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Testing and Estimation in Unstable Dynamic Models: A Case Study*

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Abstract: This paper discusses testing for parameter instability and estimation of time-varying parameters in the context of the Engle-Granger (1987) procedure. It reviews several developments in testing, in particular the new test by Bai, Lumsdaine and Stock (1991) for use in vector autoregression and error-correction models; it gives an account of the Kalman filter estimation technique; and it examines a variety of methodological matters. To illustrate the methods and issues raised, an example concerning the estimation of regional exployment multipliers for Northern Ireland is presented. The paper concludes with some remarks and recommendations for applied work in economics.

I INTRODUCTION

During the past few years there has been a growing literature on unit roots, cointegration and error-correction models. Most economists are well aware of the main developments in these areas and of their implications for applied work. In particular, it is known that observations on many macroeconomic variables appear to behave like difference-stationary time series, rather than stationary or trend-stationary series;[1] that it is important to test for the presence of unit roots, because if they remain undetected the

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1. See, for example, the seminal paper by Nelson and Plosser (1982) and the work by Campbell and Mankiw (1987) and Cochrane (1988).

use of standard asymptotic distribution theory when modelling may be invalid and thus give rise to spurious inferences;[2] and that if two or more variables are cointegrated, it may be feasible to model them using standard techniques in such a way that both short-run dynamics, as captured by the differences of the variables, and long-run equilibrium tendencies, as suggested by economic theory and represented by mechanisms involving the levels of the variables, are incorporated.[3] However, few may yet be aware that the way these findings are viewed is already beginning to change as a result of recent empirical and theoretical research on the detection and dating of changes in such things as the trend and drift parameters of economic series.

For example, Christiano (1988), Banerjee *et al.* (1989) and Zivot and Andrews (1989) present evidence of shifts in the mean growth rates of a large proportion of aggregate economic time series, including some European series. Perron (1989) and Rappoport and Reichlin (1989), using the assumption of known break dates, suggest that US output is better thought of as being stationary around a broken trend than as being integrated of order one. More recently, Perron and Vogelsang (1991) show that while the standard Dickey-Fuller test indicates unit roots in the real exchange rates between the US and UK, and between the US and Finland, these series are stationary if a change in their means is allowed for; and thus they rehabilitate the purchasing power parity hypothesis for these two pairs of countries. Bai *et al.* (1991) argue that if variables are cointegrated, or if the restrictions suggested by a variety of economic theories are imposed, then instability in one series may be inherited by other series and it may be possible to identify breaks using multivariate techniques, where none is apparent using univariate analyses on the individual series. Utilising the real business cycle theory of King *et al.* (1988) and post-war data for the US, they go on to demonstrate some evidence of a common slow down in real output per capita, consumption and investment; they also provide evidence for significant and more or less simul-

2. Recent surveys on testing for unit roots include Dickey, Bell and Miller (1986) and Diebold and Nerlove (1990). The spurious nature of the results from regressions involving non-stationary variables was suggested by the work of Yule (1926) and further investigated by Granger and Newbold (1974). It was not until the papers by Phillips (1986) and Park and Phillips (1988 and 1989), however, that the appropriate asymptotic distribution theory began to be developed for the case of unit roots.

3. The seminal paper on cointegration is Engle and Granger (1987) which includes a proof of the result that cointegrated variables admit an error-correction representation. Surveys of the subject include Dolado, Jenkinson and Sosvilla-Rivero (1990) and Dickey, Jansen and Thornton (1991). The use of error-correction models in applied work was pioneered by Sargan (1964) and popularised by the work on the consumption function by Davidson, Hendry, Srba and Yeo (1978). A recent example of the approach, used in a study of the UK demand for money, is Hendry and Ericsson (1991). A useful survey is provided by Alogoskoufis and Smith (1991).

taneous breaks in the output series of a number of EC countries.

Perhaps these new findings, and the increasing concern about the issue of instability in unit root econometrics, are not altogether surprising given the vicissitudes in economic, political and social conditions and the long tradition of concern for parameter instability in classical regression analysis. However, they would appear to have potentially significant consequences both for the general approach to dynamic modelling, and for the existing applied research output in this field, a feature of which has been the underlying assumption of stable univariate time series representations of variables and stable structural relationships amongst variables. The widely used two-step procedure of Engle and Granger (1987), which takes stability for granted in testing for unit roots and estimating cointegrating regressions, is a good example, notwithstanding the emphasis of some practitioners on post-sample stability testing of the derived error-correction model. Unfortunately, as Perron (1990) has shown, structural shifts in the mean levels of stationary time series bias the usual tests for a unit root towards non-rejection; and as the work of Bai *et al.* (1991) indicates, instability may pervade the variables that typically enter cointegrating relationships. There seems little doubt, therefore, that instability will be a subject of considerable importance in future dynamic modelling and time series research.

This paper focusses on testing for stability and estimation of time-varying parameters in the context of the Engle-Granger (1987) procedure. The aim is to show how attention to the possibility of instability may lead to radically different conclusions from those which would result from the application of the standard methodology. The approach is to utilise a number of formal and informal testing and estimation procedures within an illustrative case study, the study chosen being that of Bond (1990) on regional employment multipliers for Northern Ireland. Though essentially intended to raise questions of general methodological relevance, the results obtained may be of some interest in themselves.

The remainder of the paper is arranged as follows. Section II contains a brief review of some of the techniques available for detecting instability, and in particular the new procedure of Bai *et al.* (1991) for testing for instability in vector autoregressive and error-correction models. Section III sketches the state-space model and the use of the Kalman filter for estimating time-varying parameter models. Section IV outlines the case study and presents the results of selected tests and estimates for static (cointegrating) regressions and dynamic (error-correction) models. Section V discusses some points on modelling methodology arising from the case study; and Section VI contains some concluding remarks.

II TESTS FOR INSTABILITY

There is a wide variety of procedures available for testing for instability in econometric models. Most of them were developed for use in the static general linear model with stationary variables, though many of these are approximately (asymptotically) valid in dynamic linear models involving lagged dependent variables. The techniques developed recently tend to be of more general applicability, but few have been developed specifically for use in the presence of non-stationary variables.

The first, large group includes the log likelihood ratio procedure suggested by Quandt (1960); the Chow (1960) test and its generalisation based on the use of several moving regressions, which Brown *et al.* (1975) refer to as the homogeneity test; the Farley-Hinich (1970) test and the similar procedure based on time-trending regressions outlined in Brown *et al.* (1975); the techniques based on the use of cumulative sums (cusums) and cusums of squares of a set of recursive residuals proposed by Brown *et al.* (1975), and the least squares variant of the cusum of squares test developed by McCabe and Harrison (1980); the tests against stationary AR(1) and random walk behaviour of coefficients, such as those of LaMotte and McWhorter (1978), Tanaka (1983), Nicholls and Pagan (1985) and Watson and Engle (1985); and the point optimal tests of King (1987), Shively (1988) and Brooks (1991).

The recent group includes the proposals of Zivot and Andrews (1990), Bai *et al.* (1991), Andrews and Ploberger (1991), and a noteworthy test for use in models with non-stationary regressors by Hansen (1990). Each of these procedures is based on the behaviour of an indicator derived from a certain sequence of statistics, in the tradition of many tests for structural change when the change points are unknown. Thus Zivot and Andrews suggest examining a minimal t type statistic; Hansen proposes a mean Lagrange multiplier statistic; Andrews and Ploberger derive an asymptotically optimal procedure using an average exponential Wald test; and Bai *et al.* use various functions of Wald F type statistics.

The procedures used in the case study reported in Section IV below are the Chow test, the McCabe-Harrison cusum of squares (MH) test and the Bai-Lumsdaine-Stock (BLS) test. The Chow test is very well known and widely used, being available in most econometrics packages. In this paper, sequences of 1-step ahead and N-step back Chow tests are employed. The MH test, while less widely used, simply requires the standardised cusums of squares of ordinary least squares (OLS) residuals to be plotted as a diagram, with pairs of parallel lines superimposed by which to judge the likelihood of instability. The construction of these lines and the precise form of the test criterion are straightforward, and are described in the original MH paper. By contrast, the

BLS test is both relatively complex and little known. Therefore, there follows a brief outline of the BLS procedure.

The BLS Test

Let y_t denote an $n \times 1$ vector of I(0) time series variables, $d_t(k) = 1(k)$, where $1(\cdot)$ is a structural break indicator function which takes on the value 0 when $t<k$ and the value 1 when $t \geq k$, and X_{t-1} be a vector of stationary variables that do not depend on the break date k. The system of equations considered is

$$y_t = \mu + \lambda d_t(k) + \sum_{j=1}^{p} A_j y_{t-j} + BX_{t-1} + u_t, \tag{1}$$

where y_t, μ, λ and u_t are $n \times 1$, $\{A_j\}$ are $n \times n$, and the roots of $(I - A(L)L)$ are assumed to be outside the unit circle. This model incorporates a number of interesting special cases. For example, with $y_t = \Delta Y_t$ and X_{t-1} omitted, it is a vector autoregressive specification (VAR), applicable when the series are integrated, possibly with a changing drift, but not cointegrated; with $y_t = \Delta Y_t$ and $X_{t-1} = \gamma' Y_{t-1}$, it is a vector error-correction model (VECM); and with $n = 1$, the familiar univariate version of these alternative representations emerges.

In order to cast the sequence of statistics underpinning the BLS procedure in its most general form, it is convenient to write equation (1) more compactly as

$$y_t = (I \otimes Z_t')\beta + u_t, \tag{2}$$

where $Z_t' = (1, d_t(k), y_{t-1}', \ldots, y_{t-p}', X_{t-1}')$, $\beta = \mathrm{Vec}(D')$ and $D = (\mu, \lambda, A_1, \ldots, A_p, B)$, and to note that, for a given k, the OLS estimator of β is

$$\hat{\beta}(k) = \left[\sum_{t=1}^{T}(I \otimes Z_t Z_t')\right]^{-1} \sum_{t=1}^{T}(I \otimes Z_t Z_t')y_t. \tag{3}$$

Now the null hypothesis of stability, in the sense that there is no break in the mean, is assessed by reference to the sequence of F statistics testing $\lambda = 0$, namely,

$$\hat{F}(k) = \left[S\hat{\beta}(k)\right]' \left[S\left(\hat{\Omega}_u \otimes \left(T^{-1}\sum_{t=1}^{T} Z_t Z_t'\right)^{-1}\right)S'\right]^{-1} \left[S\hat{\beta}(k)\right],$$

$$k = k^* + 1,\ k^* + 2, \ldots, T - k^*, \tag{4}$$

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where $S = I \otimes s$ and $s = (0, 1, 0, \ldots, 0)$, $\hat{\Omega}_u$ is the estimated variance-covariance matrix of u_t, $k^* = [\delta T]$ where δ is a "trimming" parameter such that $0 < \delta < 1$ and $[\cdot]$ denotes the integer part. Denoting the stochastic F statistic process as $F_T(\delta) = \hat{F}([\delta T])$, BLS propose three test statistics: the maximum Wald statistic, the mean Wald statistic and the log exponential Wald statistic, defined, respectively, as

$$\text{Sup–W:} \qquad \sup_{\delta \epsilon (\delta^*, 1-\delta^*)} F_T(\delta) \tag{5}$$

$$\text{Mean–W:} \qquad \int_{\delta^*}^{1-\delta^*} F_T(\delta) d\delta \tag{6}$$

$$\text{Exp–W:} \qquad \ln\left\{\int_{\delta^*}^{1-\delta^*} \exp[F_T(\delta)] d\delta\right\}. \tag{7}$$

Using the known form of the limiting distribution of the Wald F statistic process $F_T(\delta)$, BLS derive the limiting distributions of the statistics (5)-(7) by means of the continuous mapping theorem and tabulate selected critical values. Fuller technical details are given in Theorem 1 of Bai *et al.* (1991, pp. 12-13).

Although the Chow and MH tests are strictly valid only in the case of fixed regressor models, they are approximately valid in dynamic models. Some support for the use of the cusum of squares test in dynamic models comes from McCabe (1987), though it should be pointed out that Ploberger and Kramer (1985) have shown that the technique may have very little power in certain cases. For testing against instability in the ECM formulations in Section IV, therefore, the BLS test will be the essential benchmark. In the presence of non-stationary variables, the behaviour of both the Chow and MH procedure is unknown. In employing them in the context of cointegrating regressions, therefore, it must be stressed that they are not being used as formal testing procedures, but merely as informal aids to diagnosis. Finally, the three chosen tests for instability are supplemented in the case study by an examination of the recursively estimated coefficients of the various models used.

III ESTIMATING VARYING PARAMETERS: THE KALMAN FILTER

If the parameters in an economic model are not stable over time, problems arise for many techniques, as mentioned in Section I. In particular, the Engle-Granger estimation procedure and related techniques in unit root econometrics are not strictly applicable. An alternative approach is to use a state-space formulation and the Kalman (1960) filter to estimate the time-varying parameters. Although this approach is much used by engineers, and by Bayesians in time series analysis, it has never played a central role in applied economics.[4] It is therefore briefly outlined in this section.

State-space models focus on m unobservable state variables which change over time, and on how these variables relate to certain other variables which can be observed. If x_t is the $m \times 1$ vector of state variables and y_t the $n \times 1$ vector of observables at time t, then the relationship between the two may be written as the measurement equation

$$y_t = Z_t x_t + V_t u_t, \quad t = 1, 2, \ldots, T, \tag{8}$$

where Z_t and V_t are fixed matrices of order $n \times m$ and $n \times r$, respectively. The observational disturbance u_t is assumed to be independent with zero mean and variance-covariance matrix H_t. Although x_t is not directly observable, its evolution is assumed to be governed by its random initial value, x_0, together with the variance-covariance matrix of x_0, and a process defined by the state or transition equation

$$x_t = G_t x_{t-1} + R_t w_t, \quad t = 1,2,\ldots,T, \tag{9}$$

where G_t and R_t are fixed matrices of order $m \times m$ and $m \times g$, respectively, and w_t is a $g \times 1$ vector of state disturbances with zero mean and variance-covariance matrix Q_t. The disturbances in both the measurement and transition equations are taken to be serially uncorrelated, uncorrelated with each other, and uncorrelated with x_0 for all time periods.

A feature of this multivariate state-space formulation is that it subsumes a range of models as special cases and thus provides great flexibility in tailoring models to special circumstances. The problems with the approach relate to model identification and to estimating the state vector x_t and the unknown parameters G_t, R_t, H_t and Q_t. Given values for the parameters, recursive estimation of x_t is possible using the Kalman filter. In essence, this is a two-stage recursive process that involves predicting the state vector for a given time period, and then updating this prediction by incorporating new information for that time period via the measurement equation.

Suppose that $\hat{x}_{t-1}$ is the optimal estimator of x_{t-1} at time t–1, with

$$\hat{x}_{t-1} - x_{t-1} \sim WS(0, P_{t-1}), \tag{10}$$

where WS means in the wide sense. Then the prediction equations are

4. For some examples of state space modelling in economics see Harrison and Stevens (1976), Shumway and Stoffer (1982), Harvey and Todd (1984) and Kitagawa and Gersch (1984).

$$\hat{x}_{t|t-1} = G_t \hat{x}_{t-1} \tag{11}$$

and

$$P_{t|t-1} = G_t P_{t-1} G_t' + R_t Q_t R_t', \quad t = 1,2,\dots,T, \tag{12}$$

while the updating equations are

$$\hat{x}_t = \hat{x}_{t|t-1} + P_{t|t-1} Z_t' J_t^{-1} \left(y_t - Z_t \hat{x}_{t|t-1} \right) \tag{13}$$

and

$$P_t = P_{t|t-1} - P_{t|t-1} Z_t' J_t^{-1} Z_t P_{t|t-1}, \quad t = 1,2,\dots,T, \tag{14}$$

where $J_t = Z_t P_{t|t-1} Z'_t + V_t H_t V_t'$.

This estimation method is much simplified if, as is often the case in practice, the matrices G_t, R_t, H_t and Q_t are taken to be time-invariant. For example, in a case of special interest, the measurement equation of the state-space model may be written as

$$y_t = Z_t \beta_t + u_t, \; t = 1,2,\dots,T, \tag{15}$$

where $V_t = I$ and $H_t = H \forall t$; and the transition equation may be written as

$$\beta_t = G_t \beta_{t-1} + w_t, \; t = 1,2,\dots,T, \tag{16}$$

where $R_t = I$ and $Q_t = Q \forall t$. This is clearly the multivariate regression model with time-varying parameters. If $G_t = I \forall t$ and $Q = 0$, the classical model emerges with stable parameters over time, and the Kalman filter is equivalent to applying recursive OLS to the equations. For the purposes of the case study in the following section, the more general time-varying parameters interpretation is used, though only for the single-equation case where $n = 1$.

The one remaining question concerns the estimation of the parameters required for the use of the Kalman filter. Several maximum likelihood techniques are available, if it is assumed that x_0, $u_1, \dots, u_T$ and $w_1, \dots, w_T$ are jointly normal and uncorrelated (vector) variables. A common likelihood, sometimes called the quasi-conditional likelihood, is the innovations form of Schweppe (1965). As this is highly non-linear in the unknown parameters, the usual procedure is to fix x_0 and develop a set of recursions for the log likelihood function and its first two derivatives; then a Newton-Raphson algorithm may be used successively to update the parameter values until the log likelihood is maximised. A simple method of starting the recursions is to use OLS on the first k observations on y_t and Z_t to obtain estimates of x_0 and P_0, and start the filter at $t = k + 1$. Because this is equivalent to putting $x_1 = x_2$

$= \ldots = x_k$, it is appropriate only if the parameters are known to be stable over the first k time periods, as is assumed when recursive OLS is applied to the classical form of Equation (15). Another approach is based on the expectation-maximisation (EM) algorithm of Dempster *et al.* (1977) as adapted to the state-space model by Shumway and Stoffer (1982). This proceeds by successively maximising the current conditional expectation of the log likelihood function of the complete unobserved data $x_0, u_1, \ldots, u_T, w_1, \ldots, w_T$, given $y_1, y_2, \ldots, y_T$.

In Section IV, the quasi-conditional likelihood is adopted. As the model there is a single equation one, and as a random walk formulation is used for the transition equation ($G_t = I$), maximisation of this likelihood is equivalent to minimising

$$\sum_{t=1}^{T} \left[\frac{\left(y_t - Z_t\beta_{t-1}\right)^2}{\sigma_t^2} + \ln \sigma_t^2 \right], \tag{17}$$

where $\sigma_t^2 = H_t + Z_t(P_{t-1} + Q_t)Z_t'$, recalling the notation of Equations (15) and (16) and noting that for $n = 1$, H_t is a scalar, Z_t is $1 \times m$, and both P_t and Q_t are $m \times m$.

IV CASE STUDY

In the paper by Bond (1990), the concepts of cointegration and error-correction models were used in the estimation of regional employment multipliers for Northern Ireland from quarterly data for the period June 1978 to December 1986. Since parametric stability of the models employed was assumed throughout, this work provides a useful basis for the present case study, the first aim of which is to illustrate the use of the techniques described in Sections II and III. The stability analysis of Bond's findings is extended, however, by means of an additional data set for the earlier period June 1959 to June 1971.

By way of background, the following brief sketch of Bond's study is given. The employment in the region was split into three categories: base or export employment (X) which produces output foı consumption on a wider market; non-base or service employment (S) which is mainly concerned with servicing the local economy; and employment in the autonomous sector (A) which is there to meet national requirements. All industries at the 1980 two-digit standard industrial classification (SIC) level were allocated to one of these groups, with some experimentation being carried out to determine whether construction would be in the non-base or autonomous category; details of the

Table 1(a): *Allocation of SICs to Economic Base Categories*

Group		*SICs*	*Description*	
Base		11,12,14	Coal and Petroleum Products	
		25,26	Chemicals and Man-made Fibres	
		24	Mineral Products	
		32	Mechanical Engineering	
		33,34	Office Machinery & Electrical Engineering	
		21,22,31, 35-38	Miscellaneous Engineering	
		43	Linen & Textiles	
		44,45	Clothing, Footwear, Leather & Fur	
		46	Timber & Furniture	
		47	Paper etc.	
		48,49	Rubber Products & Other Manufacturing	
Autonomous		1,2,3	Agriculture, Forestry & Fishing	
		23	Mining & Quarrying	
		91	Public Administration & Defence	
		92	Sanitary Services	
		93	Education	
		95	Health	
		97	Recreation	
	***	50	Construction (Grouping 1 only)	***
Non-Base		16,17	Gas, Electricity &Water	
		81,82	Banking & Insurance	
		83,84,85	Business Services	
		61-66	Distribution	
		71-77	Transport	
		79	Communications	
		94	Research & Development	
		96	Other Services	
		98	Personnel Services	
	***	50	Construction (Grouping 2 only)	***

Table 1(b): *Allocation of 1968 SICs*

Group		*SICs*	*Description*	
Base		3	Food, Drink and Tobacco	
		7,8,11	Mechanical, Instrument and Electrical Engineering	
		10	Ship Building	
		13	Textiles	
		15	Clothing and Footwear	
		5,6,12,14,16-19	Rest of Manufacturing	
Autonomous		1	Agriculture, Forestry and Fishing	
		2	Mining and Quarrying	
	***	20	Construction (Grouping 1 only)	***
		25	Professional and Scientific	
		27	Public Administration and Defence	
Non-base		21	Gas, Electricity and Water	
		22	Transport and Communications	
		23	Distributive Trades	
		24	Insurance, Banking and Finance	
		26	Miscellaneous Services	
	***	20	Construction (Grouping 2 only)	***

allocations are given in Table 1(a). The three derived data series, their differences, as well as the total employment series, were subjected to Dickey-Fuller (DF), augmented DF (ADF) and cointegrating regression Durbin-Watson (CRDW) unit root tests. The results of these tests are given in Table 2(a); as can be seen, they suggest that the levels series are probably integrated of order one, I(1), while the differences are I(0). Several specifications for the static cointegrating regression of S on X and A were estimated, with the residuals being tested for a unit root in each case; details of the specifications, together with the regression coefficient estimates and the unit root test results are given in Table 3(a). Overall from these results it would appear that the specifications which exclude the seasonal dummies are most likely to be cointegrated, CI(1,1).

Table 2(a): *Tests for Unit Roots in Basic Series*

Model	*Series*	*CRDW*	*DF*	*ADF*
Both:	Total	0.07	–0.32	–2.49
	ΔTotal	1.13	–4.23	–1.97
	Base	0.02	–1.02	–2.30
	ΔBase	1.88	–3.65	–1.65
Grouping 1:	Non-base	0.27	–4.21	–2.59
	ΔNon-base	1.88	–5.35	–2.40
	Autonomous	0.39	–2.40	–2.16
	ΔAutonomous	0.85	–5.16	–2.41
Grouping 2:	Non-base	0.28	–1.49	–2.54
	ΔNon-base	1.61	–4.77	–2.55
	Autonomous	0.07	–3.43	–0.63
	ΔAutonomous	1.26	–6.72	–2.00

Table 2(b): *Tests for Unit Roots in Logs of Basic Series*

Model	*Series*	*CRDW*	*DF*	*ADF*
Both:	Base	0.34	–2.06	–3.10
	Δ^4Base	2.24	–7.94	–3.10
Grouping 1:	Non-base	0.61	–3.39	–2.05
	Δ^4Non-base	2.72	–9.91	–3.52
	Autonomous	0.02	–1.01	–1.06
	Δ^4Autonomous	2.42	–8.63	–2.94
Grouping 2:	Non-base	0.32	–2.47	–1.90
	Δ^4Non-base	2.50	–8.77	–2.97
	Autonomous	0.43	–0.40	–0.00
	Δ^4Autonomous	na	na	na

na: not available due to singularities in estimation procedure.

Table 3(a): *Cointegrating Regression Results*

Equation Specifications

Specification	*Explanatory Variables*
1	Constant, Base Employment Autonomous Employment, Seasonal Dummies
2	Constant, Base Employment, Autonomous Employment
3	Base Employment, Autonomous Employment, Seasonal Dummies
4	Base Employment, Autonomous Employment

Specification	*Base*	*Autonomous*	*CRDW*	*DF*	*ADF*
Grouping 1: 1	–0.16	0.72	0.34	–2.31	–1.50
2	–0.16	0.75	0.52	–2.79	–2.03
3	–0.17	0.87	0.36	–2.18	–1.42
4	–0.17	0.87	0.54	–2.65	–1.96
Grouping 2: 1	0.40	0.87	0.40	–1.90	–2.13
2	0.40	0.90	0.56	–2.41	–2.76
3	0.39	0.82	0.39	–1.91	–2.17
4	0.39	0.83	0.55	–2.45	–2.85

Table 3(b): *Basic Levels Results (Constant term included) 1959-1971*

Grouping	*Base*	*Autonomous*	*CRDW*	*DF*	*ADF*
1	–0.18	0.07	0.75	–3.60	–1.71
2	–0.30	0.20	0.69	–2.93	–1.03

Bond's investigation of cointegration was extended by Bond and Harrison (1992) by the application of the Johansen (1988) procedure to both the levels and the logarithms of the variables, using a three period lag structure for the required VAR model. The results of this test, given in Table 4(a), lend support to Bond's findings by suggesting that the variables in his formulations for the second industry grouping in particular, as well as their logarithms, are co-integrated, and also that there is a single cointegrating vector in each case. A dynamic ECM was estimated by Bond for each of his various specifications, using the residuals from the regressions referred to in Table 3(a) as the observations on the error-correction term. Finally, though the adopted general-to-specific modelling strategy was employed independently for each specification, the derived parsimonious representation of the underlying data generation process was the same for all. The coefficient estimates and associated t statistics are given for each specification in Table 5(a).

Table 4(a): *Johansen Procedure Results 1978-1986*

Specification		H_0: *Not more than 'n' Cointegrating Regressions* 0	1	2	*Coefficients* Non-base	Autonomous
Grouping 1	levels	–27.4	–10.1	–0.6	0.12	–0.69
	logs	–26.4	–8.3	–0.6	0.14	–1.0
Grouping 2	levels	–61.7	–19.0	–3.3	0.08	1.3
	logs	–62.0	–20.1	–3.7	0.06	1.2

Table 4(b): *Results of Johansen Procedure 1959-1971*

Specification		H_0: *Not more than 'n' Cointegrating Regressions* 0	1	2	*Coefficients* Non-base	Autonomous
Grouping 1	levels	–48.1	–26.2	–9.5	–0.27	–0.47
	logs	–50.2	–28.3	–10.9	–0.59	–0.41
Grouping 2	levels	–40.2	–17.0	–1.9	–0.01	–1.2
	logs	–42.5	–19.1	–3.9	–0.06	–0.62

Table 5(a): *ECM Results*

Specification	$\Delta^4 X_t$	$\Delta^4 A_t$	$\Delta^4 S_{t-1}$	EC_{t-4}	R^2	*DW*
Grouping 1:						
1	0.12	0.37	0.50	–0.15	0.90	1.76
	(2.49)	(4.22)	(5.25)	(–2.93)		
2	–0.09	0.34	0.55	–0.33	0.89	1.96
	(–1.96)	(3.76)	(6.02)	(–2.36)		
3	–0.11	0.40	0.50	–0.40	0.89	1.72
	(–2.31)	(4.07)	(4.97)	(–2.66)		
4	–0.09	0.36	0.55	–0.30	0.89	1.93
	(–1.87)	(3.70)	(5.74)	(–2.19)		
Grouping 2:						
1	0.15	0.67	0.49	–0.72	0.85	1.09
	(3.96)	(2.94)	(4.93)	(–4.30)		
2	0.14	0.61	0.52	–0.64	0.85	1.26
	(3.61)	(2.59)	(5.10)	(–3.80)		
3	0.15	0.67	0.48	–0.73	0.87	1.10
	(3.99)	(2.98)	(4.91)	(–4.39)		
4	0.14	0.61	0.51	–0.66	0.85	1.29
	(3.65)	(2.63)	(5.08)	(–3.92)		

Table 5(b): *Preliminary ECM Results 1959-1971*

	$\Delta^4 X_t$	$\Delta^4 A_t$	$\Delta^4 S_{t-1}$	EC_{t-4}	R^2	DW
1	–0.003	0.15	0.22	–0.68	0.32	1.07
	(–0.03)	(1.68)	(1.20)	(–4.35)		
2	–0.05	0.11	0.45	–0.31	0.35	1.58
	(–0.51)	(1.06)	(3.22)	(–2.43)		

For the purposes of the present study, a logarithmic specification for the second industry grouping is chosen, namely,

$$\log S_t = a_0 + a_1 \log X_t + a_2 \log A_t + u_{1t}$$
$$\Delta^4 \log S_t = b_0 + b_1 \Delta^4 \log X_t + b_2 \Delta^4 \log A_t + b_3 \Delta^4 \log S_t + b_4 \hat{u}_{1(t-4)} + u_{2t}, \quad (18)$$

where Δ^4 is the four period difference (i.e. $\Delta^4 X_t = X_t - X_{t-4}$) and $\hat{u}_{1(t-4)}$ is the error-correction term. The estimates of a_1 and a_2 from (18) are the estimates of the long-run elasticities, and the estimates of b_1 and b_2 are short-run elasticities. The results in Table 3(a) suggest that the long-term multipliers are likely to be about 1.4 for base employment and 1.8 for autonomous employment, respectively, while those in Table 5(a) indicate that the corresponding short-term multipliers are about 1.14 and 1.6, respectively. The elasticity estimates from (18) are given in Table 6(a) and values for the various regional employment multipliers are easily derived from these. The investigation of stability commenced by replicating Bond's analysis and re-estimating model (18) using quarterly employment data for the earlier period mentioned above.[5] Unfortunately, whilst it is internally consistent, this further series is based, not on the 1980, but on the 1968 SIC. Therefore, while the allocation of industries to the base, service and autonomous sectors is similar to that adopted for the 1978-86 sample, unavoidable differences occur due to the different structure of the 1968 SIC; details of the allocations in this case are given in Table 1(b). It follows that the results, too, are not directly comparable with those for the more recent period, so care is needed in their interpretation. These results include the values of unit root test statistics in Table 2(b); the cointegrating regression estimates in Table 3(b); the values of the Johansen test statistics in Table 4(b); the parameter estimates for the dynamic ECM models in Table 5(b); and the estimated coefficients of model (18) in Table 6(b).

The prime feature of these new results is their disappointing quality compared with the original ones. The results of the unit root tests on the basic series and their differences are similar, but the results for the static regres-

5. The bulk of the calculations were done using PC-GIVE. However, MICROFIT was used for cusum of squares calculations, and RATS for the maximum likelihood computations used for the Kalman filter estimation.

Table 6(a): *Coefficient Estimates for Equation (18), 1978-86*

Static Equation	a_0		a_1		a_2	R^2	*CRDW*
	–0.14		0.25		0.87	0.63	0.60
	(–0.19)		(6.21)		(3.85)		
Dynamic Equation	b_0	b_1	b_2	b_3	b_4	R^2	*DW*
	–0.003	0.12	0.77	0.50	–0.64	0.85	1.31
	(–0.82)	(2.46)	(2.49)	(4.79)	(–3.78)		

Table 6(b): *Coefficient Estimates for Equation (18), 1959-71*

Static Equation	a_0		a_1		a_2	R^2	*CRDW*
	5.80		–0.30		0.20	0.38	0.69
	(7.98)		(–2.12))		(5.02)		
Dynamic Equation	b_0	b_1	b_2	b_3	b_4	R^2	*DW*
	0.001	–0.05	0.09	0.45	–0.31	0.35	1.56
	(0.12)	(–0.51)	(0.38)	(2.92)	(–2.36)		

t-values in parentheses.

sions indicate poor fits (R^2 values of 0.14 and 0.37 for the two equations) and very different numerical values for the coefficients (including some negative values) and their associated long-run multipliers. The Johansen test also suggests negative coefficients, and the possibility of two cointegrating regressions for the industry groupings, while the dynamic models have low coefficients of determination and very few significant t statistics associated with the parameter estimates.

The preliminary indications were therefore that there appear to be significant differences in the structure of the models for the 1959-1971 and 1978-1986 periods, and that within the former period the data pose particular problems for estimation. On reflection, it may not be surprising that a parametrically stable model does not fit the data well in the earlier period; for the 1960s was a period of considerable change, with a strong regional policy being pursued from about 1963 onwards, relaxation of the "Imperial Contribution" leading to an influx of additional public monies in the pursuit of proportionality, and the increase in general political instability. Thus it was decided to subject the two sample periods to more formal checks for structural change.

The Chow F test and MH cusum of squares test were applied to both the static and dynamic models. The results for the 1-step ahead variant of the Chow test are given in Figure 1, those for the N-step back variant are given in Figure 2, and those for the MH test are plotted in Figure 3. The BLS test was applied only to the dynamic ECM formulations for which it is intended,

Figure 1: *Chow Test 1978-1986*

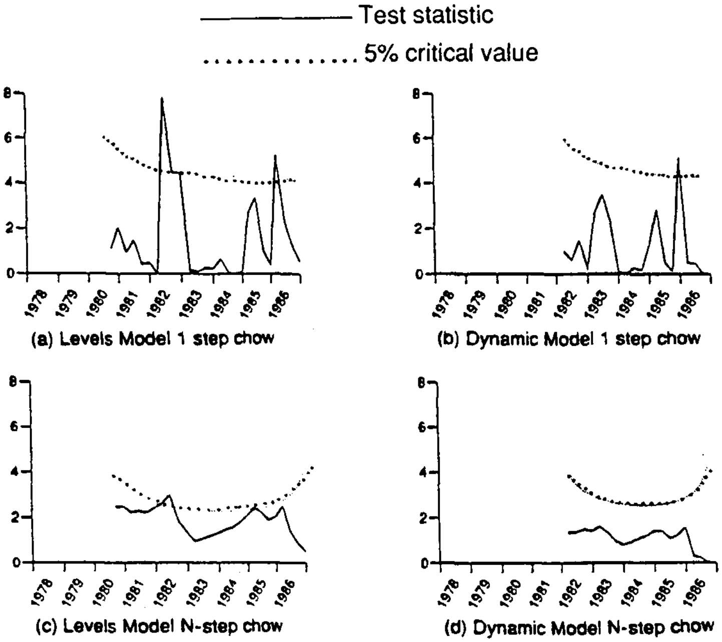


Figure 2: *Chow Test 1959-1972*

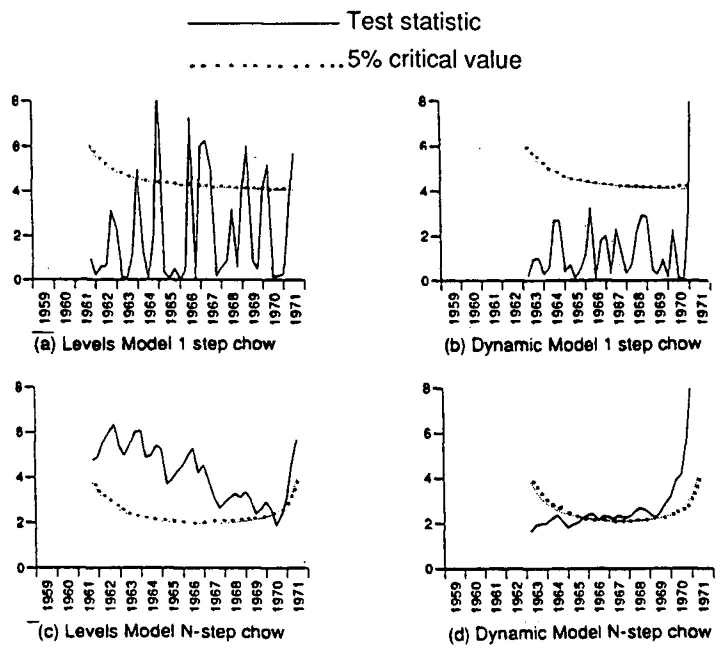


Figure 3: *MH CUSUM Test*

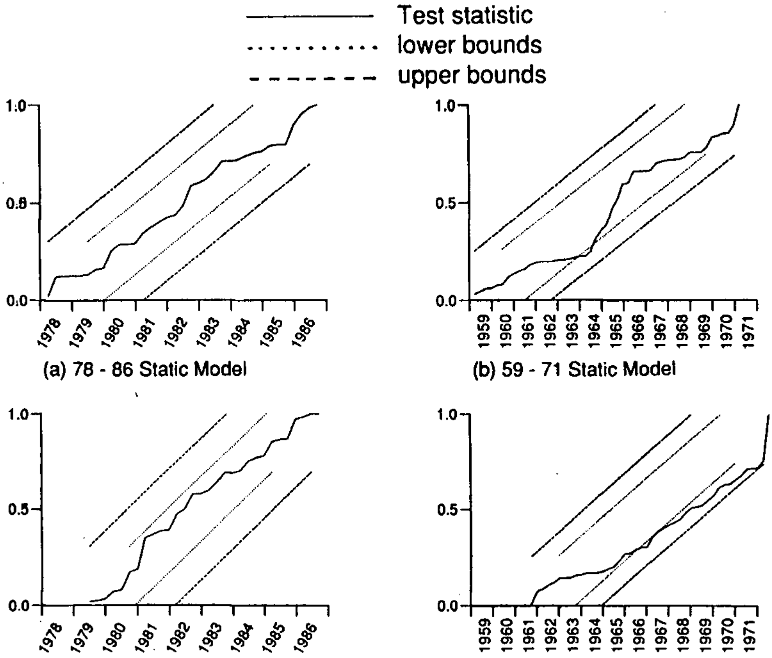
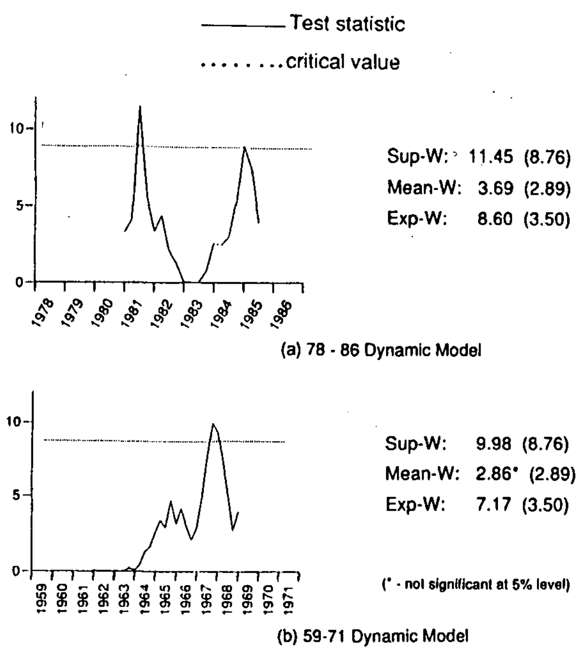


Figure 4: *BLS-F(k) Test*



all three variants (Sup-W, Mean-W and Exp-W) being used. Figure 4 presents the plots of the sequences of Wald F statistics for the two sample cases, full details of the corresponding range of BLS test statistics and the required critical values.[6] The nominal 5 per cent significance level was used throughout for all test procedures. Although the MH test is insignificant for both the static and dynamic models using the 1978-1986 data, and the N-step back Chow test for the dynamic model using the same sample, there is in general very strong evidence of instability in both periods. The evidence for the 1959-1971 period seems overwhelming, with all tests indicating shifts round about 1964-1965 and/or 1968-1969. The weight of evidence for the 1978-1986 period suggests breaks round about 1982 and possibly in 1986.

In the light of these findings it was decided to estimate the static equation in (18) as a time-varying parameters model for each of the two periods, using the Kalman filter as described in Section III. However, the quasi-conditional likelihood (17) was maximised using a grid search, rather than the more standard technique, because the sensitivity of the Johansen test to slight variations in the starting point suggested that the first few periods also need to be modelled. Utilising the OLS estimates of the appropriate covariance matrices from the full sample as the starting values for P_t and H_t in each case, and the symmetric matrix

$$Q_t = Q = \begin{bmatrix} 0 & & \\ 0 & 0.5 & \\ 0 & -0.25 & 0.5 \end{bmatrix}, \tag{19}$$

the search grid used the range of starting values $a_0 = 0$, $a_1 = 0$ to 1.5, and $a_2 =$ 0 to 1.5. The assumption of positive elasticities seems quite reasonable, but could be relaxed if required. To ease the computational burden, estimation was only undertaken for $G_t = I \forall t$, which implies that the a_i follow a random walk.

Tables 7(a) and 7(b) and Figures 5 and 6 provide the details of the resulting Kalman filter estimates. The mean values of a_1 and a_2 for the 1978-86 sample are 0.22 and 0.81, respectively, which are similar to the values obtained in the original static regression; the values of the CRDW and the DF statistics in this case are both better than in the static regression, though that of the ADF statistic is not. For the 1959-1971 period, the mean values of

6. When examining the results it should be borne in mind that both the Chow tests and the BLS tests require a number of sample observations for the purpose of initialization of the calculations. Specifically, both variants of the Chow test use the first 9 data points in the static model and the first 11 in the dynamic model; the BLS test, with a trimming parameter of 0.15, requires the first and last 7 points for the 1959-71 sample and the first and last 5 points for the 1978-1986 sample.

a_1 and a_2 are 0.85 and 0.17, respectively, which are radically different from the estimates obtained in the static regression. Moreover, there is noticeably more variation in the estimates of the time-varying parameters for this period than there is in the time-varying estimates for the 1978-1986 period.

Table 7(a): *Summary of Varying Parameter Specification (Grouping 2) 1978.2-1986.4*

	Base	*Autonomous*	*CRDW*	*DF*	*ADF*
mean	0.22	0.81	2.18	–6.3	–2.54
standard dev.	0.0007	0.002			
s.d. mean	0.0001	0.0003			

Table 7(b): *Summary of Varying Parameter Specification (Grouping 2) 1959-1971*

	Base	*Autonomous*	*CRDW*	*DF*	*ADF*
mean	0.84	0.17	2.31	–8.1	–2.65
standard dev.	0.005	0.005			
s.d. mean	0.0007	0.0007			

Figure 5: *Beta Coeff. Base Industries*

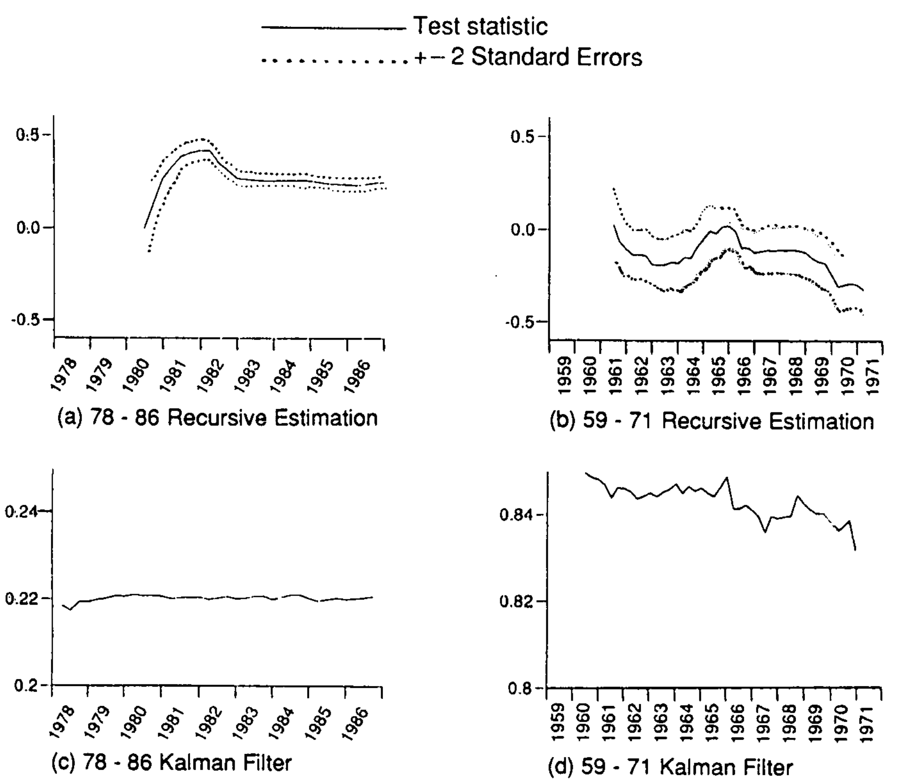
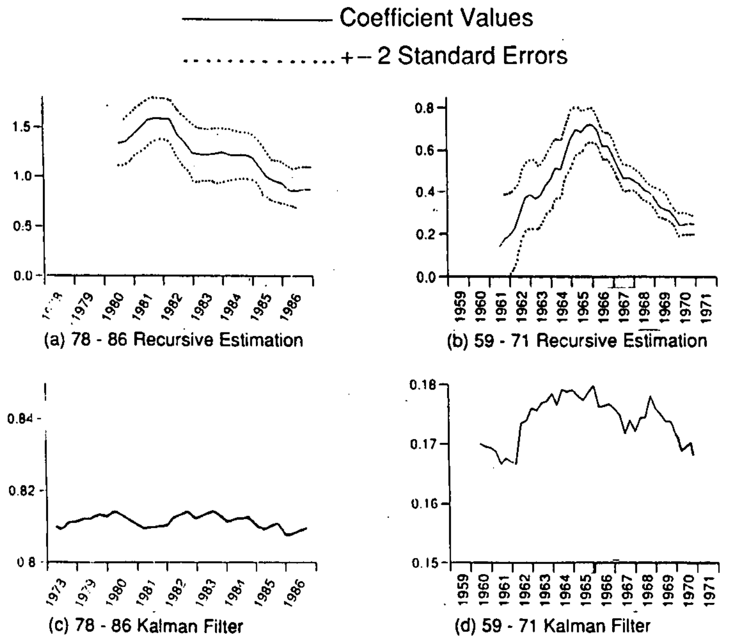


Figure 6: *Beta Coeff. Autonomous Industries*



Finally, recursive OLS estimates of the parameters in both the static and dynamic model were computed for the two sample periods. Full details are not reported, but the recursive estimates of the coefficients in the static model are shown in Figures 5 and 6 for comparison with the Kalman filter results. Once again the findings betray instability round about 1965 in the earlier period, and round about 1982 and 1985 in the later period. It is noteworthy that the recursive estimates tend to mimic the behaviour of the Kalman filter estimates, though not surprisingly their range of variation is very much greater.

V DISCUSSION

Despite the apparently satisfactory results of Bond (1990), the study reported in Section IV provides much evidence of parameter instability in both of the periods examined. Interestingly, the "benchmark" BLS test tends to perform relatively poorly in the earlier period when instability appears to have been more problematical. On the other hand, the Engle-Granger procedure, and the ECM to which the BLS test is designed to be applied, yields very poor results for the earlier period. These findings seem to raise a number of methodological questions which are discussed briefly in this section.

First, the incidence of instability would appear to create a number of serious difficulties for the Engle-Granger procedure. Checking for unit roots is impaired in the sense that there will be an increased likelihood of non-rejection when using standard tests, although as was mentioned previously, Perron (1990) has proposed a unit root test which allows for the possibility of structural shifts. Difficulties arise for the rapid convergence results of Stock (1987) which underpin the two-step procedure, and for the Johansen test. There are problems surrounding the parameter estimates, not least of which is the interpretation of the coefficients from an unstable cointegrating regression as relating to a long-run equilibrium situation. The mechanical use of the Engle-Granger approach would appear to be alright if the parameters of the model are stable, but otherwise it would seem to be fraught with a variety of serious pitfalls. There is therefore a clear need for tests against instability to be undertaken when this approach is employed.

Unfortunately, there may also be problems in implementing stability tests. Attention has already been drawn to the use of the Chow and MH tests when variables are I(1). There may also be problems with the BLS test when variables are I(1) but not cointegrated, which may explain the relatively poor performance of this test in the context of an unsatisfactory ECM for the 1959-1971 period, as mentioned above. In such a case it might be worthwhile exploring the use of the BLS procedure in the context of a VAR model rather than an ECM.

However, why assume either model and test for stability within it? The question arises: why not begin by adopting a more general variable parameter model and see if stability is an adequate hypothesis before proceeding to use, say, the Engle-Granger method? This would appear to be entirely in the spirit of the popular general-to-specific methodology. While it may sound reasonable to many practitioners, there are further difficulties with this approach. One concerns the estimation problem. There is no general varying coefficient maximum likelihood estimator for dynamic models. The Kalman filter can be used, as illustrated in this study, but even that is subject to restrictions, such as those involved in the need to specify the form of the transition equation. Moreover, having obtained estimates via the Kalman filter, it is not clear how stability may be tested formally; no procedure appears to be available for this.

Another question arises, namely, should researchers be advised to do both; i.e. estimate dynamic ECMs and test for stability, and examine the results of estimating time-varying parameter models? The whole exercise might proceed in the manner of exploratory data analysis (EDA), a technique advocated strongly by many statisticians, with full advantage being taken of available computing power to provide more information, much of which may be presented in the form of plots and diagrams. Although it may elicit the charge of data-mining, the inclination of the present authors is to favour this type of analysis in the hope that it would produce a general consensus of results. Incidentally, such an approach might imply that consideration be given to the use of neural network theory in economic modelling, since this is intended to simulate, albeit crudely, the sort of trial and error processes envisaged.[7]

VI CONCLUSION

This paper has drawn attention to the current concern about testing for instability in time series econometrics. The new test for structural breaks in dynamic error-correction models due to Bai, Lumsdaine and Stock has been described, as has the Kalman filter technique for estimating time-varying parameter models. These procedures, and several others, have been applied to a model developed to estimate employment multipliers for Northern Ireland, using two samples of quarterly data. Though the study was essentially illustrative in nature, highlighing the need for caution when applying the concepts of cointegration, the findings of extensive instability and radically different elasticity estimates between the two periods may be of some interest in themselves and could provide the basis for further research. More important, perhaps, is the fact that the case study gives rise to a number of general methodological questions concerning the Engle-Granger approach to modelling dynamic economic relationships. These issues have been discussed briefly, and a suggestion has been made to make greater use of the exploratory approach to data analysis in economic modelling.

Three main conclusions emerge from the paper. First, applied economists should consider using more general tests for unit roots, such as that of Perron, which allow for the possibility of structural breaks. Second, there is a need for more robust estimation and testing procedures in cointegrating regression. It may be that the new canonical cointegrating regression technique proposed by Park (1992) may shortly provide a means of coping better with cointegrated models with structural instability. Third, economists should perhaps reconsider what some would see as an over-reliance on parametric techniques, fostered by the ready availability of powerful computer software, and embrace more the spirit of exploratory data analysis and consider examining the use of recent developments in the field of neural network theory.

7. For an account of neural network theory and its relevance to econometrics see Kuan and White (1991).

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