Labour Supply and Commodity Demands: An Application to Irish Data*

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Abstract: Annual Irish data are used to estimate a model which allows for the joint determination of commodity demands and labour supply. Consumer preferences are modelled by a cost function of the Gorman polar form which permits exact linear aggregation over individuals with different money wage rates. Separability between goods and leisure is rejected by the data. Labour supply is found to be a positive function of the wage rate.

I INTRODUCTION

Over the last ten years there has been considerable interest in the joint modelling of labour supply and commodity demands. Abbot and Ashenfelter (1976) use a variety of models to estimate the joint demands for goods and leisure from annual US time series data. Ashenfelter (1980), using an augmented LES, estimates a model in which the commodity demands of unrationed consumers are combined with those of consumers who are restricted in their choice of leisure (labour supply). Blundell and Walker (1982) estimate similarly rationed and unrationed demands derived from a generalisation of the LES suggested by Deaton and Muellbauer (1981). Using UK cross section data Blundell and Walker decisively reject the hypothesis of separability between goods and leisure as does Barnett (1979) using US times series data.

If separability between goods and leisure is not a sustainable hypothesis then parameter estimates derived from a model which either imposes it or which excludes leisure from the preference ordering will be inconsistent and will result in incorrect inferences. Rejection of separability also has important implications for optimal tax theory which requires knowledge of the substitute/complement patterns between goods and leisure.

The primary objective of the present paper is to utilise annual Irish data for the period 1953 to 1983 to estimate a model which allows for the joint determination of commodity demands and labour supply and which permits an explicit test of the separability hypothesis. As such, our objective is somewhat limited. We do not, for example, include demographic factors or distinguish between male and female labour supply decisions and the rôle of children. Nor do we consider the possibility that some consumers may be rationed in their choice of leisure hours.[1] An adequate treatment of these aspects of household behaviour requires the use of cross-section data or panel data. Unfortunately we do not have direct access to cross-section and Irish panel data do not exist. Being restricted to a time series context we consequently address only those questions which the data may be reasonably expected to deal with.

A further restriction on our approach is the consideration that while individuals may be reasonably assumed to face a common set of commodity prices it is totally implausible to assume that they sell their labour at a common wage. Given that wage rates differ over individual consumers the use of aggregate time series data requires that we work with demand functions that permit aggregation in money wages and non-labour incomes. In order to comply with this restriction we confine our attention to the case of exact linear aggregation and model behaviour by a cost, or expenditure function, of the Gorman polar form. This type of cost function is somewhat restrictive in that it implies that preferences are quasi-homothetic. Thus the Engel curves are linear in full income. The requirement of linear aggregation combined with time-series data eliminates more flexible functional forms such as Deaton and Muellbauer's Almost Ideal Demand System (AIDS). Section II outlines the model and discusses its properties. The estimates are presented in Section III while Section IV discusses the paper's main conclusions. An Appendix is included which gives definitions and sources of the data used.

II THEORY

Consider an individual who is free to allocate a fixed time endowment T between labour supply h and leisure $q_0 = T - h$. Corner solutions are ignored. In addition the consumer allocates money income among the elements of an n-commodity vector q at exogenous prices p. Full expenditure per period is given by:

$$x = wq_0 + p'q \qquad (1)$$

where w is the money wage rate or the opportunity cost of leisure. We also con-

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1. For a comprehensive treatment of these issues see Blundell and Walker (1982).

strain total commodity expenditure per period to equal the sum of labour and non-labour incomes so that $p'q = wh + y$, where y denotes non-labour income. Substitution into (1) gives the full income budget constraint:

$$x = wT + y \tag{2}$$

To derive the consumer's demand system it is first necessary to define a preference ordering over q_0 and q. Throughout this paper we assume that preferences are intertemporally weakly separable and describe within period preferences by the utility function:[2]

$$u = v(q_0, q) \tag{3}$$

and model the individual as choosing leisure and commodities so as to maximise (3) subject to (2) in each period. Alternatively, we may think of the individual as choosing (q_0, q) in each period so as to minimise the full cost of achieving the utility level given by the solution to the primary problem. Hence we can define the cost (or expenditure) function for this problem as:

$$c(w,p,u) = \min(wq_0 + p'q: \text{s.t. } v(q_0,q) = u) \tag{4}$$

By Shepard's Lemma the compensated or Hicksian labour supply and commodity demands are:

$$h = T - c_0(w,p,u) \tag{5}$$

$$q_i = c_i(w,p,u), \quad i = 1 \ldots n. \tag{6}$$

where c_0 c_i are partial derivatives of the cost function with respect to w and p_i respectively. Inverting (4) to give the indirect utility function $u = f(w,p,x)$ and substituting for u in (5) and (6) gives the uncompensated or Marshallian demands:

$$h = T - g_0(w,p,x) \tag{7}$$

$$q_i = g_i(w,p,x), \quad i = 1 \ldots n. \tag{8}$$

With time series data it is desirable to choose a functional form for (4) that allows exact linear aggregation over w and y. Muellbauer (1981) shows that exact linear aggregation over n consumers requires that the "micro" and "macro" cost functions be of the Gorman polar form:

$$c_h(w_h,p,u_h) = a_h(p) + w_h d(p) + w^k b(p)^{(1-k)} u_h; \quad h = 1 \ldots n \tag{9}$$

$$c(w,p,u) = a(p) + wd(p) + w^k b(p)^{(1-k)} u \tag{10}$$

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2. The assumption of weak separability is almost universal in empirical demand studies. We note that in an explicit intertemporal model y equals the change in wealth minus interest income, or savings from labour income. See McCurdy (1983).

where $a(p) = (1/n)\Sigma a_h(p)$ $w = (1/n)\Sigma\ w_h u = (1/n)\Sigma u_h$ and $a(p)$ and $b(p)$ are homogeneous of degree 1, and $d(p)$ is homogeneous of degree zero. If leisure and total commodity expenditure are both normal then $0 < k < 1$. The cost function (10) yields the following uncompensated demands:

$$wh = w(T - d(p)) - kM \tag{11}$$

$$p_i q_i = p_i a_i(p) + w p_i d_i(p) + p_i(1 - k)b_i^*(p)M \tag{12}$$

Where $M = wT + y - a(p) - wd(p)$ and $b_i^*(p) = b_i(p)/b(p)$. Note that the labour hours supply function does not permit linear aggregation over individuals with differing wage rates. We therefore choose to work with a labour income function and to express commodity demands in expenditure terms.[3]

Apart from yielding a tractable demand system (10) also permits a relatively simple test for separability. Goldman and Uzawa (1964) show that goods and leisure are weakly separable if $c_{i0} = zc_{iu}$ where z is a constant for all $i = 1 \ldots n$. (See section (b) of the Appendix). In the case of (10) separability holds when $d_i(p) = 0$ for all i.

III   ESTIMATES

Estimation of the system (11)–(12) requires that we first specify appropriate functional forms for $a(p)$, $d(p)$ and $b(p)$. We choose:

$$a(p) = \sum_i \sum_j a_{ij}(p_i p_j)^{1/2}; \qquad a_{ij} = a_{ji}$$

$$d(p) = d_0 \prod_i p^{di}; \qquad \Sigma\ d_i = 0 \tag{13}$$

$$b(p) = b_0 \prod_i p^{bi}; \qquad \Sigma\ b_i = 1$$

As in the Linear Expenditure system the terms $a(p)$ and $d(p)$ may be interpreted as minimum expenditures on goods and leisure respectively. For example the minimum quantity of good i purchased per period (at zero utility) is given by:

$$q_i = \sum_j a_{ij}(p_j/p_i)^{1/2} + wd_0 d_i \prod p^{di}/p_i \tag{14}$$

Hence minimum quantities are specified as general functions of prices rather than as constants as in the LES. However, whereas the additivity of the LES implies that substitution of one good for another operates through income effects

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3. Deaton and Muellbauer (1981) show that if the cost function is specified so that h is linear in w then the resulting commodity demands are non-linear in w.

only, the use of a second order flexible functional form for a(p) and a non-constant expression for d(p) permits more general substitution effects between goods and between goods and leisure. Given (13) the cost function is a generalised form of the LES and reduces to the latter when the off-diagonal terms in a(p) and the d_i terms $(1 = 1 \ldots n)$ are zero. In this case preferences would be separable and additive.[4] Using (13) gives the following labour supply and commodity demands:

$$wh = w(T - d_0 \Pi p^{di}) - kM \qquad (15)$$

$$p_i q_i = \sum_j a_{ij}(p_i p_j)^{1/2} + wd_0 d_i \Pi p^{di} + (1 - k)b_i M; \quad i = 1 \ldots n. \qquad (16)$$

$$M = wT + y - \sum_i \sum_j a_{ij}(p_i p_j)^{1/2} - wd_0 \Pi p^{di} \qquad (17)$$

Separability between goods and leisure requires that $d_i = 0$ for all $i = 1 \ldots n$ and can be tested using the likelihood ratio test.

To estimate the demand system (15)–(16) we used annual observations on seven classes of non-durable expenditures over the period 1951 to 1983.[5] A full description of the data and their sources are given in the Appendix. Our measures of the wage and labour supply are average earnings and numbers employed. This is necessitated by the lack of a suitable economy-wide series on hours worked. In order to remove the singularity of the system the labour supply equation was deleted and the remaining seven expenditure functions were estimated using the appropriate maximum likelihood procedures in the SHAZAM package. Parameter estimates are presented in Table 1. As data are not available for the time endowment we treated T as a parameter in the estimation. As a rough check on the estimate for T we require that wT be greater than labour income. The estimate for T meets this condition at all sample points. The model was also estimated with separability imposed and a likelihood ratio test used to test the hypothesis that goods and leisure are separable. The statistic 2DLR, equal to twice the absolute difference between the respective log likelihood functions, is distributed as chi-square with six degrees of freedom and indicates a decisive rejection of the separability hypothesis with a critical level of less than .001.[6]

4. Note that minimum leisure hours equal d_0, a constant, under separability.

5.   Initial results derived from a data set which included durables proved unsatisfactory. We note that it is not uncommon for empirical demand studies to exclude durables. See Blundell and Walker (1982), Anderson and Blundell (1983) and Attfield and Browning (1985). Strictly, durables are assumed to be separable from non-durables and leisure.

6.   See Silvey (1970) for a discussion of the properties of the likelihood ratio test. The critical level is the significance level at which the null hypothesis is just rejected.

Table 1: *Parameter Estimates*

	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	a_{i7}	b_i	d_i
Food	569.2	-195.5	104.1	-152.4	138.9	-309.9	-90.9	0.259	-4.44
	(9.5)	(3.8)	(2.2)	(7.6)	(2.8)	(3.9)	(1.8)	(26.4)	(3.4)
Alcohol*		311.1	46.2	-40.9	124.8	-431.0	210.5	0.140	-0.03
		(7.8)	(1.6)	(1.7)	(3.1)	(6.8)	(4.2)	(10.4)	(0.1)
Clothing			107.5	40.2	-184.3	80.1	5.2	0.065	4.47
			(2.1)	(1.8)	(3.5)	(2.8)	(5.6)	(12.0)	(4.0)
Fuel				154.6	-6.7	-5.5	-21.0	0.066	0.06
				(6.9)	(0.2)	(0.2)	(0.7)	(16.2)	(0.1)
Housing					-61.1	179.9	-283.6	0.085	5.83
					(0.8)	(2.1)	(4.4)	(8.60)	(4.6)
Services						105.1	128.1	0.245	-5.11
						(1.5)	(1.3)	(18.8)	(3.5)
Other Goods							347.2	0.134	-0.78
							(3.5)	(—)	(—)

$T = 1962.2 \ (5.9)$; $k = 0.536 \ (6.15)$; $d_0 = 1.657 \ (1.40)$; 2DLR = 79.714.
Figures in parentheses are t-statistics.
*Including tobacco.

The parameter estimates appear to be reasonably "sensible". Leisure and total commodity expenditure are both normal as are the individual commodity groups. Various elasticities, computed at the sample means, are presented in Table 2. The computation of the various elasticities is outlined in the Appendix. Both the uncompensated and compensated labour supply wage elasticities are positive indicating an upward sloping supply curve. This result was replicated at 30 out of 31 sample points, the exception being 1983 which gave an uncompensated wage elasticity of -0.027. All commodities and leisure are unambiguous net substitutes which imply that all commodities and labour supply are net complements. Leisure is a gross substitute for fuel, housing and other goods but a gross complement for the remaining commodity groups. Table 3 presents own-price elasticities based on parameter estimates when separability is imposed on the system. With the exception of services the absolute values of these elasticities are lower than those computed from the unrestricted system suggesting the possibility of a systematic bias in the restricted model.

IV   CONCLUSIONS

This paper presented a model of consumer behaviour in which decisions on commodity demands and labour supply are taken jointly. As such it constitutes an advance on previous empirical studies of Irish consumer behaviour by O'Riordan (1975) and McCarthy (1977) which exclude leisure hours from the

Table 2: *Elasticities*

	Commodities				
	Own Price		Wage Rate		Non-Labour
	Uncomp.	Comp.	Uncomp.	Comp.	Income
Food	-.383	-.262	.695	.514	.171
Alcohol*	-.477	-.409	.683	.509	.164
Clothing	-.324	-.292	.643	.537	.137
Fuel	-.401	-.371	.981	.783	.234
Housing	-.795	-.756	1.334	1.031	.286
Services	-.753	-.693	.620	.461	.149
Other Goods	-.415	-.301	1.247	.939	.316

	Labour Supply Elasticities	
	Uncomp.	Comp.
Food	.262	-.177
Alcohol*	.188	-.101
Clothing	.008	-.055
Fuel	-.009	-.477
Housing	-.035	-.075
Services	.022	-.097
Other Goods	-.057	-.161
Wage Rate	.185	.721

*Including tobacco.

Table 3: *Elasticities – Separability*

	Own Price	
	Uncomp	Comp
Food	-.284	-.162
Alcohol*	-.394	-.327
Clothing	-.211	-.177
Fuel	-.373	-.341
Housing	-.236	-.186
Services	-.979	-.908
Other Goods	-.187	-.079
Labour Supply	.210	.752

*Including tobacco.

preference ordering. The paper's principal conclusion is that the data decisively reject the hypothesis of inseparability between goods and leisure. The estimates also suggest that the uncompensated labour supply curve is upward sloping rather than backward bending. Although these results appear plausible there are several reasons for treating them with some caution. First separability is an

hypothesis about individual preferences so that its rejection in tests based on aggregate data may not have a clear interpretation. Second the functional forms used are somewhat restrictive. Although separability and additivity are not imposed, homogeneity and symmetry are. We note that this is the price which has to be paid for exact linear aggregation over individuals with different wage rates. Third, the model assumes that individuals are free to allocate their time between goods and leisure. A more realistic approach may be one in which some individuals are rationed in their choice of leisure hours. However, as we have shown elsewhere exact linear aggregation over rationed and unrationed consumers is only possible under one of the following restrictive assumptions.[7] (i) Preferences are weakly separable between goods and leisure. (ii) All rationed individuals are rationed at the same level. (iii) All unrationed individuals sell their labour at the same wage. In the case of labour supply the assumptions of a common ration or a common wage are implausible and we wish to test for separability, not to impose it.

Finally, the approach taken in this paper is that dictated by theory. We recognise that certain aspects of the model may be restrictive. However, given the constraints imposed by time-series data, alternative approaches are probably no less restrictive.

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7. See Murphy and Thom (1986).

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APPENDIX

(a) Data

The data consist of annual Irish data for the period 1951-1983. The series on commodity expenditures are taken from various issues of *National Income and Expenditure*. Prices are computed as the ratio of expenditures at current prices to expenditures at constant (1975) prices.

Time-series of economy-wide data on hours is unavailable for Ireland. This did not present any problems for estimation as the labour supply equation is expressed in income terms and is deleted to satisfy the adding-up constraint. As a series on hours is unavailable labour supply is measured as numbers employed and the wage rate as annual earnings.

(b) Separability

Define G and H to be subgroups containing commodities (i = 1 ... n) and leisure (0) respectively. Goldman and Uzawa (1964) show that weak separability between goods and leisure requires that:

$$c_{i0} = z^* q_{ix} q_{0x}$$

for i = 1 ... n. Where c_{i0} is the Slutsky substitution term, q_{ix} and q_{0x} are the slopes of the Engel curves for goods (i = 1 ... n) and leisure and z^* depends on G and H but is independent of i. In terms of the cost function $q_{ix} = c_{iu}/c_u$. The above condition can be recast as:

$$c_{i0} = z c_{iu}$$

where $z = z^* c_{0u}/(c_u c_u)$. The derivative c_u is the inverse of the marginal utility of income and is therefore independent of i under utility maximisation.

(c) Elasticities

The uncompensated own price and wage elasticities are computed in the usual way — i.e., by differentiating (15) - (17) wrt p_i and w multiplying through

by the appropriate price/quantity ratio. The own price compensated elasticities are obtained for the Slutsky decomposition:

$$E^c = E + w_i e_i$$

where E^c is the compensated elasticity, E is the uncompensated elasticity, w_i is the budget share and e_i is the Marshallian income elasticity. Note that the second term on the RHS reduces to $(1 - k)b_i$ and does not depend on the estimate for T. To compute the labour supply wage elasticity we used the elasticity of labour income with respect to w. From (11) and using $x = wT + y$:

$$E_{(wh),w} = (1 - k)(T - d(p))(w/wh)$$

For hours the elasticity is:

$$E_{h,w} = -1 + E_{(wh),w}$$

The wage rate is computed as the net of tax non-agricultural wage bill divided by non-agricultural employment. Non-labour income is obtained as a residual from the budget constraint. That is: $y = p'q - wh$, where wh is the total wage bill.