Inflation, Taxation, Capital Markets and the Demand for Housing in Ireland*

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Abstract: The objective of this paper is to examine the effect of inflation on the quantity of housing demanded where mortgage interest payments are tax deductable and where capital markets set a limit upon the amount which can be borrowed to purchase a house. It is illustrated theoretically that the answer to this question depends upon the potential housebuyer's marginal tax rate. A micro simulation model is then constructed to examine the likely numerical effects on demand of different rates of inflation. The results indicate that demand, on the part of a typical potential buyer, should increase in response to a lowering of the inflation rate. The simulation model also indicates that the real value of tax savings *increase* in response to a lower inflation rate. These conclusions differ substantially from what would be obtained in other economies whose tax laws are similar in intent. The difference is attributable to the ceiling on tax deductable interest payments in Ireland. A subsidiary objective of the paper is to examine the tax cost of current government measures designed to encourage house purchase. It is illustrated that the tax rate at which households may deduct interest payments could be reduced with only a minor effect on housing demand.

I INTRODUCTION

The role which inflation plays in determining both the quantity of housing demanded and the price of the stock has received substantial attention in the literature. The role played by inflation in determining the cost of capital has been emphasised by Rosen and Rosen (1980) and Hendershott in a series of papers (1980, 1981). Arcelus and Meltzer (1973) have argued that, in the absence of borrowing constraints, if inflation changes only the nominal (but not real) interest/mortgage rate the quantity of housing demanded should remain unaffected. Schwab (1982) analysed this claim more extensively in a theoretical-cum-simulation context and illustrated the importance of the role of capital market constraints. Kearl (1979) examined a similar question in an econometric framework.

The present paper examines the interaction of inflation, borrowing constraints and taxation upon the demand for housing. Schwab found that where quantity constraints on borrowing exist, an increase in inflation would decrease demand under very plausible circumstances. However, in that paper he ignored the role played by taxes. In the first part of this paper I address the effect of a change in the inflation rate on the quantity of housing demanded in a framework where the realities of the Irish tax laws governing mortgage interest deductibility are incorporated.

With *perfect capital markets* and no provision for mortgage interest tax deductibility it is straightforward to show that the inflation-induced higher carrying costs in the early phase of a mortgage are exactly offset by the capital gains in the later stages. With tax deductibility, there is an unequivocal decrease in the cost of housing services and hence demand increases. However, with *imperfect capital markets* and tax deductibility the effect of inflation is indeterminate and thus is examined using a numerical model.

The role played by inflation is clearly of importance. It has been perceived in the past to have had a positive effect on housing demand due to the capital gains which may result. However, in the context of a formal optimising model, some doubt must be cast on such beliefs. To determine the effects of inflation numerically, I make use of a micro simulation model in which a household maximises lifetime utility. By reproducing observed data on house purchase decisions, estimated values for the parameters of the model are obtained. It is then illustrated that the effects of inflation upon demand depend upon its interaction with tax offset provisions and capital market constraints.

The second objective of the paper is to examine how the magnitude of the current government subsidy (in the form of mortgage interest tax deductibility) is altered by inflation.

The paper proceeds as follows: A theoretical two period model is developed in Section II. Here it is shown that the effects of a change in the inflation rate are indeterminate. In Section III the numerical optimisation model is developed and results are presented. Conclusions are then offered.

II THE THEORETICAL MODEL

(i) Mortgage Interest Not Tax Deductible

A utility maximising consumer, who lives for two periods, consumes the services from his stock of housing (Z) and other goods (C). His intertemporal utility function, where the rate of time preference is δ, is given by:

$$V = U(C_1, Z) + 1/(1+\delta) \ U(C_2, Z) \tag{1}$$

The consumer's real income stream is given by Y_1 and Y_2, the real interest rate is ρ, expected inflation is π,[1] the nominal interest rate is $(\rho + \pi + \rho\pi)$ and W is the desired stock of real assets at the end of the second period. P is the price of housing relative to other goods. It remains constant over the two periods and thus rises in nominal terms at a rate π. To maintain simplicity it is assumed that only interest payments are made on the mortgage and that the principal is repaid entirely at end period II. The nominal interest payment in each period is thus $(\rho + \pi + \rho\pi)$PZ and the real value of mortgage interest per pound of principal (incurred at end period I) is $R_1 = (\rho + \pi + \rho\pi)/(1 + \rho)(1 + \pi)$. The real value per pound of mortgage principal in the second period (the interest payment minus the capital gain) is $R_2 = (\rho - \pi)/(1 + \rho)(1 + \pi)$. The intertemporal budget constraint is thus:

$$Y_1 + Y_2/(1 + \rho) - C_1 - C_2/(1 + \rho) - R_1 PZ - R_2 PZ/(1 + \rho) = W/(1 + \rho)^2 \qquad (2)$$

In the case where the first period borrowing constraint $(Y_1 - C_1 - R_1 PZ = 0)$ is not binding it is straightforward to show that a change in the expected rate of inflation has no effect on housing demand. A higher inflation rate increases first period costs, but decreases second period costs by an equivalent amount in present value terms. Hence, in terms of (2), since:

$$\delta R_1/\delta\pi + \delta R_2/\delta\pi/(1 + \rho) = 0 \qquad (3)$$

the budget constraint is invariant with respect to the rate of inflation.

Where capital markets are such that the individual cannot borrow without collateral (Y_1 includes all assets) the additional constraint is introduced into the model:

$$Y_1 - C_1 - R_1 PZ \geq 0 \qquad (4)$$

This holds with equality where it is binding. In this instance Schwab demonstrates by means of a two period Slutsky equation that $\delta Z/\delta\pi < 0$ under the plausible circumstance that a relaxation of (4) would lead to increased consumption of both C and Z.

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1. This model assumes perfect foresight in that the expected and actual inflation rates are equated.

(ii) Tax Deductible Mortgage Interest

Defining the tax rate at which the consumer may deduct mortgage interest from gross income as t, the mortgage costs per pound of principal now become:

$$R_1 = \frac{(\rho + \pi + \rho\pi)(1 - t)}{(1 + \rho)(1 + \pi)} \qquad (5)$$

$$R_2 = \frac{(\rho + \pi + \rho\pi)(1 - t) - \{(1 + \pi)^2 - 1\}}{(1 + \rho)(1 + \pi)^2} \qquad (6)$$

and the present value of the sum of such costs is

$$R = R_1 + R_2/1 + \rho \qquad (7)$$

Here the nominal discount rate, or opportunity cost, remains at $(1 + \rho)(1 + \pi)$ and capital gains are untaxed.

Given the system defined by (1), (2) and (4) - (7) the following proposition results:

Proposition 1

> With mortgage interest tax deductible, the demand for housing is not homogeneous of degree zero in inflation, in perfect capital market conditions of degree zero in inflation, in perfect capital market conditions.

Proof

The simplest way to illustrate this result is to show that the budget constraint is not invariant to changes in the rate of inflation. Since π enters through R_1 and R_2 only, given that the other variables are defined in real terms, it is sufficient to show that $\delta R / \delta \pi \neq 0$. By definition:

$$\delta R / \delta \pi = \delta R_1 / \delta \pi + \delta / \delta \pi (R_2/(1 + \rho)) \qquad (8)$$

With some manipulation and defining d as $(\rho + \pi + \rho\pi)$, it can be shown that:

$$\delta R_1 / \delta \pi = \frac{(1 - t)(1 + \rho)}{(1 + d)^2} \qquad (9)$$

$$\delta / \delta \pi \{R_2/(1 + \rho)\} = \frac{-(1 + \rho)[(1 + t) + d(1 - t)]}{(1 + d)^3} \qquad (10)$$

and thus that:

$$\delta R/\delta\pi = -2t/(1+\rho)^2 \; (1+\pi)^3 \tag{11}$$

So, in the first period, higher nominal rates increase the interest cost of the mortgage. In the second period the cost is less due to the capital gains which are realised at the terminal time. However, unlike the case where mortgage interest is not tax deductible, these two effects do not offset each other. The overall effect of a change in anticipated inflation on housing cost is unambiguously negative as shown in Equation (11), and hence the demand for housing rises with an increase in the inflation rate, *ceteris paribus*.

The result is attributable to (a) the asymmetric tax treatment of capital gain and mortgage interest and (b) the asymmetric treatment of mortgage interest and interest from other sources. Sufficient conditions for tax neutrality are: (i) that all real interest payments be taxed uniformly with capital gains remaining untaxed *or* (ii) that nominal capital gains be taxed in the presence of mortgage interest tax deductibility. Condition (i) can be seen by observing that if ρ in Equation (3) is defined to be the net of tax return, neutrality holds. In case (ii) Equation (6) is augmented by the term $\pi t(2+\pi)/(1+\rho)(1+\pi)^2$. It is straightforward to show that $\partial R/\partial\pi$ then become zero. Note that no distinction is drawn between the value of the house and the value of the mortgage. Neutrality would cease to hold if the capital gains on the *house* were taxable and *mortgage* interest tax deductible. It would also not hold if capital gains were taxed upon an accrual rather than realisation basis.[2]

We next examine the effects of anticipated inflation on housing demand in the context of a binding borrowing constraint in period I.

Proposition 2

   With capital market constraints, the effect of an increase in anticipated inflation on the demand for housing is indeterminate.

This proposition is a little more complex, since an additional constraint is involved in the consumer's optimisation process. Maximising (1) subject to (2) and (4), where (4) is assumed to hold with equality, and R_1 defined by (5) and (6) yields a set of first order conditions. When totally differentiated, the effect of inflation on housing demand is given by:

$$\delta Z/\delta\pi = (\delta Z/\delta R_1)(\delta R_1/\delta\pi) + (\delta Z/\delta R_2)(\delta R_2/\delta\pi) \tag{12}$$

To sign this expression, it is useful to reformulate it as a two-period Slutsky equation:

$$\delta Z/\delta\pi = [(\delta Z^*/\delta R_1)(\delta R_1/\delta\pi) + (\delta Z^*/\delta R_2)(\delta R_2/\delta\pi)] -$$
$$PZ[(\delta Z/\delta Y_1)(\delta R_1/\delta\pi) + (\delta Z/\delta Y_2)(\delta R_2/\delta\pi)] \qquad (13)$$

where Z^* refers to the Hicksian demand with compensation taking place in the first period. This expression is evaluated in Appendix A where it is shown that both the substitution and income terms depend upon the magnitude of t. For each term the effect of inflation is to increase cost with a low value of t and to decrease it for a high value. An intuitive explanation of this result is as follows.

With a high tax rate at which the consumer can deduct mortgage interest for tax purposes, the reduction in overall housing cost caused by higher inflation is substantial, given that the full nominal interest payments may be deducted. Hence, while still facing a binding constraint in period 1 the overall housing cost reduction is so great that the consumer reduces C_1 and buys more Z *despite* the first period increase in carrying costs.

On the other hand, if the tax rate faced by the consumer is very low the result of a higher inflation rate is an inconsequential decrease in the overall cost of housing (again see Equation (11)) and thus less housing will be purchased due to the increased importance of the borrowing constraint.

Effectively, the outcome depends upon the relative magnitudes of the inflation-induced lower cost and the utility cost of the borrowing constraint. At low tax rates the capital market constraint overwhelms any benefits in the form of lower housing costs. The opposite occurs at high tax rates.

To determine the effects of changes in the current inflation rate on the demand for housing on the part of a representative house purchaser, I next make use of a lifecycle utility maximising model which replicates observed behaviour.

II NUMERICAL ANALYSIS

In this section of the paper I analyse the effects of inflation on housing demand by means of a model similar to that used by Tobin and Dolde (1971) and Alm and Follain (1984). Again the focus is upon the behaviour of a representative household, not upon the whole population.

The consumer is assumed to maximise the intertemporal utility function

$$V = \sum_{t=1}^{T} U(C_t, Z)/(1+\delta)^{t-1} \qquad (14)$$

where

$$U = -C_t^{-\alpha} - \beta Z^{-\alpha} \qquad (15)$$

Equation (15) is a CES utility function with $(-\alpha - 1)$ the elasticity of marginal utility of C. This function implies unitary income elasticities of demand. β relates the utility of housing (Z), which is the same in each period, to the utility of all other goods (C). The lifetime budget constraint is given by

$$\sum_{t=1}^{T} Y_t/(1+\rho)^{t-1} = \sum_{t=1}^{T} C_t/(1+\rho)^{t-1} + \sum_{t=1}^{T} M/(1+d)^t \qquad (16)$$

where all of the variables are as before, with M the annual mortgage payment in nominal terms net of income tax concessions, paid at the end of each period.

In choosing values for the parameters of the model I have been guided by the data for the Irish housing market for the year 1984. With a nominal mortgage rate of approximately 12 per cent and an inflation rate of 9 per cent I have set the real rate of interest (ρ) equal to 3 per cent. To ensure a desired increasing consumption stream over the lifetime, the rate of time preference is set to 1.5 per cent.

In choosing the household whose behaviour is to be modelled a choice must be made due to the segmentation of the market. In this paper I have chosen to focus upon the effects of mortgage interest deductibility and hence have *de facto* opted for non-first time buyers — since first time buyers are in addition entitled to substantial mortgage grants.

The fact that the market is composed of different groups is not, unfortunately, always represented in the published statistics and this introduces a certain element of arbitrariness in choosing base year values. For example, second time buyers are more likely to purchase a larger house and to have a higher income than first time buyers.

The average price of a mortgage financed house purchase in 1984 was approximately £35,400. The income of a representative buyer can be computed from the distribution of buyers by income group published by the Department of the Environment in "Housing Loan Statistics" (1984). By using the method described in Irvine (1984, 3.II) the average gross income of a married couple can be computed to be £11,270. Accordingly, I have chosen to model the behaviour of a household with an income of £14,000 which purchases a house valued at £40,000. The income tax brackets for 1984 indicate that average and marginal tax rates of 30 per cent and 40 per cent respectively are appropriate. Consequently, the income stream of this household is .7+ £14,000 and this is assumed to grow in real terms at a rate (g) of 3 per cent over the assumed 25 year

horizon. Mortgage interest is assumed deductible at a rate of 40 per cent.

The values of α and β are then chosen so that optimising behaviour generates a demand for a house valued at £40,000. This procedure assures that the values are purely demand determined and it may involve a bias to the extent that (e.g.) mortgage rationing exists. It also assumes that the year from which data are drawn represents an equilibrium, in the sense of having no strong cyclical elements. But this is an intrinsic feature of comparative static type models of the type used here. The purchaser is assumed to have £10,000 in capital[3], yielding a loan to value ratio of .75. Optimisation of (14) is carried out subject to the *additional* constraint that total expenditure in any time period not exceed total current income[4]

$$C_t + M_t / (1 + d) \ (1 + \pi)^{t-1} \leqslant Y_t \tag{17}$$

and that the maximum mortgage interest allowance is £2,000. M_t is a standard annuity mortgage payment net of income tax allowance. The optimal control algorithm for solving this problem is given in Appendix B.

The α parameter was set at a value of 0.5 and the resulting value of β was .34. This yields a value for the elasticity of the marginal utility of consumption equal to 1.5. Since the $\{\alpha, \beta\}$ combination yielding the base year values is not unique, different values have been tried without major differences in the results.

The planning-horizon for the potential buyer is set at 16 years, even though the term of the mortgage is 25 years. This assumption permits the household to change house more frequently than at the expiry date of the mortgage and is further reasonably consistent with the recent observed patterns. The terminal wealth target for the programme is 10 per cent of the present value of real gross income.[5]

It is also worth noting that the utility function is defined only for buyers, and not for renters. Effectively, buyers and renters are assumed to constitute different segments of the market with correspondingly different utility functions. But since the analysis examines the behaviour of a typical or median individual, rather than the whole population, changes in relative prices affect only the quantity of housing purchased and not the rental-ownership decision.

This, then, describes the functioning of the model. Given the solution algorithm, the effects of changes in any of the parameters or policy variables on the demand for housing can be examined.

3. The down payment is not modelled endogenously, nor is it assumed to vary with the inflation rate.

4. In this model, this is equivalent to assuming that borrowing can be undertaken only to purchase the house.

5. This is composed of real equity holding at the target date plus saving in non-housing form. Together these must equal the 10 per cent of real gross income target.

(i) Inflation

To illustrate how inflation operates in the model it is useful to refer to Figure 1. With a zero inflation rate the real value of the payments stream is given by R. With inflation positive, the stream is tilted (S) so that the *real* payment in the early years increases and the real payment in the later years decreases. Stream T represents yet a higher inflation rate. Because of tax provisions (in which *nominal* interest is deductible), the real present value of the payments stream represented by T is less than that of S, which in turn is less than that of R.

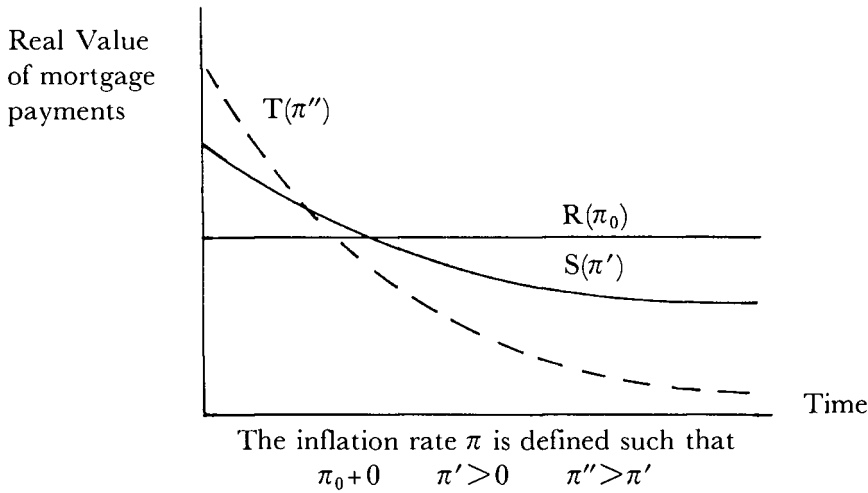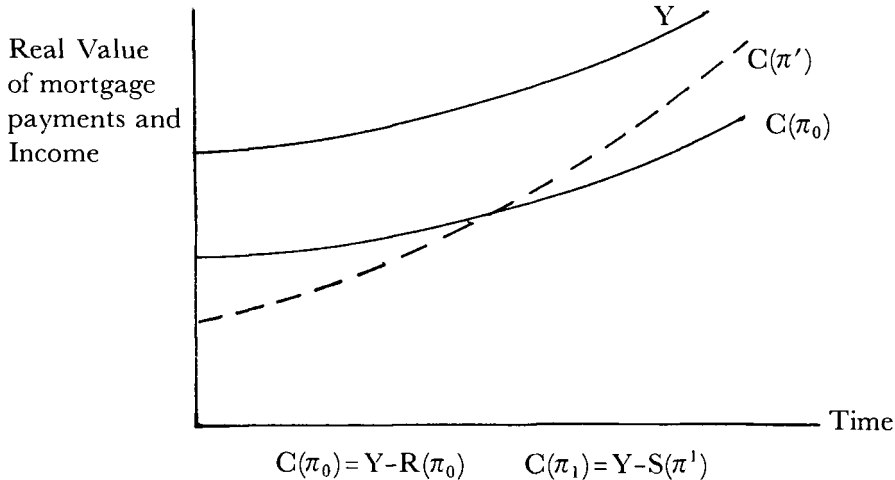Figure I: *Inflation and the Real Value of Mortgage Payments*

The inflation rate π is defined such that
$$\pi_0 + 0 \qquad \pi' > 0 \qquad \pi'' > \pi'$$

Figure II: *Inflation Effects On Consumption*

$$C(\pi_0) = Y - R(\pi_0) \qquad C(\pi_1) = Y - S(\pi^1)$$

Offsetting the utility gain associated with the lower overall cost of the higher inflation mortgage is the fact that the borrowing constraint skews lifetime consumption on all other goods (C_t) in an undesirable manner. Superimposing streams R and S on the income stream in Figure II it is clear that S squeezes consumption $C(\pi)$ in the early years relative to the later years more than R.

The choice of house size thus reflects the effects of inflation on net cost and its effects on the degree of skewness in the consumption stream. The lower cost due to higher inflation induces individuals to substitute towards housing, but the disequilibrating impact on the consumption stream acts in the opposite direction.
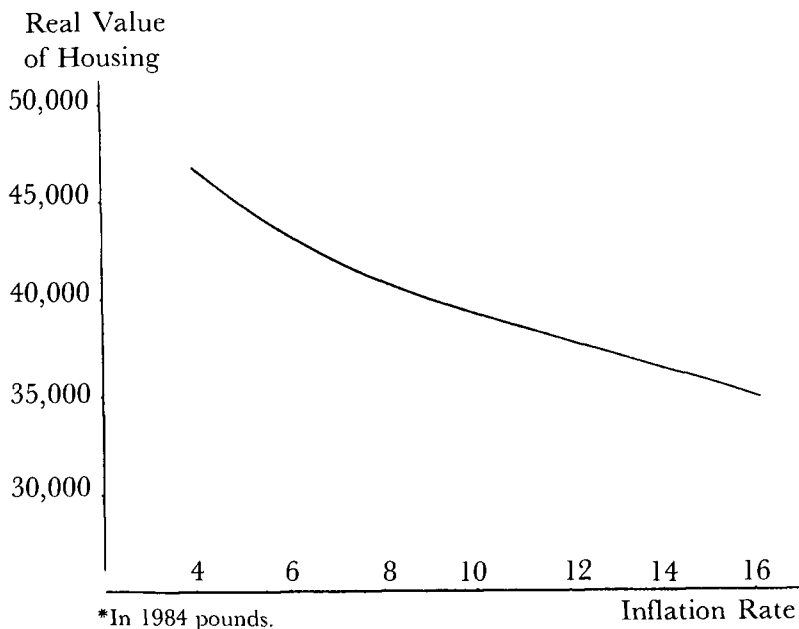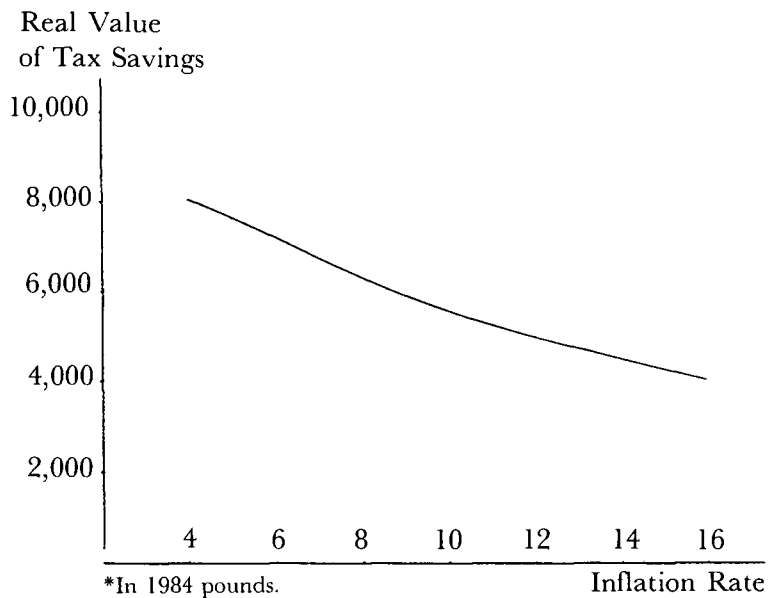
The results of the simulations are presented in Figure III. The quantity of housing demanded (in 1984 values) is given on the vertical axis corresponding to the inflation rates 4 per cent–16 per cent for a marginal tax rate of 40 per cent.

As can be seen, a lower inflation rate increases the quantity of housing demanded. Thus, if a demand is £40,000 at a rate of 9 per cent, predicted demand would be £44,630 at a rate of 5 per cent. To see why this occurs it is convenient to examine the twin effects of the inflation rate change.

The lower inflation rate decreases the tilt of the real mortgage payment stream and consequently permits the household to have a more balanced consumption stream (C). As a consequence the household tends to purchase more housing. But what now of the possible change in net cost due to the tax deductibility provisions? The key to this issue lies in recognising that with a house purchase in the range given in Figure III, the £2,000 interest deductibility limit is operative for many years of the mortgage. Furthermore this limit is defined in nominal rather than real terms. Hence, with a higher inflation rate the real present value of a given nominal stream of tax savings is reduced since these savings must be discounted by the nominal interest rate. So, in contrast to the situation where full interest payments are deductible and where, correspondingly, inflation reduces the real cost, in this instance the limit on interest deductions effectively causes house costs to increase with higher inflation rates.

This finding is in contrast to what simulations without the £2,000 limit yield. There the housing demand function is hump shaped; indicating that as inflation increases from zero the lower net cost dominates the tilting phenomenon, but at some inflation rate the tilting becomes so severe that any cost reduction due to higher inflation is not sufficient to overcome the utility losses caused by a severely uneven consumption stream (C).

To illustrate the role played by tax laws Figure IV describes the real present value of tax savings corresponding to the inflation rates and associated demands given in Figure III. This indicates that a lower inflation rate, by increasing the real value of the permissible tax deductions, increases the demand for housing. In 1984 with inflation at 9 per cent the value of the tax savings on a £40,000 house amounted to about £6,000. With inflation reduced to 5 per cent the

Figure III: *Housing Demand at Differing Inflation Rates**

Real Value
of Housing

50,000

45,000

40,000

35,000

30,000

4   6   8   10   12   14   16

*In 1984 pounds.   Inflation Rate

Figure IV: *Tax Savings for Differing Inflation Rates**

Real Value
of Tax Savings

10,000

8,000

6,000

4,000

2,000

4   6   8   10   12   14   16

*In 1984 pounds.   Inflation Rate

simulations indicate that, if all other economic conditions had remained unchanged, a medium income household would have purchased a house valued at £44,630 with a tax saving of £7,760. Thus, the increase in quantity purchased of £4,630 is attributable to an effective decrease in cost of £1,760 and to the less tilted consumption stream which is made possible by the lower inflation rate.

(ii) Taxation

Considerable speculation has existed in the past regarding the likely effects of a reduction in the rate at which mortgage interest can be offset. The model described above can be used to evaluate this question. Rather than assume a constant marginal tax rate the model was rerun with rates varying between 30 per cent and 45 per cent, assuming that the average tax rate remains unchanged. The reason for limiting the simulations to relatively narrow bands is because of the likely sensitivity of these models to major changes in policy variables (Irvine (1985)).

A reduction in the marginal tax rate to 30 per cent from 40 per cent was found to decrease real demand by 1 per cent to 3 per cent on the part of the typical household over the inflation range 4 per cent–16 per cent. This very small decrease in demand is again attributable to the fact that a £2,000 interest deduction limit is operative. So when the tax rate changes from 40 per cent to 30 per cent the maximum annual loss in nominal tax savings is a mere £200. With a 4 per cent inflation rate the real present value of the difference in tax savings over the life of the mortgage amounts to £2,085, and with a 16 per cent inflation rate to £1,013. The differences are less when considered over the assumed 16 year optimising horizon. As a consequence a minimal reduction in demand is to be anticipated.

IV CONCLUSIONS

The purpose of this paper has been to shed light on the effects of inflation on real housing demand in the context of relatively realistic capital market constraints.

It has been illustrated that while inflation may theoretically increase *or* decrease demand, depending upon how it interacts with the tax system and borrowing constraints, the particulars of the Irish tax laws ensure that higher inflation will generally decrease demand. Numerical simulation indicates that, for a representative household, a reduction in the rate of inflation from 9 per cent to 5 per cent would increase demand by approximately 11 per cent *ceteris paribus*. This is attributable to an increase in the real present value of tax savings which are fixed in nominal terms over the early and middle years of a mortgage and to a less uneven consumption stream which the reduced tilting of the mortgage pay-

ment stream makes possible. This behaviour is in contrast to what would materialise if there were no limit upon the amount of interest which could be deducted.

A secondary conclusion of the study is that the tax rate at which interest may be deducted could be reduced with no more than a very minor effect on real demand.

These conclusions must, of course, be interpreted within the confines of the model. The model is fundamentally comparative, static in nature, even though it incorporates lifecycle type decision making. As a result, its predictions are contingent upon all other influences on the demand decision remaining unchanged. So, while simulations have been presented which replicate inflation rates for 1984 and 1985, the predictions of the model capture only the change in inflation between these years.

A further consideration with models of this type is the parsimony of their parameterisation. I have argued elsewhere (Irvine (1985)) that they are suitable only for modelling relatively small changes in policy variables and that they are not appropriate for examining regime changes. It is for this reason that the simulation results have been presented for moderate variations in the inflation rate and the marginal tax rate. Finally, while the paper is micro economic in orientation and not designed to indicate how the aggregate price of housing should respond to changes in the inflation rate, it is worth comparing the reported findings for a period of falling inflation with what was observed in the 'seventies when inflation was increasing.

In that period the rise in house prices reflected increasing demand. The very strong demographic expansion represented an increased demand upon the existing and new housing stock. With a positively sloping supply function real forces undoubtedly played a part in increasing prices. Inflation itself may also have increased the demand for housing in the early 'seventies because the limits on mortgage interest deductibility were not operative until 1974. As illustrated in the text, a higher inflation rate in these circumstances may increase demand due to the lower real post-tax cost of housing. As a consequence, the findings presented here need not be viewed as being in conflict with what was historically believed to have been the possible result of inflation. The institutional circumstances have changed.

APPENDIX A

INFLATION AND HOUSING DEMAND — COMPARATIVE STATICS

(A11) and (A12) from Schwab (1982) yield:

$$\delta Z/\delta R_1 = (1+p)(1+\gamma_1/\gamma_2)\delta Z^*/\delta R_2 \tag{A1}$$

and (5)-(7) from the text yield

$$\delta R_1/\delta\pi = k\,\delta R_2/\delta\pi \tag{A2}$$

where

$$k = -(1-t)(1+\pi)/(1+t+d(1-t)).$$

The composite substitution term can then be written as

$$(\delta Z^*/\delta R_1)(\delta R_1/\delta\pi) + (\delta Z^*/\delta R_2)(\delta R_2/\delta\pi) =$$
$$[1-\{(1+d)(1-t)(1+\gamma_1/\gamma_2)/(1+t+d(1-t))\}]\ (\delta Z^*/\delta R_2)(\delta R_2/\delta\pi) \tag{A3}$$

Since $\delta Z^*/\delta R_2$ is negative, being the Hicksian substitution term, and $\delta R_2/\delta\pi$ is negative from Equation (10), the sign of the whole expression depends upon t. As $t\rightarrow 0$ the term in square parentheses tends towards $-\gamma_1/\gamma_2$ and hence the whole expression is negative; as $t\rightarrow 1$ it tends towards $+1$ and the whole expression is positive.

Likewise the composite income effect can be written as

$$-PZ[(\delta Z/\delta Y_1)(\delta R_1/\delta\pi) + (\delta Z/\delta Y_2)(\delta R_2/\delta\pi)]$$
$$= -PZ[(\delta Z/\delta Y_1) - k(\delta Z/\delta Y_2)](\delta R_1/\delta\pi) \tag{A4}$$

where k is defined by (A2) above. This can be rewritten as

$$-PZ[(\delta Z/\delta Y_1) - (1+p)(\delta Z/\delta Y_2) - 2t/\{(1-t)(1+\pi)\}(\delta Z/\delta Y_2)]$$
$$\times (\delta R_1/\delta\pi) \tag{A5}$$

Defining B as the budget constraint this becomes

$$-PZ[(\delta Z)/(\delta B) - \{(2t)/(1-t)(1+\pi)\}(\delta Z/\delta Y_2)](\delta R_1/\delta\pi) \tag{A6}$$

The sign of this again depends upon t. Clearly for small t, given that $\delta R_1/\delta \pi > 0$ (see Equation (9)) and $\delta Z/\delta B > 0$ (see Schwab (1982)) the whole expression is negative. But for large t the expression is positive since the second term in square parentheses dominates the first.

APPENDIX B

SOLUTION ALGORITHM TO CONSTRAINED LIFETIME DEMAND

The solution to the system (14)–(17) is an iterative one. The key to the solution lies in recognising that Z (once chosen) is constant throughout the life of the plan. Hence, in optimising terms it is equivalent to a parameter.

The *unconstrained* plan (i.e. (14)–(16)) is solved by choosing an optimal consumption stream C* conditional upon some value for Z. Z is then adjusted by an amount determined by the gradient of the function. The value for Z and its associated C* which maximise lifetime utility are defined as the optimum. Of course, this violates the borrowing constraint (17) given the parameter values of the model and the rate of growth in the income profile.

The *constrained* optimal plan is similarly generated by iterating over values for Z. Where the borrowing constraint is violated by the unconstrained optimal consumption stream, the optimal constrained solution is recomputed so that (17) is not violated. The optimisation assumes that the average and marginal rates of tax are constant and do not vary in response to fluctuations in the quantity of housing consumed.

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