

The Irish PAYE Personal Income Tax System*

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I INTRODUCTION

This paper describes part of the Personal Income Tax system in the context of work in progress on a model of the Irish economy. The model is a short-run one designed to quantify the effects of fiscal policy on the major economic aggregates. It is being estimated over the 21 year period 1953 to 1973. We wished to consider only those effects of fiscal policy which occur within one year of a given fiscal action. Thus, in so far as personal income tax is concerned, we need only consider the Pay As You Earn (PAYE) tax system; other personal income tax receipts in any year, such as those from self-employed persons, have been functions of income and the tax system prevailing in the previous year, or in previous years. That is because there have been lags between the period when income accrued and when it was assessed for tax, and a further lag between assessment and payment of tax. No such lag operates in the PAYE system.

The PAYE system was introduced in 1960. Therefore we examine that tax system for the period from 1960 to 1973. In the years 1953 to 1960 personal income tax receipts are exogenous to our model, being determined completely by the level of income and the tax system of the previous year, or of previous years.

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Examination of the Personal Income Tax system in Ireland has been greatly facilitated by the recent work of Dowling (1977). He constructs a detailed model of the complete Personal Income Tax base. Already features of that work have been employed to estimate personal income tax receipts for use in a macroeconomic model (Kelleher, 1976). Our attempt to analyse the PAYE system in the manner in which Dowling examines the complete Personal Income Tax system leads us to adopt a procedure different from that of Kelleher.

Throughout our sample period the PAYE system covered almost all employees except civil servants, the army and the gardai.¹ The system involves deduction of tax by the employer before payment of wages or salary to the employee. The amount of tax payable was estimated, until 1974, by application of a Standard Tax Rate to a tax base, known as *Taxable Income*. The Surtax system, which involved application of various tax rates depending on the level of income (and with significant lags) was a separate tax structure and shall not be dealt with here.²

The amount, *Taxable Income*, to which to apply the Standard Tax Rate (or, after April 1974, standard tax rates) was, and still is, estimated by making various deductions from the Gross Incomes of potential PAYE taxpayers. For the PAYE system, *Gross Income* represents the total income that comes to the attention of the Revenue Commissioners. By and large, the incomes of those who are below the "exemption limit", which is set by the level of Personal Allowances, are excluded from the definition of Gross Income. Next, under the heading "Exemptions and Deductions", are interest payments and payments under insurance policies; these payments are subtracted from Gross Income to give a figure known as *Actual Income*. From Actual Income, until the April 1974 Budget, was subtracted Earned Income Relief. That was a system which allowed a worker deduct 25 per cent of total Earned Income up to a maximum of £500, from income before assessment of tax due. The deduction of Earned Income Relief from Actual Income gave a figure known as *Assessable Income*. Personal Allowances were then, and still are,³ subtracted from Assessable Income to yield *Taxable Income*, to which the Standard Tax Rate was applied.

1. Civil servants, the army and the gardai were brought into the PAYE system in April 1976.

2. The Surtax system was abolished in 1974 and amalgamated with the Income Tax system. Thus a single Standard Tax Rate no longer exists. Instead different tax rates apply to different bands of taxable income.

3. Since Earned Income Relief, after April 1974, was zero, the concepts Actual Income and Assessable Income have been identical after that date.

II ANALYTICAL PRINCIPLES AND METHODS

Tax receipts are the outcome of a tax rate (or rates) operating on a tax base (or bases). In the case of the PAYE system, the overall base, Taxable Income, has been the outcome of the operation of several rates on several bases. Thus there have been three rates, or sets of rates, in the PAYE system in the period under review. These rates are instruments or potential instruments of fiscal policy. First, there was the rate of Earned Income Relief; secondly, there are the Personal Allowance rates and limits; finally, there was the Standard Tax Rate. Total Earned Income, as statutorily defined, was the base upon which the rate of Earned Income Relief operated. For each of the Personal Allowance rates there is a base; these bases are the number of married persons, the number of single persons, the number of widows and widowers, etc. And, as stated earlier, the Standard Tax Rate operated on the base Taxable Income.

Unfortunately it is not possible for us to follow the logic of the PAYE tax system as closely as Dowling has followed the logic of the complete Personal Income Tax system. That is because data are not available on the intermediate steps in the calculation of Taxable Income for the PAYE system. We do not know what Actual Income or Assessable Income were in the PAYE system in any year. Nor are there any published data on how much Earned Income Relief or Personal Allowances were claimed by PAYE earners in any year.

Given the limitations of our data, we have to consider what features of the tax system are of primary interest and in what way we can best analyse these. We wish to examine how tax receipts have been determined by the level of Personal Income, the level of Personal Allowances, the Standard Tax Rate and the rate of Earned Income Relief. We have to consider whether to *directly* estimate tax receipts as a function of these four or whether to estimate the tax base, Taxable Income, on which the Standard Tax Rate operated to give tax receipts.

We have to consider also how we can incorporate the instruments of fiscal policy — the Standard Tax Rate, the level of Personal Allowances and the rate of Earned Income Relief. As far as possible we have tried to respect what Balopoulos (1967) calls “a fundamental principle, namely, that the parameters of government action to be introduced ought to be pure government action parameters, i.e., parameters of action upon which the government exercises exclusive control”. A “composite” government action parameter would be a parameter the value of which depended not only on government decisions but also on “the value of a great variety of economic and demo-

graphic variables, such as the level of aggregate personal income, the pattern of income distribution, the number of taxpayers, the proportion of earned relative to total personal income, the personal circumstances of taxpayers, etc., which are outside government control". (Balopoulos, 1967, p. 14). Balopoulos points out that the use of composite government action parameters could lead to poor forecasting where a change in the tax structure or in income distribution occurs. And in a model specifically designed to serve policy purposes, an estimate of a composite government action parameter is inappropriate unless we have an estimate of the link between the composite parameter and some pure parameter of government action. "The validity of such composite estimates rests on the assumption that the underlying structure (the tax structure, income distribution, etc.) remains unchanged, but the principal objective of any economic policy model is by definition to consider changes in the structure". (Balopoulos, 1967, p. 17).

We decided to concentrate our attention on estimating the determinants of the PAYE tax *base*, rather than on estimating PAYE tax *receipts directly*. (We will obtain the ultimate determinants of receipts *indirectly*, via the tax base and the tax rate.) Kelleher has applied part of Dowling's work to derive a more *direct* estimate of total Personal Income Tax Receipts as a function of Personal Income and the level of Personal Allowances claimable. However, estimation of the determination of the tax base (from which we derive the determinants of tax receipts) has, we believe, several advantages, some of which are noted by Dowling. First, it allows us to separate the influence on tax receipts of changes in the tax base and changes in the tax rate. Secondly, it makes the functions we estimate more applicable to the Irish tax system after 1974 than would be the case if we estimated tax receipts directly. In 1974 the Standard Tax Rate was abolished and replaced by a range of rates applicable to different slices of taxable income, but the calculation of the tax base, Taxable Income, remains more or less the same as before. As Dowling notes, the use of an equation or model estimating the tax base, Taxable Income, after 1974 will depend on some exploration of the relationship between tax rates and tax base in the new tax code. This is necessary if we are to go from our estimate of the determinants of the tax base, Taxable Income, to an estimate of the determinants of tax receipts. Thirdly, estimating first the tax base rather than directly estimating tax receipts allows us to incorporate the Standard Tax Rate in our model. Very little variation occurred in the Standard Tax Rate during our sample period; thus it was found impossible to *directly* estimate tax receipts as a function of the Standard Tax Rate. It seems important, however, to include the Standard Tax Rate in our model in some way. Primarily because it was an important fiscal policy instrument. Secondly, because of the impossibility of

modelling in as much detail as we would ideally wish, the operation of Personal Allowances and Earned Income Relief in the PAYE system due to the limited nature of the data available to us. As already indicated, Kelleher used Personal Income and Personal Allowances claimable to determine tax receipts. However, Kelleher notes that "strictly speaking, other variables should be included to take proper account of the tax structure. In particular, tax rates and the rate at which Earned Income Relief could be claimed, should be included". (Kelleher, 1976, p. 2). We shall consider the extent to which it is possible to take account of those two variables.

A standard rate of tax applied throughout our sample period 1960 to 1973. Thus PAYE tax receipts, T , in any year, n , will be Taxable Income, TI , in year n multiplied by the Standard Tax Rate, t , in year n :

$$T_n = (TI_n)(t_n)$$

We want to estimate the determinants of TI , the PAYE tax base, as a function of Personal Income, Personal Allowances, and the rate of Earned Income Relief, i.e.,

$$TI = f(Y, PAL, EIR),$$

where Y = Personal Income
 PAL = Personal Allowances Claimable
 EIR = Earned Income Relief

Having estimated the determinants of PAYE Taxable Income in our sample period, it will be a simple matter to estimate PAYE tax Receipts.

The only official data available for the PAYE system are Gross Income, Tax Payable, and Tax Receipts. Otherwise the figures on PAYE are included in the official data for the complete Income Tax system. Thus we have had to adjust many of the official statistics to derive our data. The data used, along with a description of its derivation, can be found in the appendix. Before proceeding to estimation some comments on our data adjustments are appropriate. First, in relation to deriving a series for PAYE Taxable Income we note that there is virtually no lag between assessment of income and payment of tax in the PAYE system, and so Tax Payable and Tax Receipts are practically identical; we will thus assume that they are identical. Thus we do not have to choose between estimating tax liabilities and estimating tax payments. This choice is clearly crucial in models of taxation systems which contain lags.

As *ex post* identities for the PAYE system it must be the case that actual

tax yield = (tax rate) (actual tax base). So actual tax base = (actual tax yield)/ (tax rate). From the published data we know actual tax receipts, so we can divide through by the Standard Tax Rate to get the actual tax base, what we will call Taxable Income. It is our objective to find the ultimate *ex ante* determinants of this Taxable Income (in terms of behavioural equations rather than expressed as an identity) and hence, of PAYE tax receipts.

Personal Allowances operate twice in the determination of Taxable Income. Those whose incomes are below their personal allowances are exempted from income tax. Secondly, those who are actually in the tax net are entitled to claim tax-free income up to the level of their particular personal allowances. Because of the recursive nature of his model, Dowling was able to incorporate these two effects separately. This cannot be done for the PAYE system since we cannot estimate the intermediate steps in the determination of Taxable Income.

Our series PAL is a measure of the Personal Allowances *claimable* by potential PAYE taxpayers. Available data do not permit us to examine the relationship between the level of allowances *claimable* and the level of allowances *actually* claimed. If everyone were in the tax net the two would be approximately equal. The value which PAL takes in any year depends primarily on the levels at which government sets the various personal allowances but is also sensitive to demographic movements and shifts in labour force composition and employment status. These magnitudes are largely beyond government's control. This makes the measure PAL not a pure government action parameter in the sense discussed earlier; strictly speaking, a measure of the responsiveness of Taxable Income to changes in PAL does not tell us exactly how Taxable Income responds to a given change in the money values of the various personal allowances. But since our model is a short-run one, we can assume that the demographic and employment structure is constant in any given year. Thus in a short-run model PAL conforms reasonably well to the analytical principles we would like to see in fiscal policy models. (See items 5, 6 and 7 in the appendix).

EIR was claimable at 25 per cent of earned income up to a maximum claim of £500 throughout our sample period 1961-1973. First it interacted with Personal Allowances in determining whether an individual was in the tax net. A person with an income of £1,100 and Personal Allowances of £1,000 would not have been in the tax net since the excess of income over tax-free Personal Allowances was smaller than his/her entitlement to EIR at 25 per cent of £1,100, i.e., £275. Secondly, those who were in the tax net could deduct 25 per cent of their earned income from their income that was otherwise subject to tax. Again, unlike Dowling, we cannot incorporate these two effects separately.

It seems impossible to adequately take account of EIR in the PAYE system. First, because the rate of EIR was constant throughout our sample period we saw no point in attempting to use the rate itself as an explanatory variable in estimating the parameters of a function for Taxable Income. Second, the rate of EIR is the only information available on the operation of EIR in the PAYE system. The effect of a constant rate of EIR could be analysed if some data on the amount of EIR claimed each year were available; but as already indicated such data were not available. It was, however, felt that to ignore EIR would be to impute the deduction under EIR to either Personal Allowances or Personal Income.

Thus we adopted a commonsense procedure by simply deducting 25 per cent of Earned Income.⁴ That is, we scale our Personal Income series by 0.75. (All income in the PAYE system is Earned Income.) We must take note of an assumption involved in this procedure. The system of EIR contained a limit of £500 on the maximum claim. Thus a taxpayer with Earned Income of more than £2,000 could claim only EIR of £500 and would not therefore be getting relief of 25 per cent of total Earned Income. Our assumption is therefore unrealistic in as much as there were taxpayers with earned incomes over £2,000. However, EIR was abolished in 1974 and the above adjustment which we propose to adopt for it would not be made in application of the model to years after 1973.

In order to evaluate this adjustment for EIR we will also estimate a function for Taxable Income without any adjustment to our Personal Income series.

III ESTIMATION

We wish to estimate two general functions:

$TI = f(Y, PAL)$, with no adjustment for EIR, and

$TI = f(.75Y, PAL)$, with an adjustment.

We can identify some *a priori* expectations about some of the derivatives.

$$1 > \delta TI / \delta Y > 0 \quad (i)$$

$$\delta^2 TI / \delta Y^2 > 0 \quad (ii)$$

$$\delta^2 TI / \delta Y \delta PAL < 0 \quad (iii)$$

$$-1 < \delta TI / \delta PAL < 0 \quad (iv)$$

$$\delta^2 TI / \delta PAL^2 > 0 \quad (v)$$

⁴ For a description of the procedure actually applied by the Revenue Commissioners see Kelly and Carmichael, 1968, p. 230.

- (i) We obviously expect an increase in Personal Income to cause an increase in TI. However, we do not expect that an increase in Personal Income could cause a greater increase in TI. If everyone were in the tax net, $\delta TI/\delta Y$ would equal unity.
- (ii) We also expect $\delta TI/\delta Y$, the responsiveness of TI to an increase in Personal Income, to increase as Personal Income increases. As Personal Income rises, more and more people enter the tax net and further changes in Personal Income are more fully reflected in TI. If everyone were in the tax net, $\delta^2 TI/\delta Y^2$ would equal zero.
- (iii) As Personal Allowances claimable increase we expect $\delta TI/\delta Y$ to fall. An increase in PAL removes some people from the tax net, thus making TI less responsive to changes in Personal Income. If all personal incomes were very high we might expect $\delta TI/\delta Y$ to be unaffected by an increase in PAL; however, this case is likely to be outside the range of our observations. Given a certain continuity assumption condition (iii) is mathematically equivalent to $\delta^2 TI/\delta PAL\delta Y < 0$.
- (iv) We expect $\delta TI/\delta PAL$ to be negative. An increase in PAL will always reduce TI. But we do not expect that, within the ranges of our observations, a given increase in claimable Personal Allowances could reduce TI by as much as that increase in Allowances. Thus $\delta TI/\delta PAL > -1$.
- (v) We expect $\delta TI/\delta PAL$ to be increased as Personal Allowances are increased. An increase in the level of Personal Allowances removes people from the tax net. Then any given change in Personal Allowances will be reflected less fully in changes in TI; the value of $\delta TI/\delta PAL$ will have become less negative. Therefore $\delta^2 TI/\delta PAL^2 > 0$.

Several functional forms, which ideally should satisfy the more important of our *a priori* conditions, were estimated. We preferred the double-log transformation:

$$\text{LnTI} = a_0 + a_1 \text{LnY} + a_2 \text{LnPAL}$$

with or without adjustment for EIR.

It is possible that this general specification will satisfy all the *a priori* desirable conditions. Of the various alternative forms for which parameter estimates were made, the least unsatisfactory appeared to be

$$\text{TI} = a_0 + a_1 Y + a_2 Y^2 + a_3 \text{PAL}^2$$

It will be noted, however, that this form inevitably violates *a priori* condition (iii). We re-estimated each equation with the intercept constrained to be

zero: we expect that if Personal Income and Personal Allowances were zero there would be no Taxable Income. The method of estimation was ordinary least squares.

Results

In all the non-homogeneous equations estimated, the coefficient of PAL was insignificant at the 95 per cent level. Also the intercept was not significantly different from zero in most selections. These considerations, along with the fact that we expect that if Personal Income and Personal Allowances were zero there would be no Taxable Income, led us to prefer the homogeneous selections.

Turn first to the double-log equations. These satisfied all our *a priori* conditions except one part of condition (iv); as we shall later show, for some later years of our sample period, the responsiveness of Taxable Income to Personal Allowances is less than minus one. This conflicts with our understanding of how the system operates. But before examining in detail the estimates for the double-log equations we shall look at how well our best alternative satisfies our *a priori* expectations. This other functional form was an attempt to avoid the short-coming of the double-log equations, noted above, while at the same time capturing the expected non-linearity of the system. This regression, constrained to be homogeneous, was (standard errors in parentheses):

$$TI = 0.16059Y + 0.000239Y^2 - 0.00047626PAL^2 \quad (1)$$

(0.02651) (0.0000114) (0.00011419)

$$R^2 = .998 \quad DW = 1.65 \quad \text{Geary, tau} = 4$$

In this selection two of our *a priori* conditions were not fulfilled. These were condition (v) and, of necessity by virtue of the specification, condition (iii). Condition (v) is that $\delta^2 TI / \delta PAL^2 > 0$. But in equation (1) it is negative rather than positive. The coefficient of both Y and PAL are significant. In absolute terms both $\delta TI / \delta Y$ and $\delta TI / \delta PAL$ increase through our sample period as we expected they would. $\delta TI / \delta Y$ grows from 0.28 in 1961 to 0.66 in 1973. $\delta TI / \delta PAL$ goes from -0.25 in 1961 to -0.38 in 1973.

Although this equation avoids a short-coming of the double-log transformations (equations (2) and (3) below) that for some years $\delta TI / \delta PAL < -1$, it involves violation of many other of our *a priori* expectations. As already noted, the cross partial derivations, $\delta^2 TI / \delta Y \delta PAL$ and $\delta^2 TI / \delta PAL \delta Y$ are zero. The second derivative of Taxable Income with respect to Personal Allowances is negative instead of positive. In addition to these short-

comings, we are inclined to believe that this selection does not adequately capture the nonlinearity in the relationship between Taxable Income and Personal Allowances: on the basis of Dowling's estimates for the complete Personal Income Tax system a $\delta TI/\delta PAL$ of -0.25 (referred to earlier) seems unduly negative for the year 1961 and -0.38 seems not negative enough for the year 1973.

We now examine the properties of the homogeneous double-log equations – selections (2) and (3) – in more detail. These are as follows:

$$\text{LnTI} = 2.40533\text{LnY} - 1.82744\text{LnPAL} \quad (2)$$

(0.05967) (0.06518)

$$R^2 = .994 \quad DW = .99 \quad \text{Geary tau} = 3$$

$$\text{LnTI} = 2.35736\text{Ln}(.75\text{Y}) - 1.65677\text{LnPAL} \quad (3)$$

(0.05714) (0.05955)

$$R^2 = .994 \quad DW = .91 \quad \text{Geary tau} = 3$$

Equation (3) contains an adjustment for EIR. Since $\text{Ln} .75\text{Y} = \text{Ln} .75 + \text{LnY}$, and since $\text{Ln} .75$ is negative, the effect of this adjustment is to constrain the intercept to be negative in double log space, provided the coefficient associated with LnY is positive. We remark that this does *not* imply that if Personal Income were zero, then Taxable Income would be negative, only that it would be zero; for the multiplicative form of equation (3) is of course, $\text{TI} = (.75\text{Y})^{2.35736} \text{PAL}^{-1.65677}$.

We did not apply the Durbin-Watson test for autocorrelation of the residuals because that test is not tabulated for regressions with less than 15 observations. The Geary count of sign-change was used. (Geary, 1970). At the 5 per cent level and with a Geary tau of 3, as in the three equations above, we cannot reject the hypothesis that the residuals are independent.

In Table 1 below we present the values of the multipliers $\delta TI/\delta Y$ and $\delta TI/\delta PAL$ generated by our estimates. Lest our notation be misleading, we point out that it is clear from equations (2) and (3) above that $\delta^2 \text{TI}/\delta Y^2 > 0$; thus $\delta TI/\delta Y$ increases as Y increases. Yet in Table 1 $\delta TI/\delta Y$ falls in 1972. This is because we are evaluating $\delta TI/\delta Y$ at the PAL which prevailed at each date. Thus $\delta TI/\delta Y$ is to be interpreted as $\delta TI(t)/\delta Y(t)$, given $PAL = PAL(t)$, where t in this context denotes the year in question. Similar remarks apply to the other derivatives in the table.

As noted earlier some of these multipliers have values above unity for some later years of our sample period. The problem area is $\delta TI/\delta PAL$ (from

Table 1: *Responsiveness of Taxable Income to Personal Income and Personal Allowances*

Selection	$\frac{\delta TI}{\delta Y}$	$\frac{\delta TI}{\delta Y}$	$\frac{\delta TI}{\delta PAL}$	$\frac{\delta TI}{\delta PAL}$
	2	3	2	3
1961	.245	.240	-.195	-.178
1962	.274	.268	-.238	-.216
1963	.292	.286	-.267	-.242
1964	.345	.337	-.357	-.323
1965	.364	.356	-.393	-.356
1966	.392	.383	-.449	-.407
1967	.410	.402	-.491	-.446
1968	.460	.451	-.602	-.546
1969	.522	.511	-.758	-.687
1970	.619	.604	-1.021	-.922
1971	.749	.726	-1.416	-1.271
1972	.698	.689	-1.310	-1.198
1973	.811	.801	-1.719	-1.572

equation (2) where from 1969 its absolute value exceeds unity.

In no year do selections (2) and (3) violate our *a priori* condition $\delta TI/\delta Y < 1$. Applying the Standard Tax Rate to our estimates of the response in Taxable Income to changes in Personal Income gives the marginal rates of tax. For example, the value of $\delta TI/\delta Y$ for 1973, based on the estimated coefficient values from equation (2), is .811. And multiplying this by the Standard Tax Rate of 35 per cent gives a marginal tax rate, $\delta T/\delta Y$, of 28.4 per cent. (Equation (3) implies a $\delta T/\delta Y$ of 28.1 per cent in 1973.) Since the maximum rate of tax in the PAYE system was the Standard Tax Rate, we find this result quite plausible. Dowling did not estimate $\delta T/\delta Y$ for years after 1971. However, our estimates of $\delta T/\delta Y$ for PAYE earners in the earlier years, implied by our estimates of $\delta TI/\delta Y$, conform quite closely to those of Dowling for the complete Personal Income Tax system. (Dowling, 1977, p. 47). For example, Dowling estimates $\delta T/\delta Y$ to have been 6.3 per cent in 1961; our estimates for the same year are 7.8 per cent (from equation (2)) and 7.6 per cent (from equation (3)). For 1969 Dowling estimates $\delta T/\delta Y$ to have been 20.2 per cent; our estimates for the same variable are 18.3 per cent (from equation (2)) and 18 per cent (from equation (3)).

In view of the above considerations we see no *a priori* reasons to doubt the relative accuracy of our estimates of $\delta TI/\delta Y$. Our estimates of $\delta TI/\delta PAL$ for the PAYE system are reasonably close to Dowling's estimates (for the complete Personal Income Tax system) over the period 1961 to 1969. From 1970 onwards, however, our estimates for those derivatives become implausibly large. But it should be noted, in this context, that since the OLS

estimates are random variables with normal distributions, the true coefficients could lie in a range around our estimates. We have constructed confidence intervals for our estimates of selections (2) and (3). The 95 per cent confidence intervals gave very wide ranges of possible values for the true coefficients. Using estimates of the standard errors of our coefficient estimates, the standardised estimates are no longer normal but have a t-distribution. We constructed 70 per cent confidence intervals, that is, with $t = 1.088$ and 11 degrees of freedom.

For selection (2) the estimated parameter values were $a_1 = 2.40533$ and $a_2 = -1.82744$. The 70 per cent confidence intervals for these are:

$$2.34041 < a_1 < 2.47026; -1.89835 < a_2 < -1.75652$$

Combining these possible values of the coefficients a_1 and a_2 gives a very wide dispersion of possible values for our multipliers $\delta TI/\delta Y$ and $\delta TI/\delta PAL$. For instance our estimated multiplier $\delta TI/\delta Y$ in 1961 is 0.245. But if the coefficients varied within just over one standard error of our estimates then $\delta TI/\delta Y$ could be anywhere in the range from 0.112 to 0.538. For 1973 our estimated coefficients gave a $\delta TI/\delta Y$ value of 0.811. If the true coefficients varied within 1.088 standard errors of our estimates then $\delta TI/\delta Y$ for 1973 could vary from 0.32 to 2.01. Similarly the multipliers $\delta TI/\delta PAL$ could vary around the estimated values shown in Table 1. We have calculated the possible spread of $\delta TI/\delta Y$ and $\delta TI/\delta PAL$ based on the 70 per cent confidence intervals for 1961 and 1973. In Table 2 we give the estimated values of the two multipliers for 1961 and 1973 and the maximum and minimum values.

Table 2: *Taxable Income Multipliers, 1961 and 1973, Selection 2*

	$\delta TI/\delta Y$			$\delta TI/\delta PAL$		
	<i>Min</i>	<i>Estimate</i>	<i>Max</i>	<i>Min</i>	<i>Estimate</i>	<i>Max</i>
1961	.112	.245	.538	-.400	-.195	-.090
1973	.320	.811	2.010	-3.989	-1.719	-.739

Turning to selection (3), the estimated value of a_1 was 2.35736 and of a_2 was -1.65677 . The 70 per cent confidence intervals are:

$$2.95919 < a_1 < 2.41953; -1.72156 < a_2 < -1.59198$$

We have also calculated the range of possible values of the multipliers $\delta TI/\delta Y$ and $\delta TI/\delta PAL$ if the true coefficients varied within 1.088 standard errors of our parameter estimates. These are presented in Table 3.

Table 3: *Taxable Income Multipliers, 1961 and 1973, Selection 3*

	$\delta TI/\delta Y$			$\delta TI/\delta PAL$		
	<i>Min</i>	<i>Estimate</i>	<i>Max</i>	<i>Min</i>	<i>Estimate</i>	<i>Max</i>
1961	.117	.240	.494	-.340	-.178	-.092
1973	.348	.801	1.843	-3.389	-1.572	-.729

Examination of these ranges of possible values of the coefficients and multipliers shows that the dispersion of the possible coefficient values is not nearly as wide as the dispersion of the possible multipliers. It will be noticed that the spread of possible values of $\delta TI/\delta Y$ and $\delta TI/\delta PAL$ is much greater in 1973 than in 1961. Thus we can put less confidence in any given estimate of $\delta TI/\delta Y$ or $\delta TI/\delta PAL$ in 1973 than we can in estimates for 1961. We note, however, that the number of observations over which we estimated these equations is small. We note also that our values of PAL for years after 1971 may be revised when census data become available.

Out of Sample Forecasts

An important test of our preferred equations is their ability to forecast outside the sample period. At the time the present research began, 1973 was the latest year for which workable fiscal and national accounts data were available. We accordingly re-estimated the equations over 12 sets of observations (1961-72) and forecast the level of Taxable Income for the year 1973.⁵ The equations forecast well. These forecasts are presented in Table 4.

⁵ The regressions for 1961-72 were as follows:

$$\text{Ln TI} = 2.39853 \text{ LnY} - 1.82025 \text{ Ln PAL} \\ (0.07087) \quad (0.07688) \quad (2)$$

$$R^2 = .992 \quad \text{DW} = 0.96 \quad \text{Geary tau} = 3$$

$$\text{Ln TI} = 2.3555 \text{ Ln (.75 Y)} - 1.6549 \text{ Ln PAL} \\ (0.06814) \quad (0.07049) \quad (3)$$

$$R^2 = .993 \quad \text{DW} = 0.88 \quad \text{Geary tau} = 3$$

Table 4: *Taxable Income, 1973, actual and predicted, £ million*

	1. <i>Actual</i>	2. <i>Predicted</i>	3. <i>Discrepancy</i>	4. <i>3 as per cent of 1</i>
Selection 2	383.3	377.16	6.14	1.6
Selection 3	383.3	381.57	1.73	0.45

Multiplying the forecast values of TI for (2) and (3) by the Standard Tax Rate in 1973, 35 per cent, gives our forecast values of PAYE Tax Receipts for 1973. Actual receipts compare with our forecasts as follows:

Table 5: *PAYE tax receipts, 1973, actual and forecasts, £ million*

<i>Actual</i>	<i>Forecasts</i>	
134.15	Selection 2	132.0
	Selection 3	133.5

These are clearly good forecasts.

IV THE YEARS AFTER 1973

The double log equation with an adjustment for EIR, that is selection (3), forecasts outside the 1961 to 1972 sample period marginally better than selection (2) without adjustment for EIR. However, both forecast very well for the one year outside the sample period. Adopting the equation with an adjustment for EIR would imply using the coefficient values estimated in selection (3). But for the years after 1974, when EIR was abolished, we would use the double log equation without making any adjustment to the Personal Income series.

The estimate of PAYE tax receipts will be an independent variable in other equations in our model. Inside the sample period 1961 to 1973 we can get a direct estimate of tax receipts by application of the Standard Tax Rate to our estimate of Taxable Income. The tax code changes of 1974 replaced the Standard Tax Rate with a series of rates. Therefore, to get an estimate of tax receipts in years after 1973 we will need to apply some average tax rate, or rates, to our estimate of Taxable Income. The rate or rates to apply will depend on empirical investigation of the relationship between the series of tax rates and Taxable Income in the new tax code.

We have seen that our preferred equations forecast very well for one year outside the sample years 1961 to 1972. It may be wondered why we did not try to forecast Taxable Income and Tax Receipts for 1974. The changes in the tax code in 1974 were one consideration in that context. The abolition of EIR in that year causes no problems, since this could be handled by simply setting EIR at a value of zero for years after 1973. It is admitted that the change from the Standard Tax Rate to a graduated tax structure in the PAYE system would have caused difficulties. However, our principal reason for not attempting to forecast beyond 1973 is that *National Income and Expenditure 1973* was the latest such publication available when, in the Autumn of 1975, work was begun on several equations of the complete macroeconomic model. Rather than revise what we have already done, we propose to complete the model on the basis of the data available when work on the complete model was initiated. In fact, *National Income and Expenditure 1974* contained major revisions of previously published macroeconomic data. We will not attempt to re-estimate our parameters in the light of such revisions (and later revisions) until specification and estimation of the full model for the years 1953 to 1973 is completed.

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Appendix: Data

	<i>1</i> <i>PAYE</i> <i>tax</i> <i>receipts</i>	<i>2</i> <i>Standard</i> <i>tax rate</i> <i>t</i>	<i>3</i> <i>Taxable</i> <i>income</i> <i>TI</i>	<i>4</i> <i>Personal</i> <i>income</i> <i>Y</i>	<i>5</i> <i>Total</i> <i>claimable</i> <i>personal</i> <i>allowances</i>	<i>6</i> <i>PAYE</i> <i>taxpayers</i> <i>in total</i> <i>population</i>	<i>7</i> <i>Personal</i> <i>allowances</i> <i>PAL</i>
	<i>£ million</i>	<i>per cent</i>	<i>£ million</i>	<i>£ million</i>	<i>£ million</i>	<i>per cent</i>	<i>£ million</i>
1960	3.4	35.0	5.8	—	289.3	—	—
1961	9.0	31.7	25.2	275.7	309.8	85.0	262.7
1962	11.4	31.7	34.1	306.0	313.9	85.3	267.8
1963	13.2	31.7	40.3	328.7	319.5	85.5	273.2
1964	18.9	31.7	55.1	378.9	324.3	85.8	278.2
1965	21.5	31.7	65.0	403.2	329.4	86.0	283.3
1966	26.9	35.0	74.6	439.1	336.9	86.3	290.7
1967	32.1	35.0	88.0	474.9	348.9	86.3	301.1
1968	38.9	35.0	106.2	532.7	358.0	86.3	308.9
1969	48.7	35.0	132.1	616.3	373.2	86.4	322.4
1970	63.2	35.0	170.2	712.3	379.8	86.4	328.1
1971	87.9	35.0	233.5	824.8	383.4	86.4	331.2
1972	104.7	35.0	287.1	941.7	440.4	86.5	380.9
1973	144.0	35.0	383.3	1123.9	465.0	86.5	402.2

1. *PAYE Tax Receipts*

Taken from *Annual Report of the Revenue Commissioners*, various issues.

2. *Standard Tax Rate, t*

From *Annual Report of the Revenue Commissioners*.

3. *Taxable Income, TI*

This series is PAYE Tax Receipts divided by the Standard Tax Rate. It is then adjusted to a calendar year basis by taking $\frac{3}{4}$ of taxable income of fiscal year $t/(t+1)$ plus $\frac{1}{4}$ of taxable income of fiscal year $(t-1)/t$ to get Taxable Income of calendar year t .

4. *Personal Income, Y*

This is the Personal Income of PAYE taxpayers. It is derived from *National Income and Expenditure*. First we added agricultural wages and salaries plus non-agricultural wages and salaries plus net inflow of wages and salaries from abroad. This aggregate is total remuneration of employees. From this we take Central Government wages and salaries, because during our sample period, civil servants, the army and the gardai were not in the PAYE system. Central Government wages and salaries was on a fiscal year basis, so this series was adjusted to a calendar year basis before we subtracted it from Total Remuneration of Employees to give non-Central Government wages and salaries or PAYE Personal Income, which we denote by Y . Sources, *National Income and Expenditure 1969* and *National Income and Expenditure 1973*.

5. *Total Claimable Personal Allowances*

This is Dowling's series for Personal Allowances potentially claimable, adjusted to a calendar year basis (Dowling, 1977, p. 71). It is the product of the Personal Allowances for a given category of taxpayer in a given year and the number of people in that category in that year, summed over the various categories. It is defined as

$$PAL_t = \sum_i w_{it} N_{it}$$

where w_{it} is the personal allowance for category i in year t . N_{it} is the number of persons in category i in year t . The categories are the various classifications of taxpayers as defined by the tax code, e.g., married persons, single persons, widows, widowers, etc. The potential taxpaying population was defined by Dowling as the total non-agricultural labour force and dependent children, where agriculture is defined as family farm workers only. (Dowling, 1977, p. 39).

6. *PAYE Taxpayers in Total Population*

This is the percentage of Dowling's total gainfully occupied population that are classified as "non-agricultural own account worker", "employees",

“apprentices and learners” or “out of work” in the *Census of Population* classification of persons by employment status. (Dowling, 1977, p.70). This percentage is derived by summing the numbers in the above-mentioned groups, all of whom pay tax in the PAYE system, and expressing this as a percentage of Dowling’s “Total Gainfully Occupied” series. The exact percentage can only be derived for the censal years 1961, 1966 and 1971. We interpolated the percentage in inter-censal years. The figures for 1972 and 1973 may have to be adapted when census data become available next. Sources, *Census of Population*, 1961, 1966, 1971, Dowling, 1977.

7. *Personal Allowances, PAL*

This consists of the Personal Allowances claimable by potential PAYE taxpayers. It is Column 6 applied to Column 5.