An Empirical Study of the Age Structure of the Irish Population

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THE economic and social consequences of a population's age structure are far-reaching. The proportion of a country's population in the active age groups largely determines the relationship between productivity per worker and income per person. The ability of a society to provide the health, educational and recreational facilities needed by young and old people is affected by the ratio of active to dependent people in the population. Changes in the population's age structure are of considerable significance for the level of demand for numerous consumers' durable goods, and may have wide repercussions on the cyclical behaviour of GNP, as has been argued by Easterlin [cf. 5].

It is not, of course, possible to define any age groups which consist exclusively of economically active or inactive persons, but it is easy to demarcate the ages above and below which most of the population is not gainfully occupied. Traditionally, 65 has been regarded as the age at which most people retire from active participation in the labour force, and 15 the age when most enter the labour force. In family farming, however, the division between economic participation and retirement is less well-defined than in other occupations, and probably seldom occurs as early as the sixty-fifth birthday. At the other end of the life cycle, few European countries still terminate free or compulsory education at age 14. Yet, despite these qualifications, the time-honoured boundaries to the "active" age group retain their meaningfulness in present-day Ireland in as much as labour-force participation is far more probable between 15 and 64 than at younger or older ages, and the use of alternative boundaries would introduce more problems than it would solve. An exploratory discussion of the factors influencing Irish labour-force participation is contained in [12].

Ireland is well-known to possess an unusual population age structure. Some of the details and consequences of this aspect of our demographic situation have been examined by the Emigration Commission [3, paragraph 43], Kaim-Caudle [7, p. 15], O'Mahony [11, p. 9]. The most striking features of the Irish age structure are a high percentage of the population aged 65 and over, a small percentage aged 15–44, and a high percentage aged 0–14. These factors imply an unusually heavy burden of dependency (by European standards, at least), whose unfavourable welfare implications are obvious.[1]

The present paper aims at examining the mechanisms that determine the age structure of the Irish population. The main emphasis is placed on the statistical testing of a model that may facilitate prediction of changes in this age structure (although the model is not applied to the task of forecasting in this paper). The approach taken is empirical, and even pragmatic, in character.

Background

In 1841, 41·2 per cent of the Irish population was aged under 15 or over 65, while in 1966 the corresponding figure was 42·4 per cent. Behind this apparent stability in the level of total dependency lies a steady increase in the level of old dependency (from 3·1 to 11·2 per cent) and a fall in the level of young dependency (from 38·1 to 31·2 per cent). The details of this movement are set out in Table 1. As was remarked by the Emigration Commission [3, paragraph 38], in the century after 1841 the rising trend in old dependency was approximately offset by the downward trend in young dependency, so that the burden of total dependency remained fairly constant (or even fell slightly) over the period. Since 1936, however, this pattern has not been maintained and the levels of both young and old dependencies have risen together, with a resultant increase in total dependency from 37·3 per cent in 1936 to 42·4 per cent in 1966. These trends may be explained simply as a reflection of the falling rate of natural increase up to 1936, followed by a rising rate since then, and the persistent decline of the population due to low natural increase and high emigration up to the 1960's.

Ireland's unusual age structure may be assessed in a European context from Table 2. The high percentage of Ireland's population in the young dependent age group is a striking feature of the Table. It is clear that Ireland has a high, but by no means extraordinary, percentage of its population in the old dependent category. However, Ireland is more of an exception in regard to total dependency than in regard to either of its components. Only Iceland has a higher level of total dependency. The unusual aspect of Ireland's age structure is, therefore, not so much the very high level of either young or old dependency, but rather the combination of young and old dependencies that are both fairly high. The impression conveyed by Table 2 is that in general there is an inverse association

1. While it is axiomatic that a high burden of young dependency imposes a heavy load on a country's educational system, it does not always follow that this will result in low educational attainment. The state of Utah, for example, has the highest median years of school completed in the US and also the second highest percentage of its population aged less than 15, namely, 37·43 per cent. The highest young dependency figure in Ireland is only 33·7 per cent for Offaly (1966). Utah is an interesting and unusual example of an area where religious beliefs emphasize both high marriage fertility and high educational attainment.

TABLE 1: *Historical Data on Irish Dependency (26 Counties)*

Percentage of Total Population

Date	*Aged* 0–14	65 and Over	(0–14)+(65 and Over)
1841	38·1	3·1	41·2
1881	35·4	6·3	41·7
1901	30·2	6·5	36·7
1926	29·2	9·1	38·3
1936	27·6	9·7	37·3
1946	27·9	10·6	38·5
1951	28·9	10·7	39·6
1961	31·1	11·2	42·3
1966	31·2	11·2	42·4

Sources: [3, Table 8], [1, Vol. II].

TABLE 2: *International Data On Dependency: European Countries, About 1964*

Country and Year	*% of Population Aged 0–14 (1)*	*% of Population Aged Over 64 (2)*	*Total Dependency (1)+(2)*
Austria '65	23·27	13·20	36·47
Belgium '63	23·81	12·39	36·20
Czechoslovakia '64	25·70	9·54	35·24
Denmark '64	23·84	11·25	35·09
Finland '65	27·06	7·93	34·99
France '65	24·58	12·55	37·13
Germany (West) '65	22·56	11·92	34·48
Germany (East) '64	23·61	14·45	38·06
Hungary '65	23·28	10·35	33·63
Iceland '62	34·97	8·27	43·24
Ireland '61	31·18	11·19	42·37
Italy '64	24·24	9·72	33·96
Malta '65	32·91	8·89	41·80
Netherlands '65	28·26	9·57	37·83
Norway '64	24·84	11·78	36·62
Poland '62	33·13	6·12	39·25
Portugal '65	29·02	8·42	37·44
Spain '63	27·67	8·61	36·28
Sweden '65	20·94	12·68	33·62
Switzerland '64	22·71	10·44	33·15
UK: England & Wales '63	22·64	11·95	35·59
Scotland '63	25·58	10·71	36·29

Data Source: [8].

between the two dependency categories. Regression of old dependency on young dependency using the data in Table 2 yields the following result:[2]

(1)

$$X_1 = 20{\cdot}267 - 0{\cdot}372\ X_2 \qquad \bar{R}^2 = 0{\cdot}490,\ \text{S.E.E.} = 1{\cdot}445,$$
$$(9{\cdot}482)\quad (4{\cdot}598) \qquad F(1,20) = 21{\cdot}142$$

where X_1 = percentage of population aged over 64,
X_2 = percentage of population aged under 15.

The relationship is highly significant, but much of the variance in X_1 remains unexplained. Looking at the residuals (actual *minus* predicted) from the estimated equation, it is noticeable that the two largest values are for East Germany (+2·956) and Ireland (+2·508). Both of these countries, therefore, have levels of old dependency far higher than would be expected on the basis of the experience of other countries with comparable levels of young dependency. This combination of fairly high young and old dependency is, as has been seen, the reason for Ireland's high total dependency.

The negative correlation between young and old dependency found in Table 2 is to be expected: a high rate of natural increase will generally lead to a rapid rate of population growth, and thus to a low level of old dependency. This sequence of events can, however, be broken if net emigration is heavy and sustained, and in fact this has been a feature shared by Ireland and East Germany in the post-war period. When this occurs, a country can experience a high rate of natural increase for extended intervals without ever experiencing the fall in old dependency that it normally induces. This has been the sort of situation developing in Ireland since the 1930's.

Determinants of Population Age Structure: Projection Analysis

Any population that is subject to unchanging death and birth rates for long enough will eventually assume a stable age structure and growth rate [6]. This stable age structure is such that each age group is growing at the intrinsic rate of natural increase (as derived from the Net Reproduction Rate based on the birth and death rates in question). Leslie [10] has provided a procedure for obtaining the stable age distribution associated with any set of vital rates. Let A be a square

2. Throughout this paper the following conventions have been followed in presenting regression results: the $t-$ ratio (*viz.* the estimated regression slope divided by the estimated standard error of the slope) is recorded in parentheses under the relevant coefficient; the $\bar{R}^2$, the coefficient of determination corrected for degrees of freedom $\left(=1-(1-R^2)\dfrac{N-1}{N-k}\right)$, the standard error of estimate (S.E.E.) and the F–ratio with the relevant degrees of freedom, are also recorded for each equation.

matrix of order k, where $k=$ the number of age groups into which the population is divided, and let the elements of the first row of A equal the number of female children born per period to each woman in the corresponding population age group.[3]

The elements of the main sub-diagonal of A (that is, the elements $a_{i+1,\ i}$) equal the probabilities of surviving from the i^{th} age group in one period to the $(i+1)th$ age group in the next period. The age groups and periods used here are both of five years' duration. Since the oldest age group used is "65 and over", the element $a_{k,\ k}$ is equal to the probability that a person aged over 64 at the start of a period would be still alive at the start of the next period. All other elements of A are zero. If net migration is treated as equivalent to mortality, then the elements of the sub-diagonal of A may be altered to reflect various assumptions about migration.[4] Leslie shows that the stable age distribution associated with a set of birth and death rates specified in A is given by the relative size of the element of B_1, where B_1 is the characteristic vector associated with the real dominant root, λ_1, of the characteristic equation of A. In order to obtain the stable age distribution for A, I have proceeded as follows: starting with the existing population by age arranged as a k x 1 vector C, post-multiply A by C and then post-multiply A by the resultant, continuing this sequence until the age distribution of the resultant has ceased to change significantly between iterations.[5] This is of course equivalent to obtaining A^nB where n is large enough for the age distribution to have stabilized.

In order to explore the consequences for the Irish population's age structure of various combinations of birth and migration rates, four alternative specifications of A have been used. In addition to the stable age distribution, which, like the net reproduction rate, is of theoretical interest only, the age distributions associated with the first 8 iterations (that is, forty years) are presented in Table 3. These results are presented here to shed light on the determinants of the dependency ratio and *not* as predictions of the future course of Ireland's population age structure. It is, of course, easy to realize the relevance of the technique applied here to the problem of population forecasting, but we are here concerned with determinants of age structure and not population forecasting. According to Variant II, the total dependency ratio remains fairly stable over time, although the young dependency ratio rises and the old dependency ratio falls. The assumptions behind Variant II are that recent levels of vital rates will prevail into the future. It is also interesting to note that under this set of assumptions the percentage of the population aged 15–44 gains considerably at the expense of the population aged 45–64, thus increasing the fall in the population's median age.

3. All of the projections presented here deal with the female population only.

4. It is assumed throughout that the birth rate of the migrant population is identical with that of the non-migrant population, and hence no alteration of the first row of A is made when the assumptions about migration are varied.

5. The criterion for stability I have applied was that the percentage of the population in each age group should not change by more than 0·001 between iterations.

TABLE 3: *Population Age Distribution Under Various Assumptions Concerning Migration and Birth Rates*

(% in each age group female population)

Variant: *Period=5 Years*

I: *1966 Fertility, Zero Net Migration*

after *i* periods	*i=1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*Stable* *49*
0–14	29·9	29·5	30·1	32·1	33·9	34·4	33·8	32·8	33·4
15–44	33·3	35·9	37·9	37·9	36·7	36·2	36·7	37·6	37·9
45–64	25·4	23·5	21·2	19·8	20·1	21·0	22·0	22·8	21·8
65+	11·5	11·1	10·7	10·2	9·4	8·5	7·5	6·8	6·9
Total Dependency	41·4	40·6	40·8	42·3	43·3	42·9	41·3	39·6	40·3

II: *1966 Fertility, '61–'66 Net Migration*

	i=1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*42*
0–14	30·8	30·9	31·1	32·1	32·9	33·2	33·1	32·8	33·1
15–44	31·5	32·7	34·0	33·9	33·5	33·7	34·0	34·2	34·4
45–64	25·7	24·4	22·8	22·1	22·1	22·4	23·0	23·6	23·1
65+	12·0	12·0	12·1	11·9	11·5	10·8	10·0	9·5	9·3
Total Dependency	42·8	42·9	43·2	44·0	44·4	44·0	43·1	42·3	42·4

III: *1966 Fertility—25%, '61–'66 Net Migration*

	i=1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*53*
0–14	29·0	27·2	25·3	26·8	28·0	28·4	28·0	26·9	26·5
15–44	32·5	34·5	36·9	35·4	33·8	32·9	32·4	31·9	32·6
45–64	26·4	25·7	24·5	24·6	25·2	26·1	27·6	29·4	27·2
65+	12·3	12·6	13·1	13·3	13·1	12·6	12·0	11·8	13·8
Total Dependency	41·3	39·8	38·4	40·1	41·1	41·0	40·0	38·7	40·3

IV: *1966 Fertility—25%, Zero Net Migration*

	i=1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*49*
0–14	28·1	25·8	24·4	26·8	28·9	29·7	28·9	27·2	27·2
15–44	34·1	37·8	41·0	39·9	37·4	35·7	35·3	35·5	36·5
45–64	26·0	24·8	23·0	22·0	22·9	24·6	26·7	28·7	26·2
65+	11·7	11·6	11·6	11·3	10·7	10·0	9·1	8·5	10·2
Total Dependency	39·8	37·4	36·0	38·1	39·6	39·7	38·0	35·7	37·4

Variant I reflects the assumption of maintaining current birth rates and zero net migration, and, in conjunction with Variant II, allows us to examine the effects of net migration on population age structure. Contrasting Variant I with Variant II shows that emigration tends to increase the percentage of the population in the 45–64 and 65 and over age groups, but it is clear that this ageing of the population occurs almost entirely through a reduction in the percentage in the 15–44 age group. Migration appears to have little long-run impact on the percentage of the population aged 0–14, although for the first three periods (*viz.* fifteen years) there is a noticeable increase in young dependency as a result of net emigration. These results might have been expected in view of the fact that emigration always lowers the rate of population growth, but is assumed to leave the age-specific birth rates unaltered.

Variant III examines the impact of a very sizeable reduction in the birth rate upon population age structure. The birth rates used in this Variant are equal to 75 per cent of those used in I and II, so that a woman's contribution to the number of infants is lowered by a quarter in each of the reproductive age groups. When combined with the current level of net migration this fall in birth rates leads to a net reproduction rate of 0·96 (about the level now found in Japan), so that in the long run the population is not replacing itself. These assumptions, as might be expected, produce a large reduction in young dependency (which contrasts strongly with the tendency for young dependency to rise markedly under Variants I and II), and an eventual increase in old dependency. It is interesting that under these assumptions the behaviour of both dependency ratios is cyclical before the stable population structure is reached: the sharp drop in fertility is apparently responsible for the sharp period-to-period swings in young dependency seen here and in Variant IV.

In Variant IV the lower birth rates have been combined with the assumption of zero net migration. This set of assumptions eventually results in a greatly reduced burden of total dependency, although the population age structure fluctuates considerably before arriving at its stable level. The rise in the level of old dependency is curtailed here by the assumption of zero net migration, and the benefits of a lower birth rate are not offset by a tendency for population to decrease, as was the case in Variant III. It is sobering to reflect that it would be 15 years after the attainment of the assumptions of Variant IV before Ireland's total dependency ratio would fall below, for example, the current Scottish level.

These experiments support the conclusion that net migration in the long run has contributed to the Irish dependency exclusively through its impact on the level of old dependency. The migration rate was seen to have no important, long-run effect on the level of young dependency, but this result may be a consequence of the assumption that the migrant and nonmigrant populations have the same birth rates. It would be more realistic if the birth rate were altered to take account of any effect that a fall in the migration rate might have on fertility and/or marriage rates, but our knowledge of this topic is negligible. However, it might be argued that a fall in net migration to near zero would only occur in conditions that would tend to lower births rates substantially. If this is true, then Variant IV might be considered, along with Variant II, as the most relevant to the preparation of an actual forecast of future population age structure. All four Variants illustrate one important point very clearly, namely, unless Ireland's birth rate falls its burden of young dependency will not only remain high but actually tend to rise, regardless of what happens to the net migration rate.

Irish Population Age Structure in 1966

Turning from the hypothetical evolution of population age structure to the actual structure found in Ireland today, additional light may be shed on the determinants of age structure by a study of the 1966 Census of Population county data. Table 4 presents some summary information based on these data. In this Table, and henceforward in this paper, dependency is measured in relation to the active, and not to the total, population (e.g., young dependency (*YD*) equals the ratio of population aged 0–14 to population aged 15–64 multiplied by 100).

It may be seen from Table 4 that there is little difference in the national levels of dependency between the sexes, except for the somewhat higher level of old

TABLE 4: *Summary Data On Young, Old, and Total Dependency: Ireland By County, 1966*

	YD_m	YD_f	YD_{m+f}	OD_m	OD_f	OD_{m+f}	TD_m	TD_f	TD_{m+f}
National Level	54·7	53·7	54·2	17·8	21·1	19·4	72·6	74·8	73·7
Mean of County Levels	53·5	56·3	54·7	20·2	23·3	21·7	73·7	79·5	76·4
Standard Deviation	4·0	3·7	3·3	4·0	3·9	3·9	2·5	4·6	3·2
Coefficient of Variation × 100	7·5	6·5	6·1	19·8	16·9	18·0	3·4	5·8	4·2

Intercorrelations

	YD_m	YD_f	YD_{m+f}	OD_m	OD_f	OD_{m+f}	TD_m	TD_f
YD_f	0·471							
YD_{m+f}	0·879	0·834						
OD_m	—0·803	—0·108	—0·557					
OD_f	—0·849	—0·254	—0·665	0·971				
OD_{m+f}	—0·832	—0·188	—0·619	0·992	0·993			
TD_m	—0·322	0·584	0·520	0·305	0·188	0·247		
TD_f	—0·350	0·572	0·092	0·738	0·648	0·694	0·615	
TD_{m+f}	—0·095	0·640	0·290	0·627	0·516	0·572	0·847	0·940

Extreme values

	YD_m	YD_f	YD_{m+f}	OD_m	OD_f	OD_{m+f}	TD_m	TD_f	TD_{m+f}
Highest value found in County	58·6 (Carlow, (Dublin)	64·0 (Offaly)	60·8 (Offaly)	28·4 (Leitrim)	31·8 (Leitrim)	29·9 (Leitrim)	79·2 (W'Meath)	86·2 (Rosc.)	81·9 (Mayo)
Lowest value found in County	45·1 (Leitrim)	49·7 (Dublin)	47·8 (Leitrim)	11·2 (Dublin)	15·5 (Kildare)	13·9 (Dublin)	69·8 (Dublin)	66·0 (Dublin)	67·8 (Dublin)

dependency (*OD*) among females. There are a number of reasons for studying dependency separately for each sex, as should become clear from the approach adopted below, but from an economic or social viewpoint the most meaningful concept is obviously the level of dependency for both sexes combined. It is therefore worth establishing that dependency for both sexes combined is almost entirely explained in terms of dependency for each sex, and not significantly due to variations in the sex composition of the population. This may be illustrated by a regression of total dependency (*TD*) for both sexes on each of the four dependencies that contribute to it:

(2)

$$TD_{(m+f)} = -1{\cdot}265 + 0{\cdot}543\,YD_m + 0{\cdot}474\,YD_f + 0{\cdot}505\,OD_m$$
$$(1{\cdot}756) \quad (50{\cdot}245) \quad (59{\cdot}290) \quad (19{\cdot}431)$$
$$+0{\cdot}503\,OD_f \qquad \bar{R}^2 = 0{\cdot}999,\ S.E.E. = 0{\cdot}100,$$
$$(18{\cdot}704) \qquad F(4,22) = 6699{\cdot}536$$

(The $_{m,f}$ subscripts are used to indicate male, female). Although this equation is, for all intents and purposes, definitional, it is of interest to note the higher weight accorded to YD_m than to YD_f, and the extremely high $\bar{R}^2$ achieved without the introduction of a variable measuring the sex distribution.

There is greater relative variation (as measured by the coefficients of variation recorded in Table 4) in the county data for *OD* than for *YD*, while *TD* exhibits the least variation of all. From elementary statistics we know that the variance of the sum of two random variables is the sum of the variance of each variable *plus* twice the covariance of the two variables, and therefore the reason for the low relative variance of *TD* must be sought in a high, negative correlation between *YD* and *OD*. The correlation coefficients recorded in Table 4 confirm this. All of the inter-correlations between the three *OD* and three *YD* variables are negative, and those between all three *OD* variables and YD_m and YD_{m+f} are significant at the ·01 level (that is, $r \geqslant 0{\cdot}487$). The high, negative correlation between YD_m and both OD_m and OD_f, is particularly important in reducing the variance of *TD*.

The negative correlation between *YD* and *OD* found in the Irish data would have been expected on the basis of our discussion of the international data. The counterpart to equation (1) using Irish data is:

(3)

$$OD_{m+f}=61{\cdot}296-0{\cdot}723\,YD_{m+f} \quad \bar{R}^2=0{\cdot}358,\ S.E.E.=3{\cdot}128,$$
$$(6{\cdot}083) \quad -(3{\cdot}937) \quad F(1{\cdot}25)=15{\cdot}511$$

Although the regression is highly significant, much of the intercounty variation in *OD* remains "unexplained". In our discussion of (1) it was suggested that high net emigration rates might account for deviations between the actual level of *OD* and that expected on the basis of the level of *YD*. Introducing a variable to represent the long-run level of net migration defined as the net migration per 1,000 average population 1946–1966 (the simple average of the four intercensal periods) (= *NMR*) the following result was obtained:

(4)

$$OD_{m+f}=35{\cdot}372-0{\cdot}432\,YD_{m+f}+0{\cdot}738\,NMR$$
$$(4{\cdot}343) \quad (3{\cdot}243) \quad (5{\cdot}659)$$
$$\bar{R}^2=0{\cdot}713,\ S.E.E.=2{\cdot}090,\ F(2{\cdot}24)=33{\cdot}370$$

Equation (4) provides a much improved "explanation" of the variance in OD_{m+f}. It is quite interesting to show that over 70 per cent of the variation in OD_{m+f} can be accounted for in terms of variation in YD_{m+f} and *NMR*, but at the same time this relationship provides little insight into the mechanism that induces change in the population age structure. The remainder of this paper is an attempt to specify the influences on *OD* and *YD* in a more complete and meaningful fashion.

Determinants of Young Dependency

From the foregoing discussion it is clear that the birth rate is a major influence on the level of *YD*. The birth rate (*BR*) is not expected to exert all of its influence on *YD* instantaneously, but rather over a long period of time. Thus, the analysis of the relationship between *BR* and *YD* may be facilitated through the use of a distributed lag model. Since the data employed here are primarily cross-section in nature, we are constrained as to the fineness of the lag specification that may be tested: Census data are available only at five-year intervals, and this will not allow too detailed a study of the nature of the lag structure. In any event, in the present demographic context it is unlikely that the time pattern of the weights of the "independent" variables is too complicated, and a reasonable first approximation might be to constrain the weights to decay geometrically for all time periods preceding the current period. An important advantage of this specification is that it allows estimation of the weights with the introduction of a minimum of variables (and hence with maximum parsimony of degrees of freedom), namely, the current value of the "independent" variable(s) and the one-period-lagged value of the dependent variable.[6] Using this specification to explore the relationship between *YD* and *BR* yields the following result:

6. Christ [3, Chapter 7] provides a very convenient source for a summary of the properties of the simpler distributed lag models.

(5)
$$YD_m = 11{\cdot}002 + 0{\cdot}566\,(YD_m)_{-1} + 0{\cdot}663\,BR_m$$
$$(3{\cdot}656) \quad (6{\cdot}626) \quad (5{\cdot}343)$$
$$\bar{R}^2 = 0{\cdot}940,\ S.E.E. = 0{\cdot}982,\ F(2.24) = 206{\cdot}054$$

(6)
$$YD_f = 6{\cdot}450 + 0{\cdot}724\,(YD_f)_{-1} + 0{\cdot}482\,BR_f$$
$$(2{\cdot}091) \quad (11{\cdot}200) \quad (4{\cdot}222)$$
$$\bar{R}^2 = 0{\cdot}912,\ S.E.E. = 1{\cdot}087,\ F(2{\cdot}24) = 136{\cdot}012$$

where the (-1) subscript refers to lagged values, that is, variables measured in 1961. The $\bar{R}^2$ are very high as would be expected in almost any specification that includes the lagged value of the dependent variable as an explanatory variable, but the coefficients of the BR terms are also highly significant. The correlation between YD_{-1} and BR is 0·826 for males and 0·536 for females, and, given the overall closeness of fit, this is not too high to render our estimates of the individual coefficients too imprecise. Equation (5) implies a long-run impact of BR_m on YD_m of 1·528, while equation (6) implies a long-run impact of BR_f on YD_f of 1·746. In both cases over one fourth of the total eventual impact of *BR* on *YD* is felt by the end of the first period.[7] The slightly higher impact of *BR* on *YD* for females may be attributable to the lower mortality experienced in general by females. Thus, the 1960–62 Life Tables show that from every 100,000 live male births 95,963 may be expected to survive to age 15, compared with 96,797 from every 100,000 female live births.

It is possible to apply equations (5) and (6) to the study of the dynamics of population change. Subtract YD_{-1} from both sides of each equation and set $\triangle YD = (YD - YD_{-1}) = 0$ to obtain:

$$\left.\begin{array}{ll}(5a) & (YD_m)_{-1} = 23{\cdot}350 + 1{\cdot}528\,BR_m \\ (6a) & (YD_f)_{-1} = 23{\cdot}369 + 1{\cdot}756\,BR_f\end{array}\right\}\ \text{for Stable } YD$$

Thus, (5*a*) and (6*a*) divide combinations of YD_{-1} and *BR* into those that imply an increase in *YD* and those that imply a fall in *YD* over the period. If the national values of YD_{-1} and *BR* in 1966 are inserted, it may be seen that for males a reduction, and for females an increase, in *YD* between 1961 and 1966 is indicated. This "backcast" is in fact consistent with the actual outcome for the period. Looking to the next intercensal period, 1966–71, it may be seen that at 1966 levels of *YD* the level of YD_m should fall for all BR_m less than 19·9 (the actual BR_m for 1961–66 was 18·7), whereas the level of YD_f will fall only if the level of BR_f is below 17·3 for the period (the 1961–66 level of BR_f was 18·7). As far as YD_f is concerned, this conclusion is consistent with the projections of the previous section.

7. The *BR* is measured as the annual average number of male (female) births, 1961–1966, per 100 average male (female) population aged 15–64.

Equations (5) and (6) leave a not negligible proportion of the variance in *YD* unexplained. One possibility is that net migration affects *YD* by altering the ratio of young to active in the population either as a result of its selectivity of persons whose birth rate is lower than that of non-migrants, or because the ratio of active to dependent in the migrant populations differs from county to county. Information on either of these aspects of Irish migration is as yet unobtainable.[8]

As a crude test of the influence of migration on *YD* the male and female net migration rate, 1961–66 per 1,000 average population aged 15–64, (*NMR*) was introduced and the following results obtained:

(7)

$$YD_m = 10{\cdot}131 + 0{\cdot}555\,(YD_m)_{-1} + 0{\cdot}721\,BR_m + 0{\cdot}051\,NMR'_m$$
$$(3{\cdot}226)\quad(6{\cdot}433)\qquad\quad(5{\cdot}245)\qquad\quad(0{\cdot}980)$$
$$\bar{R}^2 = 0{\cdot}940,\ S.E.E. = 0{\cdot}983,\ F(3{,}23) = 137{\cdot}445$$

(8)

$$YD_f = 6{\cdot}841 + 0{\cdot}690\,(YD_f)_{-1} + 0{\cdot}542\,BR_f + 0{\cdot}039\,NMR'_f$$
$$(2{\cdot}146)\quad(8{\cdot}082)\qquad\quad(3{\cdot}584)\qquad\quad(0{\cdot}621)$$
$$\bar{R}^2 = 0{\cdot}910,\ S.E.E. = 1{\cdot}101,\ F(3{,}23) = 88{,}476$$

Thus, in both cases, the introduction of *NMR'* adds nothing to the explained variance and leaves the magnitude and significance of the other two "independent" variables substantially unaltered. This performance of the migration variable is in keeping with the conclusion that migration exerts little or no influence on the level of *YD*.

Having established that *BR* is a major source of variation in *YD* it is natural to attempt some explanation of the sources of variation in *BR*. Leser [9] has demonstrated that much of the variation in the birth rate (male *plus* female, per 1,000 total population) may be explained in terms of the age and marital status structure of the county populations. A similar analysis of the birth rate, defined as the total number of births per 1,000 females aged 15–64 (BR'_f), has been undertaken here for the year 1966. An expected birth rate (*EBR'*) was obtained by applying the national legitimate and illegitimate age-specific fertility rates to the female population by age and marital status (five year age groups) in 1966. Regression of the actual on the expected birth rate yielded this equation:

(9)

$$BR'_f = 10{\cdot}225 + 0{\cdot}913\,EBR' \qquad \bar{R}^2 = 0{\cdot}644,\ S.E.E. = 5{\cdot}870,$$
$$(1{\cdot}061)\quad(6{\cdot}939)\qquad\qquad F(1{,}25) = 48{\cdot}147$$

Over 60 per cent of the variation in the recorded birth rate in 1966 is accounted for in terms of the age–marital status structure of the female population age 15–64. The remainder of the variation in the actual birth rate is, of course, attributable to variations in marriage fertility. A measure of marriage fertility is provided

8. The survivorship method may be used to obtain estimates of net migration by age by county, although the results are not likely to be very accurate in the older age groups where mortality is high and differs significantly from county to county (see below).

by the discrepancy between the actual and expected birth rates: when this difference is positive, marriage fertility is above average and vice versa. Leser found some tendency for variations in marriage fertility to offset the variations in age and marital status structure that underlie the expected birth rate. Using the present data, the correlation between $(BR'_f - EBR'_f)$ and EBR'_f is $-0{\cdot}131$, which is negative as expected but not significant statistically. Most of the variation in marriage fertility by county is probably due to differences in the occupational-social class composition of the married population, but this topic has not been pursued further in the present paper.

Since age and marital status structure have such important influences on the birth rate, and hence on *YD*, it is natural to seek further information on the determinants of these variables themselves. It may be seen that over 75 per cent of all births occur to women aged 15–34, so that the percentage of the female population aged 15–64 that is aged 15–34 is a useful summary measure of the fertility potential of the active population's age structure. This percentage seems to change rather slowly over time, primarily in response to changes in the net migration rate. Consider the following regression:

(10)

$$Z = -0{\cdot}187 + 1{\cdot}029\, Z_{-1} + 0{\cdot}109\, \triangle NMR'_f$$
$$(0{\cdot}098)\quad (20{\cdot}932)\qquad\qquad (2{\cdot}248)$$
$$\bar{R}^2 = 0{\cdot}952,\ S.E.E. = 0{\cdot}671,\ F(1,24) = 26{\cdot}033$$

where Z = percentage of female population aged 15–64 that is aged 15–34,

$\triangle NMR_f$ = the fall in NMR_f, 1956–61 to 1961–66.

The positive coefficient of $\triangle NMR_f$ is in accordance with a priori considerations, suggesting that the larger the reduction in *NMR* between the two intercensal periods, the higher the level of Z in 1966. The coefficient of $\triangle NMR'_f$ in (10) is significant at the ·05 level ($t_{.05} = 2{\cdot}064$), but it might be questioned whether the inclusion of this variable adds anything to the explanatory power of the relationship. Equation (11) shows the results obtained when $\triangle NMR_f$ is dropped.

(11)

$$Z = -0{\cdot}879 + 1{\cdot}063\, Z_{-1} \qquad \bar{R}^2 = 0{\cdot}945,\ S.E.E. = 0{\cdot}724,$$
$$(0{\cdot}431)\quad (21{\cdot}095) \qquad F(1,25) = 444{\cdot}907$$

Obviously the improvement of (10) and (11) is slight, but it is statistically significant (the F ratio for the additional explained variance due to the inclusion of $\triangle NMR_f$ is 4·20, while the critical value of $F(1{\cdot}24)$ is 4·24 at the ·05 level).[9]

9. The *level* of the *NMR* may also be important, as was indicated by the projections of the earlier part of this paper. The distributed lag structure used here, however, makes it difficult, in this instance where the correlation between Z and Z_{-1} is so high, to obtain reliable estimates of the influence of other variables that appear a priori relevant. It is also somewhat disturbing to note that the coefficient of Z_{-1} in (10) is greater than unity, which implies increasing weights for earlier values of the other explanatory variables. More detailed (e.g. annual) time series data might have facilitated a more satisfactory specification of this relationship.

The age distribution of the active female population does not in fact vary dramatically between the counties. The coefficient of variation of Z is 7·3 per cent. On the other hand, the marriage rate varies pronouncedly. For this reason, the marriage rate is a more important determinant of EBR' than is the Z variable, and the forces that affect the marriage rate are consequently of great importance to the age structure of the population. It has proved more convenient to treat the question of the marriage rate in a separate paper, and only the broad outline of the findings of that study are relevant here. It appears that both male and female marriage rates are influenced by the sex ratio of the unmarried population: the higher the ratio of unmarried males to unmarried females in the marriageable age groups, the lower the male, and the higher the female, marriage rate, other influences equal. The sex ratio of the unmarried population, in its turn, appears to be closely related to the net migration rate, the very high male to female ratio found in many Irish counties being the result of high and sustained net emigration. The net effect of the sex ratio variable on the fertility potential of the population may not be simply estimated due to its opposite influences on each sex's marriage rate, but it is clear that Irish females' migration patterns serve to improve their marriage prospects and thus to raise EBR' as defined above.

Our findings concerning YD may be simply summarized. The main determinant of YD for both sexes has been shown to be current and past values of the BR. The intercounty variation in BR was, in turn, substantially explained in terms of variations in the age and marital status structure of the active population. The age structure of the active population was shown to depend on changes in the NMR, while the marital status structure of the population was not examined in detail in the present paper.

Determinants of Old Dependency

The active population helps determine the level of YD through its decisions on family formation as reflected in the birth rate. There is no comparable (behavioural) link between the active population and the size of the old dependent population. On the other hand, if the active population is growing rapidly, then OD (the ratio of the old to active population) will tend to be low, other things being equal. Thus the active population does influence the level of OD through its rate of increase. The other major influence on OD that suggests itself is the level of mortality among the old population: the greater life expectancy at age 65, the higher OD should tend to be if other factors are held constant. Therefore, writing the per cent change in the active population between 1961 and 1966 as $\%\triangle AP$, and the death rate per 1,000 population aged 65 and over (average of 1961–1966) as DR, the following results have been obtained:

(12)

$$OD_m = 4{\cdot}454 + 0{\cdot}973\,(OD_m)_{-1} - 0{\cdot}096\,(\%\triangle AP_m) - 0{\cdot}048\,DR_m$$
$$(2{\cdot}195) \quad (20{\cdot}691) \quad (2{\cdot}824) \quad (2{\cdot}678)$$
$$\bar{R}^2 = 0{\cdot}990,\ S.E.E. = 0{\cdot}404,\ F(3,23) = 845{\cdot}400$$

(13)

$$OD_f = 5{\cdot}713 + 0{\cdot}963\,(OD_f)_{-1} - 0{\cdot}143\,(\%\triangle AP_f) - 0{\cdot}061\,DR$$
$$(2{\cdot}526)\;(17{\cdot}303)\qquad(2{\cdot}786)\qquad(2{\cdot}472)$$
$$\bar{R}^2 = 0{\cdot}989,\; S.E.E. = 0{\cdot}422,\; F(3,23) = 747{,}150$$

All of the coefficients in these equations have the expected signs (the more rapidly the active population increases, the lower the level of *OD*; the higher the death rate in the old population, the lower the level of *OD*.), and all are significantly different from zero at at least the ·05 level ($t \geqslant 2{\cdot}069$), while those of the lagged dependent variables and $\%\triangle AP_m$ are significant at the ·01 level ($t \geqslant 2{\cdot}807$), and those of $\%\triangle AP_f$ and DR_m are significant at the ·02 level ($t \geqslant 2{\cdot}500$). The problem of collinearity between the explanatory variables is more evident in these equations than it has been up to here in this paper: the correlation between the lagged dependent variable and the $\%\triangle AP$ variable is high: $-0{\cdot}902$ for females. Despite this, the separate influences of each of these variables appears reasonably accurately estimated in (12) and (13).

Analogously to our treatment of the equations for *YD*, (12) and (13) may be used to obtain combinations of OD_{-1} and $\%\triangle AP$ that are consistent with no change in *OD*:

(12*a*)

$$(OD_m)_{-1} = 20{\cdot}258 - 3{\cdot}556\;\%\triangle AP_m$$

(13*a*)

$$(OD_f)_{-1} = 44{\cdot}55 - 3{\cdot}889\;\%\triangle AP_f$$

(The *DR* has been set at its national level, namely, 81·4 for males and 67·0 for females.) Although having approximately equal slopes, the locus of $\triangle OD = 0$ for females lies far above (i.e. to the north-east) that for males. This implies that for a given level of OD_{-1} it requires a much higher rate of increase in the female active population than in the male active population if *OD* is not to grow. Thus, taking the 1966 national values of *OD* as OD_{-1}, the OD_m will rise for all values of $\%\triangle AP_m$ less than 0·7, compared with the value of $\%\triangle AP_f$ of 6·2 that must be reached if OD_f is not to rise. The slightly steeper slope of the female equation (13*a*) would be expected because of the lower mortality of the female population.

The coefficients of the lagged dependent variable in (12) and (13) may be compared with those in (5) and (6), and it becomes clear that the effect of the independent variables on *OD* is much slower in its operation than was the case for *YD*. The long-run effect of $\%\triangle AP$ and *DR* on *OD* are $-3{\cdot}556$ and $1{\cdot}778$ for males, $-3{\cdot}889$ and $-1{\cdot}652$ for females, which are 37 and 27 times their current-period impact. That the initial impact should be so low in relation to the eventual total impact seems unreasonable, since, for example, our results imply that only 10·4 per cent of their total influence of the independent variables on OD_m would have taken effect after four periods (20 years). The high coefficients for OD_{-1} in (12) and (13) may be a reflection of the difficulty of obtaining an accurate picture of the time pattern of the response of OD to its determinants when the only time series data available are at five-year intervals. A further drawback to the specification is that it constrains the lag structure of both independent variables to be equal.

Despite these reservations concerning the interpretation of (12) and (13), it may be safely concluded that they serve to document the role of the rate of change in the active population and the death rate in the old population in the determination of OD. In the Irish context, it would be expected that the active population grows or declines primarily in response to the net migration rate. This, in fact, appears to be the case:

(14)
$$\% \Delta AP_m = 6{\cdot}839 - 0{\cdot}908\ NMR'_m \qquad \bar{R}^2 = 0{\cdot}809,\ S.E.E. = 1{\cdot}959,$$
$$(8{\cdot}473)\ (10{\cdot}542) \qquad F(1,25) = 111{\cdot}145$$

(15)
$$\% \Delta AP_f = 6{\cdot}079 - 0{\cdot}727\ NMR'_f \qquad \bar{R}^2 = 0{\cdot}817,\ S.E.E. = 1{\cdot}611,$$
$$(8{\cdot}531)\ (10{\cdot}826) \qquad F(1,25) = 117{\cdot}202$$

The death rate among the old population should vary as a result of differences in the age structure of this population. An expected death rate, EDR, may be developed by applying the five-year mortality rates taken from the 1960–62 Life Table to the county population by age in 1961 and dividing the resultant expected number of deaths by the relevant population. When this expected death rate was used to explain the actual death rate, the following results were obtained:

(16)
$$DR_m = 134{\cdot}191 - 0{\cdot}670\ EDR_m \qquad \bar{R}^2 = 0{\cdot}013,$$
$$(2{\cdot}935)\ (1{\cdot}155) \qquad S.E.E. = 5{\cdot}481$$
$$F(1,25) = 1{\cdot}338$$

(17)
$$DR_f = 60{\cdot}417 + 0{\cdot}117\ EDR_f \qquad \bar{R}^2 = 0{\cdot}036$$
$$(2{\cdot}511)\ (0{\cdot}322) \qquad S.E.E. = 3{\cdot}519\ F(1,25) = 0{\cdot}100$$

Thus, surprisingly, neither of these regressions is significant, and the expected death rate (that is, the age structure of the population aged 65 and over) does not help to explain the variations in the actual death rate. A possible reason for this outcome is the very small (relative or absolute) variance in the EDR: the variances are 3·435 and 3·629, the coefficients of variation 2·35 and 2·87 per cent, for males and females respectively. This reflects the fact that the age structure of the old population does not vary much from county to county. The actual death rates are therefore mostly a reflection of age-specific mortality in each county. In this regard the greatest source of variation is possibly the higher level of mortality in urban areas. A measure of age-specific mortality is provided by the gap between DR and EDR: when this is positive it indicates that age-specific mortality is generally higher than average in the area, and vice versa. It happens that there is a significant, negative correlation between this measure of age-specific mortality and the EDR:

(18)
$$DR_m - EDR_m = 134{\cdot}271 - 1{\cdot}671\ EDR_m \qquad \bar{R}^2 = 0{\cdot}219$$
$$(2{\cdot}937) \quad (2{\cdot}881) \qquad S.E.E. = 5{\cdot}481,$$
$$F(1,25) = 8{\cdot}301,$$

(19)
$$DR_f - EDR_f = 60{\cdot}462 - 0{\cdot}884\ EDR_f \qquad \bar{R}^2 = 0{\cdot}161,$$
$$(2{\cdot}512) \quad (2{\cdot}442) \qquad S.E.E. = 3{\cdot}519$$
$$F(1,25) = 5{\cdot}972$$

Equation (18) is significant at the ·01 level, and (19) at the ·05 level. The negative correlation indicates that there is some tendency for countries with older than average age structures to have lower than average age-specific mortality rates (confining ourselves here to the population aged 65 and over). This is most probably a reflection of the high mortality in urban areas, where the net emigration over the years has been lower than average and has tended to reduce the average age of the over 64 population.

In summary, the study of OD led immediately to the forces that determine the rate of change of the active population and the mortality conditions of the old population. The active population was shown to vary closely in response to the net migration rate, while the number of deaths occurring among those aged over 64 was seen to vary mostly in response to age-specific mortality rates, which in turn showed some tendency to be inversely associated with the average age of the old population.

Conclusion

The roles of birth, death and migration rates in the determination of the age structure of the Irish population have been studied from an empirical viewpoint in this paper. The most striking feature of Ireland's present age structure is its combination of fairly high levels of both young and old dependency, which is unusual in the light of the general tendency for the levels of these dependencies to vary inversely to each other. The explanation of this aspect of Ireland's population structure was seen to lie in the heavy and prolonged net emigration the country has experienced.

The high level of young dependency found in Ireland was seen to be a reflection of the high birth rate that prevails among the active population. The high level of old dependency was seen to arise from the slow rate of increase, or the decrease, in the active population, which in turn was a reflection of high net emigration rates.

The prospects for young, old and total dependency in Ireland may be studied under a variety of assumptions about the birth and net migration rates. Although the immediate outlook appears to be for a lower level of old dependency due to the slackening of the rate of net emigration, we saw that existing birth rates imply a rising level of young dependency, and the prospective rise in young dependency could well exceed the fall in old dependency. Thus, any significant fall in the burden of total dependency in Ireland appears contingent on a fall in the birth rate and at least no increase in the emigration rate. Since marriage rates are unlikely to fall, and in all probability will continue to rise, lower births rates will be attained only if marriage fertility declines quite substantially.

The regression results presented in this paper could be regarded as a model of the determination of young and old dependency for both sexes. Young dependency was made a function of current and past birth rates which in turn were made functions of the age and marital status composition of the active population. The age structure of the active population was seen to vary with changes in the net migration rate, while the marital status composition of the population was not explicitly studied in the present paper.

Old dependency was made a function of current and lagged rates of change in the active population and the death rate of the old population. The rate of change of the active population is very closely determined by the net migration rate, while the death rate of the old population seems to reflect mostly the prevailing age-specific death rates.

If the age-specific death rates and the various net migration rates are regarded as exogenous variables, the regression results could be taken as a model of dependency, which "explains" dependency in terms of lagged values of dependency and current and lagged values of the exogenous variables. It is readily seen that the model specified is recursive.[10] The utility of a model of this type for forecasting changes in population age structure is at once apparent, as is its limitation, namely, that the forecasts obtained are critically dependent on the values of the exogenous variables used in their preparation. In particular, reliable predictions of the net migration rate and birth rate would be essential, but elusive, prerequisites for the application of this model to the task of forecasting. The causes of variation in net migration rates have received some attention of late, and the econometric work on this subject may help in preparing population forecasts (*cf* [12]). Forecasts of birth rates could be prepared on the basis of an extrapolation of the recent upward trend in marriage rates combined with various assumptions as to the course of marriage fertility. This last variable is probably the hardest component of Irish vital rates to forecast over the immediate future, and the most crucial single factor in determining the future of the Irish dependency ratio.

10. A model is recursive if the matrix of structural coefficients of the current values of the endogenous variables may be diagonalized, so that the entries above the main diagonal are all zero. In order for Ordinary Least Squares estimation (which has been used exclusively in this paper) to yield best, linear unbiased estimates of the parameters of a recursive model it is necessary that the disturbances of the all structural equations be independently distributed.

APPENDIX

Note on the Age Structure of Net Migration, Ireland 1946–66

At various points in this paper the importance of obtaining some knowledge about the age structure of net migration became evident. For example, the population projections that were undertaken to ascertain, among other matters, the effect of migration on population age structures were crucially dependent not only on the level of total net migration but also on its distribution between the various age groups. In my earlier study on migration, it was remarked that one of the surprising aspects of the 1961–66 period was the relatively small reduction in the net migration rate among the 15–24 age group [12. p. 24]. In this Note some tentative findings regarding the age structure of net migration are presented. The findings reported here are strictly exploratory in nature.

If we could assume that a forecast of the overall net migration rate (M) were available, the task before us here is the allocation of this total among the various age groups to obtain M_i age-specific net migration rates. (In practice, the four five-year age groups between 15 and 34 have contained the bulk of Irish net migration in the post-war period, and our attention is confined to these four age groups in this Note. Thus, M refers to the total net migration rate for the population aged 15–34.) We may define N analogously with M, where N refers to the rate of natural increase of the population in the absence of net migration. (The age-specific rates, N_i, may be calculated by applying five-year survival rates to the population aged $i-1$ at the start of the period.) Since there have been four intercensal periods since the war, and age-specific migration data have now been published for all of them [2, 1966, Vol. II, pp. xvi-xvii] we can generate 16 observations on $(N_i/N)_t$ and $(M_i/M)_t$ ($i=1, \ldots 4$ (age groups): $t=1, \ldots 4$ (intercensal time periods)). In order to try to shed some light on the determinants of migration age structure, the model has been specified:

$$(A1)\ (M_i/M)_t = a_o + a_1(N_i/N)_t + a_2D_1 + a_3D_2 + a_4D_3 + u_{i\ t}$$

where, in addition to the previously defined variables, D_1, D_2 and D_3 are dummy variables for the age groups and u_{it} is the stochastic disturbance term. Thus,

$D_1=1$ if $i=1$, zero otherwise,
$D_2=1$ if $i=2$, zero otherwise,
$D_3=1$ if $i=3$, zero otherwise.

The model hypothesizes that the ratio of the net migration rate in the i^{th} age group to the total net migration rate in each time period is a function of the ratio of the i^{th} age group's rate of natural increase to the total rate of natural increase in that time period, and an intercept term that differs for each of the i age groups and thus reflects the shift effect of factors other than $(N_i/N)_t$ upon the relative rate of net migration. Since the D_i assume the same value in all time periods, these shift factors are assumed invariant over time, and this is a major limitation of the present model. The approach is thus an analysis of covariance in which the $(N_i/N)_t$ variable is a continuous variable of the kind usually found in regression analysis, while the D_i variables are categorical variables of the type found in analysis of variance. The only a priori expectation about the values of the parameters in $(A1)$ is that $a_i > O$. Estimation of $(A1)$ using the data described above for males and females separately yielded the following results:

(Males):

$$(M_i/M)_t = \underset{(2\cdot780)}{0\cdot380} + \underset{(2\cdot294)}{0\cdot345}\,(N_i/N)_t - \underset{(0\cdot121)}{0\cdot029}\,D_1 + \underset{(1\cdot651)}{0\cdot611}\,D_2 + \underset{(1\cdot317)}{0\cdot322}\,D_3 \qquad \bar{R}^2=0\cdot785,\ S.E.E.=0\cdot271,\ F(4,11)=14\cdot719$$

(Females):

$$(M_i/M)_t = \underset{(4\cdot094)}{0\cdot418} + \underset{(1\cdot620)}{0\cdot178}\,(N_i/N)_t + \underset{(1\cdot247)}{0\cdot253}\,D_1 + \underset{(3\cdot354)}{0\cdot925}\,D_2 + \underset{(1\cdot634)}{0\cdot245}\,D_3 \qquad \bar{R}^2=0\cdot858,\ S.E.E.=0\cdot204,\ F(4,11)=23\cdot595$$

First of all, it is clear that both of these regressions are overwhelmingly significant ($F(4,11)_{\cdot01}=5\cdot67$). The coefficients of the $(N_i/N)_t$ terms are of the expected sign, but only that for males is significant at the ·05 level. (That for females is significant only at the ·20 level.) There is a fairly high correlation between the N_i/N variable and D_2 for both males and females ($r=0\cdot728$ and $0\cdot727$, respectively), but otherwise the "independent" variables are not highly intercorrelated. With the exception of the coefficient of D_2 for females, and the intercept term in each equation, none of the other coefficients is significantly different from zero.

These results leave much to be desired: despite the fairly high $\bar{R}^2$s, it cannot be claimed that the parameters of the model have been at all reliably estimated. It is probably asking too much of the rather limited data used to expect satisfactory estimates of the five parameters. The results are suggestive, however, and with additional data better estimates of the parameters might be obtained. To illustrate the usefulness of such results, it is worthwhile to record the outcome of a simple experiment aimed at forecasting the age structure of net migration 1966–71. Assuming that the aggregate rate of net migration, M, would be at its 1961–66 level and calculating N and N_i from the 1966 population by age and the 1960–62 Life Table survival ratios, it is possible to use the above equations to estimate that values of M_i for 1966–71. Relating these forecast M_i to the 1966 population by age it is possible to obtain forecasts of the number of net migrants in each age group. The results of this experiment are presented in Table I. The actual number of net migrants 1961–66 are also presented for comparative purposes. Some interesting aspects of this exercise may be seen from the Table. The same aggregate net migration rate implies a smaller number of male, and a larger number of female, migrants in 1966–71 than in 1961–66. Moreover, our model suggests a fairly substantial change in the age structure of migration between the two periods. Of course, these changes in the age structure of migration are a reflection of the changing level $(N_i/N)_t$, which in turn is partly determined by past levels of net migration. Thus, the very high relative migration rate in the 15–19 age group in 1961–66 produces a low relative natural increase for the 20–24 age group in 1966–71, and this leads our model to predict a reduced percentage of total net migration in the latter age group between 1966 and 1971.

The forecasts presented in Table I are not only contingent on the accuracy of the aggregate net migration rate, M, that is used in preparing them, but also on the meaningfulness of the model specified in ($A1$) and the accuracy with which the parameters of this model have been specified. The weakest link in this chain at present appears to be the statistical testing of ($A1$). As mentioned earlier, with increased availability of data, it may be possible to strengthen this link if the model is still found to be of interest.

TABLE I: *Projected Net Migration 1966–71, and Actual Net Migration 1961–66, by Age and Sex*

	Males		Females	
Age Group	*1961–66 actual*	*1966–71 projected*	*1961–66 actual*	*1966–71 projected*
15–19	14·6	7·9	14·2	10·7
20–24	25·1	22·0	22·6	20·7
25–29	5·0	7·2	3·0	7·7
30–34	+2·0	3·1	+0·1	3·2
15–34	42·7	40·2	39·7	42·3

+ = Net immigration.

Assumption: net migration rate (population aged 15–34) equal in 1961–66 and 1966–71.

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