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The induced anisotropy of Leighton Buzzard sand

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Abstract
The induced anisotropy of Leighton Buzzard sand under generalised stress conditions was investigated using a new, automated hollow cylinder apparatus. Identical test specimens of sand were isotropically consolidated followed by anisotropic consolidation, involving different magnitudes of the three principal stresses and rotation of the major principal stress. Details of the stress paths followed are presented. The induced anisotropy of the test specimens was analysed by exploring the relationships between the deviator strain and the principal and volumetric strains. Analysis of the experimental data indicated that a predominately linear relationship exists between the deviator strain and the principal and volumetric strains for all principal stress combinations and rotation of the major principal stress. The magnitude of the intermediate principal stress was found to have a profound influence on the strain responses. When the intermediate and minor principal stresses were maintained equal, the magnitudes of all strain components were found to be proportional to the rotation of the major principal stress. However, the latter relationship disappeared and the magnitudes of the resultant strains were smaller once the intermediate principal stress increased relative to the minor principal stress.

Introduction
The deformational response of a soil to applied load depends on the magnitude and orientation of the principal stresses. Many laboratory test apparatus are limited in the range of stress states that can be applied to a soil sample. For example, the triaxial apparatus can only subject a test-specimen to axi-symmetric loading conditions. Only the hollow cylinder apparatus (HCA) and some new designs of the directional shear cell allow reliable investigation of the effects of anisotropy. Both apparatus allow independent combinations of the three principal stresses and rotation of the major-to-minor principal stress axis.

The particulate nature of soil leads to anisotropic strength and stiffness characteristics. Anisotropic behaviour can be divided into two categories (Hoque & Tatsuoka, 1998):

1. Inherent anisotropy due to the development of an anisotropic macro or micro fabric after the deposition of particulates through air or water or when compacted.
2. Induced anisotropy developed in a soil due to the application of anisotropic stress changes.

This study investigates the induced anisotropy of Leighton Buzzard sand using a HCA. The effects of rotation of the major-to-minor principal stress axis and relative change in the magnitude of the intermediate principal stress on the deformational response of the samples are investigated.

Overview of hollow cylinder apparatus

The HCA used in the present study was developed at University College Dublin, Ireland (O'Kelly, 2000; Naughton, 2002). The test apparatus subjects a 100mm outer diameter x 71mm inner diameter x 200mm high hollow cylindrical test-specimen to an inner pressure (p_i), an outer pressure (p_o), an axial load (W), and a torque (T), Figure 1a. The test apparatus is close-loop controlled allowing precise regulation of the applied loads and pressures. Figure 1b presents the resultant normal and principal stresses induced in an element of the test-specimen wall. The deformational response of the test-specimen is shown in Figure 1c.

The deformational response of the test-specimen is recorded using both internal and external instrumentation. The internal instrumentation is connected directly or is in close proximity to the test-specimen walls and measures the actual deformation occurring over a central gauge length of the test-specimen. Two proximity transducers measure the inner and outer displacements of the test-specimen walls (u_i and u_o, respectively) at sample mid height, while two modified double-axis Imperial College-type inclinometers record the axial normal (w) and circumferential shear (θ) deformations over the middle third of the test-specimen. A single-axis inclinometer is also used to record the circumferential shear deformation.

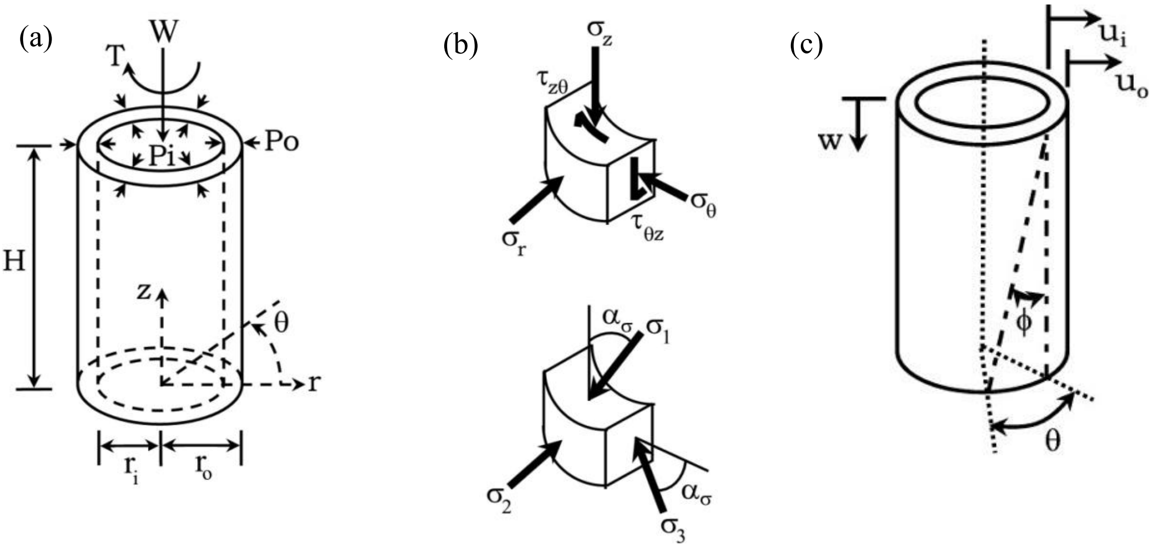

Figure 1. (a) System of pressures, axial and torsional loads applied to the test-specimen. (b) Resultant normal and principal stresses acting on an element of the test-specimen wall. (c) Deformational response of the test-specimen.

The external instrumentation determines the axial normal and circumferential shear deformations from the movement of the loading mechanisms which apply the axial load and the torque to the test-specimen. Displacements of the inner and outer test-specimen walls are determined from measured volume changes of the test-specimen itself and the inner bore cavity.

The automated test apparatus is capable of regulating the induced stresses in the test-specimen to 0.5 kPa during sample loading under either drained or undrained test conditions. The deformational response of the test-specimen is recorded using the internal and external instrumentation to a resolution of $5.3x10^{-5}$ strain and $6.3x10^{-5}$ strain, or better, respectively. A more detailed description of the HCA is given in O'Kelly & Naughton (2003). The control programmes automatically compensate for membrane penetration effects using a sample specific non-destructive method developed by Sivathayalan & Vaid (1998) and for membrane restraint effects using corrections developed by Tatsuoka *et al.* (1986).

Investigation of anisotropy

This study examined the induced anisotropy of Leighton Buzzard sand. Identical test-specimens were anisotropically consolidated by increasing the effective major-to-minor principal stress ratio, R'; reorientation of the major-to-minor principal stress axes, α_σ; and changing the magnitude of the intermediate principal stress relative to the magnitude of the major and minor principal stresses. The relative magnitude of the intermediate principal stress was quantified in terms of the intermediate principal stress parameter, b. The b parameter has a range between 0 and 1. With b = 0, the intermediate and minor principal stresses are equal, while b = 1 results in the intermediate and major principal stress being equal.

Properties of the sand

Fine-to-medium, white Leighton Buzzard sand was examined in the present study. Some physical properties of the sand are summarised in Table 1.

Table 1. Properties of the Leighton Buzzard sand examined in this study.

Property	Coefficient of uniformity	Coefficient of curvature	Specific gravity	Maximum void ratio	Minimum void ratio
Value	1.32	0.96	2.64	0.77	0.50

Sample preparation method

Hollow cylindrical test-specimens were formed using a wet-pluviation technique developed by O'Kelly (2000). Test-specimens are formed between inner and outer moulds using water-pluviation. Tapping the sides of the moulds causes the test-specimen to compact. Saturated test-specimens in very loose to loose states were formed in a single layer by depositing saturated sand through water. Pluviation has the added benefit that it replicates the sedimentation process and hence the fabric of many natural sand deposits. The preparation method provides a convenient means of studying the anisotropy of sand in the laboratory. Good repeatability of the initial properties of the test-specimens was achieved using this preparation technique (Naughton & O'Kelly, 2003).

Stress paths followed

After sample setup, the test-specimens were isotropically consolidated from an effective stress of 50 kPa at the end of saturation, to an effective stress of 100 kPa. This was to ensure all test-specimens were normally consolidated before anisotropic consolidation. The anisotropic stress paths consisted of increasing R' from R' = 1.0 to R' = 1.5, while rotating α_σ, accompanied by either maintaining b = 0 or increasing b from b = 0 to b = 0.5. Tests were designated as Ybbaa, where bb denotes the final magnitude of the b parameter with the decimal point removed and aa the magnitude of α_σ rotation at the end of anisotropic consolidation. In total, eight tests were conducted. The test-specimens had a target initial relative density of 75 %. The initial properties of the test-specimens, immediately after sample setup, are summarised in Table 2.

Table 2. Initial properties of the sand samples immediately after sample setup.

Test	Mass of dry sand in test-specimen (g)	Actual relative density (%)	Height of test-specimen (mm)	Inner radius of test-specimen (mm)	Outer radius of test-specimen (mm)
Y0000	1174.0	77.5	181.5	35.5	50.0
Y0500	1163.8	72.4	181.4	35.5	50.0
Y0030	1173.0	77.0	181.5	35.5	50.0
Y0530	1174.1	77.5	181.5	35.5	50.0
Y0060	1170.0	75.5	181.6	35.5	50.0
Y0560	1163.3	72.2	181.4	35.5	50.0
Y0090	1165.6	73.3	181.5	35.5	50.0
Y0590	1176.0	78.4	181.5	35.5	50.0

Tests Y0090 and Y0590 consisted of an instantaneous rotation of α_σ at the end of isotropic consolidation. This was achieved by starting the anisotropic consolidation stage from $\alpha_\sigma = 90^0$, b = 0 and R' = 1.

The test program resulted in four variations of anisotropic consolidation occurring:

1. The value of R' was increased with neither change in the magnitude of the b parameter or rotation of α_σ, Test Y0000.
2. The value of R' was increased and at the same time the magnitude of the b parameter was increased with no rotation of α_σ, Test Y0500.
3. The value of R' was increased with no change in the b parameter but with rotation of α_σ, Tests Y0030, Y0060, Y0090.
4. The value of R' was increased and at the same time both the magnitude of the b parameter and the rotation of α_σ, were changed, Tests Y0530, Y0560 and Y0590.

Experimental results

The effects of α_σ rotation and change in the magnitude of the b parameter on the deformational response of the test-specimens, recorded using the internal instrumentation, were analysed in terms of the principal, deviator and volumetric strains. In this study, the deviator strain is defined as:

$$\varepsilon_d = \frac{2}{\sqrt{6}}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_3)^2} \qquad \text{Eq. 1}$$

where, ε_1, ε_2 and ε_3 are the major, intermediate and minor principal strains, respectively.

A predominately linear relationship is observed between each of the principal strains and the deviator strain for all tests, Figures 2 and 3. The effect of rotating α_σ while maintaining b = 0 during anisotropic consolidation (Tests Y0000, Y0030 and Y0060) gives a consistent relationship between the principal and deviator strains. In these tests, the magnitude of the three principal and deviator strains are proportional to the rotation of α_σ. The smallest strains are observed in Test Y0000 where b = 0 and $\alpha_\sigma = 0$.

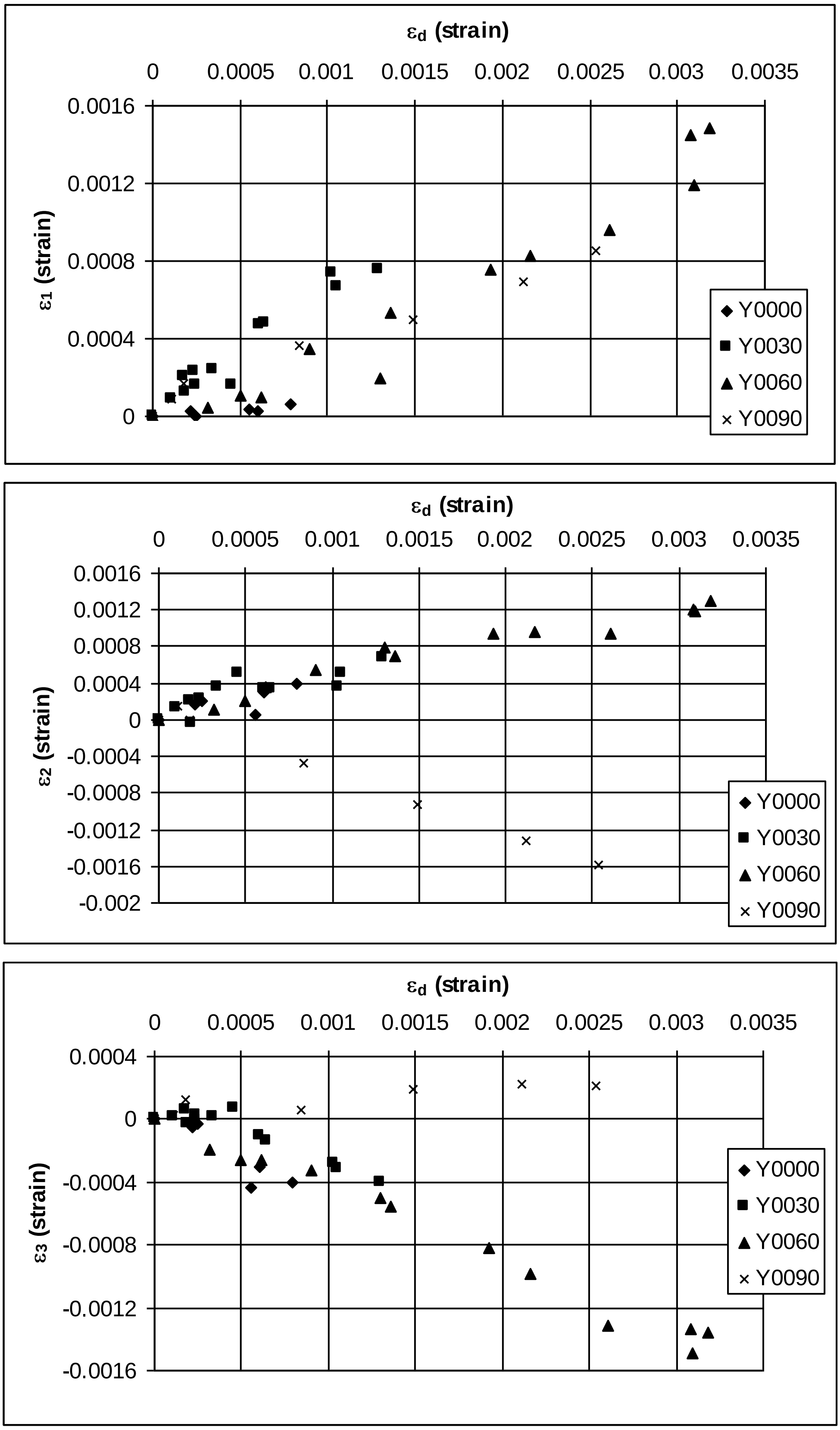

Figure 2. Relationships between the principal and the deviator strains for Tests Y0000, Y0030, Y0060 and Y0090 during anisotropic consolidation.

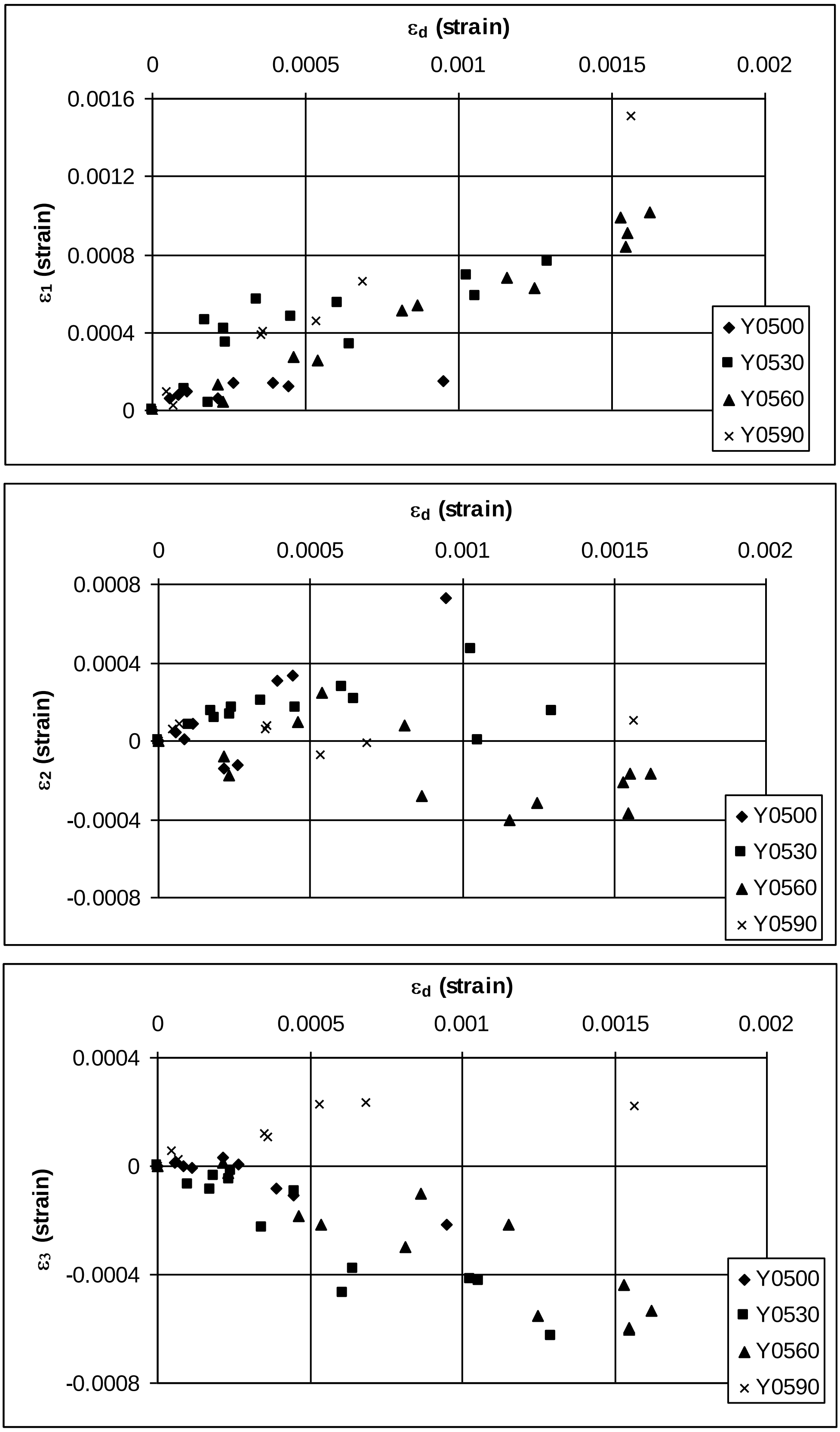

Figure 3. Relationships between the principal and the deviator strains for Tests Y0500, Y0530, Y0560 and Y0590 during anisotropic consolidation.

In Tests Y0500, Y0530, Y0560 and Y0590 a predominately linear relationship is observed between the individual principal strain components and the deviator strain but considerable scatter is evident in the strain components for individual tests. No obvious trend is evident in the data.

With the exception of Tests Y0090 and Y0590, ε_1 has the largest magnitude and is positive, followed by ε_3, which is negative. In tests where b = 0 during consolidation, the magnitude of ε_2 and ε_3 are broadly similar but opposite in sign. Extremely small changes of ε_2 are recorded, relative to the magnitudes of ε_1 and ε_3 in tests where the b parameter increased during consolidation.

In Tests Y0090 and Y0590, α_σ was not gradually rotated but rather instantaneously rotated by starting the anisotropic consolidation stage from $\alpha_\sigma = 90^0$, b = 0, R' = 1.0. The subsequent anisotropic stages resulted in a type of extension test, Figures 2 and 3, with the ε_2 and ε_3 strain components switching signs relative to comparable tests.

A similar predominately linear relationship is observed between the volumetric strain, ε_{vol}, and the deviator strain, Figures 4 and 5. Test Y0060 experiences the greatest volumetric and deviator strains, with the magnitude of ε_{vol} and ε_d for Test Y0030 both approximately 50 % of that recorded in Test Y0060. Test Y0000 experiences both the smallest volumetric and deviator strain indicating the significant effect α_σ has on the deformational response of the test-specimens. The ε_{vol} for Test Y0090 is negative indicating the sample volume increased slightly during this stage of the test.

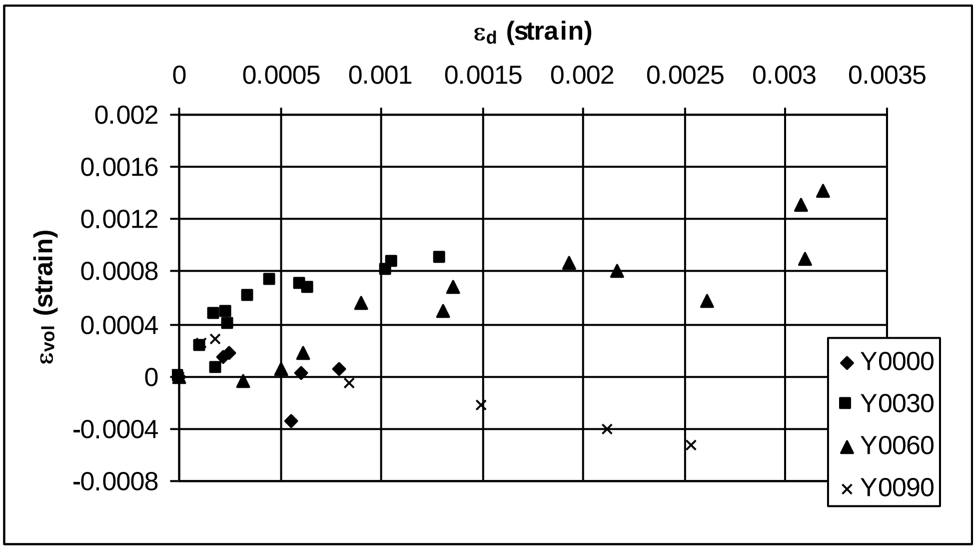

Figure 4. Relationship between the volumetric and the deviator strains for Tests Y0000, Y0030, Y0060 and Y0090 during anisotropic consolidation.

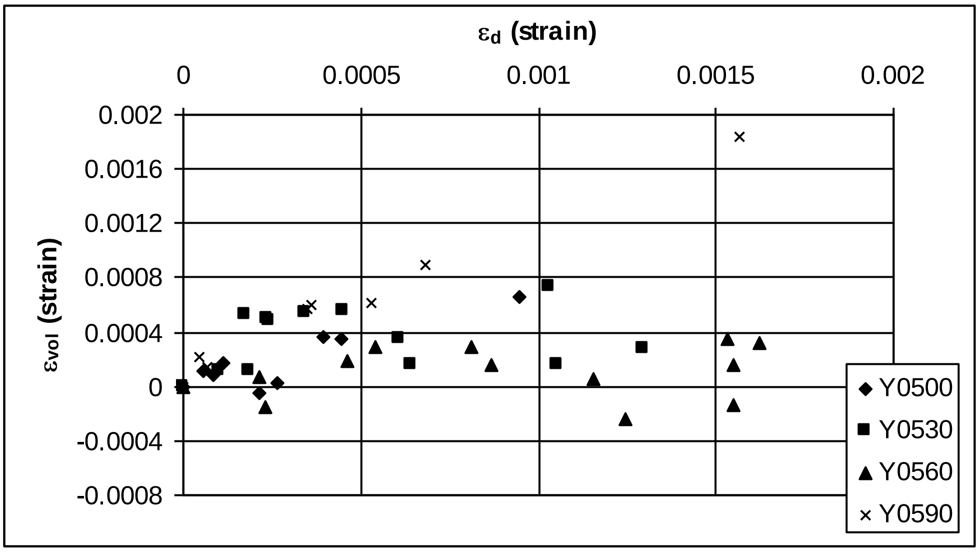

Figure 5. Relationship between the volumetric and the deviator strains for Tests Y0500, Y0530, Y0560 and Y0590 during anisotropic consolidation.

The ε_{vol} - ε_d relationship for Tests Y0500, Y0530, Y0560 and Y0590, where the magnitude of the b parameter was increased during consolidation, displays considerable more scatter than in tests where b = 0 during consolidation, Figure 5. Tests Y0530 and Y0560 both experience similar volumetric and deviator strains, within 10 % of each other, at the end of anisotropic consolidation. In Test Y0590 a positive volumetric strain implies a reduction in the sample volume as b increases from b = 0 to b = 0.5. This reduction in the sample volume is contrary to that measured in Test Y0090. In all tests the ε_{vol} strain is dominated by the magnitude of the ε_2 strain component.

Tests Y0030 and Y0530 are in the main, representative of the overall behaviour of the test-specimens during this study, Figure 6. However, some deviations from this general trend did occur in Tests Y0000 and Y0500 for ε_1 and in Tests Y0060 and Y0560 for ε_2.

A predominately linear relationship is observed between the principal strain components and ε_d in all tests. Overall the principal strain – deviator strain relationship for Tests Y0030 and Y0530 are broadly similar with positive ε_1, positive ε_2 and negative ε_3. The magnitude of ε_1 and ε_3 are broadly similar for any value of ε_d in both tests. ε_2 for Test Y0530 is lower than that for Test Y0030.

Exceptions to this general trend were Tests Y0000 and Y0500 where ε_1, and Tests Y0060 and Y0560 where ε_2, show different but consistent trends. In Test Y0060, ε_2 is negative but in Test Y0560, this strain component is positive, Figure 7. Where exceptions occurred, tests where b = 0 always followed the general trend, with tests where b increased from b = 0 to b = 0.5 displaying the opposite sign in one strain component. As with Tests Y0030 and Y0530 the magnitude of ε_2 in Test Y0560 is lower than ε_2 in Test Y0060.

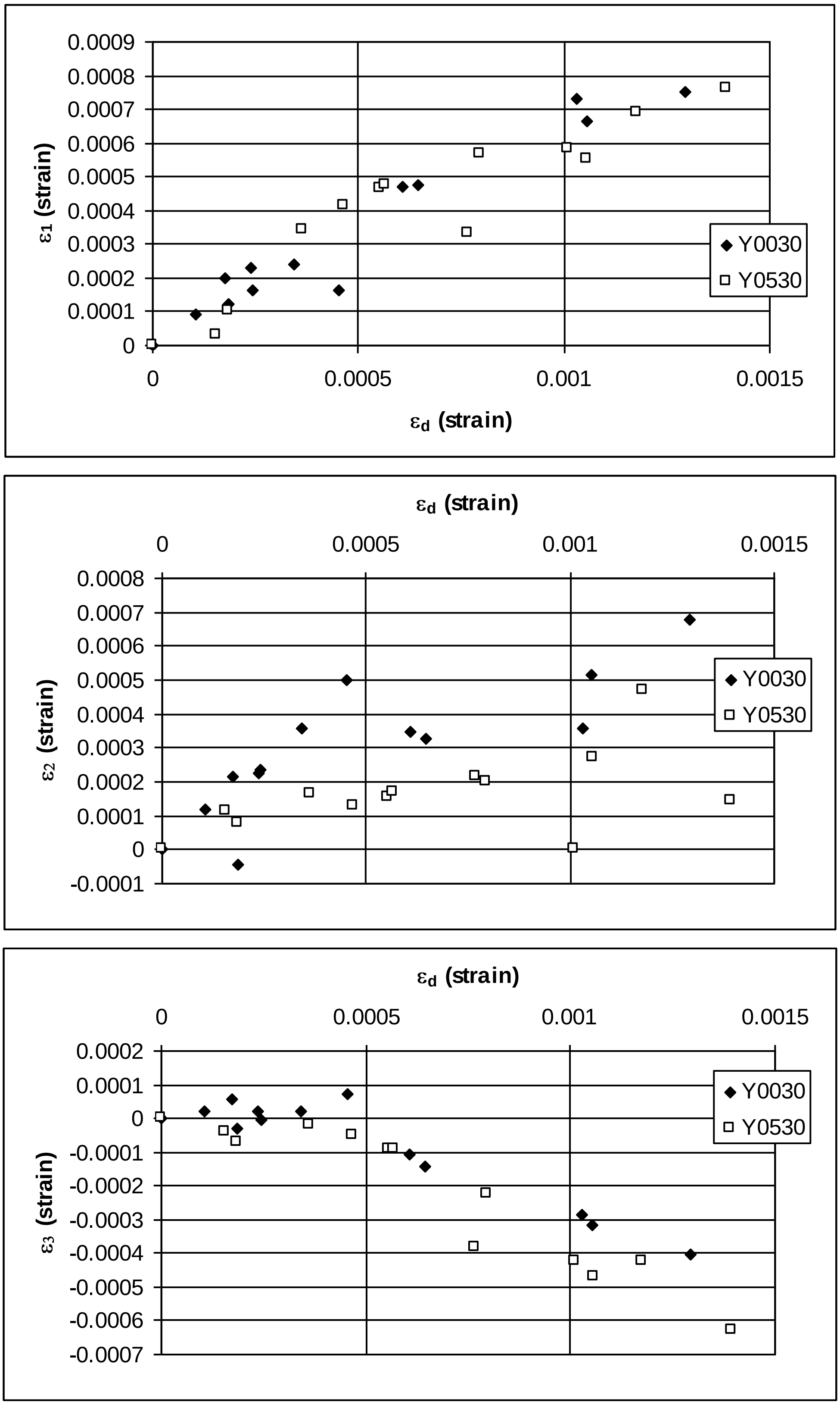

Figure 6. Relationships between the principal and the deviator strains for Tests Y0030 and Y0530 during anisotropic consolidation.

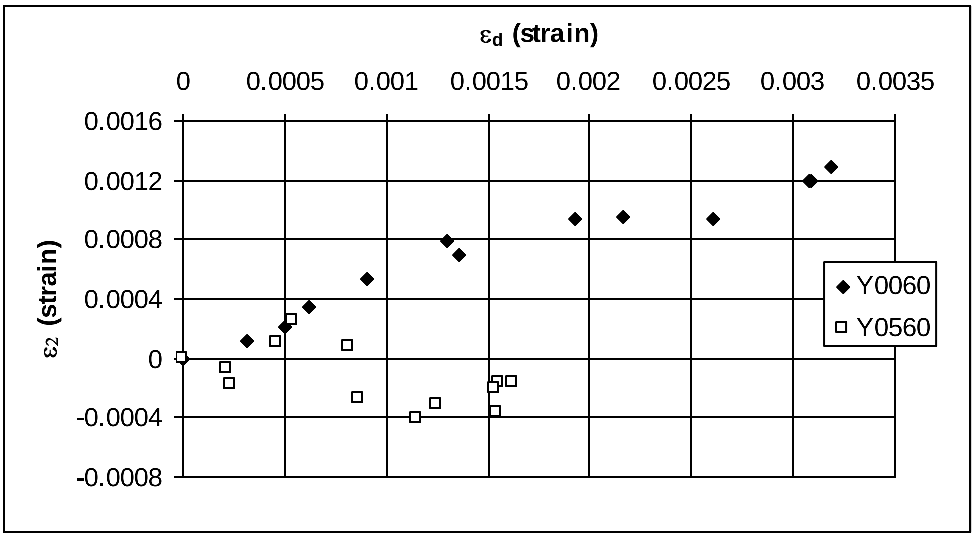

Figure 7. Relationship between ε_2 and ε_d for Tests Y0060 and Y0560 during anisotropic consolidation.

Considerable scatter is observed in the ε_{vol} - ε_d relationship for Tests Y0030 and Y0530 but the general trend is linear, Figure 8. Larger volumetric strains are observed in Test Y0030 where b = 0 than Test Y0530 where b is increased. No significant variation in the magnitude of ε_d at the end of consolidation was observed between Tests Y0030 and Y0530. The reduction in ε_{vol} in tests where the b parameter increased from b = 0 to b = 0.5, appears directly related to the reduced magnitude of ε_2 in these tests relative to tests where b = 0.

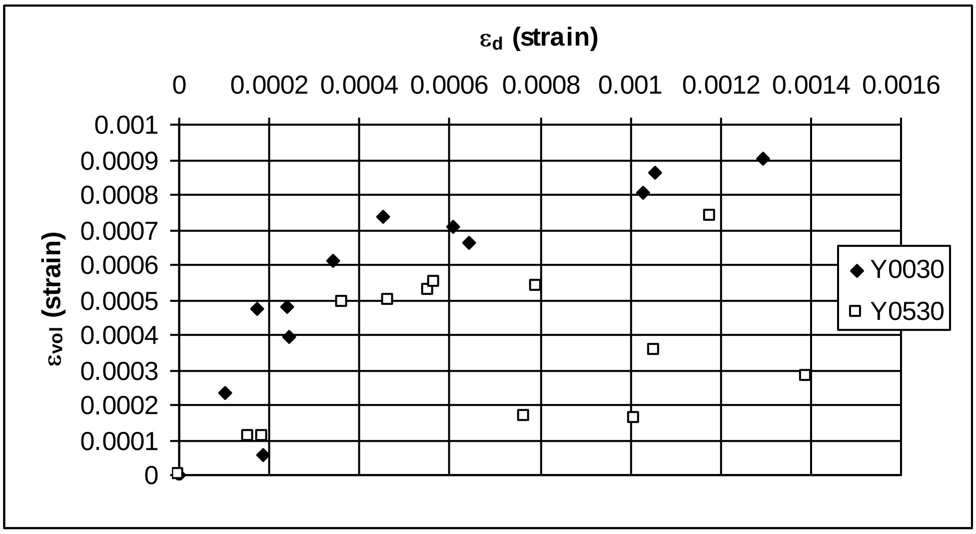

Figure 8. Relationship between the volumetric and the deviator strains for Tests Y0030 and Y0530 during anisotropic consolidation.

Discussion of results

The induced anisotropy of Leighton Buzzard sand was investigated by anisotropically consolidating identical, normally consolidated, sand test-specimens with an initial relative density of approximately 75 %. The test-specimens were all anisotropically consolidated from R' = 1 to R' = 1.5, with different rotations of α_σ and change in the magnitude of the b parameter.

In the case of test-specimens anisotropically consolidated with b = 0, the magnitude of the volumetric and deviator strains was found to be directly proportional to the rotation of the major principal stress. The same relationship was not observed in test-specimens where the magnitude of the b parameter increased to b = 0.5 during anisotropic consolidation.

The reduction in the magnitude of the intermediate principal strain component may be explained by the test-specimen experiencing near plane strain conditions in the vicinity of $b \approx 0.3$. Examining the magnitude of the effective principal stresses during anisotropic consolidation shows that the magnitude of the effective major and minor principal stresses are similar in all tests but where b increased during consolidation the magnitude of the effective intermediate principal stress remains constant, whereas in tests where b = 0 the effective intermediate and minor principal stresses are equal. The reduction in the volumetric strain when b increases during consolidation can be attributed to the magnitude of the effective intermediate principal stress remaining relatively constant during consolidation.

Zdravkovic and Jardine (2001) also reported a linear relationship between the strain components and the deviator strain in HCA tests on a silt when b = 0 during α_σ rotation. Sayao and Vaid (1996) studied Ottawa sand using a HCA and observed significant differences in the stress-strain response during drained tests for constant values of the intermediate principal stress relative to the major-to-minor principal stresses. The experimental results indicated that many geotechnical engineering problems in which plane strain conditions are dominant (i.e. where $b \approx 0.3$) may have their deformations largely overpredicated if data from the triaxial test (where b = 0) is used.

Conclusions

Analysis of the anisotropic consolidation of dense test-specimens of Leighton Buzzard sand has shown the following relationships, in terms of the deformational response of the test-specimens:

1. A predominately linear relationship is observed between both the principal and volumetric strains and the deviator strain for all tests. Tests where the magnitude of the b parameter was increased and α_σ rotated, did not produce as linear a volumetric and deviator strain relationship as other tests.
2. Overall, the deformational response of the samples displayed a complex relationship, which was dependent on the rotation of α_σ and the change in the magnitude of the b parameter.
3. The smallest volumetric and deviator strains were recorded in Test Y0000, where b = 0 and $\alpha_\sigma = 0$.
4. Tests where the magnitude of the b parameter was increased from b = 0 to b = 0.5 accompanied by α_σ rotation, displayed the smallest values of both the volumetric and

deviator strains after Test Y0000. Tests with b = 0 and where α_σ was rotated gave the largest volumetric and deviator strains at the end of consolidation.

5. In all tests, the ε_{vol} strain was dominated by the magnitude of the ε_2 strain component.
6. Tests involving rotation of α_σ with b = 0 followed the same general trend. The magnitude of both the volumetric and deviator strains at the end of consolidation were found to be proportional to the rotation of α_σ. For example, ε_d and ε_{vol} for $\alpha_\sigma = 30^0$ (Test Y0030) were found to be approximately 50 % ε_d and ε_{vol} when $\alpha_\sigma = 60^0$ (Test Y0060).
7. Tests involving rotation of α_σ and change of the b parameter display similar values of deviator and volumetric strains at the end of consolidation for all tests, irrespective of the magnitude of α_σ. The ε_{vol} - ε_d relationship is still approximately linear but the overall strain relationship between tests is not as consistent as when b = 0 during consolidation. Furthermore, ε_1, ε_2, ε_3 versus ε_d strain relationships no longer follow a general trend and considerable variation is observed between the individual principal strain components from these tests.
8. For tests where the b parameter increased during consolidation, the magnitude of ε_2 is small relative to both the ε_1 and ε_3 strains. This could be attributed to the sample experiencing plane strain conditions in the vicinity of $b \approx 0.3$. Where b increased during consolidation, the magnitude of the effective intermediate principal stress component only changes slightly, where both the effective major and minor principal stresses have similar values to tests where b = 0 during consolidation.

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