Chance in Preferential Voting Systems: An Unacceptable Element in Irish Electoral Law?*

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Abstract: The single transferable vote system of proportional representation as operated in elections to Dáil Éireann is one of the most sophisticated electoral systems, maximising the capacity of the voter to determine the outcome of an election and allowing multi-dimensional representation of party political and other interests. Its most significant drawback is the fact that it retains an element of chance in certain circumstances, although this has been reduced to insignificance in the comparable electoral systems of Tasmania, Northern Ireland and the Irish Senate. The random element may occur as a consequence of surplus distribution. If the surplus votes of an elected candidate are fewer than the number of transferable votes in the parcel being examined, Irish electoral law provides, in effect, for a random selection of papers to pass to continuing candidates. Further preferences on these papers may affect the final outcome of the election. This problem could be overcome by amendment of the electoral formula, but in an electoral system where the formula is already very complicated this would place a heavy extra burden on the agencies responsible for the election count. Technological development may, however, allow introduction of the computer in elections, and this could permit the elimination not merely of the random element but also of certain other arbitrary provisions that violate the norms of social choice theory.

I INTRODUCTION

f one assumes that the primary object of any electoral system is to produce a parliament that is representative of those who vote for it, then the yardstick by which a particular system is assessed must be its capacity to reflect voters' preferences. Few would dispute the inadequacy of the traditional plurality ("straight vote") system in this respect. In countries where this system is used (such as the United Kingdom, Canada, the United States and New Zealand) many candidates are elected on a minority vote and the results at

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national level are typically non-proportional. Attempts to remove the more outstanding anomalies of this system within single-member constituencies, such as the two-ballot system (France) or the alternative vote system (Australia, and advocated by the Fianna Fáil annual conference of 1984 for the Republic of Ireland), improve the position at constituency level by ensuring that the winning candidate has a majority of the vote locally but typically give a non-proportional result at national level. List-based systems of proportional representation, however, ensure not only that seats are distributed in multi-member constituencies on a quota basis but also normally give a proportional result at national level.

The list system of proportional representation is the one in general use in western democracies. Much more unusual is the single transferable vote (STV) form: this is used to elect the lower house of parliament in only two sovereign states (Ireland and Malta) but it is also used in direct elections to the Australian Senate and to two subordinate chambers: the House of Assembly of Tasmania and the Northern Ireland Assembly. Despite its cumbersome counting procedures and certain other drawbacks, the STV system may be seen as possessing two significant advantages over list systems of proportional representation. In the first place, it maximises the power of the individual voter. The voter is entitled to rank all candidates listed on the ballot paper; in the most liberal list systems he has a limited number of votes of equal value which he can concentrate on one or more candidates, but more typically cumulation of this kind is not permitted and the voter may not vote for candidates of more than one party. Secondly, STV-type systems seek to promote a multidimensional form of proportional representation, unlike uni-dimensional list systems which seek to secure the proportional representation only of party political loyalties. With the STV, the voter may rank candidates in terms not merely of party but also of regional, social, cultural, gender or other preferences.

Some of the practical costs of the flexibility and sophistication of the STV system as applied in Ireland are obvious: a marginally more difficult activity for the voter, a substantially more complex procedure for the returning officer and a practical limit on constituency size, which, in the absence of any compensatory mechanism, tends to lessen proportionality². A more covert, but no less significant, cost is the intrusion of a random element in the determination of the outcome of an election in certain circumstances³.

^{1.} In the United Kingdom, for instance, 64 per cent of MPs elected in the February 1974 General Election were returned on a minority vote (i.e., more voters opted for rival candidates than for them); and although the Tories "lost" the most recent British General Election in 1983 (in that their share of the poll, already a minority, dropped to 42.4 per cent from 43.9 per cent in 1979) their share of seats increased from 53.5 per cent to 61.1 per cent, and Mrs Thatcher took the view that she had been given a mandate to govern.

^{2.} For a review that covers also the broader political consequences of this type of system see Gallagher (1984).

^{3.} The original standard work on the subject drew attention to this problem at an early stage (Humphreys, 1911), as does the best recent review of the STV form of proportional representation (Lakeman, 1974, pp. 139–144).

The element of chance cannot be eliminated from electoral contests. Under the plurality or majority systems candidates may tie; under list systems, votes for two party lists may be equal, and within each list two or more candidates may each be equally entitled to a seat. In many instances, the tie-breaking mechanism is an arbitrary rather than a random one. Electoral law in continental Europe, for instance, sometimes prescribes that the eldest of two or more candidates who tie be elected. It is our object in this paper to show that in the Irish electoral system there exists a significant random element over and above this omnipresent possibility of a tied vote between candidates; to examine the probability of the outcome of an election being affected by chance; and to explore the possibility of resolving this difficulty and of eliminating certain other arbitrary procedures in the Irish electoral system that purists might find objectionable.

II RANDOM ELEMENTS IN THE IRISH ELECTORAL SYSTEM

Under the STV form of proportional representation as operated in Ireland for general elections to the Dáil, there are three occasions where ties between candidates may take place and where the electoral law provides for random selection⁴.

- (1) On any count, two or more candidates may exceed the quota by the same amount. In such an event, the surplus which is first distributed is that of the candidate with the largest number of first preference votes. If the candidates have the same number, then the first count at which they had an unequal number of votes is used, the surplus of the candidate with the largest number being distributed first. If the numbers of votes are equal on all counts, then the returning officer determines by lot which surplus will be distributed first.
- (2) On any count at which no candidate reaches the quota, two or more candidates may have the lowest number of votes. In such an event, the candidate to be eliminated is the one with the lowest number of first preference votes or, if these are equal, the candidate with the lowest number of votes at the first count at which the number of votes are unequal. If the numbers of votes are equal on all counts, then the returning officer determines by lot which candidate will be eliminated. This provision obviously governs procedure at the last count, when two candidates may tie.
- (3) On any count where a surplus is being distributed and where the surplus is less than the number of transferable papers, another more common but more trivial problem may arise. The surplus votes are allocated in proportion to the distribution of next available preferences in the parcel of votes last received by the elected candidate (i.e., all of his papers if he has been elected on the first

^{4.} The electoral regulations to which we refer were adopted by the Electoral Act, 1923 (No. 12 of 1923). Rules for the counting of votes as amended by subsequent legislation are appended to the official reports on general elections; see, for instance, Ireland (1983), pp. 59–64.

count, or all of the transferred papers that took him over the quota if he has been elected subsequently). Since the number of votes to be transferred to a given continuing candidate is not normally an integer, the election rules provide that as many fractions as necessary (in order of size) be made up to whole numbers to ensure that the full number of transfers takes place, the remaining fractions being disregarded. If a tie occurs between the last fractions, that candidate is chosen who had the largest number of votes on the first or on the earliest subsequent count; if these all tie, the returning officer determines by lot which fraction shall become a unit. As an example: if a surplus of 35 votes is to be distributed between four candidates, A, B, C and D, in the proportions 10:5:3:2, then the candidates are entitled respectively to 17.50, 8.75, 5.25 and 3.50 votes. Since the integer portions of these numbers amount only to 33, the two remaining votes go to B (whose fractional portion is the largest) and to either A or D (whose fractional portions are next largest, but tie) as determined by preferences on earlier counts or by lot.

Selection by lot, as in the three kinds of context outlined above, is a rare event, but chance frequently plays a part in determining who is to be a Dáil member. This happens in certain circumstances following a surplus distribution. When a surplus is distributed, the exercise is not merely notional: physical ballot papers are transferred. But since the surplus is usually less than the number of transferable papers, not all papers may be used; hence, there is a problem of selection. The electoral rules provide that the papers that are to be transferred physically be picked from the top of the existing bundles of transferable papers, which, given the Electoral Act's provision for an initial mixing of ballot papers, effectively gives a random selection⁵. The significance of this is that the composition of the transferred bundles might have a crucial effect on the outcome of the election. One bundle might, for instance, take another candidate over the quota. In that case the newly-elected candidate's surplus would be allocated in proportion to the distribution of next available preferences in the randomly-composed bundle which brought him to the quota; were this bundle to be composed differently (which, given a different initial mixing, it could) then his surplus would be transferred in a different manner. To illustrate this point and to underline its significance, we describe below what happened on one such occasion.

III VICTORY BY CHANCE: AN EXAMPLE FROM THE NOVEMBER 1982 GENERAL ELECTION

We take here the example of the election in Sligo-Leitrim in the November 1982 General Election, when one candidate, Ray MacSharry, defeated another,

5. The provision for mixing ballot papers dates from the Ballot Act of 1872 and was initially intended to ensure that the identity of voters could not be inferred.

John Ellis, for the last seat by chance in the sense that the latter's victory would not have been incompatible with what is known of the voters' wishes. We reproduce the results in Table 1. The constituency was to return four deputies, and Joe McCartin (FG) and Ted Nealon (FG) were elected on the third and fourth counts, respectively, following the elimination of four other candidates. On the fourth count, at which Nealon was elected on McCartin's surplus, three Fianna Fáil candidates remained: Matthew Brennan with 9,004 votes, John Ellis with 8,753 and Ray MacSharry with 8,752. The subsequent distribution of Nealon's surplus put Brennan clearly in the lead, giving him 278 extra votes, but by giving MacSharry an extra 126 to Ellis's 38 put the latter into bottom position, so that his fellow-party members were declared elected without reaching the quota. Yet, from the published results, it appears that Ellis could have won the seat with a lead of 693 votes over MacSharry.

Table 1: Results of general election, November 1982, in Sligo-Leitrim

Valid poll:46,431Number of seats:4Quota:9,287

Candidate	Count 1	Cou (Bree, Gorma inal	Finan, n elim-		nt 3 n elim- ted)	(McC	ent 4 artin's plus)	(Ne	int 5 alon's blus)
		Trans- Total fers		Trans- Total fers		Trans- Total fers		Trans- Tot fers	
Bree (-)	1,832	-1832	_	_	_		_		_
Brennan (FF)	8,167	+ 345	8,512	+ 456	8,968	+ 36	9,004	+278	9,282
Ellis (FF)	8,552	+ 160	8,712	+ 34	8,746	+ 7	8,753	+ 38	8,791
Finan (Lab)	562	- 562	_	-	_	_		_	_
Gorman (N-F	P) 180	- 180	_	~	-	_	_	_	_
Lavin (FG)	4,317	+ 241	4,558	-4,558	_	_	_	_	-
McCartin (FG	8,242	+ 277	8,519	+2,213	10,732	-1,445	9,287	_	9,287
MacSharry (F	F) 7,897	+ 676	8,573	+ 130	8,703	+ 49	8,752	+126	8,878
Nealon (FG)	6,682	+ 544	7,226	+1,662	8,888	+1,353	10,241	-954	9,287
Non-									
transferable	_	+ 331	331	+ 63	394	-	394	+512	906
Total	46,431	_	46,431	~	46,431	_	46,431	_	46,431

Elected: McCartin (Count 3) Nealon (Count 4) Brennan (Count 5) MacSharry (Count 5)

Source: Ireland, 1983, p. 44.

To clarify this point, we look in more detail in Table 2 at the last three counts. This table is based on the assumption that any count may be seen as comprising three components:

- (1) the selection of a parcel of papers for distribution on that count and the sorting of these papers according to next available preferences;
- (2) the attribution of votes to continuing candidates and the physical allocation of ballot papers to each; and
- (3) the calculation of the new aggregate vote for each candidate and declaration of the result of that count.

Table 2: Last	three counts	in Sliga-	I aitrim	aproral	election	Mariamhar	1082
Table 2. Lust	unee towns	m suvo-	сеитт.	уенетаі	election.	November	1704

Continuing	Count 3 (Lavin's votes)			Count 4			Count 5		
Candidate				(McC	artin's sur	plus)	(Nealon's surplus)		
	Papers in parcel	Votes attri- buted	Total votes	Papers in parcel	Votes attri- buted	Total votes	Papers in parcel	Votes attri- buted	Total votes
Brennan (FF)	456	456	8,968	53	36	9,004	278	278	9,282
Ellis (FF)	34	34	8,746	11	7	8,753	38	38	8,791
McCartin (FG)	2,213	2,213	10,732	-2,213	-1,445	9,287	_	_	9,287
MacSharry (FF)	130	130	8,703	73	49	8,752	126	126	8,878
Nealon (FG) Non-trans-	1,662	1,662	8,888	2,007	1,353	10,241	-1,353	-954	9,287
ferable	63	63	394	69	0	394	911	512	906

Notes: Quota = 9,287; four vacancies to be filled.

Source: Ireland, 1983, p. 44, and information supplied by the office of the Returning Officer for Sligo-Leitrim.

The three columns under the heading of each count in Table 2 refer to these three processes; the official results report the contents of Columns (2) and (3), except for the first count, where these are identical. In cases where the parcel in question is made up of papers of an eliminated candidate (as in the third count in Table 2), Column (1) is identical to Column (2). In cases where the parcel consists of votes received by a candidate deemed elected and where the reported number of non-transferable votes is greater than zero (as in the fifth count), the contents of Column (1) may be inferred from Column (2). In cases where the parcel consists of votes received by a candidate deemed elected and where no non-transferable votes are reported (as in the fourth count), the contents of Column (1) can only be obtained from the returning officer; they are not published.

It will be clear that on the third count McCartin was taken over the quota by a parcel of 2,213 papers from Lavin, leaving him with a surplus vote of 1,445. The actual distribution of next preferences among continuing candidates in the full parcel was next ascertained; of the 2,144 transferable votes in the McCartin parcel, 2,007 went to Nealon, for instance, 73 to MacSharry, and so on. Since McCartin's actual surplus was only 1,445, however, the value of each of these elements had to be reduced proportionately (each was to be multiplied by the ratio of the surplus to the number of transferable votes, 1,445/2,144 = 0.67). In

Nealon's case the new figure was 1,353, in MacSharry's 49, and so on. Nealon's 1,353 ballot papers were taken from the top of his sub-parcel of 2,007, MacSharry's from his sub-parcel of 73, and so on. The next available preferences on these 1,353 papers actually selected for physical transfer in Nealon's case have been clearly established, but a different selection of papers from his bundle of 2,007 might have given a different result, as illustrated in Table 3. The next available preferences on the 654 papers that remained in the parcel that took Nealon over the quota can not be established, but if (as is possible but highly unlikely) all of these went to Ellis and if, moreover (as is also possible but also very unlikely), these papers had actually been selected for physical transfer while Brennan and MacSharry received no further papers, then the outcome would have been as in the extreme right-hand column of Table 3. The order of election would have been Ellis-Brennan rather than Brennan-MacSharry. It will be clear that while this exact outcome is theoretically possible but in practice extremely unlikely, a less extreme bias in Ellis's favour (which would be much less improbable) could still leave him with a seat.

Table 3: Possible outcome on distribution of surplus on fifth count in Sligo-Leitrim, general election, November 1982

Next available preference	Actual distribution of Nealon's		selection n fifth count		Other possible selection		
p. g.v.cc	parcel on fifth count	Papers	Votes	Result	Papers	Votes	Result
Not known	654		_	_	_		
Brennan	278	278	278	9,282	0	0	9,004
Ellis	38	38	38	8,791	692	692	9,445
MacSharry	126	126	126	8,878	0	0	8,752
Non-transferable	911	911	512	906	661	262	656
Total	2007	1353	954	_	1353	954	_

One could, of course, pick other extreme cases: one could assume that all of the transfers in the "unknown" bundle passed to Brennan and that the two other continuing candidates received no further preferences, or one could make a similarly favourable assumption about MacSharry. In theory, then, the range of possible results in the Sligo-Leitrim election includes the following three extreme examples in the case of the last two vacancies (the figures in parentheses indicate the results of the last count, elected candidates are in italics and "NT" refers to non-transferable votes):

Brennan (9,936), Ellis (8,753), MacSharry (8,752), NT = 416 Ellis (9,445), Brennan (9,004), MacSharry (8,752), NT = 656 MacSharry (9,532), Brennan (9,004), Ellis (8,753), NT = 568

IV THE ROLE OF PROBABILITY IN DETERMINING ELECTION **OUTCOMES**

It will be clear from the last section that there is a possibility under Irish electoral law that, even if the intervention of counting or calculation errors is discounted, a repeat of the process of initial mixing, counting and distributing of ballot papers from start to finish may result in:

- (1) a different number of votes ultimately being attributed to each candidate; and in some cases
- (2) a different outcome for the election, in that a "defeated" and a "successful" candidate may exchange places.

	Count 1	. Count 2		Count N
Candidate 1	R ₁₁ S ₁₁ T ₁₁	R ₁₂ S ₁₂ T ₁₂		$R_{1n}S_{1n}T_{1n}$
Candidate 2	$R_{21} S_{21} T_{21}$	$R_{22}S_{22}T_{22}$		$R_{2n}S_{2n}T_{2n}$
			• • • •	
Candidate M	$R_{m1} S_{m1} T_{m1}$	$R_{m2}S_{m2}T_{m2}$		$R_{mn}S_{mn}T_{mn}$
Non-trans- ferable	R ₀₁ S ₀₁ T ₀₁	$R_{02} S_{02} T_{02}$		$R_{0n} S_{0n} T_{0n}$
Total	$R_1 S_1 T_1$	$R_2S_2T_2$		$R_n S_n T_n$

Note: $R_{ij} = Number of papers counted$ $S_{ij} = Number of votes attributed$ $T_{ij} = Aggregate or cumulative number of votes attributed.$

It remains to define these two events more precisely and to look at the probability of either event in given conditions. To begin, the process of counting votes in a particular constituency may be envisaged as following the lines indicated in Table 4. In this table R_{ii}, S_{ii} and T_{ii} represent, respectively, the three columns of Table 2: the number of ballot papers counted, the number of votes allocated and the aggregate number of votes for candidate I on count I, where there are M candidates and N counts. The penultimate row refers to non-transferable papers, where R_{0i} represents the total number of non-transferable papers occurring on each count and Soi and Toi represent, respectively, the reported number of non-transferable votes not effective and the aggregate number of non-transferable votes on each count. The bottom row represents the column totals. On the first count all ballot papers are counted, giving the total poll (R₁); invalid papers are separated out (R_{01}) and the number of valid papers attributed to the various candidates is counted $(R_{ij}: i = 1,M)$. The valid votes are then allocated to the appropriate candidates and the outcome of the first count is

known $(T_{il} = S_{il} = R_{il} : i=1,M)$; the total valid poll, T, is known $(T = T_1 = S_1 = R_1 - R_{01})$ and the quota, Q, is calculated:

$$Q = (T/(U+1)) + 1$$
, ignoring any remainder, (1)

where U is the number of seats to be filled. On subsequent counts either a surplus is distributed or the votes of one or more eliminated candidates are distributed. In the former cases, the surplus V_i is calculated:

$$V_{j} = T_{k,j-h} - Q \tag{2}$$

where $T_{k,j-h}$ represents the aggregrate vote of elected candidate K on count J-H the count on which he attained the quota. The ballot papers examined are those in the parcel that took K over the quota, so that:

$$R_{i} = S_{k,i-h} \tag{3}$$

In the latter cases, the ballot papers examined are those of the eliminated candidates, so that:

$$R_{j} = T_{i,j-h} \tag{4}$$

where the summation is over all candidates eliminated on the previous count. It will be noticed that Table 4 resembles an orthodox result sheet for a Dáil constituency in a general election, except that in the latter the full details regarding the distribution of papers at each count (Rii) are not given.

The random element intrudes, as we have indicated, in certain instances of surplus distribution. The procedure for distributing surplus V_i on count J is as follows: R_i is partitioned into its component parts according to next available preferences for continuing candidates, and the number of transferable papers, P_i, is calculated, where:

$$P_{i} = R_{i} - R_{0i}. \tag{5}$$

The next step depends on whether or not P_i exceeds the surplus, V_i ; thus,

if
$$P_j > V_j$$
, then $S_{ij} = R_{ij} * (V_j/P_j)$
and $S_{0j} = o$ (6)

$$\begin{array}{c} \text{and } S_{0j} = o \\ \text{if } P_j \leqslant V_j, \qquad \text{then } S_{ij} = R_{ij} \\ \text{and } S_{0j} = V_j - P_j. \end{array} \tag{6}$$

In the former case, ballot papers are selected at random from R_{ij} to make up S_{ij} ; in the latter case they are selected at random from R_{0i} to make up S_{0i} . The latter case is of no further interest, since the papers thus transferred are not subsequently used. In the former case, however, it is important that the sample drawn reflects accurately the contents of the bundle from which it was taken. (In other words, the distribution of next available preferences for continuing candidates in S_{ij} must be exactly proportional to that in R_{ij}.)

We now turn to look at the *probability* of elections being affected by chance. In the general case we require that the probability be equal to one that, in the case of any sub-parcel of papers passed to a candidate as part of a surplus distribution, the proportion of further preferences for continuing candidates be exactly equal to that in the parcel from which it was drawn. If a parcel of N votes takes a candidate over the quota, leaving him with a surplus of M, and if the parcel contains n papers that transfer later to candidate X, what is the probability that M papers selected at random from this parcel will contain exactly m papers that transfer later to candidate X? (To satisfy the criterion of fairness, of course, we require that m be the nearest integer to (n/N) * M.) This probability may be defined as follows:

$$P(m) = {M \choose m} {N-M \choose n-m} / {N \choose n}$$
 (8)

where $\binom{M}{m}$ denotes m items selected from M items. In Table 5 we tabulate the probabilities for the case where m/M = 0.5. To give a concrete example, Table 5 tells us that if 20 papers are picked at random from a parcel of 100 (in 50 of which the next available preference goes to candidate X), the probability is very small that X will get his due share (of 10 next preferences); more specifically, the probability is 0.1969, or roughly one in five. In the case of other typical selections the probability of an arithmetically just result is similarly low. The values in this table, it should be pointed out, depend on one assumption that is relatively favourable to the electoral system (that there are only two possible outcomes, preferences passing or not passing to X) and one unfavourable assumption (that the ratio between these outcomes is 1:1). As the number of possible outcomes increases, in other words, the values for probabilities corresponding to those in Table 5 drop; but as the ratio between outcomes changes the corresponding values rise.

Table 5: Probability of selecting m items of type X in a sample of size M drawn from a population of size N where there are n items of type X and m/M = n/N = 0.5

				Size of sa	mple (M)				
		10	20	50	100	200	500	1,000	2,000
Size 10	10	1.0000	_	_	_	_	_	_	_
	20	.3437	1.0000	_	_	_	_	-	_
Of 50 100	50	.2748	.2267	1.0000	_	-	_	-	_
	100	.2593	.1969	.1584	1.0000	_	_	-	_
Popu- 20	200	.2525	.1857	.1296	.1124	1.0000	_	_	_
-	500	.2486	.1798	.1183	.0890	.0727	1.0000	-	_
lation	1000	.2473	.1780	.1152	.0839	.0630	.0504	1.0000	_
	2000	.2467	.1771	.1137	.0817	.0594	.0412	.0357	1.0000
(N) 5000	5000	.2463	.1765	.1128	.0804	.0575	.0376	.0282	.0230
	10000	.2462	.1764	.1126	.0800	.0569	.0366	.0266	.0199

It will be clear, then, that in most cases it is overwhelmingly improbable that the final tally of votes will be "accurate" in constituencies where a parcel of votes is further distributed under the conditions described above. In the SligoLeitrim case, for instance, if one assumes that Nealon's parcel of 2,007 votes contains, say, 200 continuing preferences for MacSharry, the probability that a random selection of 1,353 of these will contain MacSharry's exact due share of 135 is very slight. It should, however, be pointed out that a statistically inaccurate result may not prejudice the outcome of the election in terms of who win seats. The "victory margin" of the weakest winning candidate may be taken as indicating an acceptable error margin in those cases where the random element is a factor, and Table 5 could be re-computed for any given range of error. To take the example given above, if in selecting 20 papers from a bundle of 100 (of which 50 give candidate X as next available preference) the requirement that exactly 10 of those chosen also list X next is relaxed to allow an error margin of one on either side (i.e., 9, 10 or 11 papers listing X as next available preference would be acceptable) the chances of a satisfactory outcome rise to 0.5373, or approximately 1 in 2.

On the one hand, then, the degree of arithmetical accuracy will tend to fall as the number of papers chosen at random increases, and the latter will tend to increase as the ratio of valid votes to vacancies increases (the quota would be a crude approximation of this ratio). On the other hand, the winning margin of the last candidate elected will also tend to increase as the quota increases. Thus, although the error in terms of absolute numbers of votes increases as the quota increases, the error as a percentage of the quota will tend to decrease and the probability of the outcome of an election being affected will tend to diminish.

V THE ORIGIN OF THE RANDOM ELEMENT

The rules for counting votes in Irish elections have long roots. The principle on which they were based was invented in 1855 by a Danish mathematician, C. G. Andrae, and applied to elections to the Danish Rigsraad. According to Andrae's system voters ranked candidates, the ballot papers were mixed in an urn and drawn out one by one, and votes were attributed to candidates initially according to first preferences. The random element appeared, however, when any candidate reached the electoral quota: he was immediately declared elected, and his name was ignored in any further ballot papers (Sterne, 1871, pp. 233–237). A similar system was simultaneously devised, apparently independently, by an Englishman, Thomas Hare. Hare's original scheme contained a random element identical to that of Andrae (see Hare, 1859, pp. 186–200). It should be pointed out that the random element in the original Andrae and Hare systems was rather great: it would affect any count on which a surplus was being distributed, and not merely counts subsequent to a surplus distribution.

This system aroused the interest of some intellectuals, but few further attempts at practical application were made, and all of these were in existing or former British possessions. The first such attempt was in Tasmania, where

Hare's system was introduced by the Electoral Act of 1896 on a limited basis. This represented the first stage in the elimination of the random element. The act incorporated a modification of the counting rules by Attorney-General Clark which provided that the surplus votes of an elected candidate be distributed in the same proportions as the distribution of next preferences in the parcel that took him over the quota; ballot papers would, however, in effect be selected at random for physical transfer (Piesse, 1913, pp. 60-62). Its innovatory status meant that this system was intensively studied. In particular, the surviving random element came in for much criticism. The second stage in its removal involved adoption of the "Gregory method", so called after the person who invented it in 1880. This was built into the Tasmanian Electoral Act of 1907, which extended the system to the whole membership of the House of Assembly⁶. The act provided that, when a surplus was to be distributed, all ballot papers in the parcel which brought the successful candidate over the quota be further distributed, though at appropriately reduced value (Piesse, 1913, pp. 67-71). A further refinement introduced in the electoral law of the South African Senate may be interpreted as a third stage in the removal of the random element. This sought to avoid the loss of value owing to disregard of fractions (which could be of considerable importance in a small electorate)⁷ by treating each ballot paper as if it had the value 100 (Flandin, 1914, pp. 43-56).

In Britain itself, where electoral reform was much slower to take root, the sophistication of the Gregory method was felt to be an unnecessary complication. A Royal Commission of Enquiry into electoral systems established by the British Parliament in 1909 felt that the additional accuracy gained by this refinement "would not in our opinion repay the extra labour involved" (United Kingdom, 1910). The regulations adopted for certain limited purposes and advocated by the Proportional Representation Society consequently retained this random element that remained in the original Hare-Clark rules for surplus distribution. It had been intended that this system be adopted for elections to the Senate and, in some constituencies, to the House of Commons of the Irish Parliament proposed in the Government of Ireland Act, 1914. The collapse of this measure meant that, apart from a "model election" carried out through the newspapers under the auspices of the Proportional Representation Society of Ireland in 1911, it was not until the election in 1919 in the troubled borough of Sligo that Irish people got their first opportunity of using this system. The success of this experiment was followed by the introduction of the same system for all local elections in the country by the Local Government (Ireland) Act, 1919, and for elections to the proposed Southern and Northern Houses of Commons by the Government of Ireland

^{6.} This act also replaced the Hare quota (number of valid votes divided by the number of vacancies) by the Droop quota (number of valid votes divided by one more than the number of vacancies) (Piesse, 1912, p. 6).

^{7.} For instance, in the Tasmanian House of Assembly elections of 1979 in the constituency of Franklin 137 votes were "lost" in this manner out of a total valid poll of 48,766 (Tasmania, 1980b, p. 30).

Act, 1920. The regulations then adopted, though dropped in the 1920s for local and parliamentary elections in Northern Ireland when the plurality system was re-introduced, were built into the electoral law of the Irish Free State by the Electoral Act, 1923, and remain the basic regulations for Irish local and general elections. They were further extended to govern popular elections to the Senate on the only occasion on which these took place (1925).

The Irish regulations were adopted in the knowledge of the existence of the random element; this had been pointed out forcefully by one critic of the STV system, who advocated instead a modification of the Belgian list system (Meredith, 1913, pp. 87-90)8. Account has had to be taken of this criticism in the case of Senate elections. The Senates of Southern Ireland and Northern Ireland proposed by the Government of Ireland Act, 1920, were to have elected components, but the electorate in each case would be very small. In the Northern Ireland Senate (1921-1972), for instance, 24 members were to be elected for an eight-year term (half of the membership retiring and being replaced every four years) by 52 members of the House of Commons. As elections were to be by the STV form of proportional representation, the quota would be very small (at most, 5 votes) and the random element, consequently, would be very high. A similar problem existed in the case of the Senate of the Irish Free State (elected by members of the Oireachtas except in 1925) and of the present Seanad Eireann (elected by a very small electoral college up to 1947 and by a larger one since then). The quota for election in Seanad Eireann in recent elections, for instance, has been less than 100 in the case of most panels. It was, consequently, found necessary to adopt the Gregory modification in elections to the Senates created by the Government of Ireland Act (United Kingdom, 1921), and a similar reform took place in the South (Ireland, 1927, pp. 117-119). These regulations, incorporating the Gregory modification, continue to apply in elections to Seanad Eireann. They are also advocated by the Electoral Reform Society of the United Kingdom, which has built the Gregory modification into its set of model election rules (Newland and Britton, 1976); this was adopted also in the electoral law for the Northern Ireland Assembly (United Kingdom, 1973).

It may be appropriate at this stage to summarise the measures that have been taken to reduce the random element in STV-based systems of proportional representation. Of the five most important directly-elected parliamentary assemblies that use this system, the electoral regulations in three (Dáil Éireann, the House of Representatives of Malta and the Australian Senate) retain the original Hare-Clark rules for surplus distribution and thus permit the intrusion

^{8.} The drafters of the Electoral Act of 1923 calculated the probable effects of the random element but decided that they would not justify adoption of the Gregory modification (information provided by Ernest Blythe to Noel Mulcahy in 1968).

^{9.} For reviews of the two most important systems outside Ireland, see Gregory (1971), Proctor (1980) and O'Connell (1983).

of a random element in certain circumstances where surplus votes are being distributed (Ireland, 1983, pp. 59–64; Malta, 1970, art. 11; Australia, 1980, art. 135). It should, of course, be noted that the enormous Australian electorate reduces the random element to insignificance – in New South Wales, for instance, the quota is typically in the region of half a million votes. In the two remaining cases (the Northern Ireland Assembly and the Tasmanian House of Assembly) this element is reduced to insignificance by the adoption of the Gregory procedure (United Kingdom, 1973, art. 56; Tasmania, 1980a, schedule 4); and this is the case also in indirect elections to the Irish Senate (Ireland, 1982, pp. 45–47). It should be mentioned that the additional refinement of treating each ballot paper as if it had the value 100 (adopted for the Senates of Northern Ireland and of Southern Ireland on the South African model) has been further developed in elections to the Irish Senate, where each paper is treated as if it had the value 1000; in Northern Ireland, values are instead calculated to two decimal places.

The essence of the Gregory procedure is that when a surplus is being distributed all of the votes in the parcel last received by the candidate who has been elected are passed on, though at a value reduced in proportion to the ratio between the surplus and the number of transferable votes. In other words, in the terminology used above, each continuing candidate receives a number of ballot papers equal to R_{ij} , so that $S_{ij} = R_{ij}$, but each of these papers has the value S_j/P_j . While this significantly increases the amount of paper-handling and calculation entailed in an election count, it also introduces a complication when votes of eliminated candidates are being distributed. In addition to his own first preference votes, a candidate may have received transferred votes at a variety of values. The rules provide that the parcels of an eliminated candidate be redistributed in stages in the order in which he received them; and a candidate who reaches the quota at the end of one such stage may be declared elected, before the process of vote transfer has been completed.

VI CONCLUSION: REMOVING THE RANDOM ELEMENT FROM IRESH ELECTIONS

The complexity and cumbersome counting procedures that characterise Irish electoral law are part of the price that must be paid for an electoral system that gives the voter so great a degree of flexibility in choosing between candidates and parties at little risk of vote wastage. The addition of an extra cost, in the form of the presence of a random element in certain circumstances, might seem a small burden and one that is worth bearing when the advantages of the Irish electoral system are set against the disadvantages of the predominant list system of proportional representation, of which the most obvious is the narrowing of the voter's options and the advantage given to party organisations. The plurality-based electoral systems of the major English-speaking countries, in

addition to enhancing the role of the political parties and restricting the voter's choice to a single dimension (i.e., to vote for the party or to vote for the person), provide an added measure of crudity and capriciousness, making the STV system seem almost flawless. It has been shown above that in certain circumstances that occur quite frequently in Irish elections (and where there are no counting errors) the probability of a mathematically exact reproduction of the result on a complete recount is very low, but the probability of the same general outcome (in terms of persons elected) is very high. Even this chance element can be removed with a little trouble.

The more complicated the system the less intuitively obvious its fairness to the electorate and the more difficult it is for the returning officer to administer. Procedural and counting errors and the random component in the system are tolerated, presumably, in part at least because the candidates involved, their agents and the electorate find the finer points of the election rules bewildering. Furthermore, the rules seek to minimise the risk of a different result in the event of a recount by providing for the perforation or marking of papers selected for transfer; the same papers should thus be selected in any recount, disguising the random element. Adoption of the Gregory modification, which would undoubtedly produce a fairer result, would be unlikely to lead to further public enlightenment or to lessen significantly the possibility of error. The computer would seem to offer a solution to some of these difficulties, and could also resolve some other anomalies of STV-based proportional representation.

The existence of a random element in the Irish electoral system is the consequence of a short cut intended to simplify counting procedures. For similar reasons other arbitrary decisions have been made. The most significant of these is the rule, common to all STV-type systems, that in distributing surpluses only the votes in the parcel last received by the elected candidates be considered. It has been argued that this factor renders the STV system less than satisfactory by the standards of social choice theory, in that it fails to minimise the temptation to tactical voting and to maximise equality of votes (Meek, 1969). It is, however, possible in theory to overcome these difficulties: introduction of a scaling factor for each candidate on each count (0 for eliminated candidates, 1 for continuing, non-elected candidates and a value ranging between 0 and 1 for elected candidates) would permit account to be taken of further preferences in all of the papers of all elected candidates, and would allow transferred votes to be attributed also to candidates who had already exceeded the quota (Meek, 1969; Meek, 1970). The very complicated calculations involved would present no problem in a computer-based system, which could simultaneously eliminate this arbitrary element and the random element (Woodall, 1982).

Even more radically, use of the computer would permit greater flexibility in the adoption of rules for the counting of votes. It has been pointed out that the Hare system may result in the elimination of the Condorcet winner (i.e., that candidate who, in a succession of two-candidate contests, would be preferred to every other candidate) (Nurmi, 1983), but a computer-based counting system could be fundamentally different while retaining the same voting procedure and remaining compatible with existing constitutional provisions.

It is, however, ironic that computers have been used so far only in instances where, as in the United States and Germany, the electoral system is very simple and only elementary processes of counting are required. The difficulty of devising a satisfactory input mechanism for a computer-based voting system with the STV has discouraged certain British electoral reformers from advocating too strongly the introduction of mechanical voting procedures in the United Kingdom (opponents of reform could, for instance, support the introduction of the computer but argue that this rules out use of the relatively complicated preferential voting system). In addition, provision of equipment for mechanical or electronic recording of votes in polling booths or of facilities for converting traditional ballot papers into machine-readable form at some central location might be prohibitively expensive, and the latter, which would involve the punching up or keying in of each ballot paper, might make the count even slower than it is at present (Simpson, 1981).

The computer has, nevertheless, been used in Cambridge, Mass., in STV elections, and a demonstration of its usefulness was made by Professor P. M. Quinlan during the 1968 referendum campaign on the proposed abolition of proportional representation. In order to counter the allegation that the random element was a necessary feature of the STV system, he held a mock election in Cork in which the count was carried out by computer; not alone did his programme eliminate the random element, but the error-free count lasted only half an hour¹⁰. Although Professor Quinlan's prediction that Irish election results would be produced by computers within five years has not been borne out, it is possible that technological developments may eventually make the prospect of reconciling the STV system with the computer more realistic (for instance, use of sophisticated optical readers might cut down the need for a large volume of expensive hardware).

The introduction of the paraphernalia of the computer age to Irish polling booths might seem utopian; the electorate would have to be assured that the secrecy of the act of voting would be maintained and that the possibility of fraud would be eliminated. These problems have been overcome where, as in the United States, voting machines are commonly used. At the present stage of technological and social development a more serious problem remains: the competence (objective and subjective) of the voter to use an electronic or mechanical instrument, however simple, to order candidates (though learning to use such instruments is less demanding than learning to read). Yet the shift to such a system, which puts a premium only on a willingness and ability to adapt to technological change, is surely less revolutionary than the shift from oral

voting to voting by ballot paper (1872), which put a premium on literacy. Would it be any more objectionable for a returning officer to assist computerphobes in pressing buttons than for him to assist illiterates in filling out ballot papers? If not, if prejudice against the computer could be overcome and if candidates and observers were prepared to forgo the doubtful pleasures and excitement of a long drawn-out count, then the computer might present the answer to the main technical flaw in the Irish electoral system and might pave the way for a radical re-consideration of the rules for counting votes.

REFERENCES

- AUSTRALIA 1980. Commonwealth Electoral Act 1918, Incorporating all Amendments by Legislation made to 30 June 1980, Canberra: Commonwealth Government Printer.
- FLANDIN, PIERRE-ETIENNE 1914. La Question de la Représentation proportionnelle en Angleterre et dans les Colonies Anglaises: le Vote Transférable, Paris: Librairie Dalloz.
- GALLAGHER, MICHAEL 1984. "The Political Consequences of the Electoral System in the Republic of Ireland", Paper presented to the ECPR workshop on Comparing Electoral Systems, Salzburg, 13–18 April 1984.
- GREGORY, MAURICE 1971. "The Method of Voting in Malta", *The Parliamentarian*, Vol. 52, No. 4, pp. 275–277.
- HARE, THOMAS 1859. A Treatise on the Election of Representatives, Parliamentary and Municipal, London: Longman, Brown, Green, Longmans and Roberts.
- HUMPHREYS, JOHN 1911. Proportional Representation, London: Methuen.
- IRELAND 1927. Department of Local Government and Public Health: First Report 1922-1925, Dublin: Stationery Office.
- IRELAND 1982. Seanad General Election, August 1981, and Bye-Elections to 1977-81 Seanad, Dublin: Stationery Office.
- IRELAND 1983. Election Results and Transfer of Votes in General Election (November, 1982) for Twenty-Fourth Dáil and Bye-Elections to Twenty-Third Dáil (March-November, 1982), Dublin: Stationery Office.
- LAKEMAN, ENID 1974. How Democracies Vote: A Study of Electoral Systems 4th ed., London: Faber and Faber.
- MALTA 1970. The Polling Regulations, 1939 [as amended up to 31 January 1970], Valetta: Department of Information.
- MEEK, B.L., 1969. "Une Nouvelle Approche du Scrutin Transférable", Mathématiques et Sciences Humaines, Vol. 7, No. 25, pp. 13-23.
- MEEK, B.L., 1970. "Une Nouvelle Approche du Scrutin Transférable (fin)", Mathématiques et Sciences Humaines, Vol. 9, No. 29, pp. 33-39.
- MEREDITH, J.C., 1913. Proportional Representation in Ireland, Dublin: Edward Ponsonby.
- NEWLAND, ROBERT A., and FRANK S. BRITTON 1976. How to Conduct an Election by the Single Transferable Vote, London: Electoral Reform Society.
- NURMI, HANNU 1983. "Voting Procedures: a Summary Analysis", British Journal of Political Science Vol. 13, No. 2, pp. 181-208.
- O'CONNELL, DECLAN 1983. "Proportional Representation and Intraparty Competition in Tasmania and the Republic of Ireland", Journal of Commonwealth and Comparative Politics, Vol. 21, No. 1, pp. 45-70.

- PIESSE, E.L., 1912. "The Theory of the Quota in Proportional Representation", Fapers and Proceedings of the Royal Society of Tasmania, pp. 1-30.
- PIESSE, E.L., 1913. "Bibliography of Proportional Representation in Tasmania", Papers and Proceedings of the Royal Society of Tasmania, pp. 39-75.
- PROCTOR, J.H., 1980. "The Acceptance of Proportional Representation in Malta", Parliamentary Affairs Vol. 33, No. 3, pp. 308-321.
- SIMPSON, MICHAEL 1981. "Elections and the Chip", Representation Vol. 21, No. 83, pp. 17–19.
- STERNE, SIMON 1871. On Representative Government and Personal Representation, Philadelphia: J.B. Lippincott and Co.
- TASMANIA 1980a. Electoral Act 1907 [as amended up to 1 January 1980], Hobart: Government Printer.
- TASMANIA 1980b. Report on Parliamentary Elections 1978 to 1980, Hobart: Government Printer.
- UNITED KINGDOM 1910. Report of the Royal Commission on Electoral Reform (Cmnd 5163), London: HMSO.
- UNITED KINGDOM 1921. Ireland, Government of: The Irish Senates (Proportional Representation) Order, 1921, Statutory Rules and Orders, 1921, No. 727, London: HMSO.
- UNITED KINGDOM 1973. Northern Ireland: The Northern Ireland Assembly (Election) Order 1973, Statutory Instruments, 1973, No. 890, London: HMSO.
- WOODALL, D.R., 1982. "Computer Counting in STV Elections", Representation, Vol. 23, No. 90, pp. 4-6.