Then

$$x_{k+1} = a_1 \cdot (a_2' a_2)^{k-1} \cdot (a_1' a_1)^{k-1} \cdot (a_2' x_0). \quad (11)$$

Taking $a_1$ orthogonal to $a_2$ we have $\rho(A_i) = 0 \;\forall i$, but $\{x_i\}$ diverges if $a_1, a_2$ are chosen so that $(a_2' a_2)(a_1' a_1) > 1$.

The pedagogical point made here is that time varying systems may be unstable even if $\rho(A_i) < 1 \;\forall i$. In other words, although $A_i$ may be a "stable matrix" for every $i$, (8) may represent an unstable time varying sysytem. This example thus demonstrates the pitfalls associated with the extension of results associated with time invariant systems or first-order systems to the $n$th order time varying case. Note that the matrices of the example satisfy $\|a_1 a_2'\| = \|a_1\| \cdot \|a_2\|$, which may be arbitrarily large while $\rho(a_1 a_2')$ may be arbitrarily small.

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# Comments on "On the Stability of Two-Dimensional Discrete Systems"

F. M. BOLAND

*Abstract*—**This correspondence shows that independent necessary conditions and sufficient conditions for the asymptotic stability of two-dimensional discrete linear systems presented by Ahmed [1] can be readily obtained from previous results.**

## INTRODUCTION

Recently, Fornasini and Marchesini [2] [1] have considered the problem of asymptotic stability (AS) of 2-D discrete systems. Ahmed [1] has presented a sufficient condition for AS and a necessary condition which is also sufficient in the special case of separable 2-D systems. In this correspondence the conditions presented by Ahmed are shown to be easily derived from known results.

The state-space representation of a 2-D discrete system is given by [3].

$$\begin{pmatrix} R(i+1,j) \\ S(i,j+1) \end{pmatrix} = \begin{pmatrix} A_1 A_2 \\ A_3 A_4 \end{pmatrix} \begin{pmatrix} R(i,j) \\ S(i,j) \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} U(i,j) \quad (1)$$

$$Y(i,j) = [C_1 C_2] \begin{pmatrix} R(i,j) \\ S(i,j) \end{pmatrix} + DU(i,j).$$

*Definition [2]:* Let $\Sigma$ be described by (1). The system $\Sigma$ is AS if assuming zero input ($U = 0$) and $\|X_0\|$ finite, $\|X_i\| \to 0$ as $i \to \infty$.

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[1] E. Fornasini and G. Marchesini, *IEEE Trans. Automat. Contr.*, vol. AC-24, pp. 129–130, Feb. 1979.

Here,

$$X_r = \{x(h,k) : x(h,k) \in R^n \text{ (local state space)}, h + k = r\}. \quad (2)$$

The $Z$-transform of (1) is

$$\begin{pmatrix} R(z,w) \\ S(z,w) \end{pmatrix} = \begin{pmatrix} I - zA_1 & -zA_2 \\ -wA_3 & I - wA_4 \end{pmatrix} \begin{matrix} R(0,w) \\ B(z,0). \end{matrix} \quad (3)$$

Then, $\Sigma$ is AS if and only if

$$\det \begin{pmatrix} I - zA_1 & -zA_2 \\ -wA_3 & I - wA_4 \end{pmatrix} \neq 0 \quad \text{in } U$$

$$U = \{(z,w) \in C \times C : |z| \leq 1, |w| \leq 1\}. \quad (4)$$

Applying Schurs formula [4], (4) can be expressed as

$$\varphi(z,w) = |I - zA_1|\,|I - wA_4 - zwA_3(I - zA_1)^{-1}A_2| \neq 0 \quad (z,w) \in U. \quad (5)$$

Applying Huang's stability test requires that $\varphi(z,0) = |1 - zA_1| \neq 0$ for $|z| < 1$ and by interchanging the roles of $z$ and $w$, $\varphi(0,w) = |I - wA_4| \neq 0$ for $|w| < 1$. It follows that for AS of (1) it is necessary that

$$\sigma(A_1), \text{ the spectral radius of } A_1 < 1$$
$$\text{and} \quad \sigma(A_4) < 1. \quad (6)$$

The conditions given in (6) are identical to those of Ahmed's Theorem 3. For the special case of a separable 2-D system, $A_3 = 0$, giving from (5)

$$\varphi(z,w) = |I - zA_1|\,|I - wA_4|. \quad (7)$$

The conditions (6) are seen to be necessary and sufficient for AS in this special case, Ahmed's Theorem 2 [1]. It is of interest to note that in independent work the following natural extension of Ahmed's Theorem 3 to necessary and sufficient conditions for AS of (1) has been proved.

*Theorem [5]:* The Shanks (or, equivalently, Huang) stability test for the 2-D Roesser model is equivalent to the conditions

a) $A_1$ is a stability matrix
b) $A_4$ is a stability matrix
c) All the eigenvalues of the transfer function matrix

$$Q(z^{-1}) = A_4 + A_3(z^{-1}I - A_1)^{-1}A_2$$

with $|z| = 1$ lie in the interior of the unit circle in the complex plane. To obtain conditions sufficient for the asymptotic stability of (1), Ahmed proves the following proposition.

*Proposition:* If

$$\|A_4\| + \|A_2\|\,\|A_3\|(1 - \|A_1\|)^{-1} < 1$$

or

$$\|A_1\| + \|A_2\|\,\|A_3\|(1 - \|A_4\|)^{-1} < 1 \quad (8)$$

then $\Sigma$ is asymptotically stable, where $\|A\|$ is some suitable matrix norm. We wish to note that (8) is easily obtained from the following result of Humes (see Jury [6, p. 1041]). A sufficient condition for asymptotic stability of the system $\Sigma$ is given by

$$\|A_1\| + \|A_4\| - \|A_1\|\,\|A_4\| + \|A_2\|\,\|A_3\| < 1. \quad (9)$$

## CONCLUSIONS

In this correspondence we have shown that conditions presented by Ahmed [1] for the asymptotic stability of 2-D discrete systems are an immediate consequence of known results. Furthermore, a theorem giving necessary conditions for asymptotic stability has been extended to give conditions which are necessary and sufficient.

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# Comments on "Predictive Guidance for Interceptors with Time Lag in Acceleration"

R. J. FITZGERALD

*Abstract*—**Previous solutions related to those in the above paper[1] are pointed out, and some interpretations of the results are given in terms of "zero-effort miss."**

In the above paper a solution is presented for the optimum intercept problem with a first-order-lag autopilot model. It should be pointed out that this problem was previously solved by Willems [1], using a Riccati equation approach (which the authors[1] attempted without success). Furthermore, Willems also solved the more complex problem in which the autopilot is represented by two unequal lags in cascade [2]. (Both [1] and [2] consider only the intercept problem, while the above paper[1] includes also the "rendezvous" problem.) A solution similar to that of Hecht and Troesch (again for the intercept problem only) was later published in [3].

The results obtained[1] are more satisfying when interpreted in terms of "zero-effort miss." If the control $u$ is removed, the acceleration error $A_e$ decays as

$$A_e(t) = A_e(0)e^{-t/\tau} \tag{1}$$

producing an additional velocity error

$$\Delta V_e(t) = \tau A_e(0)\,(1 - e^{-t/\tau}) \tag{2}$$

and an additional position error

$$\Delta P_e(t) = \tau A_e(0)[t - \tau(1 - e^{-t/\tau})]. \tag{3}$$

In the terminology of Hecht and Troesch, when $t \to T_g$ we have

$$\Delta P_e(T_g) = F\tau A_e(0) \tag{4}$$

so that the interceptor solution [equation (22)] can be written

$$u = \frac{-F}{H} M_Z \tag{5}$$

where $M_Z$ is the predicted zero-effort miss

$$M_Z = \delta x + T_g \delta v + \Delta P_e(T_g). \tag{6}$$

Similarly, if we define a "predicted zero-effort velocity miss" as

$$M_{VZ} = \delta v + \Delta V_e(T_g) \tag{7}$$

the rendezvous solution [equation (25)] can be written in the form

$$u = -K_1 M_Z - K_2 M_{VZ} \tag{8}$$

where

$$K_1 = \frac{F}{D}(F + ET_g/2) \tag{9}$$

and

$$K_2 = \frac{1}{D}(F^3/2 + EH). \tag{10}$$

Thus, both guidance laws consist of gains applied to predicted zero effort misses—which is logical, since the optimum control must be zero if this will result in zero miss. Furthermore, the form of (5) and (8) makes the inclusion of target maneuver immediately obvious: we simply add to $M_Z$ and/or $M_{VZ}$ the miss contributions which will result from the expected future time history of the target acceleration. This procedure yields the same solutions as are found by solving the problem with the target acceleration included in the initial formulation, as was done, for example, in [4].

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# Comments on "On Parameter and Structural Identifiability: Nonunique Observability/Reconstructibility for Identifiable Systems, Other Ambiguities, and New Definitions"

YVES LECOURTIER AND ERIC WALTER

*Abstract*—**This correspondence makes critical remarks on the terminology presented in the above paper[1] and proposes another one, based on classical vocabulary.**

In the above paper[1], DiStefano and Cobelli proposed a set of identifiability definitions for deterministic models. While we do agree with the necessity of finding some universally accepted definitions in order to clarify the existing jungle, we do not think that the proposed terminology is adequate. It is our opinion that the authors have not fully reached their proclaimed goal of avoiding any useless "special jargon."

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