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Real Exchange Rates, Co-Integration and Purchasing Power Parity: Irish Experience in the EMS

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Abstract: Dickey-Fuller and Co-Integration techniques are used to test the hypothesis that co-movements in Irish nominal exchange rates and relative prices are consistent with the implications of Purchasing Power Parity. The data reject PPP between Ireland and the US. Results from Irish/UK and Irish/German data are less decisive against the possibility that linear combinations of the nominal exchange rate and corresponding relative prices are stationary series.

I INTRODUCTION

The volatility of real exchange rate movements during the last 10 to 15 years has led to widespread scepticism about the ability of the standard purchasing power parity (PPP) model to adequately explain co-movements in nominal exchange rates and relative prices of internationally traded goods. Although there is general agreement that PPP does not appear to hold in the short and intermediate runs, there remains considerable disagreement over the validity of PPP in the longer run. For example, Dornbusch (1976) and Ainzenman (1986) attempt to rationalise deviations from parity in terms of commodity markets characterised by slow price adjustment interacting with flexible asset markets. These sticky price, or overshooting, models permit sustained deviations from parity but typically maintain PPP as a valid long-run hypothesis. However, Roll (1979) and Alder and Lehman (1983) have developed theoretical models, based on efficient international capital markets,

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which suggest that PPP is violated in the long run. These authors also present econometric evidence supporting the hypothesis that real exchange rates follow a random walk, implying that shocks have infinitely long-lived effects and that there is no tendency to revert to parity in the long run.

In an Irish context, Walsh (1983 and 1988) has documented the volatility of real exchange movements since the foundation of the European Monetary System (EMS) in 1979. For example, four years after Ireland's decision to participate in the EMS Walsh commented that the Irish experience "... may be seen as another example of a significant and relatively enduring change in real exchange rates, to be added to the list that has already been compiled by ... Frenkel (1981) ... it implies that the SOE view of inflation does not fit the facts over the short to medium run in Ireland." (Walsh, 1983, p. 178). The significance of this statement lies in the fact that it was the SOE view of inflation, or PPP, which provided the theoretical base for Ireland's decision to participate in the EMS. In the period immediately preceding the formation of the EMS, sterling had been a relatively weak high inflation currency. Hence by breaking the parity link with sterling and fixing the nominal value of the Irish pound within a quasi-fixed exchange rate system based on the D-Mark, Ireland anticipated a decline in domestic inflation as the price level converged towards the EMS average. Further, if sterling continued to depreciate against EMS currencies (UK inflation remained high) then PPP would ensure that Ireland did not lose competitiveness against the UK in the sense that any nominal appreciation against sterling would be compensated for by an offsetting rise in the UK/Irish relative price.

These expectations were, however, unrealised during the initial years. Not only did British and Irish inflation rates remain relatively high, but sterling appreciated in nominal terms relative to participating currencies, with the consequence that Ireland experienced competitive gains *vis-à-vis* Britain and lost competitiveness within the system. Note that PPP implies that the appreciation of sterling should have been accompanied by a relative decline in the British inflation rate thereby maintaining Ireland's competitive position against the UK. Further, when the British inflation rate eventually declined it was accompanied by a nominal depreciation of sterling which eroded the initial competitive gains to the point, in March 1983 and again in August 1986, at which Ireland decided to devalue against the other EMS currencies in order to restore a competitive position against the UK. Hence the apparent failure of nominal exchange rates and relative prices to move in accordance with the predictions of PPP has led to a policy dilemma in the sense that Ireland has to balance its commitment to EMS parities with the objective of maintaining a reasonable level of competitiveness against its most important trading partner, the UK.

However, it is important to note that an adjustable peg policy within the EMS is not necessarily inconsistent with PPP. For example, in the context of a Dornbusch-type sticky price model, the speed of adjustment towards parity may be so slow as to justify direct intervention designed to moderate the extent to which the nominal exchange rate overshoots its long-run equilibrium level. In this case the underlying trend will be towards parity and realignments of the nominal exchange may be interpreted as measures which smooth the adjustment process. On the other hand, if the real exchange rate follows a random walk, as suggested by Alder and Lehman for example, then there will be no tendency towards reversion and realignments of the nominal rate will not necessarily move the real exchange rate towards its parity level.

Hence the purpose of this paper is to utilise time series data on Irish pound real exchanges against sterling, the dollar and the D-Mark to test the hypothesis that the behaviour of these series is inconsistent with the long-run implications of PPP. These implications along with possible tests for PPP are outlined in the next section. Section III presents results from univariate and co-integration tests on real exchange rates against Sterling, the D-Mark and the Dollar. The paper's principal conclusions are summarised in the final section.

II TESTS FOR PPP

I define the real exchange rate as the product of the nominal spot exchange rate and the ratio of foreign to domestic prices. Working in logarithms the real exchange rate may be expressed as:

$$q_{jt} = s_{jt} + r_{jt} \tag{1}$$

where s_j is the domestic currency price of currency j and r_j is the ratio of the price level in country j to the domestic price level. PPP permits q_j to deviate from parity in the "short run" but requires convergence to an equilibrium in the "long run". In a stochastic context, this long-run convergence to an equilibrium value can be interpreted as implying that the generating mechanism for the real exchange rate is a stationary process with a time invariant mean and variance. For example, in the stochastic linear purchasing power model:

$$s_{jt} = \alpha + \beta r_{jt} + \sum_{0}^{\infty} \lambda_i \upsilon_{t-i} \tag{2}$$

where υ_t is a random disturbance with a zero mean and constant variance. PPP will hold in its absolute form, $E(q_t = 0)$, if $(\alpha,\beta) = (0, -1)$ or in its relative form, $E(q_t = \alpha)$, if $(\alpha, \beta) = (\alpha, -1)$ where α is a non-zero constant.
So long as $\Sigma \lambda_i \upsilon_{t-1}$ is a stationary stochastic process with λ_i approaching zero

at "large" i then PPP will hold in its absolute form, $E(q_t = 0)$, if $(\alpha,\beta) = (0, -1)$ or in its relative form, $E(q_t = \alpha)$, if $(\alpha,\beta) = (\alpha, -1)$ where α is a non-zero constant. Autocorrelation in the innovations may be exploited in the short run forecasts, but fluctuations in the real exchange rate around its expected, or equilibrium value will be purely temporary with the actual value of the real exchange rate returning to $E(q)$ in the long run.

If the real exchange can be modelled as a stationary process then it is said to be integrated of order zero, or $q \sim I(0)$. That is, q is stationary in its level. If, on the other hand, q_j follows a random walk then the mean and variance will be undefined and the series will be non-stationary with deviations from parity increasing over time. Note that in this case q is stationary in first differences and is said to be integrated of order one, or $q \sim I(1)$. Hence a possible test of long-run PPP is that $q_t \sim I(1)$ against the alternative that it is I(0). PPP requires that the null be rejected.

Dickey and Fuller (1981) provide an appropriate test of the hypothesis that a series x_t is I(1) against the alternative that it is I(0). In the Augmented Dickey-Fuller regression:

$$\Delta x_t = \alpha + \gamma T + \theta x_{t-1} + \sum_{1}^{m} \gamma_i \Delta x_{t-i} + \epsilon_t \qquad (3)$$

where T denotes a time trend and m is large enough to ensure that the residuals are white noise, x_t is I(1) without drift if we cannot reject the hypothesis that $\alpha = \gamma = \theta = 0$ or x_t is I(1) with drift if $\gamma = \theta = 0$. If, for example, the first hypothesis is maintained then the appropriate modelling procedure for x_t is an AR(m) process which is integrated of order one, or a random walk without drift.

However the requirement that a linear combination of s_t and r_t be a stationary series does not necessarily imply that the constituent series must themselves be stationary. If the nominal exchange rate and relative price term are individually I(0) then it is generally true that linear combinations such as those implied by either absolute or relative PPP will also be I(0). However, it is possible that s_t and r_t may each be I(1), but there exists a linear combination of these variables which is a stationary I(0) series. If this is the case then PPP holds in the more general sense that a shock will cause s_t and r_t to drift apart initially but converge towards a stationary equilibrium in the longer run.

Engle and Granger (1987) formalise the idea of variables sharing an equilibrium relationship in terms of co-integration between time series.[1] To illustrate, consider two series x and y both of which are I(d).[2] Linear com-

1. Taylor and McMahon (1988) use co-integration techniques to test for PPP in the 1920s.
2. That is, both x and y are stationary after differencing d times.

binations of x and y will, in general, also be I(d). However it is possible that there exists a vector (α^*, β^*) such that the combination:

$$z_t = x_t - \alpha^* - \beta^* y_t \tag{4}$$

is I(d-b) b > 0. If (α^*, β^*) exists then z_t may be interpreted as an equilibrium error and x_t and y_t are said to be co-integrated of order (d, b), or CI(d,b). Hence, if d=b=1 the existence of (α^*, β^*) implies that the equilibrium error is I(0) and the combination (4) is a stationary process. Engle and Granger (1987) also show that when x_t and y_t are co-integrated then there exists an error correction model (ECM) of the form:

$$\begin{aligned} \Delta x_t &= \alpha_1 + \gamma_1 z_{t-1} + \sum_1 \gamma_{xi} \Delta x_{t-i} + \sum_1 \mu_{xi} \Delta y_{t-1} + \epsilon_{1t} \\ \Delta y_t &= \alpha_2 + \gamma_2 z_{t-1} + \sum_1 \gamma_{yi} \Delta x_{t-i} + \sum_1 \mu_{yi} \Delta y_{t-1} + \epsilon_{2t} \end{aligned} \tag{5}$$

where one of γ_1, γ_2 is non-zero. Note that if x and y are both I(1) and are also CI(1, 1) then every term in (5) will be I(0). If, on the other hand, x and y are not CI(1, 1) then z will not be I(0) and will have no place in (5). For example, if x and y are CI(1, 1) then z_t represents the deviation from equilibrium in period t, and the ECM determines the proportion of the disequilibrium which is corrected in period t+1. Hence, if z_t is I(0) with, say, $E(z_t) = 0$ then x and y will eventually converge to an equilibrium. However, if x and y are not C(1, 1) then they cannot share an equilibrium relationship and the error correction term z will have no place in (5). It follows that we can accept PPP in the sense that it predicts a long-run equilibrium relationship between s_t and r_t if these variables are found to be co-integrated with an I(0) equilibrium error.

An appropriate sequence of tests for co-integration requires the following steps. First, the Dickey-Fuller test outlined above can be used to test the hypothesis that the individual series on s_t and r_t are I(1). Second, given that s_t and r_t are both I(1), Stock (1987) shows that if the series are co-integrated then an OLS regression of s_t on r_t (or r_t on s_t) provides an efficient estimator of the co-integration vector. Third, the hypothesis of no co-integration requires that the residuals from the co-integrating regressions are a non-stationary process. In what follows I present the results of three tests on the null hypothesis that $u_t \sim I(1)$, where u_t are the residuals from the co-integrating regression.

(i) An augmented Dickey-Fuller test based on the OLS regression:

$$\Delta u_t = \beta u_{t-1} + \sum_1^m \gamma_i \Delta u_{t-1} + \epsilon_t \tag{6}$$

If $u_t \sim I(1)$ then the OLS estimator for β in (6) should be insignificantly different from zero, with the rejection region consisting of relatively large negative values for the ratio of β to its estimated standard error. However, as Engle and Granger (1987) point out, the Dickey-Fuller (1981) test statistics are inappropriate when the co-integrating parameters are unknown and have to be estimated. As in the standard Dickey-Fuller case, the test statistic will not have a t-distribution under the null. Hence Engle and Granger provide critical values for the ratio of the OLS estimator for β to its standard error based on the assumption that the null is true.

(ii) An alternative test for co-integration, suggested by Sargan and Bhargava (1983), is that the Durbin-Watson statistic from the co-integrating regression is significantly different from zero. If, for example, the residuals follow a random walk with a first-order autocorrelation coefficient equal to unity then the DW statistic should be approximately zero. Hence low values for the DW favours acceptance of the null hypothesis of no co-integration.[3]

Note that the DW and ADF tests may have low power against plausible alternatives such as a stationary autoregressive process with an autocorrelation coefficient close to unity. Hence the second stage estimation of the ECM which uses the residuals from the prior level regressions provides an additional check on the co-integration hypothesis. The relatively low power of DW and ADF tests is due to the fact that they attempt to distinguish between series that have no random walk component and series that have a random walk component. In the former $E(u_{t+k})$ is independent of shocks to u_t, while a unit shock at time t increases $E(u_{t+k})$ by one unit in the case of a random walk. However, as Cochrane (1988) shows, any series which is first difference stationary can be decomposed into a stationary, or temporary component, and a random walk, or permanent, component. In tests for a unit root, such as DW and ADF, the null assumes that the temporary component is zero and tests for the existence of the latter. Consequently these tests will have difficulty in distinguishing between a stationary series and a series in which the random walk component is relatively small. If the series is stationary without a random walk component then it will exhibit complete reversion to a constant expected value. On the other hand, if the series contains a random walk component then it may exhibit partial mean reversion, with the dependence

3. Engle and Granger (1987) give critical values for the DW statistic generated from Monte Carlo simulations.

of $E(u_{t+k})$ on shocks at time t depending on the relative importance on the random walk component. The third test on the residuals of the co-integrating regressions, discussed below, attempts to assess the importance of the permanent component in u_t.

(iii) A variance ratio test based on Cochrane (1988). If u_t follows the random walk model:

$$u_t = u_{t-1} + \epsilon_t \tag{7}$$

where $\epsilon_t \sim (0, \sigma^2)$ the variance of the first difference is simply σ^2, and the variance of $(u_t - u_{t-k})$ is $k\sigma^2$, so that the ratio:

$$VR = (1/k)VAR(u_t - u_{t-k})/VAR(\Delta u_t) \tag{8}$$

should equal one for all values of k. If, on the other hand, u_t is a stationary process with a small random walk component, then VR should decline with k. That is, the greater the decline in VR the smaller the random walk, or permanent, component in u_t.

III RESULTS

This section presents results from both univariate tests based on Equation (3) and from the co-integration tests outlined above. Table 1 gives the Augmented Dickey-Fuller (ADF) statistics on univariate tests (Equation 3) for three real exchange rates – the Irish pound against sterling, the US dollar and the German mark. The statistics Φ_2 and Φ_3 test the hypotheses that the real exchange rate follows a random walk without drift (Φ_2) and with drift (Φ_3). Note that under the null hypothesis that x_t is I(1) these statistics will not have the standard F-distribution. However, critical values are given in Dickey and Fuller (1981). The data are monthly and the sample period is 1980(1) to 1987(12). The nominal exchange rate series are taken from the Central Bank of Ireland *Quarterly Bulletin* (various issues) and the prices are indices of wholesale prices based on 1980 = 100 taken from the OECD *Main Economic Indicators*.

The ADF statistics in Table 1 reject the null hypothesis that the Irish pound/sterling real exchange rate is I(1) at the 5 per cent level but accept it for the real exchange rate against the dollar. Tests on the D-Mark real exchange rate accept the random walk hypothesis at the 5 per cent level but reject it at the 10 per cent significance level. Hence while the sterling rate appears to be a stationary series, as implied by PPP, the other series are non-stationary and at variance with the predictions of PPP. Results for the D-Mark are, however, inconclusive.

Table 1: *Augmented Dickey-Fuller Statistics*

Real Exchange Rates

	Φ_2	Φ_3
Sterling	5.629	8.329
Dollar	3.151	4.707
D-Mark	4.545	6.429

Notes: Φ_2 is the Dickey-Fuller (1981) statistic for H_o: $\alpha = \gamma = \beta = 0$. Critical values are 6.50(1%), 4.88(5%) and 4.16(10%).
Φ_3 is the Dickey-Fuller (1981) statistic for H_o: $\gamma = \beta = 0$. Critical values are 8.73(1%), 7.44(5%) and 5.47(10%).

Tables 2 to 5 give the results of co-integration tests for real exchange rates for the Irish pound against sterling, the US dollar and the German mark. Table 2 gives Augmented Dickey-Fuller statistics (1981) on the null hypothesis that each series is I(1) while Table 3 gives the results from ADF and DW tests on the co-integrating regressions. The variance ratio tests and error correction models are given in Tables 4 and 5 respectively. The results can be summarised as follows.

Table 2: *Augmented Dickey-Fuller Statistics*

Nominal Exchange Rates and Relative Prices

	Exchange Rate		*Relative Price*	
	Φ_2	Φ_3	Φ_2	Φ_3
Sterling	4.228	6.290	1.952	2.927
Dollar	4.222	6.053	4.314	1.921
D-Mark	4.380	2.933	8.079	4.900

Note: Critical values are as in Table 1.

Table 3: *DW and ADF Tests on Co-integrating Regressions*

	Sample Period: 1980(1)-1987(12)		
Currency	*Regression*	*Test Statistics*	
Sterling:	$s_t = 0.093 - 1.032r_t$	DW = 0.205	ADF = −3.001
	$r_t = 0.004 - 0.450s_t$	DW = 0.135	ADF = −2.380
Dollar:	$s_t = -0.605 - 1.408r_t$	DW = 0.060	ADF = −0.426
	$r_t = -0.325 - 0.390s_t$	DW = 0.038	ADF = 0.625
D-Mark:	$s_t = -1.325 - 0.847r_t$	DW = 0.293	ADF = −2.971
	$r_t = -1.503 - 1.115s_t$	DW = 0.289	ADF = −3.207

Notes: Critical values for DW are 0.511(1%), 0.386(5%) and 0.322(10%).
Critical values for ADF are 3.77(1%), 3.17(5%) and 2.84(10%). ADF is a test on $\beta = 0$ in the regression: $\Delta u_t = \beta u_{t-1} + \sum_1^4 \beta_i \Delta u_{t-1}$.

Table 4: *Variance Ratio Tests*

Residuals from Co-integrating Regressions

	Sterling		*D-Mark*	
Dep. Var.	s_t	r_t	s_t	r_t
k				
2	1.26	1.13	1.01	1.01
	(0.04)	(0.03)	(0.03)	(0.03)
3	1.39	1.17	0.90	0.89
	(0.08)	(0.05)	(0.03)	(0.03)
4	1.45	1.15	0.74	0.72
	(0.11)	(0.07)	(0.03)	(0.03)
5	1.43	1.09	0.62	0.60
	(0.14)	(0.08)	(0.03)	(0.02)
6	1.36	1.03	0.66	0.63
	(0.15)	(0.09)	(0.04)	(0.03)
7	1.28	0.98	0.69	0.68
	(0.16)	(0.09)	(0.05)	(0.05)
8	1.23	0.97	0.71	0.68
	(0.17)	(0.11)	(0.06)	(0.05)
9	1.18	0.95	0.67	0.64
	(0.18)	(0.11)	(0.06)	(0.05)
10	1.11	0.91	0.65	0.62
	(0.17)	(0.11)	(0.06)	(0.05)
20	0.41	0.79	0.61	0.58
	(0.04)	(0.17)	(0.11)	(0.09)
30	0.35	0.87	0.39	0.37
	(0.05)	(0.32)	(0.06)	(0.06)
40	0.29	0.84	0.10	0.09
	(0.05)	(0.40)	(0.01)	(0.01)
50	0.30	0.91	0.11	0.12
	(0.06)	(0.58)	(0.01)	(0.01)

Note: Figures in parentheses are Bartlett standard errors estimated as $(4k/3T)^{.5}[k^{-1}\mathrm{var}(u_t-u_{t-k})]^2/[\mathrm{var}(\Delta u_t)]^2$. The variance of k differences is estimated as: $\sum_{k+1}^{T}[(u_t - u_{t-k}) - \sum_{k+1}^{T}(u_t - u_{t-k})/(T-k)]^2/(T-k)$ with a small sample correction of $T/(T-k+1)$ for degrees of freedom. See Cochrane (1988).

Table 5: *Error Correction Model*

	Sterling+		*Sterling*++		*D-Mark*	
Dep. Var.	Δs_t (1)	Δr_t (2)	Δs_t (3)	Δr_t (4)	Δs_t (5)	Δr_t (6)
z_{t-1}	-.116 (2.197)	-.081 (2.651)	-.021 (0.270)	-.059 (2.780)	-.132 (3.562)	-.226 (0.690)
Δs_{t-1}	.446 (4.059)		.362 (3.237)		.414 (4.619)	
Δs_{t-3}		-.084 (1.889)		-.068 (1.474)		
Δs_{t-4}						-.314 (2.886)
Δs_{t-6}					.325 (2.410)	
Δr_{t-3}	-.436 (1.659)		-.260 (1.008)			-.033 (0.306)
Δr_{t-5}					.258 (2.818)	
Δr_{t-6}					-.364 (3.955)	
R^2	.158	.167	.108	.164	.333	-.147
DW	2.110	2.013	2.017	2.086	1.966	2.048
SEE	.019	.008	.020	.031	.008	.012
CHOW	0.437	0.942	0.120	0.301	18.960*	3.450
LM(12)	6.952	8.792	9.562	7.446	9.903	25.789*
ARCH(12)	3.009	18.682	3.498	19.897	2.763	16.148
FEXC	0.644	0.733			0.974	0.726

Notes: z is the residual from the co-integrating regression.

+ Uses residuals from appropriate co-integrating regression – i.e., s_t on r_t in Column 1 and r_t on s_t in Column 2.

++ Uses residuals from alternative co-integrating regression – i.e., r_t on s_t in Column 3 and s_t on r_t in Column 4.

( ) are t-statistics.

CHOW (parameter stability), LM (Lagrange multiplier test for serial correlation) and ARCH (autoregressive-conditional heteroscedasticity test) are given as χ^2. FEXC is an F-test on the hypothesis that excluded lags on Δs and Δr in the unrestricted VAR are jointly zero.

* indicates significance at the 5% level.

First, the Dickey-Fuller statistics in Table 2 appear to accept the hypotheses that the individual series on nominal exchange rates and relative prices can be modelled as random walks. Second, the Sargan-Bhargava DW tests on the co-integrating regressions in Table 3 unambiguously accept the hypothesis of no co-integration for all three real exchange rates. Over the full sample period the DW statistics are insignificantly different from zero in all cases, indicating that the residuals from the co-integrating regressions are AR(1) with a unit first order autocorrelation coefficient – that is, a random walk. Third, the ADF tests on the residuals of the co-integrating regressions give weak support for co-integration on Irish/UK and Irish/German data but reject co-integration between the Irish/dollar nominal exchange rate and the ratio of US to Irish prices. Given that there is some conflict between the results of these tests it is relevant to note that the Engle and Granger (1987) simulations suggest that the ADF test has greater power against plausible alternatives.[4]

Fourth, the variance ratio tests, Table 4, suggest that the random walk components of the residuals from the D-Mark co-integrating equations are small relative to the temporary components. In each case 1/k times the variance of the k differences settles down to approximately 10 per cent of the variance of month to month changes suggesting that these series are dominated by stationary, or temporary, components. However, whereas the residuals from the OLS regression of the sterling nominal exchange on relative prices also exhibits a significant stationary component, the relative price innovations are consistent with a random walk. That is, the ratio of 1/k times the variance of k differences to the variance of first differences appears to be insignificantly different from unity up to k equal to 50 months. Hence, the VR test tends to reject the null of no co-integration on the Irish/German data, but accept it on the Irish/UK data. Hence the DW tests are decisive against co-integration while the ADF statistics are inconclusive for both sterling and the D-Mark. VR tests, on the other hand, support D-Mark co-integration but not for sterling.

Table 5 presents the results from error correction models for the series on nominal exchange rates and relative prices for sterling and the D-Mark. In each case the ECM was selected by first estimating an unrestricted vector autoregression with first differences of s_t and r_t regressed on s_{t-1} and r_{t-1} and six lags on Δs_t and Δr_t. The error correction models were then specified to include significant lags on Δs_t and Δr_t and the lagged residuals from the co-integrating regression – the error correction term. The CHOW, LM and ARCH statistics are diagnostic checks for parameter stability, autocorrelation and heteroscedasticity, respectively, and FEXC is an F-test on the hypothesis that the coefficients on the omitted lags from the vector autoregression are

4. As the results presented so far unambiguously reject PPP on Irish/US data, the dollar terms are dropped from the subsequent analysis.

jointly zero. Note, however, that the co-integrating regressions for sterling, Table 3, differ significantly with the choice of dependent variable. For example, normalising the sterling relative price equation on s_t gives a slope coefficient of -2.222 or more than twice the absolute value of the slope coefficient in the nominal exchange rate regression. Doing the same normalisation on the D-Mark relative price regression gives a slope coefficient of -0.896 which is virtually identical to the corresponding coefficient in the alternative co-integrating regression. The significance of this result is that the equilibrium solution for the sterling error correction models (Δ terms = 0) will differ according to which error correction term is used. Hence the ECMs for sterling are estimated both ways. Columns 1 and 2 in Table 5 give the ECMs for the sterling nominal exchange rate and relative price using the residuals from their respective co-integrating regressions while Columns 3 and 4 use the residuals from the alternative regression.[5]

When assessing the significance of the error correction term it is important to remember that the "t" statistic will be invalid under the null that z is I(1). Hence, Banerjee *et al.* (1986) suggest, on the basis of a simulation study, a rejection region consisting of "t" statistics in excess of three. On this criterion the data appear to identify an ECM for the D-Mark nominal exchange rate, although there is evidence of parameter instability, but not for the corresponding relative price term, which can be interpreted as implying that the latter is exogenous even though the variables may be co-integrated in the sense that an ECM can be found. For example, in the ECM for the D-Mark exchange rate, Column 5 of Table 5, a fall in the equilibrium error z_{t-1} (real exchange rate overvalued relative to parity) is associated with a significant positive change in s_t (depreciation of the nominal exchange rate) but has no impact on the adjustment of relative prices. However, the data cannot identify an ECM for either the Irish pound/sterling nominal exchange rate or for the corresponding relative price term. Further, when the sterling error correction terms are reversed, Columns 3 and 4, z_{t-1} becomes insignificant in the nominal exchange rate equation implying that the variation in r_t which is not attributable to s_t plays no role in the dynamics of the latter.

Finally, an important result from Engle and Granger (1987) is that variables which are co-integrated must not only obey an ECM, but data generated from the latter must be co-integrated. Hence an additional test on the ECM is to relax the coefficient restrictions implied by the co-integrating regression and to include s_{t-1} and r_{t-1} as separate regressors in the dynamic specification. If the variables are co-integrated then intuitively we might expect the equilibrium solution to these unrestricted ECMs to be similar to the estimated

5. That is Column 1 uses the residuals from s regressed on r, while Column 3 uses the residuals from r regressed on s, etc.

co-integrating regression in Table 3. Table 6 gives estimates of unrestricted models.[6] The statistics F1 and F2 are F-tests on the hypotheses that the coefficients on the constant, r_{t-1} and s_{t-1} (F1) and the coefficients on s_{t-1} and r_{t-1} (F2) are jointly zero. These restrictions are equivalent to imposing a zero co-integrating vector on the data and, with one exception, they are rejected thereby giving support for the ECM – that is, the significance of the lagged levels indicates that these terms have a role in explaining the dynamics of nominal exchange rates and relative prices.

The bottom of Table 6 gives the implied co-integrating equations (Δ terms = 0) for the unrestricted ECMs. Whereas both the estimated and implied co-integrating regressions for the D-Mark are similar, and the equations for r_t are approximately equal to the inverse of the s_t equations, this is not so for the sterling equations. As a final check on the ECM I computed the Dickey-Fuller (1981) Φ_1 and ADF statistics using the computed residuals from the co-integrating equations implied by the unrestricted models.[7] These statistics are given in the last two rows of Table 6. The ADF statistics accept the hypothesis that these residuals are I(1) in all cases. However, the Dickey-Fuller Φ_1 statistic rejects the null of a random walk for the residuals derived from the co-integrating vector for the D-Mark nominal exchange rate.

IV SUMMARY AND CONCLUSIONS

The major conclusions arising from this paper may be summarised as follows. Univariate tests based on Equation (3) support PPP between Ireland and the UK, but reject the hypothesis on Irish/US and Irish/German data, although the latter accepts the PPP hypothesis at the 10 per cent significance level. However, a major defect of these tests is that they impose an $(\alpha, -1)$ constraint on the equilibrium relationship between the nominal exchange rate and relative prices. In terms of the co-integration approach this is equivalent to imposing a prior restriction on the co-integrating vector which, of course, may not exist. When this constraint is relaxed, and PPP is defined more generally as a theory which simply suggests a long-run equilibrium relationship between the nominal exchange rate and the corresponding relative price term, the co-integration analysis fails to find a unique long-run relationship between the Irish/UK series.

6. Note that the regressions in Table 6 include a constant because this is included in the co-integrating regressions. Constant terms in the ECM in Table 5 were insignificantly different from zero.

7. Φ_1 is a test on $(\alpha, \beta) = (0, 1)$ in $x_t = \alpha + \beta x_{t-1}$. Critical values are: 6.70(1%), 4.71(5%) and 3.86(10%).

Table 6: *Error Correction Model – II*

	Sterling		D-Mark	
Dep. Var.	Δs_t	Δr_t	Δs_t	Δr_t
Con.	.021	.033	-.154	-.040
	(2.986)	(1.198)	(2.987)	(0.729)
s_{t-1}	-.132	-.057	-.115	-.025
	(2.550)	(2.709)	(2.988)	(0.670)
r_{t-1}	-.024	-.083	-.099	-.027
	(0.326)	(2.729)	(3.209)	(0.811)
Δs_{t-1}	.424		.382	
	(3.888)		(3.874)	
Δs_{t-3}		-.066		
		(1.437)		
Δs_{t-4}				-.166
				(1.616)
Δs_{t-6}			.273	
			(1.862)	
Δr_{t-3}	-.507			-.247
	(1.935)			(2.347)
Δr_{t-5}			.292	
			(2.899)	
Δr_{t-6}			-.320	
			(3.053)	
R^2	.182	.159	.314	.080
DW	2.095	2.066	1.927	2.413
SEE	.019	.008	.008	.009
CHOW	1.549	3.170	34.993*	2.295
LM(12)	7.669	7.456	8.901	19.135*
ARCH(12)	4.332	18.672	2.897	11.909
F1	3.246*	2.952*	4.014*	7.824*
F2	4.867*	4.412*	5.205*	0.524
Φ_1	3.595	1.590	5.899*	2.966
ADF	-2.483	-2.238	-2.365	-2.129

Notes: F1 is an F-test on the hypothesis that the constant and the coefficients on s_{t-1} and r_{t-1} are jointly zero. F2 is an F-test on the hypothesis that the coefficients on s_{t-1} and r_{t-1} are jointly zero.

* indicates significance at the 5% level.

Implied co-integrating equations are:

Sterling:	$s_t = .159 - .185r_t$:	$r_t = .036 - .686s_t$
D-Mark:	$s_t = -1.339 - .861s_t$:	$r_t = -1.481 - .926s_t$

Co-integration tests on Irish/German data, on the other hand, are slightly less decisive against PPP. ADF and variance ratio tests on the residuals from the co-integrating regressions are consistent with the hypothesis that these series have significant stationary components. There is some evidence that the data can identify an appropriate error correction model and the estimates are reasonably robust against the direction of the test. That is, the equilibrium solution is similar regardless of whether the error correction model is unconstrained or restricted by the prior level co-integrating regression. However, as the ECMs for the D-Mark nominal exchange rate display evidence of parameter instability, the apparent significance of the error correction term must be treated with caution.

Hence it would be difficult to argue that the EMS has been a success in the sense that there is conclusive evidence that co-movements in the Irish pound/D-Mark nominal exchange rate and the corresponding relative price term are consistent with the predictions of PPP, namely long-run convergence to an equilibrium parity value. Further, the failure to find a similar relationship between the sterling nominal exchange rate and the ratio of UK to Irish wholesale prices, suggests that PPP does not hold between Ireland and its most important trading partner. If this is the case then we must take considerable care in assessing the long-run implications for the real exchange rate, and hence competitiveness against the UK, of shocks to the nominal exchange rate such as Irish realignments within the EMS.

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