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Notes and Comments

A Note on Demand Elasticities for Energy Imports

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Abstract: Annual Irish data on fuel imports are used to estimate inter-fuel substitution models. Both a four- and a three-input classification are employed in the estimation of a translog cost function. Restrictions implied by neo-classical production theory are imposed and statistically tested. The concavity and monotonicity conditions of the cost functions are also examined. Both models reject the neo-classical restrictions and their performances with regard to concavity and monotonicity conditions are mixed. Both cross- and own-price elasticities of demand for the fuel imports are also reported.

I. INTRODUCTION

Irish energy imports comprised over 64 per cent of the economy's primary energy requirements in 1982 and in the same year accounted for over 8 per cent of GDP. There has been significant change in the composition of energy imports over the past two decades. The purpose of this note is to analyse the effects of relative price changes on the structure of energy imports. This study estimates an energy import cost function for Irish data using a flexible functional form and reports estimates for both own and cross price demand elasticities for energy imports.

It is assumed that energy imports are weakly separable from domestic energy.[1] Clearly, there is no strong theoretical justification for this treatment. The approach adopted is largely dictated by the absence of Irish energy data and, in particular, energy price data. The *Trade Statistics of Ireland* (see below) provides information on imported fuels and the data used in this note are obtained exclusively from this source.

[1]The assumption of weak separability of imported energy and domestic energy is not explicitly tested here.

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II. METHODOLOGY

The introduction of flexible functional forms has facilitated the estimation of production structures which impose few *a priori* restrictions on demand elasticities. A twice differentiable production function which is homothetically weakly separable in its inputs (one of which is imported energy) is assumed to exist.[2] As Fuss (1977) points out aggregates only exist under the assumption of weak separability.[3] Further, the existence of aggregates which are homothetic in their inputs permits a two-stage optimising procedure. Firstly, firms choose the optimal mix within each aggregate and then the optimal levels for each aggregate. Homotheticity is a necessary and sufficient condition for the optimality of such a two-stage procedure. This note confines itself to the first stage as applied to the import energy aggregator.

Expressing the import energy aggregator as

$$Z = f(z_1, \ldots z_n) \tag{1}$$

where the z_is represent imported energy inputs. Following Shephard's duality theorem (1953) the cost function may be expressed as

$$C = g(p_1, \ldots p_n, Z) \tag{2}$$

with the subscripts as above. This duality theorem ensures that the cost function parameters describe a set of parameters that characterise an underlying production possibility set. In this note the analysis focuses on the cost function assuming that the import energy prices facing the Irish economy are exogenous.

The transcendental logarithmic functional form (translog) provides a second order local approximation to any twice differentiable cost function. For a more extensive treatment of the translog in this particular context see Halvorsen (1977), Fuss (1977), Pindyck (1978) or Berndt and Wood (1979). The logarithmic differentiation of the translog cost function with respect to input prices yields, via Shephard's lemma, input demand functions which may be expressed as share equations for the i imported energy imputs. The cost shares denoted by M_i may be written as

$$M_i = a_i + \sum_{j=1}^{n} \beta_{ij} \log p_j + \beta_{zi} \log Z, \tag{3}$$

where $i, j = 1, \ldots, n$.

[2]Hicks-neutral technical progress is assumed to exist.
[3]See Berndt and Christensen (1973).

Adding-up and neo-classical production theory imply the following set of parameter restrictions

$$\sum_{i=1}^{n} a_i = 1, \tag{4a}$$

$$\sum_{j=1}^{n} \beta_{ij} = 0, \tag{4b}$$

$$\beta_{zi} = 0, \tag{4c}$$

$$\beta_{ij} = \beta_{ji}. \tag{4d}$$

The first two restrictions of (4a) and (4b) ensure adding-up, (4c) represents the set of restrictions implied by homotheticity. This restriction is equivalent to assuming that the optimal imported energy cost shares are independent of total import energy cost. The final set of restrictions in (4d) ensure that symmetry is fulfilled and also guarantee Allen symmetry of the cross partial elasticities.

The Allen partial elasticities of substitution (AES) derived from the translog cost function are of the form

$$\sigma_{ii} = \frac{\hat{\beta}_{ii} + \hat{M}_i^2 - \hat{M}_i}{\hat{M}_i^2} \tag{5}$$

$$\sigma_{ij} = \frac{\hat{\beta}_{ij} + \hat{M}_i \hat{M} j}{\hat{M}_i \hat{M}_j} \tag{6}$$

where $i,j = 1, \ldots, n$, the $\hat{M}_i$s are the fitted cost shares and the $\hat{\beta}_{ij}$s are the estimated parameters. The own and cross price demand elasticities are related to the AES and may be written respectively as

$$\eta_{ii} = \sigma_{ii} \hat{M}_i \tag{7}$$

$$\eta_{ij} = \sigma_{ij} \hat{M}_j \tag{8}$$

III. DATA

Section three of the *Trade Statistics of Ireland* provides data on fuel imports broken down by fuel type and expressed in value and quantity terms. The

December issue of each year contains annual estimates of the previous year's imports. Various issues of this publication and its predecessor, *The Irish Trade Statistics*, are used to obtain a consistent value and quantity series from 1960 to 1983.

Since both a four-input and a three-input model are estimated all imported fuels are allocated intitially across four (coal, crude oil, refined oil and manufactured gas) and then three broad classifications (coal, a composite of crude and refined and manufactured gas). Both a value and a constant 1975 price series are constructed from the data for the broad classifications used. The cost shares used in the estimation are derived from the above value series. The price series used in the estimation are the implicit price deflators with base 1975 equal to unity (see Appendix).

IV. ESTIMATION

The system of input demand equations described by (3) is estimated for both a four-input and a three-input model. Restriction (4b) (linear homogeneity input prices) is imposed yielding cost share equations as linear functions of input price ratios. In order to allow for errors in cost minimising behaviour, etc., error terms are appended and the estimating equations are given by

$$M_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \log\left(\frac{p_i}{p_j}\right) + \beta_{zi} \log Z + u_i \qquad (9)$$

where all the variables[4] are as before with u_i an error term.

Since the dependent variable are the cost shares and since both the actual and the predicted values of the cost shares will sum to unity, the sum of the error terms across the equations will sum to zero at each observation point. This implies that the covariance matrix of the disturbance terms, Ω, is singular and non-diagonal. Thus, one equation (gas imports) is dropped in both models and the disturbance column vector, $u(t)$, is specified as identically normally distributed with mean vector zero and a non-singular covariance matrix. The choice of omitted equation is purely arbitrary since a maximum likelihood estimator is used. Barten (1969) has shown that with maximum likelihood the parameter estimates are invariant as to the excluded equation.

The unestimated parameters are calculated from the restrictions in (4a) and (4b). Since these restrictions are linear in the parameters no problem attaches to the calculation of their respective variances. Following Pindyck (1978) the estimated variances of the elasticities are calculated as $var(\eta_{ii}) = var(\hat{\beta}_{ii})/\hat{M}_i^2$ and $var(\eta_{ij}) = var(\hat{\beta}_{ij})/\hat{M}_i^2$.

Finally, Government policy with respect to Whitegate resulted in a diminu-

[4]The Z variable is the total cost of the energy imports under consideration deflated by a share weighted composite price index.

tion of Ireland's capacity to import crude oil. This was matched by an equally large increase in the level of refined imports. Thus, a dummy variable with ones for 1981 and 1982 is used in the four-input case. To ensure adding-up dummy coefficients are estimated for all three equations in the four-input model.

V. STATISTICAL TESTS

The performance of both the four-input model (coal, crude, refined and gas) and the three-input model (coal, oil and gas) is examined by first testing the homotheticity restriction of (4c) and, then, the symmetry restrictions of (4d).[5] A standard likelihood ratio test distributed as χ^2 is employed for these tests. The regularity conditions of concavity and monotonicity are also checked. The former condition implies a negative relationship between input price and demand with the latter condition implying that free disposal prevails (i.e., an increase in inputs cannot decrease production).

In terms of the four-input model the three restrictions implied by homotheticity are imposed and tested. The resultant test statistic is 18.63 and with a critical value of 11.34 at the 1 per cent level of significance these restrictions are rejected. However, for the purposes of this analysis, homotheticity is imposed and the symmetry restrictions of (4d) are subsequently tested.[6] The test statistic is 70.45 with three degrees of freedom indicating a decisive rejection of the symmetry hypothesis. However, since symmetry is a requirement for the existence of a well behaved cost function this study imposes the cross-equation restrictions despite this rejection.

A check for monotonicity is provided by examining the fitted values of the cost shares at each data point to establish whether they are positive and their sum is not larger than unity. The fitted values satisfy both these criteria for the four-input case. Jorgenson and Lau (1975) provide a statistical test for monotonicity which consists of examining the intercept terms to ascertain whether they are positive and significant. All the a_is are positive and significant at the 1 per cent level in the four-input case (see Table 1).

Concavity of the cost function requires that the Hessian of the cost function be negative semi-definite. The Hessian may be expressed as

$$H_{ij} = \left(\frac{\hat{M}_i}{p_i}\right) \left(\frac{\hat{M}_j}{p_j}\right) \quad \sigma_{ij} \tag{10}$$

where $i,j = 1, \ldots, n$.

[5]Homotheticity is a necessary but not a sufficient condition for the technology to exhibit constant returns to scale.

[6]As Mizon (1977) points out the test of the symmetry restrictions is not an independent test as a consequence of the encountered rejection.

Table 1: *Parameter estimates for four-input translog*

Coefficient	*Estimate*	*Standard Error*
a_1	.1849**	.0263
a_2	.4112**	.0264
a_3	.3890**	.0478
a_4	.0148**	.0023
β_{11}	-0.0864	.0792
β_{22}	.1235	.1390
β_{33}	-0.0675	.2593
β_{44}	.0064	.0049
β_{12}	-0.0623	.0658
β_{13}	.1502	.1299
β_{14}	-0.0014	.0064
β_{23}	-0.0694	.1753
β_{24}	.0083	.0124
β_{34}	-0.0133	.0146

** and * denote significance at the 1 per cent and 5 per cent levels respectively and 1=coal, 2=crude oil, 3=refined oil and 4=gas.

Since the translog is a local approximation, concavity must be verified at each observation point. The Hessian of the cost function is a square matrix of dimensions $n \times n$ and rank $n - 1$ (as this rank condition was imposed when one equation was deleted in estimation). Thus, the condition for negative semi-definiteness is satisfied if the $n - 1$ eigenvalues associated with the Hessian are all negative (the n^{th} eigenvalue being obviously zero). This condition for negative semi-definiteness of the Hessian is met at only nine of the twenty-four observation points in the sample. Significantly, the negativity condition is upheld for the years 1974 to 1980 inclusive (with 1970 and 1983 providing the other two years for which the condition is also upheld). The eigenvalues at the means of the data are 0.1292, -4,0905 and -1,7376.

There are, however, grounds for believing that crude and refined oil should not be treated as separate inputs. The existence of limited refining capacity and the fact that crude oil is not used as a direct energy input lend support to this view. Following Berndt and Wood (1979) a linear separability restriction is imposed and tested to establish whether a condition for the consistent aggregation of crude and refined oil is upheld. The likelihood ratio test value for this restriction is 3.798 with four degrees of freedom and the associated critical value for the χ^2 is 13.28 at the 1 per cent level of significance.[7] Thus, it is clear that the

[7]The restriction in this case is given by $\beta_{12} = \beta_{13} = \beta_{24} = \beta_{34} = 0$.

data cannot reject a necessary condition for the consistent aggregation of crude and refined oil.

Therefore, it was deemed appropriate to re-do the exercise using a three-input model aggregating crude and refined oil.[8] The same set of statistical tests were carried out as in the four-input case. First, the two homotheticity restrictions are imposed and tested yielding a likelihood ratio test value of 32.844 and, consequently, a rejection of the hypothesis under consideration.[9] As in the above case, homotheticity is imposed and the one symmetry restriction associated with the three-input case (i.e., $\beta_{12} = \beta_{21}$) is tested and rejected with a χ_1^2 value of 32.658 recorded.

However, the necessary requirements for monotonicity are met by the fitted values assumed by the cost shares. The positive and highly significant values adopted by the intercept terms (see Table 2) provide statistical confirmation of the monotonicity condition.

Concavity is examined, again, at each observation point by calculating the eigenvalues of the Hessian matrix in (10). The condition for negative semi-

Table 2: *Parameter estimates for three-input translog*

Coefficient	*Estimate*	*Standard Error*
a_1	.16424**	.02444
a_2	.81883**	.02236
a_3	.01692	.00265
β_{11}	.14839	.10125
β_{22}	.10679	.08968
β_{33}	.01149*	.00541
β_{12}	-0.12185	.09635
β_{13}	-0.02654**	.00744
β_{23}	.01506**	.00751

** and * denote significance at the 1% and 5% levels respectively and 1=coal, 2=oil and 3=gas.

[8] A composite oil price index based on the relative weights of crude and refined in the newly defined oil category was constructed for use in the three-input case.

[9] It has been pointed out by a referee that the treatment of the total energy cost variable, Z, as exogenous may be inappropriate. It should be assumed endogenous and an instrumental variable estimator employed in estimation. Using three-stage least squares (3SLS), however, does not produce coefficient estimates that are invariant to the deleted equation. Ideally, an iterative three-stage least squares (I3SLS) estimator should be employed. Due to the unavailability of such a routine, however, fitted values from an ordinary least squares (OLS) regression of the Z variable on a number of instruments (GNP, industrial output, a measure of capacity utilisation obtained from the Central Bank of Ireland and the prices of the energy inputs themselves) were used in FIML regressions for both the three and four-input models and the homotheticity restrictions were re-tested using a likelihood ratio test. The R^2s for the instrument equations were 0.92 and 0.90 for the four- and three-input models respectively. The likelihood ratio tests values are 10.786 and 23.134, respectively. Thus, the homotheticity findings are reversed for the four-input model but are unaffected for the three input model when the endogeneity of the Z variable is taken into consideration.

definiteness is not upheld by the data at any of the observation points. The eigenvalues recorded at the means of the data are 0.0661 and -0.8945.

It is clear from this section that there is very little evidence in favour of either a well-behaved cost function or cost-minimising behaviour. The decisive rejection of homotheticity in terms of the three-input model casts doubt about the validity of the assumed production structure and its implicit assumption of weak separability between domestic and imported energy.

VI. PARAMETER ESTIMATES AND DEMAND ELASTICITIES

Tables 1 and 2 record the parameter estimates for the four- and three-input models, respectively, with homotheticity and symmetry imposed. In terms of the former model only the intercept terms are in any way significant. In contrast, the parameter estimates for the latter model are, in general, well determined with six of its nine estimated coefficients significant at either the 1 per cent or the 5 per cent level.[10]

Demand elasticities for the two models appear in Tables 3 and 4. All the elasticities are evaluated at the means of the data and should be interpreted as partial elasticities since the energy import aggregate is constrained to be constant.

All the own-price elasticities derived from the four-input model exhibit *a priori* the correct negative sign. Only the own-price elasticities of coal and gas are in any way significant (the former at the 1 per cent level and the latter at the 10 per cent level). The evidence of Table 3 indicates that imported coal is the most

Table 3: *Demand elasticities for the four-input translog*

Fuel	*Coal*	*Crude*	*Refined*	*Gas*
Coal	-1.3378**	-0.0220	.5335	.0782
	(0.4955)	(0.2543)	(0.3108)	(0.4608)
Crude	-0.0113	-0.2387	.1983	.9484**
	(0.4116)	(0.5372)	(0.4192)	(0.0479)
Refined	1.3418	.2180	-0.5986	-0.4950
	(0.8130)	(0.6775)	(0.6198)	(1.0508)
Gas	.0060	.0414	-0.0167	-0.5330
	(0.0401)	(0.0479)	(0.0350)	(0.3531)

Standard errors are in parentheses with the rows giving the effects of changes in price on the quantities demanded of the relevant fuels.
* and ** denote significance at the 1 per cent and 5 per cent level respectively.

[10]The joint significance from zero of the dummy coefficients was tested using a likelihood ratio test. The value of the χ_3^2 in this case was 29.334 indicating rejection of the null hypothesis.

Table 4: *Demand elasticities for the three-input translog*

Fuel	*Coal*	*Oil*	*Gas*
Coal	.0503	.0278	—1.8872**
	(0.6103)	(0.1201)	(0.6760)
Oil	.0895	-0.0625	1.9767**
	(0.5806)	(0.1118)	(0.6825)
Gas	-0.1398**	.0347**	-0.0895
	(0.0499)	(0.0094)	(0.4917)

Standard errors are in parentheses with rows giving the effects of changes in price on the quantities demanded of the relevant fuels.
** and * denote significance at the 1 per cent and 5 per cent levels respectively.

responsive to a change in its own price, recording an elasticity value in excess of unity. The signs adopted by the cross-price elasticities allow a distinction to be made in terms of substitutes and complements. Of the six pairs of cross-price elasticities associated with the four-input case, two are interpreted as complements (coal and crude and refined and gas) and four (coal and gas, crude and gas, coal and refined and crude and refined) as substitutes.

In general, the magnitude of the elasticities obtained from the three-input model are considerably lower than those obtained from estimating with the wider input classification. Four of the nine demand elasticities derived in this case are significant at the 1 per cent level as Table 4 shows. However, none of the own-price elasticities is significant and one,coal, exhibits a perverse positive sign. The classification of the fuels into substitutes and complements appears sensitive to the aggregation of crude and refined. In contrast to the four-input case, coal and gas are now interpreted as complementary fuels. The signs assumed by the remaining two pairs (oil and coal and gas and oil) indicate substitute inputs. Nevertheless, these findings might be interpreted as being more consistent with one's priors than the results suggested by the four-input model.

VII. CONCLUSIONS

The coefficient and elasticity estimates associated with the three-input translog are, in general, more well determined than those obtained from the four-input model. For reasons outlined above the former model might also be regarded as being the more appropriate specification of the two. However, in terms of satisfying the restrictions implied by neo-classical production theory neither of the models could have been said to have performed with distinction. Though treating the cost of total imported energy as endogenous (see footnote 9) was found to reverse the rejection of homotheticity in the four-input case, no such reversal was found to occur in the context of the three-input model. This suggests, at worst, that the assumption of a production function homothetic and weakly separable in domestic and imported energy inputs for the three-input model is suspect. The evidence presented in this note could not be considered strong enough to form the basis of any policy proposals and, thus, no such proposals are presented.

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DATA APPENDIX

The data used in the estimation were obtained from the *Trade Statistics of Ireland* (various issues). The coal category comprises all div. 32 (i.e., including anthracite and lignite). The crude oil category is then 33300, div. 33. Refined oil consists of items 33411, 33412, 33419, 33421, 33429, 33430 and 33440 (all in div. 33). Manufactured gas is composed of all the items in div. 34. Tables A.1 and A.2 give respectively the cost shares of the energy imports and the prices based at 1975=1.

Table A.1: *Cost shares for the imported fuels*

Year	*Coal*	*Crude*	*Gas*	*Refined*
1960	.34417	.44026	.00620	.20937
1961	.33984	.40051	.00294	.25671
1962	.32940	.43637	.00521	.22902
1963	.33049	.44669	.00643	.21639
1964	.30792	.45576	.00765	.22867
1965	.28946	.48272	.00652	.22130
1966	.29251	.35233	.01165	.34351
1967	.22966	.49829	.00616	.26589
1968	.22005	.48857	.01035	.28103
1969	.20438	.45719	.01259	.32584
1970	.19567	.44109	.01134	.35190
1971	.14649	.45001	.01179	.39171
1972	.15861	.37044	.01640	.45455
1973	.12997	.31191	.01693	.54119
1974	.07526	.38318	.01014	.53142
1975	.06522	.39620	.01627	.52231
1976	.05092	.28582	.02406	.63920
1977	.07261	.34832	.02557	.55350
1978	.08254	.31584	.02523	.57639
1979	.08417	.22961	.02217	.66405
1980	.07652	.28763	.02841	.60744
1981	.09403	.10435	.02981	.77181
1982	.09080	.09448	.03145	.78318
1983	.09891	.20909	.03683	.65517

Table A.2: *Price indices for energy imports with base 1975=1*

Year	*Coal*	*Crude*	*Gas*	*Refined*
1960	.23485	.19000	.39166	.30694
1961	.23601	.18537	.35704	.35203
1962	.25219	.17986	.35998	.28199
1963	.27318	.18001	.36751	.26216
1964	.29804	.18205	.36996	.35647
1965	.30403	.17590	.38119	.22993
1966	.29747	.16886	.37941	.22571
1967	.29340	.17720	.39811	.19747
1968	.31094	.20712	.38149	.21960
1969	.33299	.21003	.38188	.18999
1970	.36791	.20629	.32655	.21651
1971	.41470	.24147	.33392	.24450
1972	.47559	.23446	.38263	.24813
1973	.46020	.23626	.36101	.29476
1974	.78742	.83893	.65474	.97633
1975	1.00000	1.00000	1.00000	1.00000
1976	1.11329	1.31205	1.30812	1.30349
1977	1.40523	1.54712	1.52272	1.42627
1978	1.55362	1.41740	1.49369	1.35881
1979	1.77630	1.66243	1.64149	2.17379
1980	2.25311	2.74006	2.59511	2.84780
1981	3.07512	3.82681	3.14111	3.90172
1982	3.23247	4.80573	3.49911	4.26670
1983	2.98724	4.84501	4.15345	4.40063