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Speculative Rational Expectations' Exchange Rate Dynamics with Risk Averse Wealth Owners and a Monetary Authority Averse to Exchange Rate Variability

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I INTRODUCTION

In the analysis to follow an important aspect of speculative exchange rate movements is examined in isolation from macroeconomic feedback effects. The over-riding objective here is to investigate the effect of adherence to an interest rate rule by the monetary authority when international speculators behave in a way such that there is full rational expectations with respect to both the mean and variance of the future exchange rate.

The model is most relevant when there is free capital mobility across international capital markets; a small domestic capital market relative to competing foreign capital markets and when the domestic monetary authority uses the interest rate as a means of adhering to a dirty fixed exchange rate mechanism. A cautious monetary authority raises the domestic interest rate above the foreign rate so as to allow a risk premium for the expected variance around the expected future exchange rate. The effect of adhering to an interest rate rule of this sort (when speculators are well informed about the rule), results in a less variable but weaker exchange rate the more cautious the monetary authority tries to be.

The weakness in the exchange rate occurs because although the monetary authority is more cautious, this in itself reduces the expected exchange rate variability to such an extent that a lower interest rate premium is expected.

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This in turn reduces the demand for domestic treasury bills at any given exchange rate. Since the supply of treasury bills is assumed constant there is a jump depreciation in the expected equilibrium exchange rate.

The origin of the present analysis is an elegant paper by Begg (1984), where there is presented a rational expectations model of equilibrium bond pricing in which risk averse lenders make portfolio choices between short and long assets so that expectations are self fulfilling in both the mean and variance of future bond prices. The introduction of the expected variance into the analysis results in a non-linear difference equation for the evolution of bond prices (which when appropriately transformed is the logistic equation).

Begg showed that for appropriate levels of uncertainty and of risk aversion the equilibrium bond price would be locally unstable so that a unique rational expectation equilibrium bond price would exist. Begg associated financial panic with parameter values for which the equilibrium bond price was stable since this would imply that there existed an infinite amount of possible paths to equilibrium. Van der Ploeg (1986), showed that the same model could give rise to unique non-convergent forward looking rational expectations paths in the form of limit cycles or erratic trajectories.

Specifically, the bond pricing model considers a portfolio choice over two assets one of which has a return with certainty by way of a capital gain on a one-period treasury bill. The other asset is a conventional bond the capital gain on which is uncertain. This formulation allows all of the uncertainty of the expected return on the portfolio to be due to the uncertain capital gain on the bond. In the exchange rate model presented here it is assumed that foreign wealth holders make a portfolio choice between the one-period treasury bills of two countries which have certain capital gains in own-currency terms. This makes it possible to attribute all of the uncertainty of the expected return on the international portfolio to the uncertain future exchange rate.

II ANALYSIS

It is assumed that the small country monetary authority sets its one-period interest rate on treasury bills above the foreign rate by a fixed proportion of the expected variance of the end of period exchange rate.

$$\text{Thus: } r_t = r^* + \gamma \hat{\sigma}^2_{e,t+1}.$$

(A circumflex hereafter represents the expectations operator.)

It is assumed also that the future exchange rate has some random fluctuation around its expected future value.

$$\text{Thus: } e_{t+j} = \hat{e}_{t+j} + U_{e,t+j}.$$

$$\text{Where } U_{e,t+j} \text{ is iid}(0,\sigma_c^2).$$

(The expected variance of the exchange rate will be endogenously determined.)

We consider the portfolio choice of a foreign wealth holder between a return with certainty on their own country's treasury bill and an uncertain return on the treasury bill issued by the small-country monetary authority. Valued in foreign currency terms the return on such a portfolio would actually be:

$$i_t = a_t(e_{t+1} - e_t) + a_t(r^* + \gamma\hat{\sigma}^2_{e,t+1})e_{t+1} + (W - a_t e_t)r^*.$$

Here the exchange rate is the foreign currency cost of domestic currency. a_t is the quantity demanded of small-country treasury bills which are valued at a unitary price. The redemption value of the treasury bills is set at the beginning and this determines the interest rate for the period. W is wealth assumed constant over time.

The first term in the equation gives the capital gain/loss on the portfolio. The second term is the end of period value of the interest return on a_t. The third term is the return on remaining wealth invested in foreign treasury bills.

The expected return on the portfolio is:

$$\hat{i}_t = a_t(\hat{e}_{t+1} - e_t) + a_t(r^* + \gamma\hat{\sigma}^2_{e,t+1})\hat{e}_{t+1} + (W - a_t e_t)r^*.$$

We can now proceed to calculate the variance of the return as follows:

$$i_t - \hat{i}_t = a_t(1 + r^* + \gamma\hat{\sigma}^2_{e,t+1})(e_{t+1} - \hat{e}_{t+1}).$$

Squaring and taking expectations we obtain:

$$\hat{\sigma}^2_{i,t} = a_t^2(1 + r^* + \gamma\hat{\sigma}^2_{e,t+1})^2\hat{\sigma}^2_{e,t+1}.$$

Now suppose foreign wealth holders have the following expected utility function with respect to the return and variance associated with a given portfolio.

$$\hat{U} = \hat{i}_t - (1/2)b\hat{\sigma}^2_{i,t}.$$

(The parameter b is a constant coefficient of absolute risk aversion.)

The portfolio choice problem of foreign wealth holders is therefore to choose the amount of small-country treasury bills consistent with maximising expected utility. Formally this is:

$$\text{Max}(\text{wrt}\, a_t)\ a_t(\hat{e}_{t+1} - e_t) + a_t(r^* + \gamma\hat{\sigma}^2_{e,t+1})\hat{e}_{t+1} + (W - a_t e_t)r^*$$
$$- (1/2)ba_t^2(1 + r^* + \gamma\hat{\sigma}^2_{e,t+1})^2\hat{\sigma}^2_{e,t+1}.$$

The first order condition gives:

$$(\hat{e}_{t+1} - e_t) + (r^* + \gamma\hat{\sigma}^2_{e,t+1})\hat{e}_{t+1} - e_t r^* - ba_t(1 + r^* + \gamma\hat{\sigma}^2_{e,t+1})^2\hat{\sigma}^2_{e,t+1} = 0.$$

Which on rearranging gives:

$$e_t = \frac{(1 + r^* + \gamma\hat{\sigma}^2_{e,t+1})}{(1 + r^*)}(\hat{e}_{t+1} - ba_t(1 + r^* + \gamma\hat{\sigma}^2_{e,t+1})\,\hat{\sigma}^2_{e,t+1}).$$

This equation describes the demand, a_t, for the small-country treasury bills. Alternatively, if we assume that the monetary authority keeps the supply of treasury bills constant, this equation then describes the equilibrium exchange rate as a function of both the expected future rate and the expected future variance.

We can now proceed to derive the expected future variance. Recall that in any given period the present period's interest rate on treasury bills is known since it is simply the difference between the issue and redemption prices. Assuming that the monetary authority follows its interest rate rule with some random fluctuation and that foreign interest rates remain constant over time, we can conclude that:

$$r_{t+1} = r^* + \gamma\hat{\sigma}^2_{e,t+2} + Z_{t+1}.$$

Where exogenously determined Z_{t+j} is iid $(0,\sigma_z^2)$.

Agents in time t will, therefore, hold the following expectation for e_{t+1}:

$$\hat{e}_{t+1} = \frac{(1 + r^* + \gamma\hat{\sigma}^2_{e,t+2})}{(1 + r^*)}(\hat{e}_{t+2} - ba_t(1 + r^* + \gamma\hat{\sigma}^2_{e,t+2})\hat{\sigma}^2_{e,t+2}).$$

However, actual e_{t+1} will be:

$$e_{t+1} = \frac{(1 + r^* + \gamma\hat{\sigma}^2_{e,t+2} + Z_{t+1})}{(1 + r^*)}(\hat{e}_{t+2} - ba_t(1 + r^* + \gamma\hat{\sigma}^2_{e,t+2})\hat{\sigma}^2_{e,t+2}).$$

Thus subtracting we obtain:

$$e_{t+1} - \hat{e}_{t+1} = Z_{t+1}(\hat{e}_{t+2} - ba(1 + r^* + \gamma\hat{\sigma}^2_{e,t+2})\hat{\sigma}^2_{e,t+2})$$

or

$$e_{t+1} - \hat{e}_{t+1} = Z_{t+1}\left[\frac{1 + r^*}{(1 + r^* + \gamma\hat{\sigma}^2_{e,t+2})}\right]\hat{e}_{t+1}.$$

Squaring and taking expectations gives:

$$\hat{\sigma}^2_{e,t+1} = \sigma^2_z \left[\frac{(1 + r^*)^2}{(1 + r^* + \gamma\hat{\sigma}^2_{e,t+2})^2}\right](\hat{e}_{t+1})^2.$$

In this equation the time label on the information set is understood to be period t. The equations for future periods may be based upon the information set of any previous time period. This is possible because the driving variable Z_{t+j} is iid and agents learn nothing from the realised disturbance which can help them revise beliefs about the future. We may therefore write down the two interacting equations of the system as follows:

$$\hat{e}_{t+j} = \frac{(1 + r^* + \gamma\hat{\sigma}^2_{e,t+j+1})}{(1 + r^*)}(\hat{e}_{t+j+1} - ba_t(1 + r^* + \gamma\hat{\sigma}^2_{e,t+j+1})\hat{\sigma}^2_{e,t+j+1})$$

and

$$\hat{\sigma}^2_{e,t+j} = \sigma^2_z \left[\frac{(1 + r^*)^2}{(1 + r^* + \gamma\hat{\sigma}^2_{e,t+j+1})^2}\right](\hat{e}_{t+j})^2.$$

In equilibrium:

$$\hat{e}^*_t = \frac{ba(1 + r^* + \gamma\hat{\sigma}^2_e)^2}{\gamma}.$$

Substituting this into the equation for the equilibrium variance gives a quadratic of the form:

$$-\sigma_z^2(1+r^*)^2b^2a^2(\hat{\sigma}_e^2)^2 + \left(1 - \left[\frac{2\sigma_z^2(1+r^*)^3b^2a^2}{\gamma}\right]\right)\hat{\sigma}_e^2$$

$$-\frac{\sigma_z^2(1+r^*)^4b^2a^2}{\gamma^2} = 0.$$

By the implicit function theorem we obtain:

$$\frac{\delta\hat{\sigma}_e^2}{\delta\gamma} = \left(\frac{\dfrac{-2\sigma_z^2(1+r^*)^3b^2a^2}{\gamma^2}\left[\hat{\sigma}_e^2 + \dfrac{(1+r^*)}{\gamma}\right]}{1 - 2\sigma_z^2(1+r^*)^2b^2a^2\left[\hat{\sigma}_e^2 + \dfrac{(1+r^*)}{\gamma}\right]}\right).$$

It will be shown below that the sign of the denominator will be positive (and the derivative therefore negative), for most relevant solutions to the qudratic above. To see this notice that to achieve non-complex roots in the variance quadratic, the following condition must hold:

$$1 > \frac{4\sigma_z^2(1+r^*)^3b^2a^2}{\gamma}.$$

Rewriting the denominator of the derivative as

$$1 - \frac{4\sigma_z^2(1+r^*)^3b^2a^2}{\gamma}\left[\frac{\gamma\hat{\sigma}_e^2}{2(1+r^*)} + \frac{1}{2}\right]$$

then, if

$$\gamma\hat{\sigma}_e^2 \leqslant (1+r^*)$$

and if complex roots are excluded then the entire derivative is unambiguously negative. The left hand side of the inequality is simply the domestic interest rate premium and this is very unlikely to equal or exceed $(1 + r^*)$.

Furthermore, the partial derivative of the expected equilibrium exchange rate with respect to the parameter of caution is:

$$\frac{\delta\hat{e}^*}{\delta\gamma} = ba\left(\frac{-(1+r^*)^2}{\gamma^2} + 2(1+r^*+\gamma\hat{\sigma}_e^2)\left[\frac{\delta\hat{\sigma}_e^2}{\delta\gamma}\right] + (\hat{\sigma}_e^2)^2\right).$$

Here, the middle term within the parentheses is negative if the partial derivative of the equilibrium expected variance with respect to the parameter of caution is negative. This implies that if the absolute value of the last term is less than or equal to that of the first term then the entire derivative is negative. This in fact reduces to the same condition sufficient to ensure that:

$$\frac{\delta \hat{\sigma}_e^2}{\delta \gamma} < 0 \quad \text{namely: } \gamma \hat{\sigma}_e^2 \leqslant (1 + r^*) \Rightarrow \frac{\delta \hat{e}^*}{\delta \gamma} < 0.$$

Finally, to obtain a rational expectations solution for this system we need to have:

$$-1 < \frac{\delta e_{t+j}}{\delta e_{t+j+1}}, \frac{\delta \sigma^2_{e,t+j}}{\delta \sigma^2_{e,t+j+1}} < 0$$

with the derivatives evaluated when the expected exchange rate and its expected variance are at their equilibrium values. As in the bond pricing model it can be shown that for large enough values of the risk aversion parameter, the supply of treasury bills and variance of the driving variable r, the system either gives rise to what Begg termed "financial panic", or to unique but non-convergent solutions of a chaotic nature similar to the Van der Ploeg case.

To conclude, it should be clear that if speculators are interested in the stance of a monetary authority viz. exchange rate stability and if the monetary authority uses the interest rate rule suggested above, then a policy trade-off will exist between equilibrium exchange rate variability and the equilibrium exchange rate level.

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