPP co-integration using both Irish/UK and Irish/German data, though they speculate that it may apply between Irish and a linear combination of UK and German prices and nominal exchange rates.[3] Despite such econometric work, Irish economists still typically treat the traded sector as price-taking. The issue

2. We test a null of non-co-integration by applying standard unit root tests to the residuals from the regression. In general, the critical values are not the same as in an ordinary unit root test because the residuals are generated. But, if the parameters of the co-integrating regression are constrained in line with PPP theory then this reduces to an ordinary unit root test applied to the real exchange rate, implemented using ordinary critical values because no parameters have been estimated.

3. They were unable to test their hypothesis in the EMS period because critical values were not available at that time.

has considerable policy relevance because the most often cited economic reason for Ireland's membership of the EMS and the strength of resistance to devaluation is the expectation that Ireland can link its inflation rate to the low German level by linking the Irish £ to the D-Mark. We are thus motivated to re-appraise the application of the techniques of non-stationarity time-series analysis to the PPP issue in Ireland, looking in particular at the impact of asset market factors.

There is a profusion of alternative models of exchange rate determination, but most are based on the assumption that PPP holds in the long run. Typically this assumption is augmented by uncovered interest parity, which however does not have strong empirical support either. For instance, the leading overshooting model of Dornbusch (1976), making these two assumptions, forecasts that the rate of change of the real exchange rate is proportional to the expected real interest differential.

Our aim is to test for PPP co-integration and it is hence not appropriate for us to start our analysis with any model imposing this hypothesis. We do not, therefore, start from any model imposing PPP in the long-run, but, instead, use a variant of the much simpler Mundell-Fleming model. Suppose that the current account surplus with a foreign country is a function of the real exchange rate and that the capital account surplus is a function of the interest differential. Neglecting accommodating transactions, the current and capital account surplus must be of equal magnitude and opposite sign. This implies a relationship between the real exchange rate and interest differential. Notwithstanding the simplicity of the economic argument, such a relationship has been explored in several recent empirical papers. Johansen and Juselius (1992) augment the simple PPP co-integrating system by domestic and foreign interest rates in an application with UK/foreign countries (trade-weighted) data.[4] Because there are five variables in this system (two prices, two interest rates and the exchange rate), the analysis must allow for multiple co-integrating vectors. Potentially, there could be as many as four co-integrating vectors. The approach to inference in this case, used in Johansen and Juselius (1992), is the pseudo-maximum-likelihood procedure introduced in Johansen (1988) and extended in Johansen (1991a). It is our goal, in this paper, to apply this approach to Irish/UK and Irish/German data. So, using these two data sets, we consider co-integration within the vector (p_f, p_d, e, i_f, i_d) where i is the short-term interest rate.[5] In Figures 1 and 2, real exchange rates and interest differentials are plotted for both data

4. Other applications of this idea can be found in Johansen (1991b) and Juselius (1991).

5. Economic theory might lead us to use the log of one plus the interest rate (see Dornbusch, 1976), which is however numerically close to the interest rate.

sets.[6] Our work is also related to Honohan and Conroy (1993) and Walsh (1993) who investigate co-integrating relationships between real exchange rates and foreign and domestic interest rates: we investigate if the impact of those interest rates on real exchange rates can account for the rejection of PPP co-integration.

The remainder of this paper is set out as follows. In Section II, we briefly review co-integration and the Johansen procedure. Technical aspects are confined to the Appendix. Using our two data sets, in Section III we determine the orders of integration of the individual series, the co-integrating rank and estimate the co-integrating space. In Section IV we test hypotheses specifying linear restrictions on the co-integrating space. The analysis presented considers certain long-run time-series properties of this vector, namely, the determination of co-integrating rank and the co-integrating relations and no attempt is made to construct a complete econometric model. Section V contains a brief conclusion.

II CO-INTEGRATION AND THE JOHANSEN PROCEDURE

The field of co-integration has been extraordinarily active in recent econometric research, both on the theoretical and applied fronts. For this paper, it will suffice to define a series to be I(d) if it is stationary and invertible in dth differences where d is any non-negative integer. Two or more I(d) series are then said to be co-integrated of order (d, b) and with co-integrating rank s, if there exist s linearly independent linear combinations of those series which are I(d – b): the most common case in the literature is co-integration of order (1, 1), which we refer to simply as co-integration.

Any linear combination of one or more co-integrating vectors is trivially a co-integrating vector too. We must seek to estimate s linearly independent vectors which span the co-integrating space (i.e., the set of all co-integrating vectors). In the case where $s=1$, if we normalise the first element of the co-integrating vector to unity, then it is unique and can be estimated by least squares. If p series co-integrate, then $0 < s < p$, so that if $p = 2$ this is the only possible form of co-integration. For higher p, it is important to consider the case $s > 1$. In this case, least squares will estimate only an arbitrary linear combination of a set of linearly independent co-integrating vectors, even after normalising the first element to unity. This is because any linear combination of two or more co-integrating vectors is a co-integrating vector too. To estimate basis vectors for the co-integrating space, we can use the pseudo-

6. The real exchange rates have been transformed linearly so as to fit in the same graphs as the interest differentials, and are not in any meaningful units.

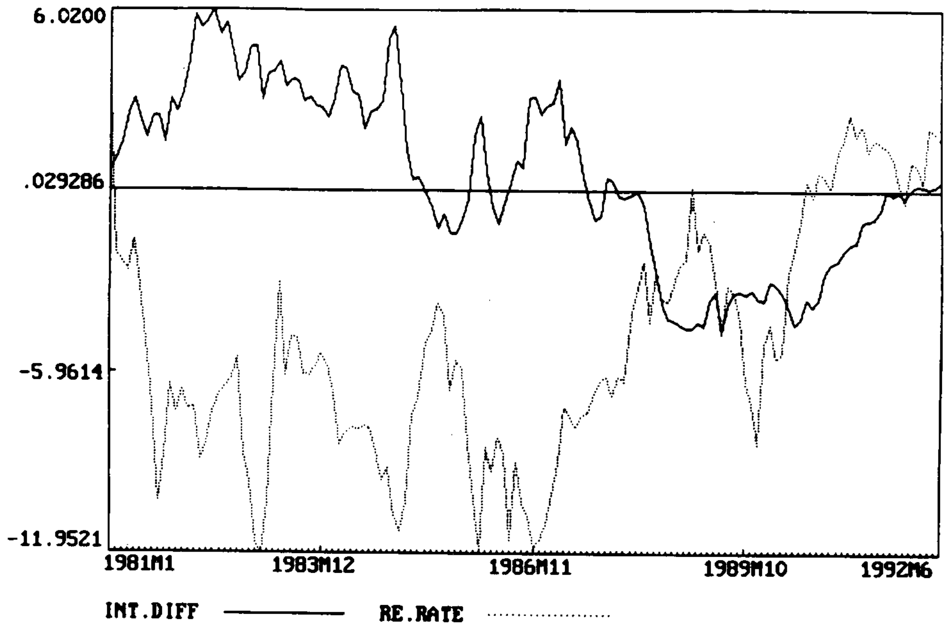

Figure 1: *Interest Differential and Real Exchange Rate (UK Data)*

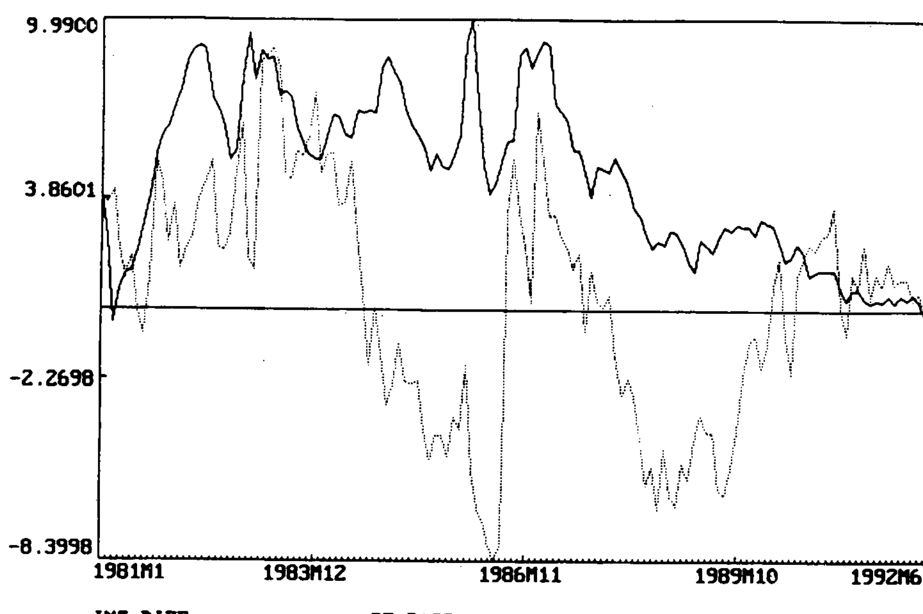

Figure 2: *Interest Differential and Real Exchange Rate (German Data)*

maximum-likelihood[7] procedure first proposed in Johansen (1988) and also presented, with some extensions, in Johansen (1991a). Well defined links exist between co-integrating systems and Error Correction Models (ECMs) specified in Granger's Representation Theorem (Engle and Granger, 1987). Suppose x_t is an I(1) p-vector with an Error Correction Model (ECM) of the form

$$\Delta x_t = \Pi(L)\Delta x_{t-1} + \delta\alpha' x_{t-k} + \nu + \varepsilon_t \tag{2.1}$$

where $\Pi(L)$ is the matrix lag polynomial $\Pi_1 L + \ldots \Pi_{\kappa-1} L^{k-1}$, $k < \infty$, α is a pxs matrix of full column rank, ε_t is a p-vector i.i.d. process and $x_{1-k} \ldots x_0$ are fixed. Then x_t is co-integrated with co-integrating rank s and the columns of s constitute a basis for the co-integrating space. This ECM is the basis of the Johansen procedure which estimates all the parameters of (2.1) (notably the co-integrating vectors in α) by pseudo-maximum-likelihood. We may then test linear homogeneous restrictions imposed on α and construct two tests of the null $H_0: s = s_0$, one against the alternative $s = s_0 + 1$ (the maximal eigenvalue test) and the other against the alternative $s > s_0$ (the trace test). We sketch details of the procedure in the Appendix. Notice however that we test a null that $s = s_0$ against an alternative specifying more co-integrating vectors, generalising the well-known fact that we detect co-integration by testing a null of non co-integration. To determine the co-integrating rank of a system we test H_0 for each s_0 from 0 to $p - 1$, using either of the two tests.

III EMPIRICAL WORK: ESTIMATING THE CO-INTEGRATING SPACES

A step which comes logically prior to estimating the co-integrating space is to determine the orders of integration of the individual series. For instance, if we hypothesise a set of series all to be I(1), when in fact even just one is I(0), then a linear combination of the series will obviously be I(0), even though they may be entirely unrelated, a situation we might term spurious co-integration. Inference in the simple Johansen procedure is based on the assumption that all the variables in x_t are I(1), although (2.1) can be augmented by I(0) variables (see Johansen, 1991a). The theory for the case where some co-integrating series are I(2) is more complicated (Johansen (1991c)). Our strategy is first to test all series for unit roots and, for those series for which the null is not rejected, we then proceed to test the differences of the

7. We refer to the method as pseudo-maximum-likelihood as it is a maximum-likelihood method only under Gaussianity. We make no Gaussianity assumption, so the estimators yielded may or may not be maximum-likelihood, but will always be least generalised variance estimators.

series for a unit root. Accordingly, we may categorise each series as I(0), I(1) or I(2).

In testing for a unit root in a univariate series y_t, we use the simple Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests which test the null H_0: $\beta = 1$ against the alternative $\beta < 1$ in an autoregression of the form

$$y_t = \beta y_{t-1} + u_t \qquad (3.1)$$

In the ADF test, with q lags, u_t is specified to be a stationary AR(q) process while the DF test requires u_t to be iid. Separate versions of the tests exist which augment (3.1) with an intercept term or both intercept and trend terms.

Our data consists of 8 series, as follows:

(i) i_d: Irish 3-month interest rates.
(ii) i_u: UK 3-month Treasury Bill rates.
(iii) i_g: German 3-month interbank interest rates.
(iv) p_d: Log Irish wholesale price index.
(v) p_u: Log UK manufacturing output price index.
(vi) p_g: Log German wholesale price index.
(vii) e_u: Log UK spot exchange rate in Irish currency terms.
(viii) e_g: Log German spot exchange rate in Irish currency terms.

Our observations are monthly, from January 1981 to June 1992 inclusive. Series are taken from various issues of OECD Main Economic Indicators and the UK CSO *Annual Abstract of Statistics*. Our data cover virtually the entire EMS period. We however deliberately omit all consideration of the recent instability within the EMS since this will not help to clarify the properties of the system in a long-run steady-state equilibrium. We use wholesale price indices (essentially traded sector prices) rather than consumer price indices because Ireland has only quarterly consumer price data whereas all other series are monthly and because these are the indices used in Thom (1989), in common with most of the literature in this area.

The DF and ADF tests (with 1-6 lags) were applied to all series with and without an intercept in (3.1).[8] The results are given in Table 1. Considering these results, we clearly cannot reject the null of a unit root, except perhaps in the cases of p_d and e_g. In the case of p_d, we have a strong prior that a price index cannot be a stationary process and indeed the null is not rejected in any

8. In principle, we would wish to use the more complicated semi-parametric unit root test proposed in Phillips (1987). However, we have a strong prior that all our series have unit roots, except for the interest rates. The DF and ADF procedures very clearly fail to reject the unit root hypothesis for these series.

Table 1: *DF/ADF Tests on Series Levels Without (With) Intercept*
Critical value is –1.95 (–2.88)

	0 Lags	1 Lag	2 Lags	3 Lags	4 Lags	5 Lags	6 Lags
p_d	5.51	3.35	2.89	2.21	1.78	1.28	1.21
	(–7.73)	(–4.91)	(–4.58)	(–3.64)	(–3.50)	(–2.95)	(–2.70)
i_d	–0.77	–0.75	–0.73	–0.83	–0.86	–0.85	–0.97
	(–1.63)	(–2.20)	(–1.89)	(–1.96)	(–1.91)	(–1.87)	(–1.61)
p_u	19.20	7.34	5.11	3.79	3.24	3.48	2.89
	(–4.87)	(–3.83)	(–3.53)	(–2.82)	(–2.66)	(–2.85)	(–2.61)
i_u	–0.80	–0.65	–0.59	–0.57	–0.57	–0.68	–0.84
	(–1.83)	(–2.33)	(–2.25)	(–2.05)	(–2.04)	(–1.94)	(–2.27)
e_u	–1.80	–1.45	–1.54	–1.54	–1.64	–1.45	–1.36
	(–2.64)	(–2.36)	(–2.51)	(–2.44)	(–2.52)	(–2.18)	(–2.01)
p_g	1.00	0.60	0.45	0.36	0.18	0.08	0.06
	(–1.76)	(–1.62)	(–1.38)	(–1.39)	(–1.69)	(–2.01)	(–2.08)
i_g	–0.24	–0.77	–2.13	–1.37	–1.42	–1.33	–1.09
	(–0.85)	(–1.48)	(–2.11)	(–1.83)	(–1.86)	(–1.95)	(–1.78)
e_g	–3.22	–2.62	–2.68	–2.44	–2.48	–2.08	–1.98
	(–2.53)	(–2.05)	(–2.14)	(–2.20)	(–2.35)	(–2.15)	(–2.28)

tests without intercepts nor in the ADF test with 6 lags and an intercept. In the case of e_g, although the null is rejected in all tests without intercept, it is not rejected in any tests with intercept.[9] We conclude that all series have unit roots.

The same tests were applied to the differences of the three price series as there is some debate as to whether prices are I(1) or I(2) variables (i.e., whether the inflation rate is I(0) or I(1)). The results are given in Table 2 and the unit root nulls are rejected in all cases except for marginal non-rejection in some ADF tests on p_u without intercept (coupled with rejection in all the tests with intercept). We conclude that all series are I(1).

The Johansen procedure with drift both in the data generating process and in the statistical model was then applied to the Irish/UK and Irish/German data.[10] We set $k = 4$ in both cases. Normally, in the literature, smaller values of k are chosen in order to obtain a parsimonious model. Our choice of k means that there are 100 parameters governing the short-run dynamics of

9. For both p_d and e_g, the unit root null was not rejected in any ADF tests with both an intercept and a trend.

10. We experimented with adding some other variables into the system, such as a dummy to control for the 1985 oil price drop, but concluded that such an augmentation of the system was unnecessary.

Table 2: *DF/ADF Tests on Series Differences Without (With) Intercept Critical value is –1.95 (–2.88)*

	0 Lags	*1 Lag*	*2 Lags*	*3 Lags*	*4 Lags*	*5 Lags*	*6 Lags*
p_d	–7.99 (–8.97)	–5.85 (–6.69)	–4.44 (–5.00)	–3.39 (–3.85)	–2.65 (–2.92)	–2.68 (–2.89)	–2.79 (–3.00)
p_u	–3.98 (–8.99)	–2.49 (–6.04)	–1.99 (–4.50)	–1.73 (–3.86)	–1.79 (–4.11)	–1.52 (–3.43)	–1.62 (–3.94)
p_g	–9.08 (–9.07)	–7.41 (–7.40)	–5.88 (–5.87)	–4.05 (–4.04)	–3.17 (–3.15)	–2.95 (–2.93)	–3.06 (–3.03)

the system (with 690 observations), although we make no attempt to interpret these parameters and just use them to try to control for short-run dynamics. The reason why we choose this value of k is that our results are not robust to reducing k below 4 (while they are robust to increasing it further). We choose the smallest value of k such that our main conclusions are robust to increasing it.

Inference on α, in the Johansen procedure, only requires the partial sums of the u_ts to converge to a Brownian motion. Granted that the u_ts are I(0) (as implied by the co-integration), this imposes only minimal regularity conditions on the disturbance terms. These are highly technical mixing conditions, for which we do not know of any tests. In particular, and in contrast with inference in the ordinary linear regression model, serial correlation and heteroskedasticity have no implications for inference on α. Our estimators may not be efficient if u_t is not iid, but we are not aware of any work in the theoretical econometric literature which extends the Johansen procedure to this case. For these reasons, we do not apply the usual diagnostic tests to the disturbance terms.

The tests for co-integrating rank are reported in Tables 3(a) and (b). With both Irish/UK and Irish/German data, the maximal eigenvalue and trace procedures imply different conclusions, indicating that s = 1/s = 3 respectively, for both data sets. In such cases, graphical examination of the co-integrating residuals can be useful. These residuals can be adjusted for short-run dynamics. If β is a column of α, then a plot of $\beta' R_{1t}$ should appear to be roughly stationary and if it looks clearly non-stationary, then we must infer that α has lower column rank than we had hypothesised, where R_{1t} is as defined in the Appendix. From examination of the plots of co-integrating residuals, we conclude that s = 2 for both data sets. The co-integrating residuals, adjusted for short-run dynamics, are plotted in Figures 3 and 4.

Table 3(a): *Maximal Eigenvalue Tests*

H_0	*Irish / UK Data*	*Irish / German Data*	*5% Critical Values*
s = 0	51.55	40.23	33.46
s = 1	19.72	24.60	27.07
s = 2	18.28	19.68	20.97
s = 3	10.99	10.00	14.07
s = 4	2.71	2.23	3.76

Table 3(b): *Trace Tests*

H_0	*Irish / UK Data*	*Irish / German Data*	*5% Critical Values*
s = 0	103.29	96.72	68.52
s = 1	51.69	56.50	47.21
s = 2	31.97	31.90	29.68
s = 3	13.70	12.22	15.41
s = 4	2.71	2.23	3.76

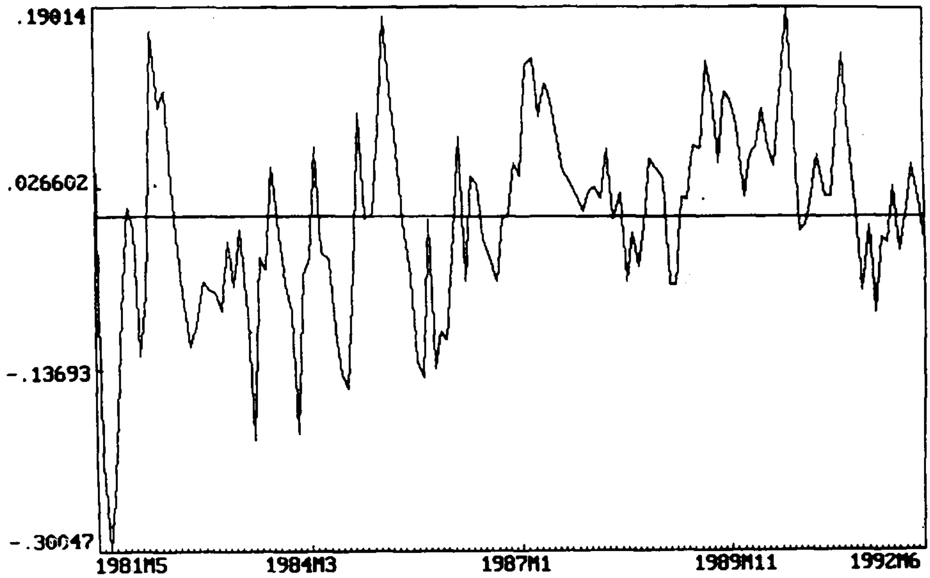

Figure 3(a): *Residuals of Vector 1 Adjusted for Short-run (UK Data)*

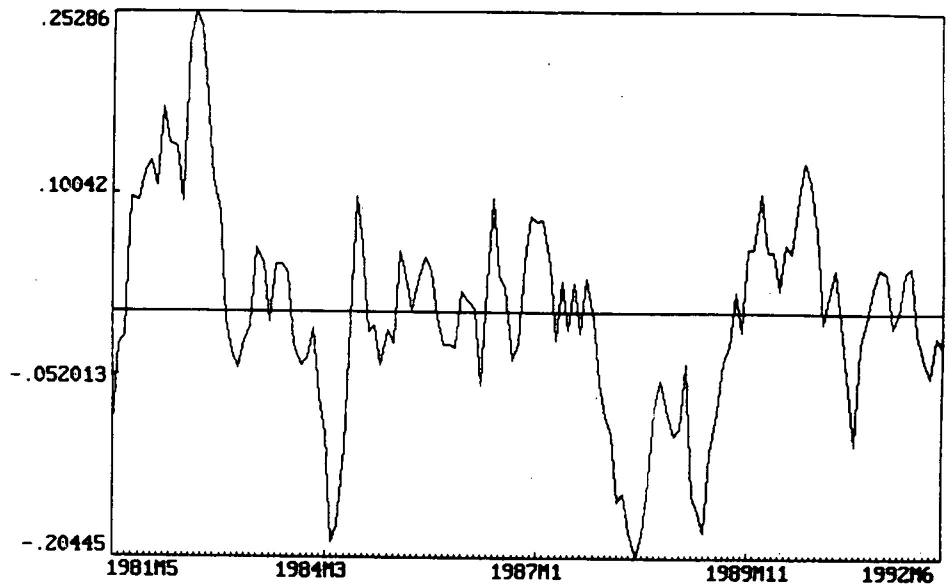

Figure 3(b): *Residuals of Vector 2 Adjusted for Short-run (UK Data)*

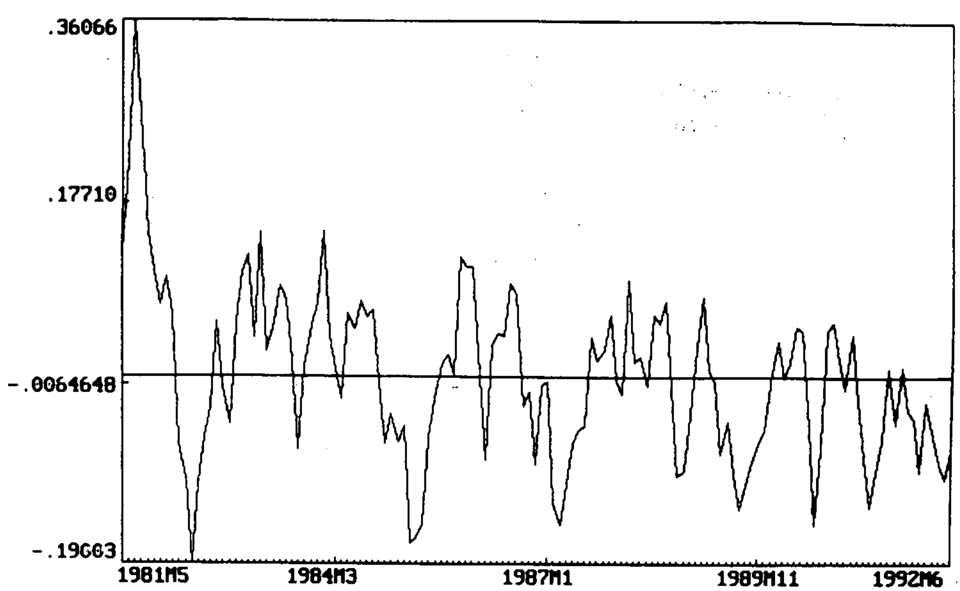

Figure 4(a): *Residuals of Vector 1 Adjusted for Short-run (German Data)*

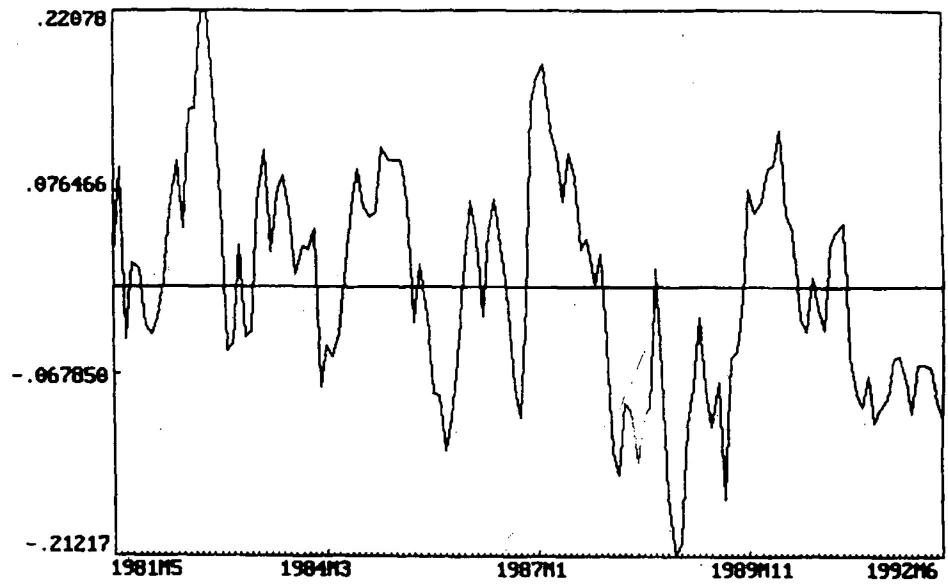

Figure 4(b): *Residuals of Vector 2 Adjusted for Short-run (German Data)*

Table 4 gives vectors spanning the co-integrating space, with the first element of each normalised to unity, given the above choices of s, with both data sets. These vectors, *as well as any linear combination of them*, constitute our estimated co-integrating vectors.

Table 4: *Co-integrating Vectors*

	UK Data: Vector 1	*UK Data: Vector 2*	*FRG Data: Vector 1*	*FRG Data: Vector 2*
p_f	1.00	1.00	1.00	1.00
p_d	–2.28	–6.35	–1.87	–5.53
e	–1.86	7.49	–0.66	–7.11
i_f	–0.27	0.44	–0.00023	–0.11
i_d	–0.0040	0.56	–0.03	0.47

IV EMPIRICAL WORK: STRUCTURAL HYPOTHESIS TESTS

Given our estimate of the co-integrating space, we test a sequence of hypotheses pertaining to one/all co-integrating vector(s) as listed below. These hypotheses are nested conforming to the widely accepted general to specific modelling strategy:

(i) H_1: The co-integrating vector(s) have the form (1, –1, –1, –a, b), meaning that the long-run relationship(s) have the PPP form augmented by the two interest rates (theory predicting that $a, b \geq 0$). This implies that the equilibrium relation is $p_f - p_d - e - ai_f + bi_d = 0$.

(ii) H_2: The co-integrating vector(s) have the form (1, –1, –1, –a, a), meaning that the long-run relationship(s) have the PPP form augmented just by the interest rate differential. This implies that the equilibrium relation is $p_f - p_d - e - a\,(i_f - i_d) = 0$.

(iii) H_3: The co-integrating vector(s) have the form (1, –1, –1, 0, 0), meaning that simple PPP co-integration holds and does not need to be augmented by interest rate factors. This implies that the equilibrium relation is $p_f - p_d - e = 0$.

The restrictions can be imposed either on all basis vectors (meaning that we are imposing it on all elements of the co-integrating space) or we can test the restriction applied to a single co-integrating vector. H_1 applied to both co-integrating vectors yields test statistics of 24.88 and 21.86 using Irish/UK and Irish/German data respectively which, compared with the χ^2 on 4 degrees of freedom null limiting distribution, both correspond to negligible p-values. Our strategy is hence to test if each restriction in the above sequence can be applied to an element of the co-integrating space and the results are reported in Table 5. For both data sets we reject H_3, but do not reject H_1 and H_2. Had we chosen lower values of k we would have rejected H_2 (while not rejecting H_1). It would be very hard to find an economic rationale for this. However, it seems to result from a purely statistical problem, namely that the short-run dynamics cannot be adequately controlled for with k<4.

Table 5: *Test Statistics (P-values)*

	Irish / UK Data		*Irish / German Data*	
H_1	0.62	(43.2%)	1.18	(27.8%)
H_2	1.26	(53.2%)	2.12	(34.7%)
H_3	14.81	(0.2%)	12.35	(0.6%)

Granted that H_2 describes one basis vector in both co-integrating spaces, we are obviously motivated to test the stationarity of the interest rate differential or, equivalently, to test the null that another basis vector of the co-integrating space is (0, 0, 0, 1, –1). If this restriction is satisfied, we must conclude that the PPP relation does hold in the long run, but that if we fail to correct for short-run interest rate effects, we will incorrectly reject PPP co-integration.

A direct test for a unit root in the interest differentials can be conducted using the usual ADF procedures, as shown in Table 6. Economics gives us a strong prior that these differentials should be I(0), but, in Table 6, we clearly fail to reject the unit root nulls. The power of these tests is however notoriously low, and our data cover a rather short period. We therefore also test the restriction that (0, 0, 0, 1, –1) is an element of the co-integrating space and we do not reject this null for either data set. The test statistics are 3.83 and 5.3 for the Irish/UK and Irish/German data respectively, which we compare with χ^2 distributions on 3 degrees of freedom. The statistical evidence on the order of integration of the interest differentials is not consistent, but the properties of the tests and our economic prior lead us to conclude that they are both I(0).

Table 6: *DF/ADF Tests on Interest Differentials Without (With) Intercept*

Critical value is –1.95 (–2.88)

	0 Lags	*1 Lag*	*2 Lags*	*3 Lags*	*4 Lags*	*5 Lags*	*6 Lags*
German Data	–1.08	–1.21	–0.93	–1.02	–0.95	–0.87	–0.78
	(–1.43)	(–2.15)	(–1.99)	(–1.91)	(–1.82)	(–1.59)	(–1.19)
UK Data	–1.51	–1.76	–1.61	–1.63	–1.63	–1.55	–1.39
	(–1.53)	(–1.77)	(–1.61)	(–1.60)	(–1.60)	(–1.53)	(–1.38)

V CONCLUSION

Our analysis of the co-integrating relationship between Irish and foreign interest rates, prices and exchange rates gives strong evidence for the existence of a long-run equilibrium relation between these variables using either Irish/UK or Irish/German data. In both cases the co-integrating space appears to be two-dimensional (i.e., there are two linearly independent long-run relations among our set of variables) and one of these equilibrium relations seems to be of the PPP form augmented by the interest differential, but we strongly reject any further restrictions that we impose on this relation, including that of simple PPP. If we repeat the analysis using just the prices and exchange rates in the co-integrating vector then we do not reject the null that the co-integrating rank is zero, provided that enough lags are included, with either data set. We however also find the interest differential to be another co-integrating vector. This leaves us with a paradox: PPP does not hold as a long-run relationship without augmentation by the interest differential and yet the interest differential seems itself to be I(0).

Non-stationary time-series analysis has become enormously popular in applied macroeconometrics, but this has arisen mainly in the context of the US business cycle debate where data spanning over a century are available. Even with data covering long periods, a mean-reverting process can be nearly observationally equivalent to one that is not mean-reverting, provided that any mean-reversion is slow enough. Large data sets of high frequency data are of limited value unless they cover a long period. The rejection of PPP co-integration found by Thom (1989), Callan and Fitz Gerald (1989) and ourselves can be interpreted simply as a consequence of the low power of our tests and the short span of Ireland's experience in the EMS. Although we fail to reject a unit root null in the real exchange rate series, Wright (1993), in a related paper, argues that they are much "less non-stationary" (in a precise sense, defined in that paper) than the constituent price and nominal exchange rate series. We can interpret the effect of adding the interest differential into the relationship as removing some I(0) effects that are causing us incorrectly to reject the existence of a co-integrating relationship involving foreign and domestic prices and the nominal exchange rate outright. Incorrect inference may be made about long-run relations if the researcher fails adequately to control for short-run effects.

REFERENCES

BRADLEY, JOHN, 1977. "Lags in the Transmission of Inflation", *The Economic and Social Review*, Vol. 8, No. 2, pp. 149-154.

CALLAN, TIM, and JOHN FITZ GERALD, 1989. "Price Determination in Ireland: Effects of Changes in Exchange Rates and Exchange Rate Regimes", *The Economic and Social Review*, Vol. 20, No. 2, pp. 165-188.

CORBAE, DEAN, and SAM OULARIS, 1988. "Co-integration and Tests of Purchasing Power Parity", *Review of Economics and Statistics*, Vol. 70, pp. 508-511.

DORNBUSCH, RUDIGER, 1976. "Expectations and Exchange Rate Dynamics", *Journal of Political Economy*, Vol. 84, pp. 1,161-1,176.

DORNBUSCH, RUDIGER, 1987. "Purchasing Power Parity", in J. Eatwell, M. Milgate and P. Newman (eds.), *The New Palgrave: A Dictionary of Economics*, London: Macmillan.

ENGLE, ROB. F., and CLIVE W.J. GRANGER, 1987. "Co-integration and Error Correction: Representation, Estimation and Testing", *Econometrica*, Vol. 55, pp. 251-276.

HONOHAN, PATRICK, and CHARLES CONROY, 1994. "Sterling Movements and Irish Pound Interest Rates", *The Economic and Social Review*, this issue, pp. 201-220.

JOHANSEN, SOREN, 1988. "Statistical Analysis of Co-integrating Vectors", *Journal of Economic Dynamics and Control*, Vol. 12, pp. 231-254.

JOHANSEN, SOREN, 1991a. "Estimation and Hypothesis Testing of Co-integrating Vectors in Gaussian Vector Autoregressive Models", *Econometrica*, Vol. 59, pp. 1,551-1,580.

JOHANSEN, SOREN, 1991b. "An I(2) Co-integration Analysis of Purchasing Power

Parity Between Australia and the United States", Working Paper No. 231, Australian National University.

JOHANSEN, SOREN, 1991c. "A Statistical Analysis of Co-integration of I(2) Variables", University of Copenhagen, Preprint.

JOHANSEN, SOREN, and KATARINA JUSELIUS, 1992. "Testing Structural Hypotheses in a Multivariate Co-integration Analysis of the PPP and the UIP for UK", *Journal of Econometrics*, Vol. 53, pp. 211-244.

JUSELIUS, KATARINA, 1991. "Long-run Relations in a Well Defined Statistical Model for the Data Generating Process: Co-integration Analysis of the PPP and UIP Relations between Denmark and Germany", in J. Gruber (ed.), *Econometric Decision Models: New Methods of Modelling and Applications*, New York: Springer-Verlag.

PHILLIPS, PETER C.B., 1987. "Time Series Regression With a Unit Root", *Econometrica*, Vol. 55, pp. 277-301.

RAO, C.R., 1973. *Linear Statistical Inference and Its Applications*, New York: John Wiley.

THOM, RODNEY, 1989. "Real Exchange Rates, Co-integration and Purchasing Power Parity: Irish Experience in the EMS", *The Economic and Social Review*, Vol. 20, No. 2, pp. 147-163.

WALSH, BRENDAN, 1993. "Credibility, Interest Rates and the ERM: The Irish Experience 1986-92", *Oxford Bulletin of Economics and Statistics*, forthcoming.

WRIGHT, JONATHAN H., 1993. "Quantifying the Non-Stationarity in Irish Real Exchange Rates", *The Economic and Social Review*, Vol. 25, No. 1, pp. 109-119.

APPENDIX

This Appendix deals with technical issues involved with the pseudo-maximum-likelihood estimator of the ECM[11]

$$\Delta x_t = \Pi(L)\Delta x_{t-1} + \delta\alpha' x_{t-k} + v + \varepsilon_t.$$

We begin by concentrating out the Π_is and v by regressing Δx_t and x_{t-1} onto $(1, \Delta x_{t-1} \ldots \Delta x_{t-k})'$ yielding the partial residuals R_{0t} and R_{1t} respectively. So we can now write

$$R_{0t} = \delta\alpha' R_{1t} + \varepsilon_t$$

and the pseudo-maximum-likelihood estimator (PMLE) of δ and α minimises the criterion function

$$\Lambda(\alpha, \delta) = \left|\Sigma_{t=1}^{T} \varepsilon_t \varepsilon_t'\right| = \left|\Sigma_{t=1}^{T} (R_{0t} - \delta\alpha' R_{1t})(R_{0t} - \delta\alpha' R_{1t})'\right|.$$

11. This ECM and version of the Johansen procedure, with unrestricted v, allows for drift in x_t and is used in the empirical work in the paper. Several other variants on the procedure exist (see, e.g. Johansen (1991a)).

Define the four pxp product moment matrices

$$S_{ij} = \Sigma_{t=1}^{T} R_{it} R'_{jt}; \; i, j = 0, 1.$$

Holding α fixed, the PMLE of δ is obtained by an OLS regression of R_{0t} on $\alpha' R_{1t}$. So, we can write the conditional PMLE of δ as

$$\tilde{\delta}(\alpha) = S_{01}\alpha(\alpha' S_{11}\alpha)^{-1},$$

further concentrating our criterion function to

$$\Lambda^*(\alpha) = \left| S_{00} - S_{01}\alpha\tilde{\delta}(\alpha)' - \tilde{\delta}(\alpha)\alpha' S_{10} + \tilde{\delta}(\alpha)\alpha' S_{11}\alpha\tilde{\delta}(\alpha)' \right|$$
$$\therefore \Lambda^*(\alpha) = \left| S_{00} - S_{01}\alpha(\alpha' S_{11}\alpha)^{-1}\alpha' S_{10} \right|.$$

It is a well-known property of the determinant of a partitioned matrix (Rao, 1973) that

$$\begin{vmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{vmatrix} = \left| \alpha' S_{11}\alpha \right| \left| S_{00} - S_{01}\alpha(\alpha' S_{11}\alpha)^{-1}\alpha' S_{10} \right|$$
$$= \left| S_{00} \right| \left| \alpha' S_{11}\alpha - \alpha' S_{10}(\alpha' S_{00}\alpha)^{-1} S_{01}\alpha \right|$$

$$\therefore \arg\min_\alpha \Lambda^*(\alpha) = \arg\min_\alpha \frac{\left|\alpha' S_{11}\alpha - \alpha' S_{10} S_{00}^{-1} S_{01}\alpha\right|}{\left|\alpha' S_{11}\alpha\right|}$$

The minimisation is now a special case of a standard problem (Rao, 1973), the solution to which yields as the PMLE of α that matrix the columns of which are the eigenvectors corresponding to the s largest (generalised) eigenvalues in the (generalised) eigenvalue problem

$$\left|\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}\right| = 0.$$

Let λ_m be the mth largest generalised eigenvalue solving this problem.

Johansen (1988) constructs tests of linear homogeneous restrictions on α and of co-integrating rank. We refer to these tests as pseudo-likelihood-ratio tests as they have a likelihood ratio interpretation under Gaussianity and consider these tests, in turn. If we wish to test p linear homogeneous restrictions on the co-integrating vectors, we can either test the null that some element of the co-integrating space satisfies the restriction or that all co-integrating vectors satisfy it. In either case, standard asymptotic χ^2 inference is used, with the limiting χ^2 random variable having ps degrees of freedom or p+1–s degrees of freedom in the two respective tests. In testing for co-integrating rank, the two available procedures are the maximal eigenvalue test, which tests $H_0: s = s_0$ against the alternative $s = s_0 + 1$ by comparing J_0 with the distribution of the smallest eigenvalue of a certain stochastic matrix, Q and the trace test, which tests H_0 against the alternative $s > s_0$ by comparing J_1 with the sum of the eigenvalues of Q (which is just the trace of that matrix), the test statistics being defined as

$$J_0 = T\lambda_{s_0+1}; \; J_1 = T\Sigma_{m=s_0+1}^{p}\lambda_m.$$

For further details, including the definition of Q, the reader is referred to Johansen (1988).